Quantized vortices in pionic superfluid

A Virtual Tribute to Quark Confinement and the Hadron Spectrum 2021

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Main Topics

- Hyperon polarization and Axial Vortical Effect
- Spin polarization: phases vs dissipation in the cores of quantized vortices
- Vortices from vortex sheets
- Vortex rings: jets and local polarization
- Critical energy of vortices emergence

Where is the fastest possible rotation and acceleration?

- Non-central heavy ion collisions (Angular velocity ~ c/Compton wavelength) $\Omega \sim \frac{c}{\lambda_p} = \frac{m_p c^2}{\hbar}$
- ~25 orders of magnitude faster than Earth's rotation $\eta_{\rm rot} = \frac{\Omega}{\omega_\oplus} = \frac{c}{R_A} \cdot \frac{T_\oplus}{2\pi} = \frac{1}{2\pi} \cdot \frac{cT_\oplus}{R_A} \approx 10^{27}$
- Differential rotation – vorticity
- P-odd :May lead to various P-odd effects (Chiral magnetic/vortical effects)
- Acceleration: even larger ratio with the gravity of Earth

$$\eta_{\rm acc} = \frac{c}{R_A} \cdot \frac{c}{g_\oplus} = \eta_{\rm rot} \frac{2\pi c}{T_\oplus g_\oplus} \approx 10^{30} \qquad \eta_{\rm acc} = \frac{c}{R_A} \cdot \frac{c}{g_\oplus} = \frac{c^2}{v_\oplus^2} \cdot \frac{R_\oplus}{R_A}$$

Effective field theory: Anomalies

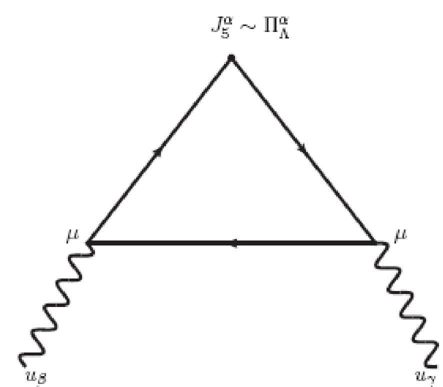
4-Velocity is also a GAUGE FIELD (A. Sadofyev, V. Shevchenko, V.I. Zakharov)

$$e_j A_\alpha J^\alpha \Rightarrow \mu_j V_\alpha J^\alpha$$

Triangle anomaly leads to polarization of quarks and hyperons
(Rogachevsky, Sorin, OT ’10)

Analogous to anomalous gluon contribution to nucleon spin
(Efremov,OT’88)

4-velocity instead of gluon field!



Anomalies due to chemical potential and temperature

Induced axial charge

$$c_V = \frac{\mu_s^2 + \mu_A^2}{2\pi^2} + \frac{T^2}{6}, \quad Q_5^s = N_c \int d^3x \, c_V \gamma^2 \epsilon^{ijk} v_i \partial_j v_k$$

Chemical potential im HIC is rapidly decreasing with energy

T-dependent term is related to holographic gravitational anomaly (K. Landsteiner et al.)

Coefficient (pi) ?

Rotated and accelerated frame: Wigner function and Zubarev density operator

G. Prokhorov, V. Zakharov,OT ’19:

Imaginary chemical potential due to acceleration appears!

$$\langle j_\mu^5\rangle = \frac{\omega_\mu + i\,\mathrm{sgn}(\omega a) a_\mu}{2(g_\omega - i g_a)} \int \frac{d^3p}{(2\pi)^3} \times \{ n_F(E_p - \mu - g_\omega/2 + i g_a/2) - n_F(E_p - \mu + g_\omega/2 i g_a/2) + n_F(E_p + \mu - g_\omega/2 + i g_a/2) - n_F(E_p + \mu + g_\omega/2 - i g_a/2)\} + \mathrm{c.c.},$$

$$\rho = \frac{7\pi^2 T^4}{60} + \frac{T^2 a^2}{24} - \frac{17 a^4}{960\pi^2} = 2\int \frac{d^3p}{(2\pi)^3}\left(\frac{|\mathbf{p}| + ia}{1 + e^{\frac{|\mathbf{p}|}{T} + \frac{ia}{2T}}} + \frac{|\mathbf{p}| - ia}{1 + e^{\frac{|\mathbf{p}|}{T} - \frac{ia}{2T}}}\right) + 4\int \frac{d^3p}{(2\pi)^3}\frac{|\mathbf{p}|}{e^{\frac{2\pi|\mathbf{p}|}{a}} - 1} \qquad (T > T_U)$$

in red: *modifications* compared to the *Wigner function*

- In the first integral, the **acceleration** enters as an **imaginary chemical potential** $\pm\frac{ia}{2}$ [G.P., O. Teryaev, V. Zakharov, Phys. Rev. D 98, no. 7, 071901 (2018)].

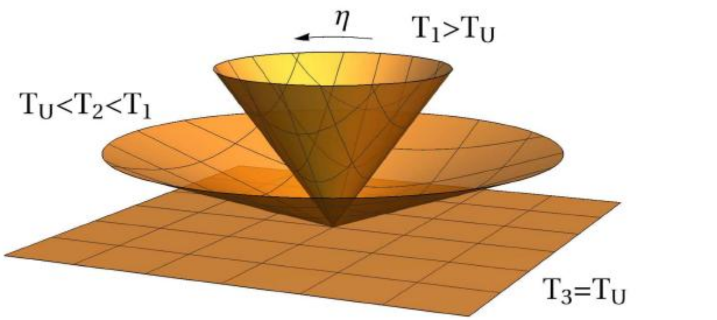
Emergent conical geometry from density operator: Prokhorov, OT, Zakharov, 1911.04545&JHEP

Results for energy density of thermal system in Minkowski space coincide with the early known for the space with conical singularity (e.g. cosmic strings)

$$\rho_{s=0} = \frac{\pi^2 T^4}{30} + \frac{T^2|a|^2}{12} - \frac{11|a|^4}{480\pi^2},$$
$$\rho_{s=1/2} = \frac{7\pi^2 T^4}{60} + \frac{T^2|a|^2}{24} - \frac{17|a|^4}{960\pi^2}$$

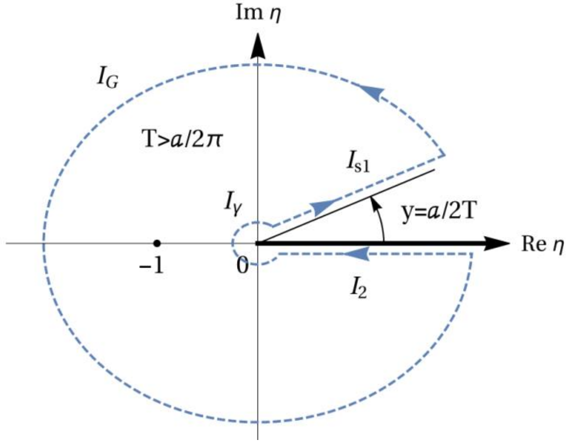
Energy density turns to zero for $T=T_U=a/(2\pi)$ (~"physical conditions of renormalization". also simple explanation of coefficient)

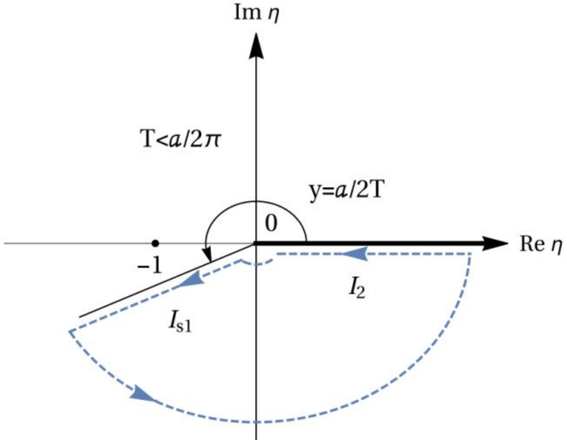
Instability for high accelerations



Normally $T>T_U$

Fast acceleration without thermalization: instability





EP ~ fall to BH?

Censorship: Origin for fast thermalization?

STAR, Nature 548 (2017) 62-65 (talk of A.

Observable for AVE: polarization

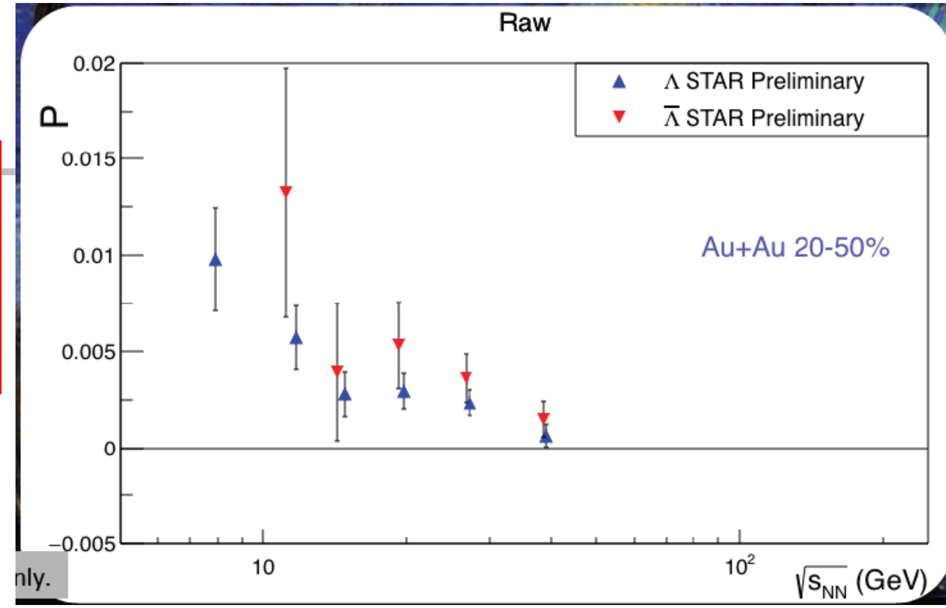
O. Rogachevsky, A. Sorin, O. Teryaev
Chiral vortaic effect and neutron asymmetries in heavy-ion collisions
PHYSICAL REVIEW C 82, 054910 (2010)

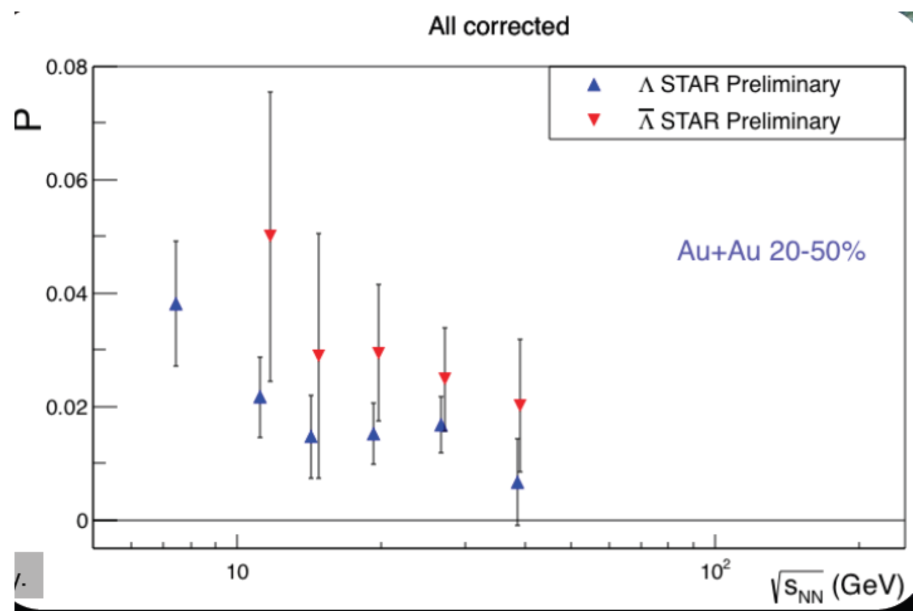
One would expect that polarization is proportional to the anomalously induced axial current [7]

$$j_A^\mu \sim \mu^2 \left(1 - \frac{2\mu n}{3(\epsilon + P)}\right) \epsilon^{\mu\nu\lambda\rho} V_\nu \partial_\lambda V_\rho, \qquad (6)$$

where n and ϵ are the corresponding charge and energy densities and P is the pressure. Therefore, the μ dependence of polarization must be stronger than that of the CVE, leading to the effect's increasing rapidly with decreasing energy.

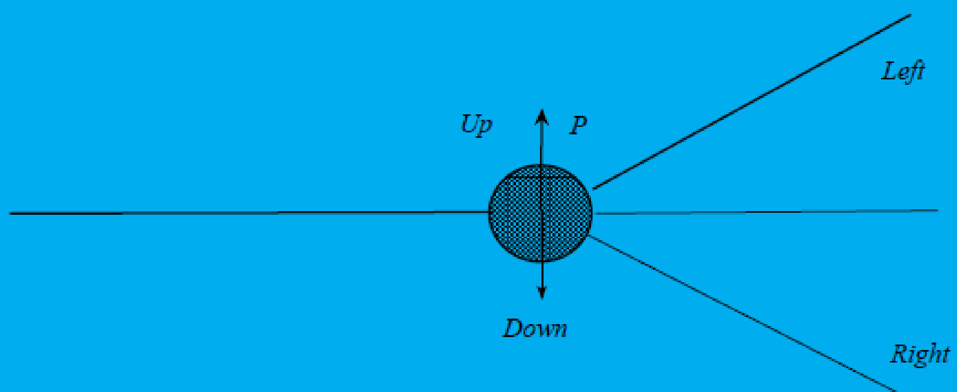
This option may be explored in the framework of the program of polarization studies at the NICA [17] performed at collision points as well as within the low-energy scan program at the RHIC.





Single Spin Asymmetries

Simplest example - (non-relativistic) elastic pion-nucleon scattering $\pi\vec{N} \to \pi N$



$M = a + ib(\vec{\sigma}\vec{n})$ $\vec{n}$ is the normal to the scattering plane.

Density matrix: $\rho = \frac{1}{2}(1 + \vec{\sigma}\vec{P})$,

Differential cross-section: $d\sigma \sim 1 + A(\vec{P}\vec{n})$, $A = \frac{2Im(ab^*)}{|a|^2+|b|^2}$

SSA

- Parity conservation – normal to scattering plane
- Interference – LS coupling
- T conservation – absorptive phases
- What is the counterpart for heavy ions? Suggestion: dissipation (cf. Montenegro, Tinti, Torrieri'17)
- QCD for hadrons – quark-gluon correlations : twist 3

Baryons in confined phase: vortices in pionic superfluid (V.I. Zakharov, OT:1705.01650 and PRD)

Pions may carry the axial current due to quantized vortices in pionic superfluid (Kirilin,Sadofyev,Zakharov'12)

$$\frac{\pi_0}{f_\pi} = \mu \cdot t + \varphi(x_i) \qquad \partial_i \varphi = \mu v_i \qquad \oint \partial_i \varphi dx_i = 2\pi n$$

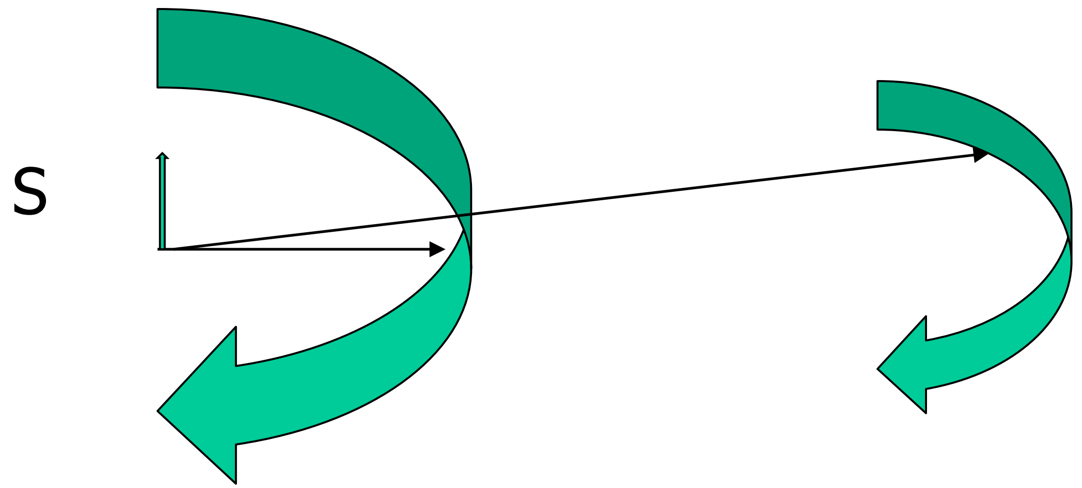
$$T_{0i} \sim \mu_5 \partial_i \tilde{\varphi}. \qquad \lim_{q_i \to 0, \omega \equiv 0} < T_{0i}, T_{0k} > \sim \mu_5^2 \frac{q_i q_k}{q_i^2}$$

$$j_5^\mu = \frac{1}{4\pi^2 f_\pi^2} \epsilon^{\mu\nu\rho\sigma} (\partial_\nu \pi^0)(\partial_\rho \partial_\sigma \pi^0)$$

Suggestion: core of the vortex- baryonic degrees of freedom - polarization

Core of quantized vortex

Constant circulation – velocity increases when core is approached



Helium ($v < v_{sound}$) bounded by intermolecular distances

Pions ($v<c$) –> (baryon) spin in the center

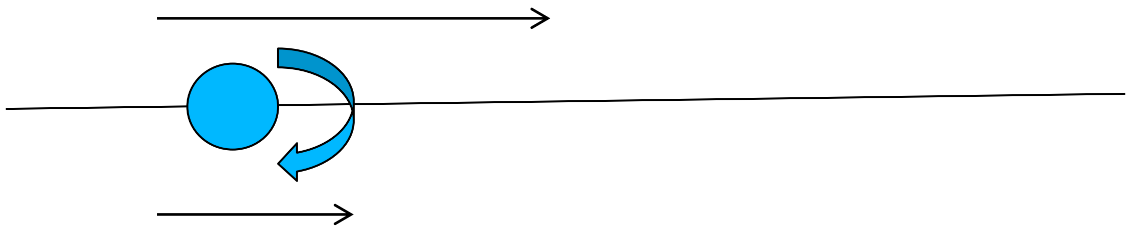
Baryon spin

- Kinematical requirement of spin appearance – similar to "historical" arguments: v~c at Compton wavelength and v>>c at classical radius required for orbital momentum
- Baryons emerge as UV cutoff
- Heavy degrees of freedom: dissipation

Where vortices emerge?

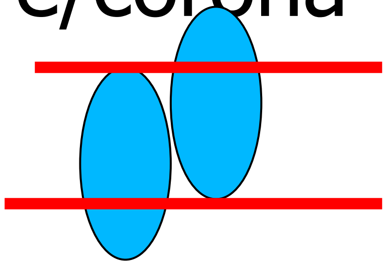
Liquid helium: layers with relative velocity

(Feynman, Statistical Mechanics, 11.9)

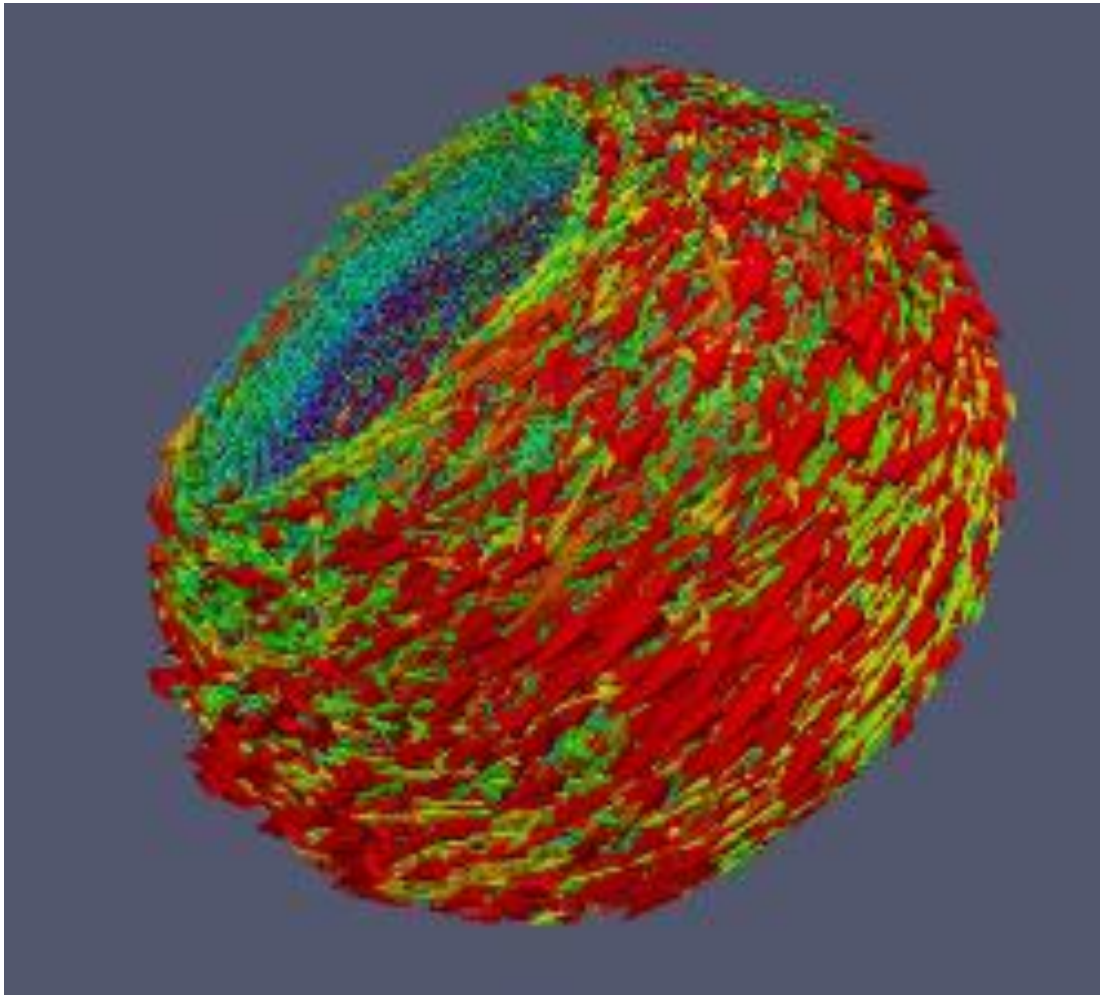


exp (im Δv R) = 1

Similar situation in HIC: core/corona border



Kinetic model (Baznat, Gudima, Sorin, OT'1507.04652 and PRC): Vortex sheet (cf. Yu. Ivanov et al.: 3-fluid hydro)



Matching classical and quantum vorticity (new)

Quantized vortex: $v(r) = 1/\mu r$

$\Omega q(r) = 1/\mu r^2$

Necessary condition:

$\Omega_{Cl}(r) > \Omega q(r)$

Cf. : constant angular velocity for the bowl with liquid helium

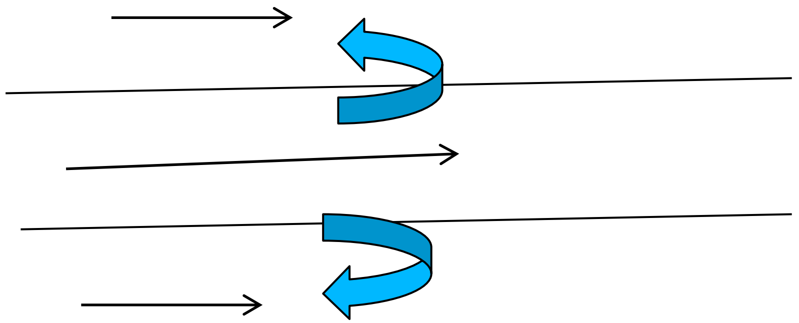
$\Omega > \ln(R/a) / m R^2$

Vortex sheet: transverse distance

$r \rightarrow r_T$

Vortex rings in helium: jets (c.f. Lisa, Serenone, Torrieri et al.'21 : HD for HIC)

Liquid helium



May explain local polarization

Critical velocity

Liquid helium: v > ln (d/a) / d m

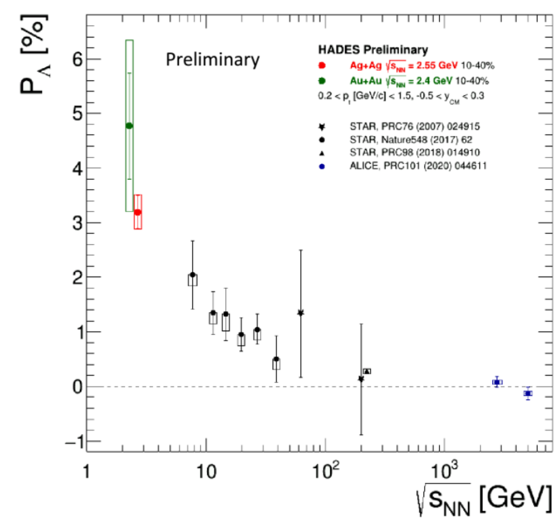
Rings: d – transverse size of jet

Tubes: --------//------- vortex sheet

Relativistic case $E_{п}$ > ln (d/a) / d

Polarization should

decrease abruptly!



Conclusions/Outlook

Duality between Effective

field theory

Statistics

Geometry

Vortex tubes/rings : similarity of global/local polarization to liquid helium physics

Matching “classical” and quantum vorticity

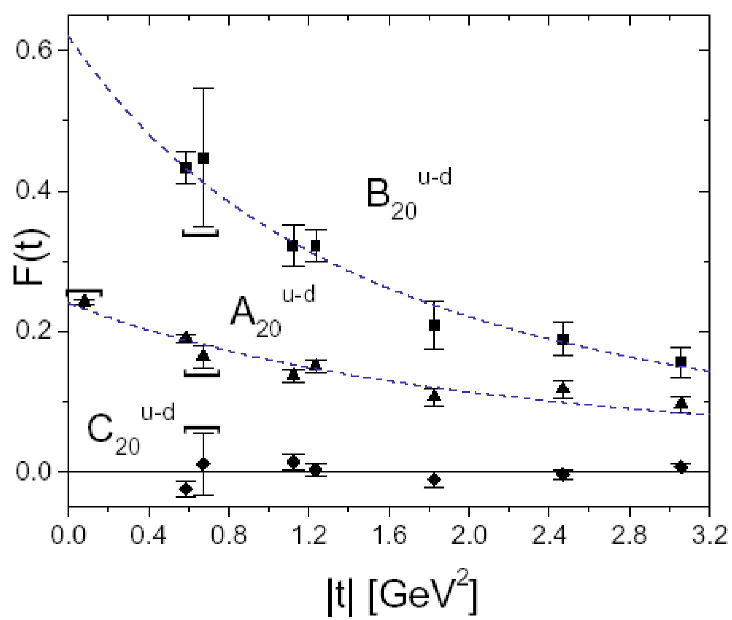
Critical energies: probe of emergent phenomena

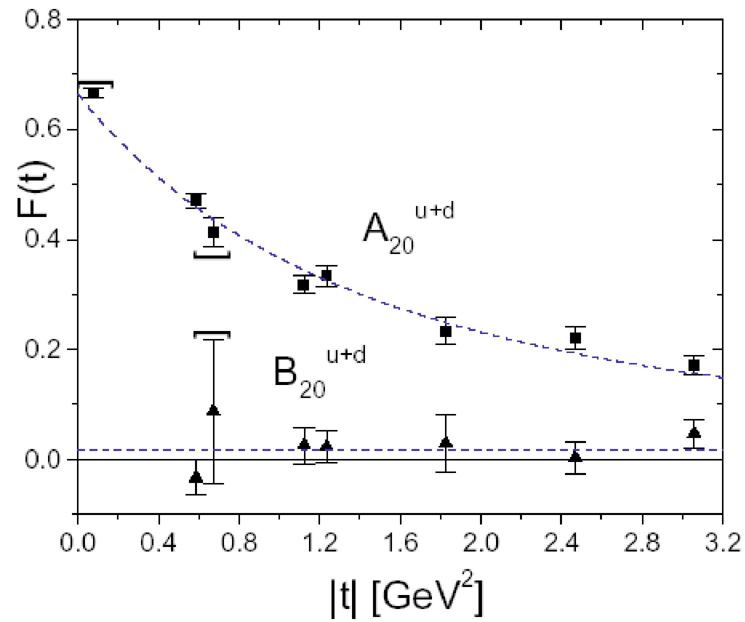
Detailed quantitative calculations to be done

BACKUP

Generalization of Equivalence principle

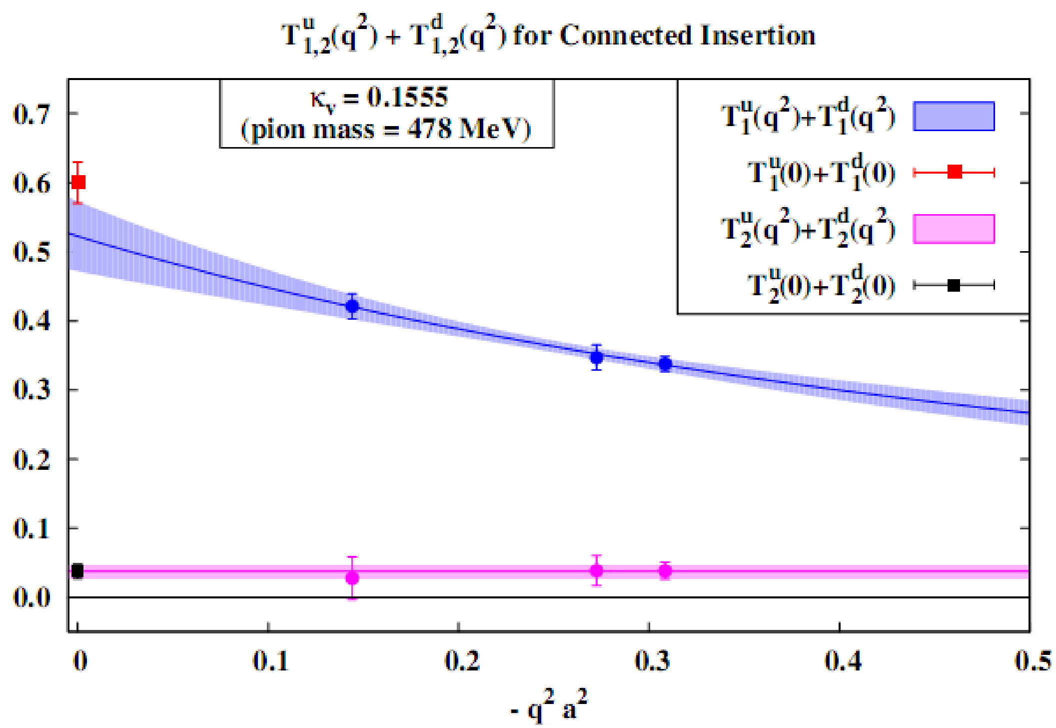
- Various arguments: AGM ≈0 separately for quarks and gluons – most clear from the lattice (LHPC/SESAM)





Recent lattice study (M. Deka et al. Phys.Rev. D91 (2015) no.1, 014505)

- Sum of u and d for Dirac (T1) and Pauli (T2) FFs



Extended Equivalence Principle=Exact EquiPartition

- In QED, pQCD – violated (Brodsky et al)
- Reason – in the case of ExEP- no smooth transition for zero fermion mass limit (Milton, 73)
- Conjecture (O.T., 2001 – prior to lattice data) – valid in NP QCD – zero quark mass limit is safe due to chiral symmetry breaking
- Gravityproof confinement? Nucleons are not broken even by black holes (“swampland”) ?
- Support by recent observation of smalness of (nucleon “cosmological constant”) Cbar

Spin 1 EMT and inclusive processes

- Forward matrix element ->density matrix
- Contains P-even term: tensor polarization $S^{\alpha\beta}$
- Symmetric and traceless: correspond to (average) shear forces
- For spin ½: P-odd vector polarization requires another vector (q) to form vector product

SUM RULEs

- Efremov,OT’81 : zero sum rules:
- 1st moment: also in parton model by Close and Kumano (90)
- 2nd moment (forward analog of Ji’s SR)
- Average shear force (compensated between quarks and gluons)
- Gravity and (Ex)EP (zero average shear separately for quarks and gluons) – OT’09

Manifestation of post-Newtonian (Ex)EP for spin 1 hadrons

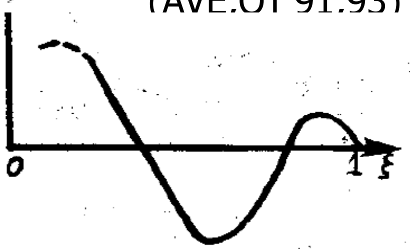
- Tensor polarization - coupling of EMT to spin in forward matrix elements - inclusive processes
- Second moments of tensor distributions should sum to zero

$$A_T = \frac{\sigma_+ + \sigma_- - 2\sigma_0}{3\bar{\sigma}}$$

$$\int_0^1 C_i^T(x)dx = 0$$

$$\langle P,S|\bar{\psi}(0)\gamma^\nu D^{\nu_1}...D^{\nu_n}\psi(0)|P,S\rangle_{\mu^2} = i^{-n}M^2 S^{\nu\nu_1}P^{\nu_2}...P\nu_n \int_0^1 C_q^T(x)x^n dx$$

(AVE,OT'91,93)



$$\sum_q \langle P,S|T_i^{\mu\nu}|P,S\rangle_{\mu^2} = 2P^\mu P^\nu(1-\delta(\mu^2)) + 2M^2 S^{\mu\nu}\delta_1(\mu^2)$$

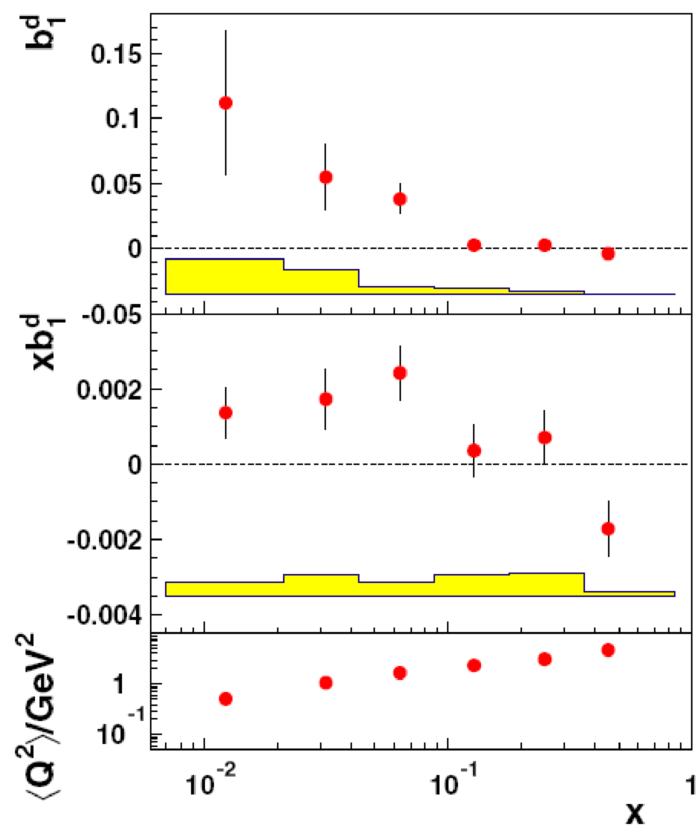
$$\langle P,S|T_g^{\mu\nu}|P,S\rangle_{\mu^2} = 2P^\mu P^\nu \delta(\mu^2) - 2M^2 S^{\mu\nu}\delta_1(\mu^2)$$

$$\sum_q \int_0^1 C_i^T(x)xdx = \delta_1(\mu^2)$$ =0 for ExEP

HERMES – data on tensor spin structure function

PRL 95, 242001 (2005)

- Isoscalar target – proportional to the sum of u and d quarks – combination required by (Ex)EP
- Second moments – compatible to zero better than the first one (collective glue << sea)



Where else to test?

- COMPASS
- EIC
- DY@J-PARC:
- However: ET'81-any hard process ("multi-messenger")
- Possibility: hadronic tensor SSA@NICA

Tensor polarized beams

- Opportunity: NICA@JINR with polarized hadronic beams
- Polarized deuterons is easier to accelerate: no depolarizing resonances
- SPD: J/Ψ + hadronic SSA

Vector vs Tensor SSA

- Vector: A = (σ(+)-σ(-))/(σ(+)+σ(-))

  Tensor:

  A_T = (σ(+)+σ(-)-2σ(0))/(σ(+)+σ(-)+σ(0))

$$A_T = \frac{d\sigma(+) + d\sigma(-) - 2d\sigma(0)}{d\sigma(+) + d\sigma(-) + d\sigma(0)} \sim \frac{\sum_{i=q,\bar{q},g} \int d\hat{\sigma}_i \delta_{Ti}(x)}{\sum_{i=q,\bar{q},g} \int d\hat{\sigma}_i f_i(x)}$$

- Inclusive hadron (pion,kaon…) production: (T-odd) vector SSA may be also excluded by summing σ(L)+ σ(R)

Polarization directions

NICA – transverse is easier

Lonfitudinal: enhanced like longitudinal vector polarization

For tensor polarization: diagonal components are not independent (also property of shear)

$$\sum_i \rho^i_{00} = 1; \quad \sum_i S_{ii} = 0.$$

Transverse polarization generates also dominant LL components

Shear: viscosity?!

Hadronic matrix element~ fluid (EoS!)?

$T^{\mu\lambda} = (e+p)\, v^{\mu} v^{\lambda} - p\, g^{\mu\lambda}$

$V^{\mu} = P^{\mu}/M$: correct normalization but no coordinate dependence

Another suggestion:

$V^{\mu} = (P^{\mu} + a(t)\, k_T^{\ \mu}) / (M^2 + a^2(t)\, k_T^{\ 2})^{1/2}$

Viscosity: $\sim \eta\ p^{[\mu} \Delta^{\lambda]}$

Naïve T-oddness: phases

Viscosity in GDA channel

- Possibility to study gravitational FFs in time-like region (Kumano, Song,OT'18)
- Viscosity (new!):will correspond to Exotic $J^{PC}=1^{-+}$ meson (already studied: Anikin, Pire, Szymanowski,OT, Wallon'06)
- πη pairs observation instead of π π required
- Smallness of viscosity: related to smallness of T-odd GPDs and exotic GDAs ?!

Conclusions

- Equivalence principle: modeling of strongest ever gravitational fields in HIC
- Dissipation is required to produce polarization
- Holographic approach to viscosity: graviton polarization?
- Shear viscosity: relation to exotic hybrid mesons?

BACKUP

Another appearance of T-oddness in EMT: Burkardt SR

T-invariance : antisymmetry of twist 3 gluonic pole matrix element nullifies its contribution to EMT

BUT Pole prescription (dynamics!) provides ("T-odd") symmetric part (OT'14)!

$$\sum \int \int dx_1 dx_2 \frac{T(x_1, x_2)}{x_1 - x_2 + i\varepsilon} = 0$$

SR:

$$\sum \int dx T(x, x) = 0$$

Also EP!

ExEP: approximate validity separately for quarks and gluons: smallness of deuteron Sivers function

D-term interpretation: Inflation and annihilation

- Quadrupole gravitational FF

$$\langle P+q/2|T^{\mu\nu}|P-q/2\rangle = C(q^2)(g^{\mu\nu}q^2 - q^\mu q^\nu) + \ldots$$

- Moment of D-term – positive
- Vacuum – Cosmological Constant $\langle 0|T^{\mu\nu}|0\rangle = \Lambda g^{\mu\nu}$

- 2D effective CC – negative in scattering, positive in annihilation

$$\Lambda = C(q^2)q^2$$

- Similarity of inflation and Schwinger pair production – Starobisnky, Zel'dovich
- Was OUR Big Bang resulting from one graviton annihilation at extra dimensions??! Version of "ekpyrotic" ("pyrotechnic") universe

C vs Cbar

- Cancellations of Cbars – negative pressure (cf Chaplygin gas)
- Cancellation in vacuum; Pauli (divergent), Zel’dovich (finite)
- Flavour structure of pressure: DVMP!

Unphysical regions

- DIS : Analytical function – polynomial in $1/x_B$ if $1 \leq |X_B|$

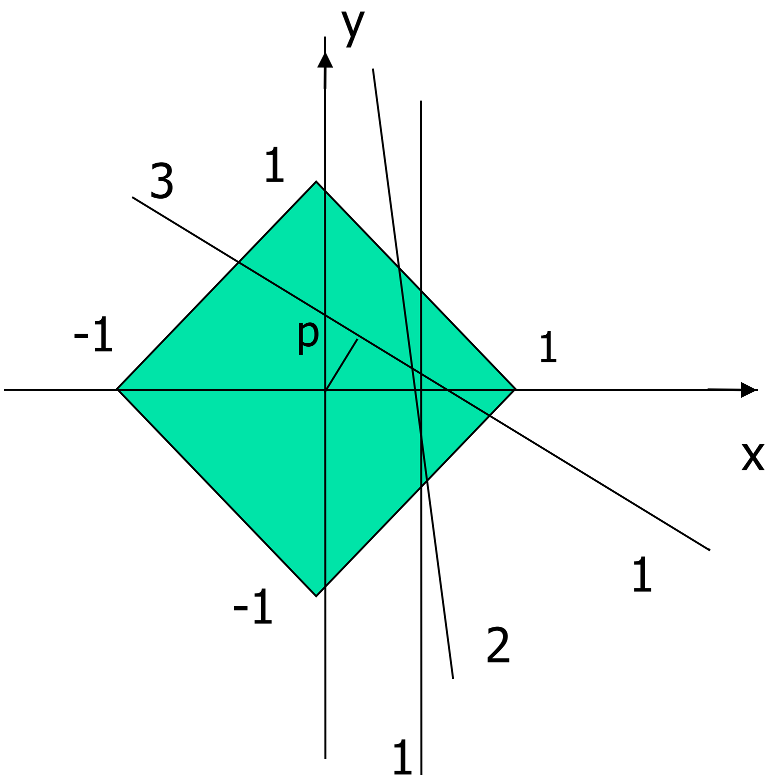
$$H(x_B) = -\int_{-1}^{1} dx \sum_{n=0}^{\infty} H(x) \frac{x^n}{x_B^{n+1}}$$

- DVCS – additional problem of analytical continuation of $H(x,\xi)$
- Solved by using of Double Distributions Radon transform

$$H(z,\xi) = \int_{-1}^{1} dx \int_{|x|-1}^{1-|x|} dy (F(x,y) + \xi G(x,y)) \delta(z - x - \xi y)$$

Double distributions and their integration

- Slope of the integration line- skewness
- Kinematics of DIS: $\xi = 0$ ("forward") - vertical line (1)
- Kinematics of DVCS: $\xi < 1$ - line 2
- Line 3: $\xi > 1$ unphysical region - required to restore DD by inverse Radon transform: tomography

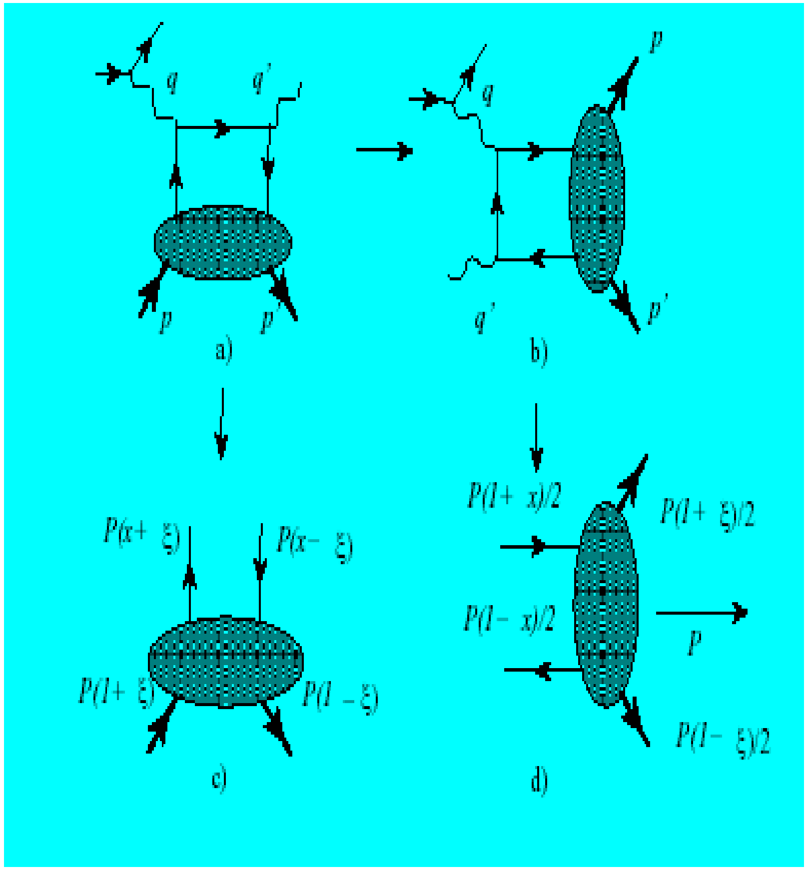


$$f(x,y) = -\frac{1}{2\pi^2}\int_0^\infty \frac{dp}{p^2}\int_0^{2\pi} d\phi|cos\phi|(H(p/cos\phi + x + ytg\phi, tg\phi) - H(x + ytg\phi, tg\phi)) =$$

$$= -\frac{1}{2\pi^2}\int_{-\infty}^\infty \frac{dz}{z^2}\int_{-\infty}^\infty d\xi(H(z + x + y\xi, \xi) - H(x + y\xi, \xi))$$

Crossing for DVCS and GPD

- DVCS -> hadron pair production in the collisions of real and virtual photons
- GPD -> Generalized Distribution Amplitudes
- Duality between s and t channels (Polyakov,Shuvaev, Guzey, Vanderhaeghen)



GDA -> back to unphysical regions for DIS and DVCS

- Recall DIS

$$H(x_B) = -\int_{-1}^{1} dx \sum_{n=0}^{\infty} H(x) \frac{x^n}{x_B^{n+1}}$$

- Non-positive powers of x_B

- DVCS

$$H(\xi) = -\int_{-1}^{1} dx \sum_{n=0}^{\infty} H(x,\xi) \frac{x^n}{\xi^{n+1}}$$

- Polynomiality (general property of Radon transforms): moments - integrals in x weighted with x ^{n} - are polynomials in 1/ ξ of power n+1
- As a result, analyticity is preserved: only non-positive powers of ξ appear

Holographic property - II

- Directly follows from double distributions

$$H(z,\xi)=\int_{-1}^{1}dx\int_{|x|-1}^{1-|x|}dy(F(x,y)+\xi G(x,y))\delta(z-x-\xi y)$$

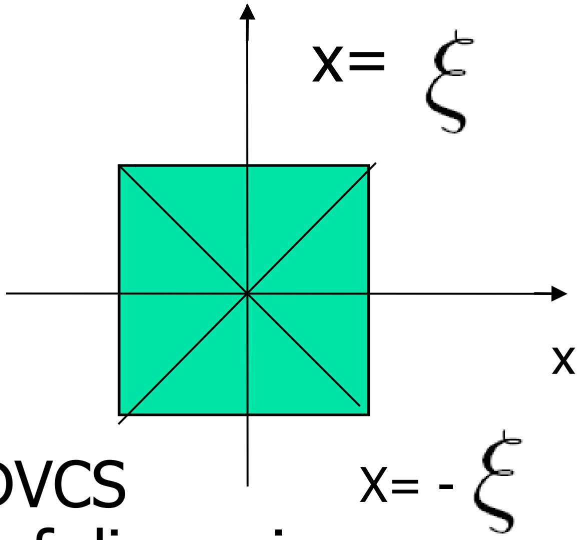
- Constant is the SUBTRACTION one - due to the (generalized) Polyakov-Weiss term G(x,y)

$$\Delta\mathcal{H}(\xi)=\int_{-1}^{1}dx\int_{|x|-1}^{1-|x|}dy\frac{G(x,y)}{1-y}$$

$$= \int_{-\xi}^{\xi}dx\frac{D(x/\xi)}{x-\xi+i\epsilon}=\int_{-1}^{1}dz\frac{D(z)}{z-1}=const$$

Holographic property - III

- 2-dimensional space -> 1-dimensional section!
- Momentum space: any relation to holography in coordinate space ?!



- Strategy (now adopted) of GPD's

studies: start at diagonals

(through SSA due to imaginary part of DVCS amplitude) and restore by making use of dispersion relations + subtraction constants

Pressure in hadron pairs production

- Back to GDA region
- -> moments of H(x,x) - define the coefficients of powers of cosine!– 1/ξ
- Higher powers of cosine in t-channel – threshold in s -channel
- Larger for pion than for nucleon pairs because of less fast decrease at x ->1
- Stability defines the sign of GDA and (via soft pion theorem) DA: work in progress

$$\mathscr{H}(\xi) = -\int_{-1/\xi}^{1/\xi} dx \sum_{n=0}^{\infty} H(x,\xi) \frac{x^n}{\xi^{n+1}}$$

$$= -\int_{-1/\xi}^{1/\xi} dx \sum_{n=0}^{\infty} H(x,x) \frac{x^n}{\xi^{n+1}} + \Delta\mathscr{H}.$$

Analyticity of Compton amplitudes in energy plane (Anikin,OT'07)

- Finite subtraction implied

$$\mathrm{Re}\mathcal{A}(\nu,Q^2)=\frac{\nu^2}{\pi}\mathcal{P}\int_{\nu_0}^{\infty}\frac{d\nu'^2}{\nu'^2}\frac{\mathrm{Im}\mathcal{A}(\nu',Q^2)}{(\nu'^2-\nu^2)}+\Delta \qquad \Delta=2\int_{-1}^{1}d\beta\,\frac{D(\beta)}{\beta-1}$$

$$\Delta^p_{\mathrm{CQM}}(2)=\Delta^n_{\mathrm{CQM}}(2)\approx 4.4, \qquad \Delta^p_{\mathrm{latt}}\approx\Delta^n_{\mathrm{latt}}\approx 1.1$$

- Numerically close to Thomson term for real proton (but NOT neutron) Compton Scattering!

- Duality (sum of squares vs square of sum; proton: 4/9+4/9+1/9=1)?!

Loss of stability?

- D=0 -> extra node required (cf tensor distribution - Efremov,OT- mechanical analogy – c.m. and c.i.)
- Smooth decrease – two extra nodes
- ++++------------------
- +++++++------+++++--------------
- J=2 (Talk of Barbara Pasquini, comment by Maxim – zeros of Bessel functs?!)

BACKUP

Is D-term independent?

- Fast enough decrease at large energy ->

$$\mathrm{Re}\,\mathcal{A}(\nu) = \frac{\mathcal{P}}{\pi}\int_{\nu_0}^{\infty} d\nu'^2 \frac{\mathrm{Im}\,\mathcal{A}(\nu')}{\nu'^2-\nu^2} + \mathrm{C}_0$$

$$\begin{aligned}\mathrm{C}_0 &= \Delta - \frac{\mathcal{P}}{\pi}\int_{\nu_0}^{\infty} d\nu'^2 \frac{\mathrm{Im}\,\mathcal{A}(\nu')}{\nu'^2} \\ &= \Delta + \mathcal{P}\int_{-1}^{1} dx \frac{H^{(+)}(x,x)}{x}.\end{aligned}$$

- FORWARD limit of Holographic equation

$$\begin{aligned}\Delta &= \mathcal{P}\int_{-1}^{1} dx \frac{H^{(+)}(x,0)-H^{(+)}(x,x)}{x} \\ &= 2\mathcal{P}\int_{-1}^{1} dx \frac{H(x,0)-H(x,x)}{x},\end{aligned}$$

$$\mathrm{C}_0(t) = 2\mathcal{P}\int_{-1}^{1} dx \frac{H(x,0,t)}{x}$$

"D – term" 30 years before…

- Cf Brodsky, Close, Gunion'72 (seagull ~ pressure) – but NOT DVMP
- D-term – a sort of renormalization constant
- May be calculated in effective theory if we know fundamental one
- OR
- Recover through special regularization procedure (D. Mueller)?

Vector mesons and EEP

- J=1/2 -> J=1. QCD SR calculation of Rho's AMM gives g close to 2.

- Maybe because of similarity of moments
- $g-2=\langle E(x)\rangle$; $B=\langle xE(x)\rangle$
- Directly for charged Rho (combinations like p+n for nucleons unnecessary!). Not reduced to non-extended EP:

EEP and AdS/QCD

- Recent development – calculation of Rho formfactors in Holographic QCD (Grigoryan, Radyushkin)
- Provides g=2 identically!
- Experimental test at time –like region possible

EEP and Sivers function

- Sivers function – process dependent (effective) one
- T-odd effect in T-conserving theory- phase
- FSI – Brodsky-Hwang-Schmidt model
- Unsuppressed by M/Q twist 3
- Process dependence- colour factors
- After Extraction of phase – relation to universal (T-even) matrix elements

EEP and Sivers function -II

- Qualitatively similar to OAM and Anomalous Magnetic Moment (talk of S. Brodsky)
- Quantification : weighted TM moment of Sivers PROPORTIONAL to GPD E (OT'07, **hep-ph/0612205**): $x f_{T}(x) : xE(x)$
- Burkardt SR for Sivers functions is then related to Ji's SR for E and, in turn, to Equivalence Principle

$$\sum_{q,G} \int dx x f_{T}(x) = \sum_{q,G} \int dx x E(x) = 0$$

EEP and Sivers function for deuteron

- EEP - smallness of deuteron Sivers function
- Cancellation of Sivers functions – separately for quarks (before inclusion gluons)
- Equipartition + small gluon spin – large longitudinal orbital momenta (BUT small transverse ones –Brodsky, Gardner)

Another relation of Gravitational FF and NP QCD (first reported at 1992: **hep-ph/9303228**)

- BELINFANTE (relocalization) invariance :

decreasing in coordinate –

$$M^{\mu,\nu\rho} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} J^5_{S\sigma} + x^\nu T^{\mu\rho} - x^\rho T^{\mu\nu}$$

smoothness in momentum space

$$M^{\mu,\nu\rho} = x^\nu T_B^{\mu\rho} - x^\rho T_B^{\mu\nu}$$

- Leads to absence of massless pole in singlet channel – U_A(1)

$$\epsilon_{\mu\nu\rho\alpha} M^{\mu,\nu\rho} = 0.$$

- Delicate effect of NP QCD

$$(g_{\rho\nu} g_{\alpha\mu} - g_{\rho\mu} g_{\alpha\nu}) \partial^\rho (J^\alpha_{5S} x^\nu) = 0.$$

- Equipartition – deeply related to relocalization invariance by QCD evolution

$$q^2 \frac{\partial}{\partial q^\alpha} \langle P|J^\alpha_{5S}|P+q\rangle = (q^\beta \frac{\partial}{\partial q^\beta} - 1) q_\gamma \langle P|J^\gamma_{5S}|P+q\rangle$$

$$\langle P,S|J^5_\mu(0)|P+q,S\rangle = 2MS_\mu G_1 + q_\mu (Sq) G_2,$$

$$q^2 G_2|_0 = 0$$