Inferring the dense nuclear matter equation of state with neutron star tides

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N. Andersson and P. P. (2020) PRD **101**, 083001; (2021) MNRAS **503**, 533
A. Passamonti, N. Andersson, and P. P. (2021) MNRAS **504**, 1273

A Virtual Tribute to Quark Confinement and the Hadron Spectrum

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SAPIENZA UNIVERSITÀ DI ROMA

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UKRI Science and Technology Facilities Council

The Love number

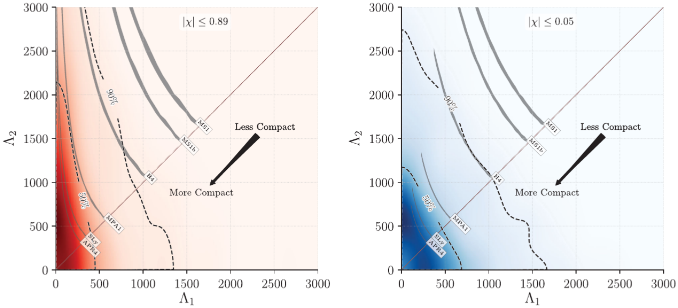
- Tidal deformability is parametrised through the *Love number* k_l

l-th multipole moment $= k_l \times$ (l-th harmonic of tidal potential)

- The Love number increases with equation of state "stiffness"
  - Can be extracted from GW data analysis and constrain the equation of state

Stiffer equation of state ⇔ Less compact ("puffier") star ⇔ Easier to deform

GW170817 tidal deformability constraints



Credit: Abbott *et al.* (2017) PRL **119**, 161101

$$\Lambda = \frac{2k_2}{3}\left(\frac{Rc^2}{GM}\right)^5$$

- But: k_l is defined through the *equilibrium tide* approximation
  - Equilibrium tide: tidally-perturbed star is always in hydrostatic equilibrium
  - Dynamical tide: non-instantaneous fluid response, oscillation mode resonances

The tidal problem

- Star perturbed by tidal potential U of a companion star, inducing tidal perturbation $\boldsymbol{\xi}$

$$\left.\begin{array}{ll} \text{Euler equation:} & \ddot{\boldsymbol{\xi}} + \dfrac{\nabla \delta p}{\rho} - \dfrac{\nabla p}{\rho^2}\delta\rho + \nabla\delta\Phi = -\nabla U \\ \text{Poisson equation:} & \nabla^2 \delta\Phi = 4\pi G \delta\rho \end{array}\right\} k_l = \frac{\delta\Phi(R)}{2U(R)}$$

- **Full solution:**
  - $\boldsymbol{\xi} = [\xi_r(r)\hat{\boldsymbol{e}}_r + \xi_h(r)\nabla]\, Y_l^m(\theta,\phi)e^{im\omega t}$, where ω is the orbital frequency
  - Mainly interested in quadrupole components of the tide: $l = 2$, $m = -2, 0, 2$

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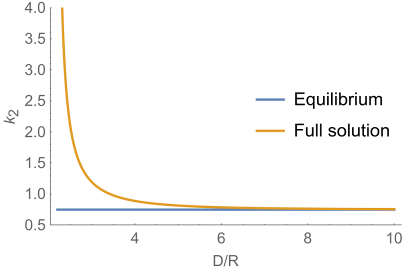
- **Equilibrium tide:** Set $\omega \to 0$ and derive tidal response
- Simplest possible neutron star model: incompressible star ($\rho = \text{const.}$)

– Equilibrium tide: $k_l = \dfrac{3}{4(l-1)}$

– Full solution: $k_l^{\text{eff}} = k_l\left[1 - \dfrac{(m\omega)^2}{\omega_f^2}\right]^{-1}$

with $\omega_f^2 = \dfrac{2l(l-1)}{2l+1}\dfrac{GM}{R^3}$ (f-mode)

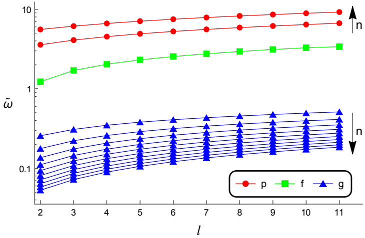


- *Effective* Love number contains f-mode resonances at $\omega = \pm\omega_f/m$

Mode expansion

- For other models, $\boldsymbol{\xi}$ has to be expanded with respect to stellar *oscillation modes*

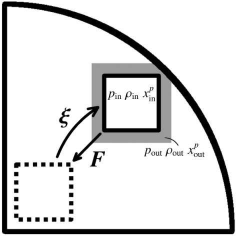
$$\boldsymbol{\xi} = \sum_n a_n \boldsymbol{\xi}_n, \quad \ddot{a}_n + \omega_n^2 a_n = -\int \boldsymbol{\xi}_n^* \cdot \nabla U \rho \mathrm{d}^3\boldsymbol{r}$$



- Fluid (polar) modes:
  - f-modes: fundamental oscillations ($n = 0$)
  - p-modes: acoustic oscillations
  - g-modes: buoyancy oscillations

  } $n > 0$ (applies to p-modes and g-modes)

- *Buoyancy* is present if nuclear reaction timescale > perturbation timescale



Credit: Herbrik and Kokkotas (2017) MNRAS **466**, 1330

- β reactions (Urca reactions):

$$n\,(+\,n) \rightarrow p\,(+\,n) + e^- + \bar{\nu}_e, \quad p\,(+\,n) + e^- \rightarrow n\,(+\,n) + \nu_e$$

- Relevant timescales:

$$t_m \sim 2\left(\frac{10^9\ \mathrm{K}}{T}\right)^6 \text{ months} \quad \text{(modified Urca reactions)}$$

$$t_d \sim 20\left(\frac{10^9\ \mathrm{K}}{T}\right)^4 \text{ sec} \quad \text{(direct Urca reactions)}$$

- During last stages of inspiral, composition is "frozen"

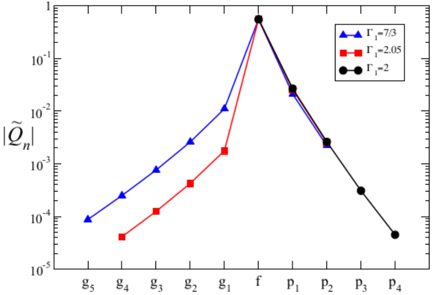
$$k_l = -\frac{1}{2} + \frac{2\pi}{2l+1}\sum_n \frac{Q_n^2}{\omega_n^2}\left[1 - \omega_n^2 \frac{\xi_{h,n}(R)}{\xi_{r,n}(R)}\right]^{-1}$$

$$Q_n \sim \int \boldsymbol{\xi}_n^* \cdot \nabla U \rho \mathrm{d}^3\boldsymbol{r}$$

Overlap between the mode and the tide

$\Gamma_1 = 2$		$\Gamma_1 = 2.05$		$\Gamma_1 = 7/3$	
mode	k_l	mode	k_l	mode	k_l
f	0.27528	f	0.27055	f	0.24685
$+p_1$	0.25887	$+p_1$	0.25526	$+g_1$	0.26115
$+p_2$	0.26021	$+p_2$	0.25653	$+p_1$	0.25052
$+p_3$	0.26015	$+g_1$	0.25878	$+g_2$	0.25556
		$+g_2$	0.25960	$+p_2$	0.25653
		$+g_3$	0.25993	$+g_3$	0.25856
		$+g_4$	0.26008	$+g_4$	0.25944
				$+g_5$	0.25983
	9×10^{-4}		7×10^{-4}		3×10^{-4}

Results for $\Gamma = 2$



- Equation of state: $p = K\rho^\Gamma$
- Buoyancy arises when $\Gamma_1 - \Gamma \neq 0$,

with $\Gamma_1 = \left(\frac{\partial \ln p}{\partial \ln \rho}\right)_{\text{composition}}$

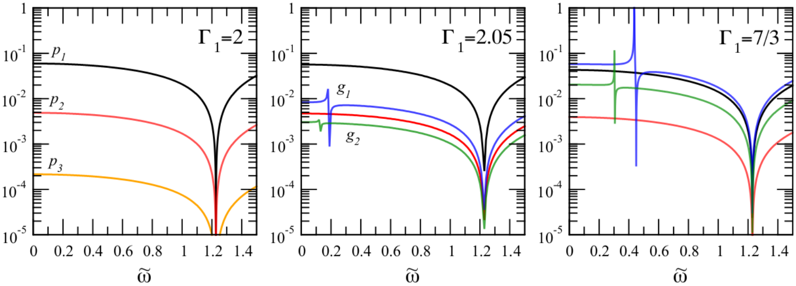
- Love number converges to its expected value ($k_2 \approx 0.259909$)
- Equilibrium tide is oblivious to composition gradients

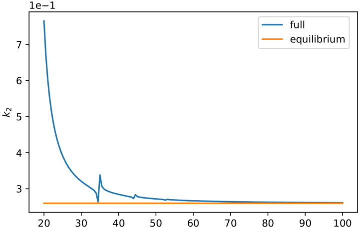
Mode expansion

Full solution

$$k_l^{\mathrm{eff}} = -\frac{1}{2} + \frac{2\pi}{2l+1}\sum_n \frac{Q_n^2}{\omega_n^2 - (m\omega)^2}\left[1 - (m\omega)^2\frac{\xi_{h,n}(R)}{\xi_{r,n}(R)}\right]\left[1 - \omega_n^2\frac{\xi_{h,n}(R)}{\xi_{r,n}(R)}\right]^{-1}$$

Mode contributions to the Love number (relative to f-mode)





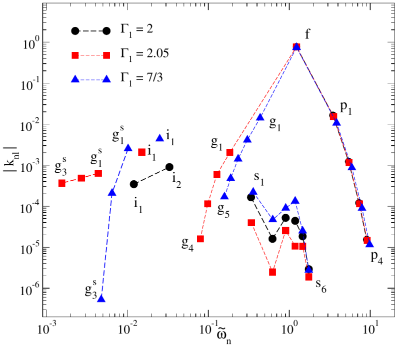
- Contribution of each mode is large near its resonance
- f-mode prevails, as expected
- Composition dependence enters at $\sim 10^{-2}$
- Possibly within reach of 3G detectors

- Crust surface layer introduces elastic stresses and new modes

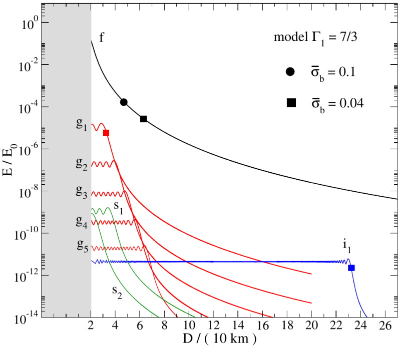
  Euler equation: $$\ddot{\xi}^i + \frac{\nabla^i \delta p}{\rho} - \frac{\nabla^i p}{\rho^2}\delta\rho + \nabla^i \delta\Phi + \frac{\nabla_j \sigma^{ij}}{\rho} = -\nabla^i U$$

  – s-modes: shear oscillations
  – i-modes: oscillations at the crust-core interface
- Modes still form an orthonormal basis (same arguments apply)
- f-mode contributes the most (again) to the Love number

Mode contributions to the Love number



Mode energy during the inspiral



- Resonant excitation of modes can *fracture the crust* [Tsang *et al.* (2012) PRL **108**, 011102]
  – Crust breaking strain is exceeded for i-mode and f-mode during the inspiral

A phenomenological model

- Account only for f-mode contribution (other modes treated as a "systematic error")

$$k_l^{\text{eff}} = k_l \left[1 - \frac{(m\omega)^2}{\omega_f^2}\right]^{-1} + \frac{(m\omega)^2}{\omega_f^2}\left[\frac{1}{2} - \frac{\omega_f^2}{GM/R^3}\frac{\xi_h(R)}{\xi_r(R)}\left(k_l + \frac{1}{2}\right)\right]\left[1 - \frac{(m\omega)^2}{\omega_f^2}\right]^{-1}$$

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- Can we meaningfully "extend" to relativity?
  - Assume expression is similar for relativistic stars
  - Use *universal relation* between ω_f and k_l

$$\omega_f = a_0 + a_1 y + a_2 y^2 + a_3 y^3 + a_4 y^4, \quad y = f(k_l)$$

[Chan *et al.* (2014) PRD **90**, 124023]

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- Replace $\frac{\xi_h(R)}{\xi_r(R)} \equiv \frac{\epsilon}{l}$
  - Incompressible stars: $\epsilon = 1$
  - Appropriately stiff equations of state: $\epsilon \sim 1$

$\Rightarrow$ One-parameter expression for k_l^{eff}

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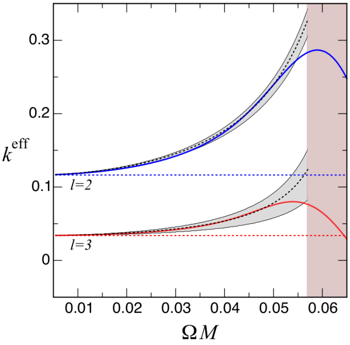
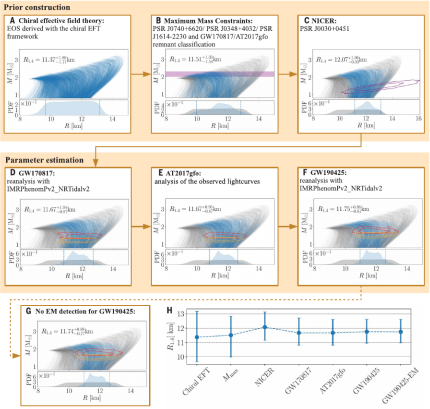


$\epsilon = 0.85 - 0.90$
Precise match: $\epsilon = 0.875$

- "Matches" results from studies where dynamical effects are incorporated in the effective-one-body framework [Hinderer *et al.* (2016) PRL **116**, 181101; Steinhoff *et al.* (2016) PRD **94**, 104028]

Summary

- Tidal effects on GW signal can constrain dense matter equation of state
- Dynamical effects are missed from equilibrium tide description
  - Oscillation mode contributions become important near *resonances*
  - Near merger, f-mode dominates
- *Effective tidal deformability* is slightly affected by neutron star composition (at % level)
- Phenomenological prescription for the dynamical tide works surprisingly well
  - Useful for inexpensive implementation of tidal effects in GW data analyses



- Ongoing efforts:
  - GWs (LIGO, Virgo, KAGRA)
  - EM observations (counterparts; NICER)
  - Experiments (PREX II)

Multi-messenger equation of state constraints

Credit: Dietrich *et al.* (2020) Science **370**, 1450

Additional material: Future tasks

- Accounting for superfluidity and superconductivity
  - Additional modes
- Effects of spin
  - Star is no longer spherical (employ slow-rotation approximation)
  - Zeeman-type mode frequency splitting for different azimuthal numbers m
  - Polar-axial mode coupling, even at leading order in rotation Ω
  - Usual mode decomposition leads to couplings among equations of motion (expand in phase space instead)
- Relativistic extension
  - Modes do not form a complete set

Additional material: The tidal problem

- Perturbed hydrodynamic equations in the rotating frame:

$$\text{Continuity:} \quad \delta\rho + \nabla\cdot(\rho\boldsymbol{\xi}) = 0,$$

$$\text{Euler:} \quad \ddot{\boldsymbol{\xi}} + 2\boldsymbol{\Omega}\times\dot{\boldsymbol{\xi}} + \frac{\nabla\delta p}{\rho} - \frac{\nabla p}{\rho^2}\delta\rho + \nabla\delta\Phi = -\nabla U,$$

$$\text{Poisson:} \quad \nabla^2\delta\Phi = 4\pi G\delta\rho,$$

$$\text{Laplace:} \quad \nabla^2 U = 0,$$

$$\text{Equation of state:} \quad \frac{\delta\rho}{\rho} = \frac{1}{\Gamma_1}\frac{\delta p}{p} - \boldsymbol{\xi}\cdot\boldsymbol{A},$$

$$\text{where} \quad \boldsymbol{A} = \frac{\nabla\rho}{\rho} - \frac{1}{\Gamma_1}\frac{\nabla p}{p} \quad \text{and} \quad \Gamma_1 = \left(\frac{\partial\ln p}{\partial\ln\rho}\right)_{\text{ad}}$$

- Equilibrium tide solution (*ignoring rotation*):

$$\delta p = -\rho(\delta\Phi + U),$$

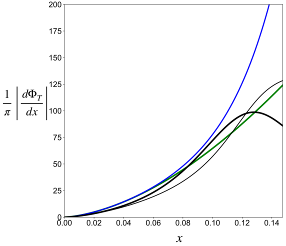
$$\delta\rho = \frac{\mathrm{d}\rho}{\mathrm{d}r}\frac{\delta\Phi + U}{g},$$

$$\xi_r = -\frac{\delta\Phi + U}{g}, \quad \xi_h = \frac{1}{l(l+1)r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\xi_r\right), \quad \nabla\cdot\boldsymbol{\xi} = 0,$$

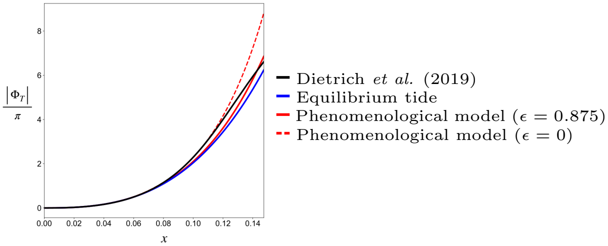
$$\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\frac{\mathrm{d}\delta\Phi}{\mathrm{d}r}\right) - \frac{l(l+1)}{r^2}\delta\Phi - 4\pi G\frac{\mathrm{d}\rho}{\mathrm{d}r}\frac{\delta\Phi + U}{g} = 0,$$

$$\text{where} \quad g = \frac{\mathrm{d}\Phi}{\mathrm{d}r}$$

- Simple and inexpensive alternative for GW data analysis
  - Numerical relativity simulations necessary to describe dynamics near merger...
  - ...but computationally costly to incorporate in detector templates
- Tidal contribution to GW phase: $\dfrac{\mathrm{d}\Phi_T}{\mathrm{d}x} = -\dfrac{65}{2^5}\dfrac{k_2 x^{3/2} f(x)}{(M/R)^5}$, $x = (M\omega)^{2/3}$ $\quad (c = G = 1)$



Comparison of various waveform models



Comparison of phenomenological relation with waveform model of Dietrich *et al.* (2019) PRD **100**, 044003

- Noticeable differences between different models
- Dynamical tide produces subradian change to GW phasing
- Detectable by 3G detectors

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