RUB

A Virtual Tribute to Quark Confinement and the Hadron Spectrum, August, 2 2021

Deuteron charge and quadrupole form factors in chiral effective field theory

Arseniy Filin

Institut für Theoretische Physik II, Ruhr-Universität Bochum, Germany

in collaboration with

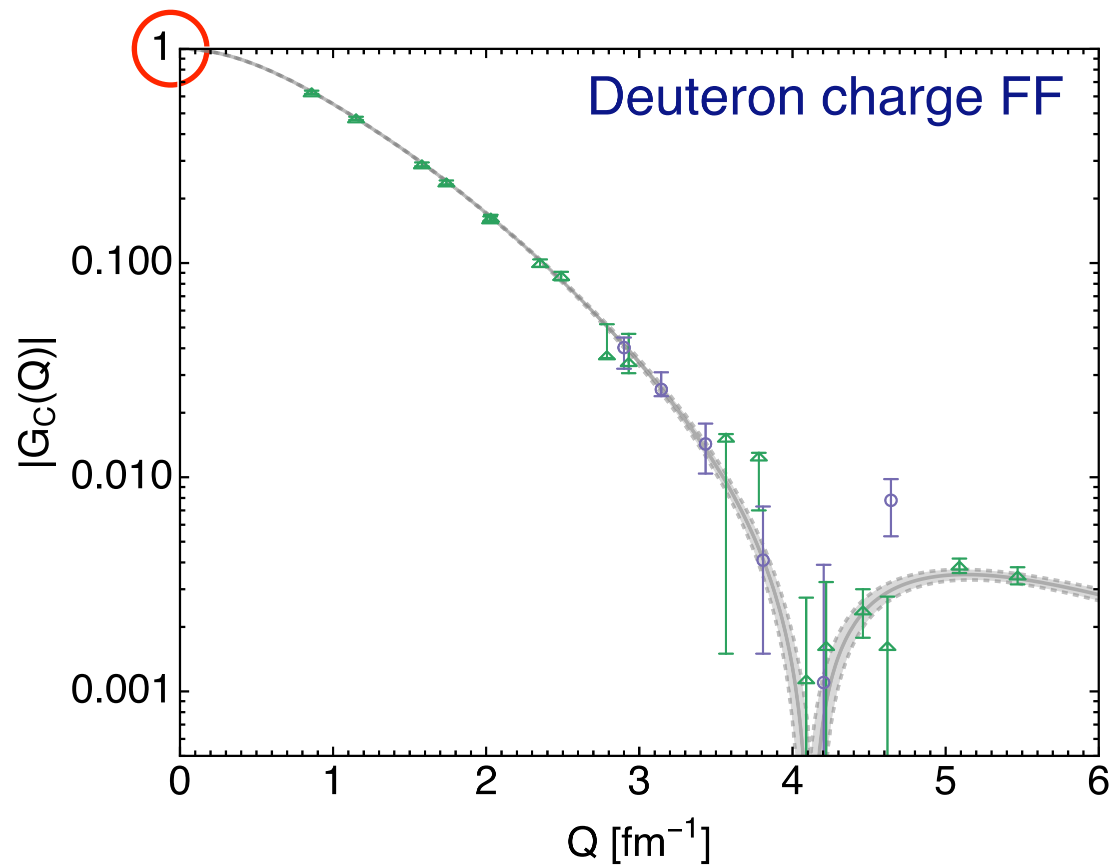
D. Möller, V. Baru, E. Epelbaum, H. Krebs, and P. Reinert

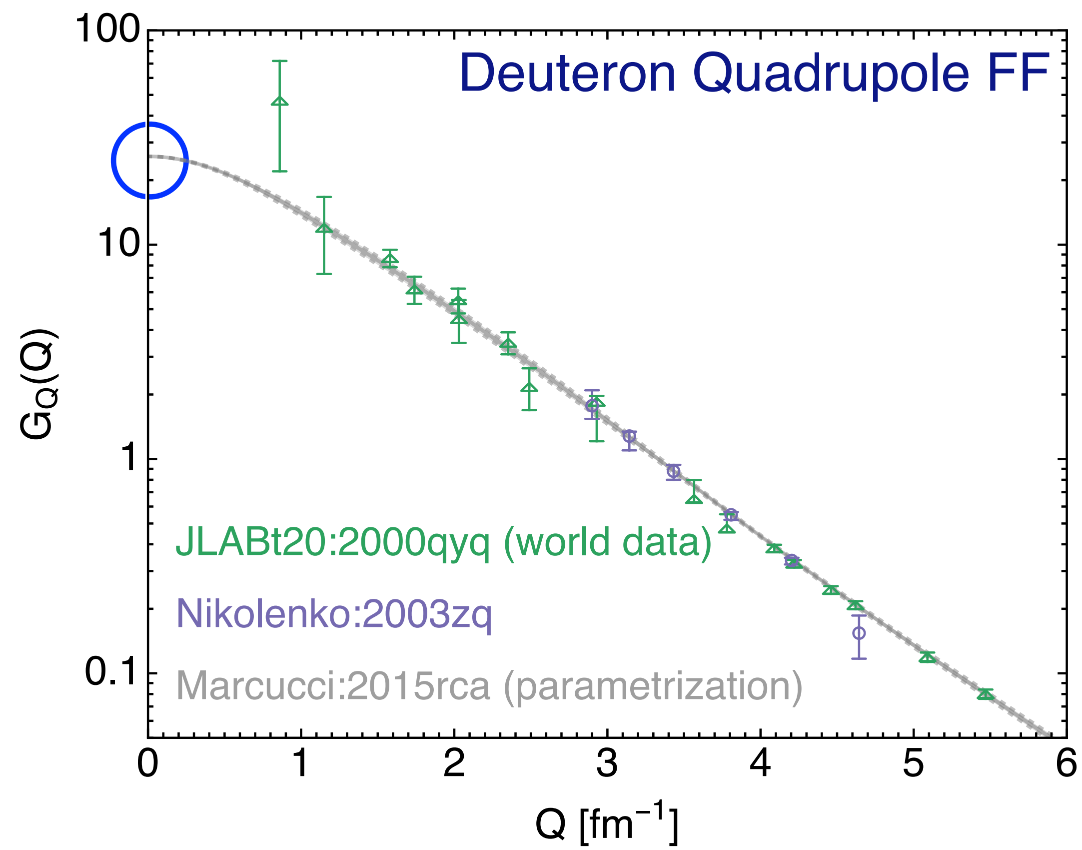
PRL124 082501 (2020)
Phys.Rev.C 103, 024313 (2021)

Deuteron charge and quadrupole form factors

Deuteron charge $G_C(Q^2)$ and quadrupole $G_Q(Q^2)$ form factors:

- parametrize the interaction of a deuteron with electro-magnetic probe
- Lorentz-invariant functions of the four-momentum transfer Q^2





- at Q=0 related to the deuteron charge radius and deuteron quadrupole moment:

$$r_d^2 = (-6)\frac{\partial}{\partial Q^2} G_C(Q^2)\Big|_{Q=0} \qquad\qquad Q_d = G_Q(0)/m_d^2$$

- belong to the most extensively studied observables in nuclear physics

Why make a new chiral EFT study of the deuteron form factors?

- experimental precision and theoretical accuracy are increased tremendously!

Motivation to study deuteron form factors

- Progress in atomic spectroscopy

Lamb-shift measurements muonic deuterium and muonic hydrogen + QED corrections

precise extraction of proton and deuteron charge radii

Pohl:2013yb, CREMA:2016idx, Pohl:2016glp, ...

Spectroscopy of atomic hydrogen Beyer:2017gug, Bezginov:2019mdi, …

Hydrogen-deuterium 1S-2S isotope shift + QED corrections

very precise extraction of deuteron-proton charge radii difference

$$r_d^2 - r_p^2 = 3.82070(31) fm^2$$

Jentschura et al. PRA 83 (2011)
Pachucki et al., PRA 97, 062511 (2018)

- Advances in chiral effective field theory (χEFT)

High-accuracy chiral NN forces (N^4LO^+) in χEFT Reinert:2020mcu PRL 126, 092501 (2021)

nearly perfect description of pp and pn scattering data up to pion production threshold

semi-local momentum-space (SMS) regulator - preserves long-range interactions

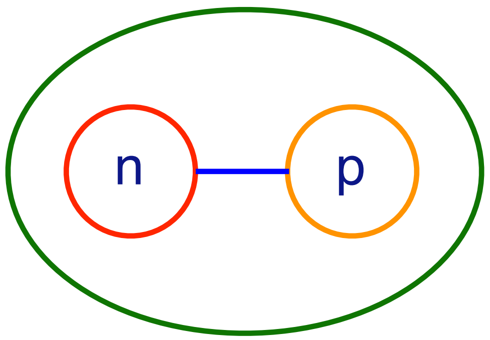
Chiral electromagnetic currents (N^3LO + partially N^4LO) in χEFT

Derived and regularized consistently with chiral NN forces

New methods to quantify chiral expansion truncation errors:

Epelbaum et al. EPJA 51 (2015)
Furnstahl et al. PRC 92, 024005 (2015).
Melendez et al. PRC 96, 024003 (2017).
Wesolowski et al. J. Phys. G 46, 045102 (2019).
Melendez et al. PRC 100, 044001 (2019).

Motivation to study deuteron form factors



Deuteron form factors depend on

- properties of proton
- properties of neutron
- deuteron structure

Deuteron charge radius: $$r_d^2 = r_{str}^2 + r_p^2 + r_n^2 + \frac{3}{4m_p^2}$$

Advances in χEFT allow a high-accuracy calculation of deuteron structure effects

Combining with precise spectroscopic data $(r_d^2 - r_p^2) = 3.82070(31) fm^2$

(atomic spectroscopy + QED corrections)

Jentschura et al. PRA 83 (2011)

Pachucki et al., PRA 97, 062511 (2018)

New method of determination of neutron charge radius:

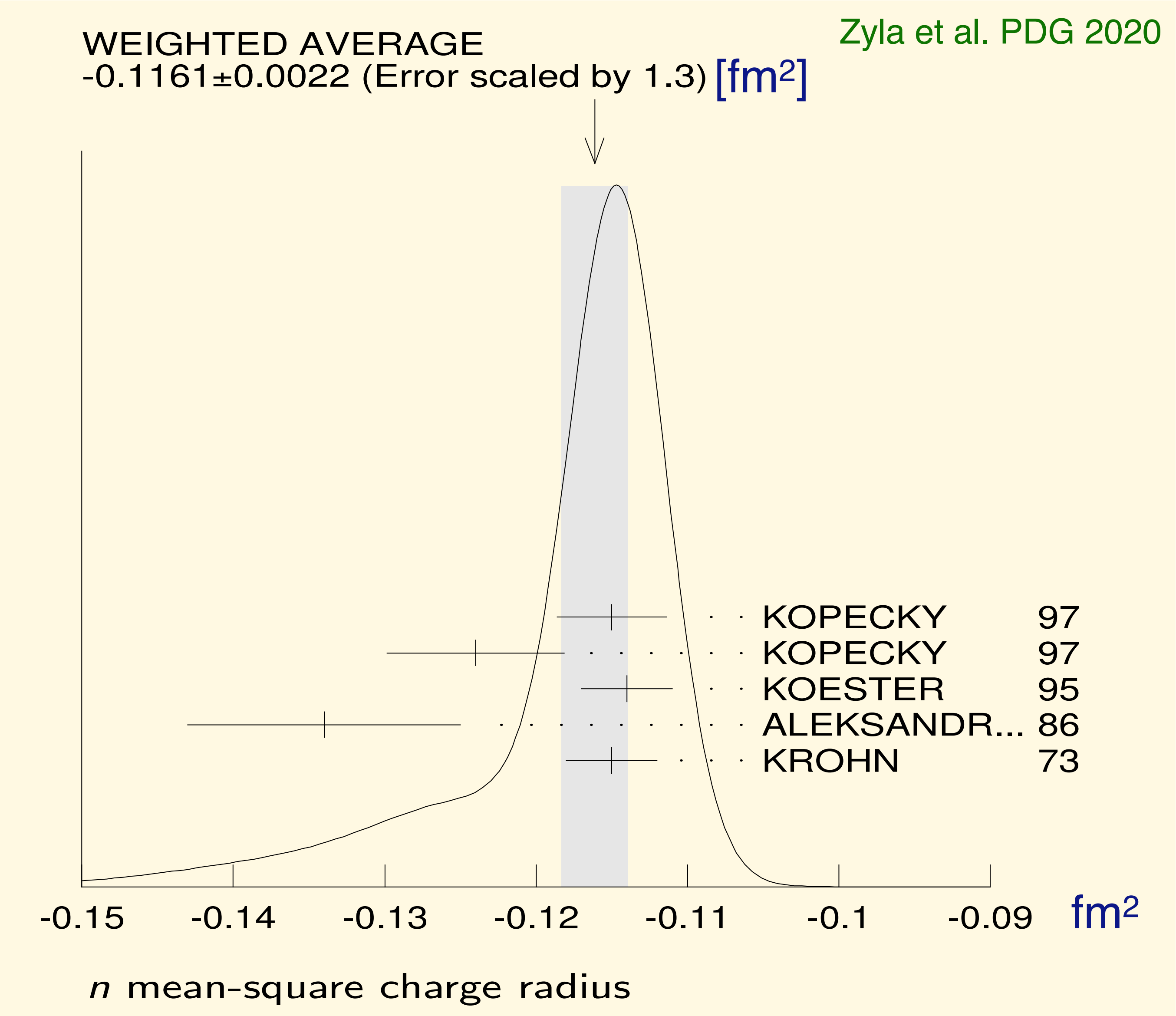
$$r_n^2 = (r_d^2 - r_p^2) - \frac{3}{4m_p^2} - r_{str}^2$$

χEFT calculation of deuteron form factors

Neutron charge radius determination

Neutron charge radius PDG2020 weighted average $r_n^2 = -0.1161(22) fm^2$ Zyla et al. (PDG2020)

- Based on neutron-electron scattering length measurements on ^{208}Pb, ^{209}Bi, and other heavy nuclei
- Spread between determination on Pb and Bi is larger than (scaled) error
- Uncertainty may be underestimated

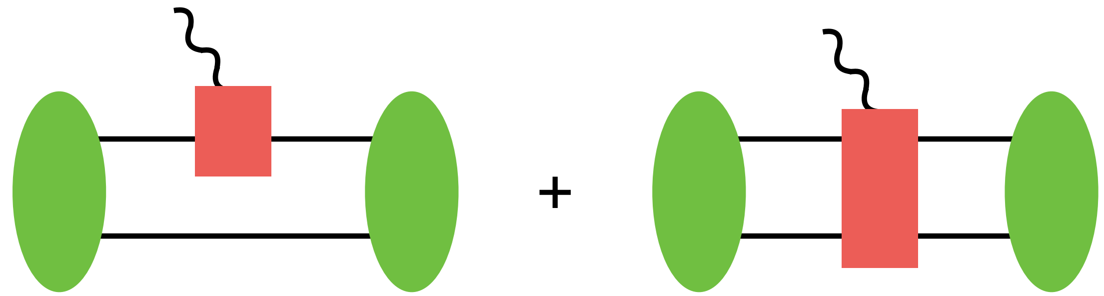


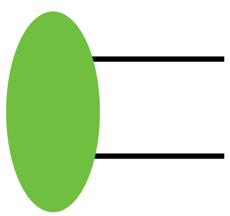
New method of determination:

Neutron charge radius can be extracted from deuteron-proton charge radii difference

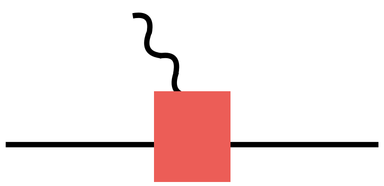
χEFT calculation of deuteron form factors

Form factors are obtained by convolution of deuteron wave function and charge density operator (1N, 2N)

$G_{C/Q}(Q) =$ 



Deuteron wave function - solution of the Schrödinger Eq. with chiral semi-local momentum-space (SMS) NN potential LO - N^4LO^+

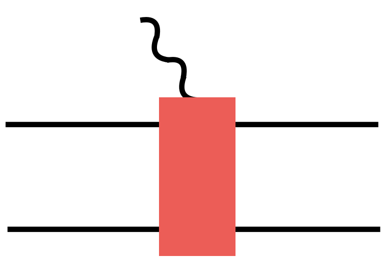


1N charge-density operators LO - N^3LO

$$\rho_{1\mathrm{N}} = e\left(1 - \frac{\boldsymbol{k}^2}{8m_N^2}\right) G_E(\boldsymbol{k}^2) + ie\frac{2G_M(\boldsymbol{k}^2) - G_E(\boldsymbol{k}^2)}{4m_N^2}(\boldsymbol{\sigma}\cdot\boldsymbol{k}\times\boldsymbol{p}).$$

not expanded; using modern parameterizations of nucleon FF:

Phenomenological: Ye:2017gyb Dispersion theory: Belushkin:2006qa, Lin:2021umz



2N charge-density operators: one-pion-exchange (N^3LO) + contact (N^4LO)

Derived and regularized consistently with SMS NN potential
regulators, cutoff values, and other conventions - same as in SMS NN potential

Depend on two N^4LO low-energy constants - we fit them to $G_C(Q)$ and $G_Q(Q)$ data

Predictions for deuteron structure radius and quadrupole moment

Scheme of the FF calculation:

- Pre-calculate contributions $G_x^i(Q)$ from all charge-density operators: i = {main, SO, DF, Boost, OPE, CT}

$$G_x(Q) = G_x^{LO}(Q) + G_x^{SO}(Q) + G_x^{DF}(Q) + G_x^{Boost}(Q) + G_x^{OPE}(Q) + (A + B + C/3)G_x^{CT1}(Q) + C\,G_x^{CT2}(Q)$$

x = {C,Q}

- Fit two low-energy constants (A+B+C/3) and (C) to experimental data for G_C and G_Q

fit includes theoretical errors from truncation and from nucleon FFs

$$\chi^2 = \sum_i \frac{\left(G_C^{th}(Q_i^2; A+B+C/3; C) - G_C^{exp}(Q_i^2)\right)^2}{\Delta G_C(Q_i^2)^2} + \sum_i \frac{\left(G_Q^{th}(Q_i^2; A+B+C/3; C) - G_Q^{exp}(Q_i^2)\right)^2}{\Delta G_Q(Q_i^2)^2}$$

Our predictions: (using fitted low-energy constants)

Deuteron structure radius $r_{str}^2 = r_m^2 + r_{SO}^2 + r_{DF}^2 + r_{OPE}^2 + (A + B + C/3)r_{CT1}^2 + C\,r_{CT2}^2$

$$r_{str}^2 = 3.8925 fm^2$$

Deuteron quadrupole moment $Q_d = Q_0 + Q_{SO} + Q_{DF} + Q_{OPE} + (A + B + C/3)Q_{CT1} + C\,Q_{CT2}$

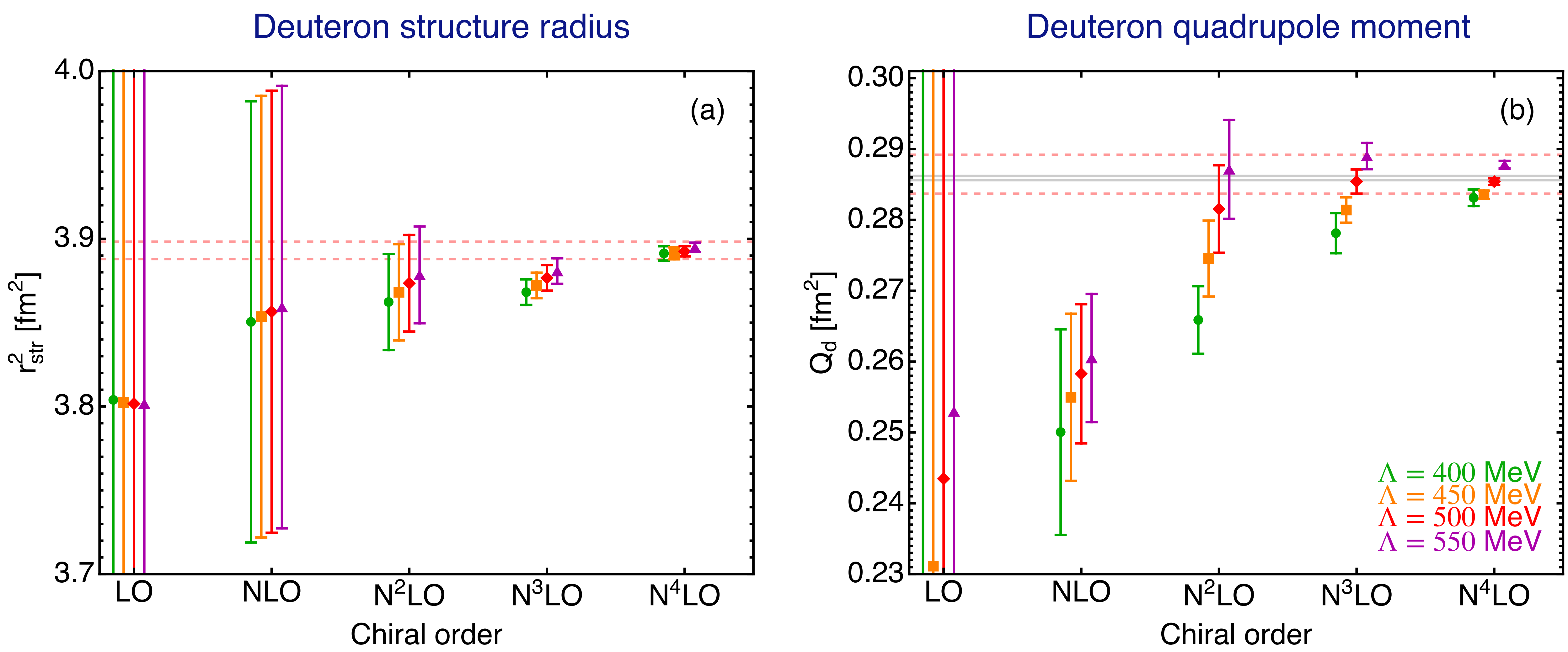
$$Q_d = 0.2854 fm^2$$

Next step: uncertainty analysis

Chiral expansion: convergence and truncation error

To estimate chiral expansion truncation error:

- repeat the calculation of G_C, G_Q, r_{str}, and Q_d for each chiral order (LO-N^{4}LO)
- use Bayesian model to estimate truncation errors Epelbaum et al. EPJA 56, 92 (2020)



red band = our prediction + total uncertainty

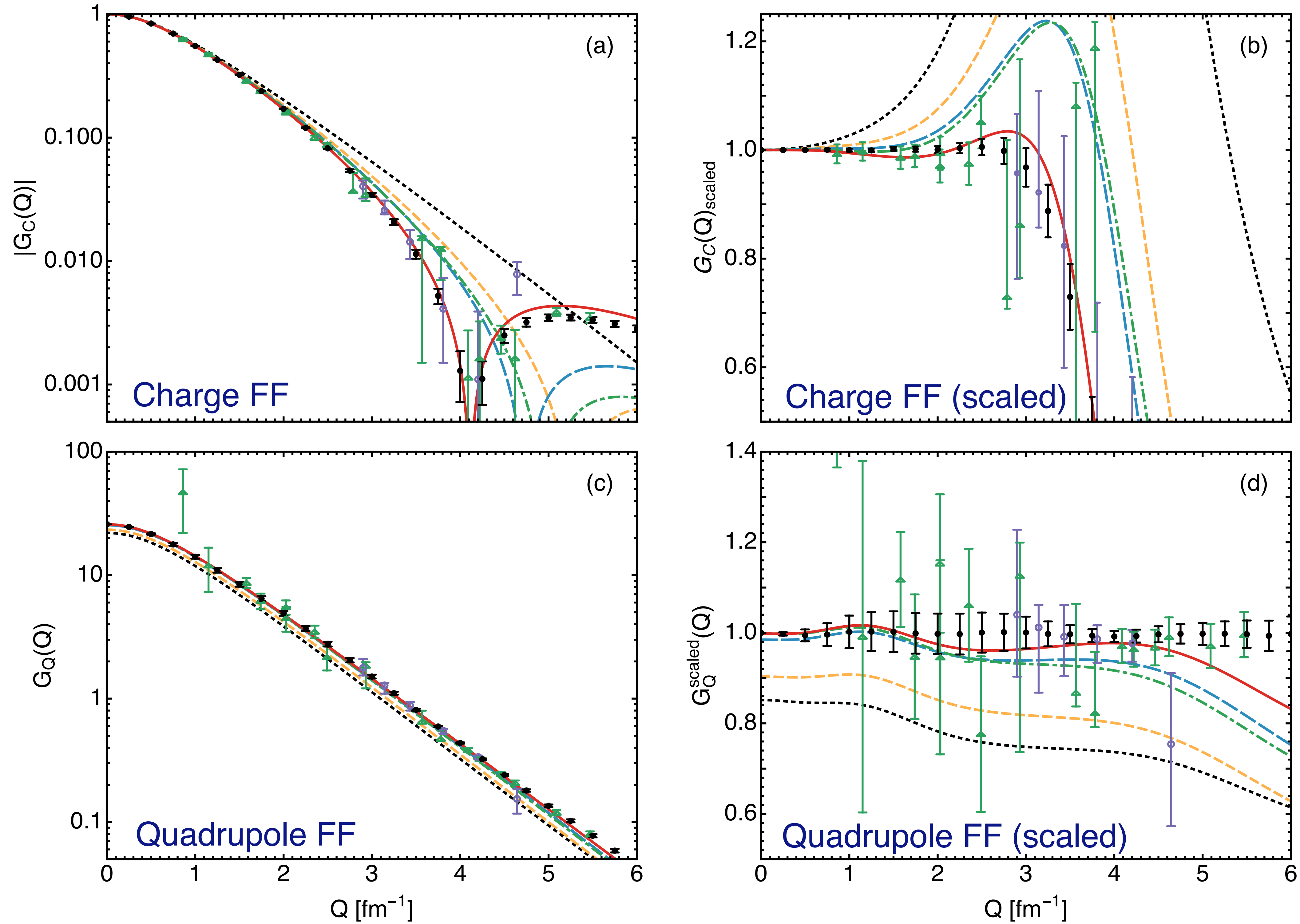
gray band = empirical value of Q_d

Chiral expansion of r_{str}, and Q_d converges well

Cutoff dependence of structure radius is smaller than truncation error

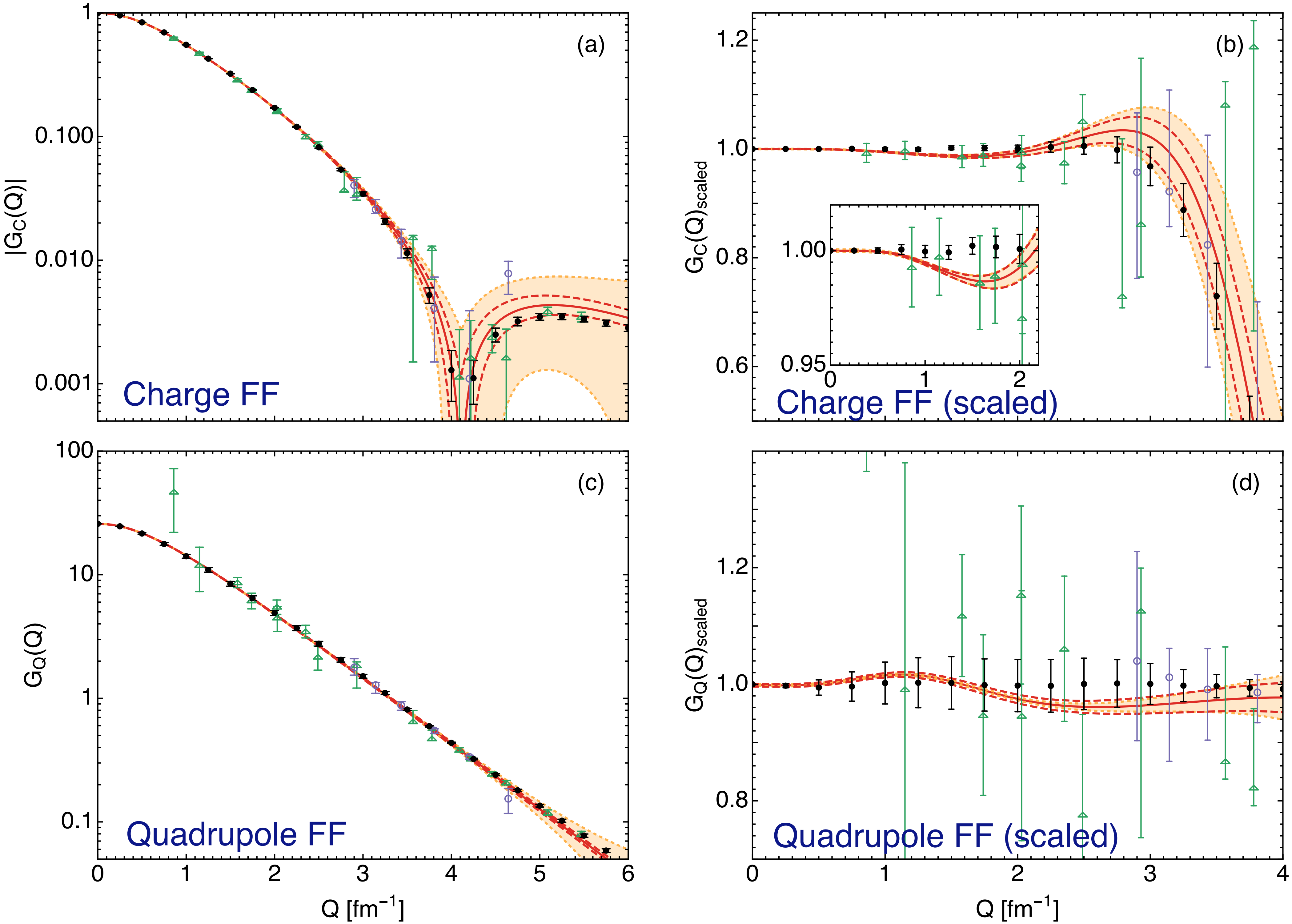
Chiral expansion: convergence of the deuteron form factors

Convergence pattern: LO; NLO; N^2LO; N^3LO; N^4LO^+



World data collection: JLABt20:2000qyq + Nikolenko:2003zq; Parameterization (black dots): Marcucci:2015rca

Chiral expansion truncation error for deuteron form factors

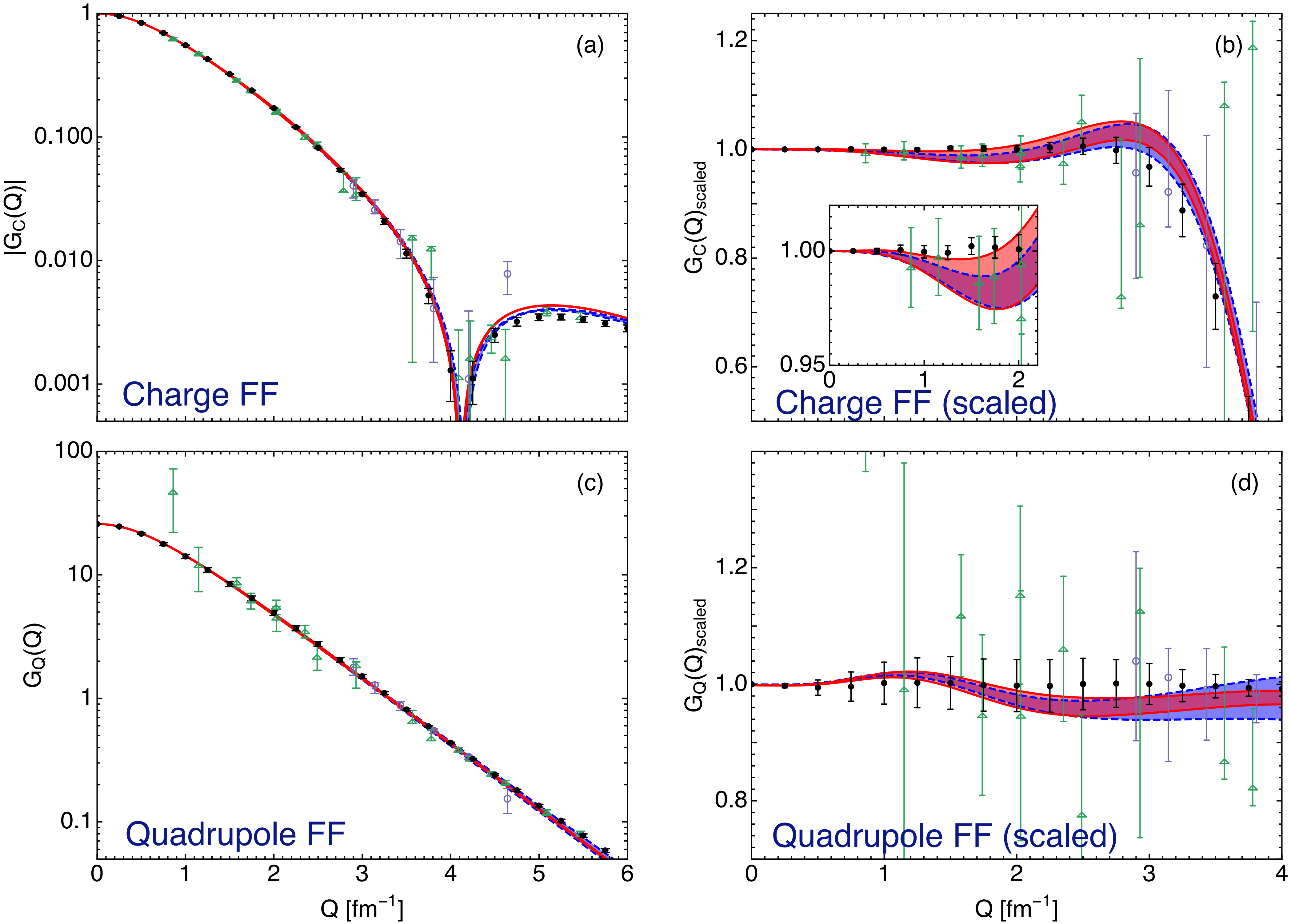


Orange band:
N⁴LO truncation error

Red band:
1σ statistical error from the fit of N4LO LECs to deuteron FF data

World data collection: JLABt20:2000qyq + Nikolenko:2003zq; Parameterization (black dots): Marcucci:2015rca

Uncertainty from nucleon form-factor parametrization



Red band:
phenomenological
parametrization
by Ye et al.
updated to
CODATA2018
proton charge radius
Ye:2017gyb

Blue band:
dispersion-theory based
parametrization
by Belushkin et. al
Belushkin:2006qa

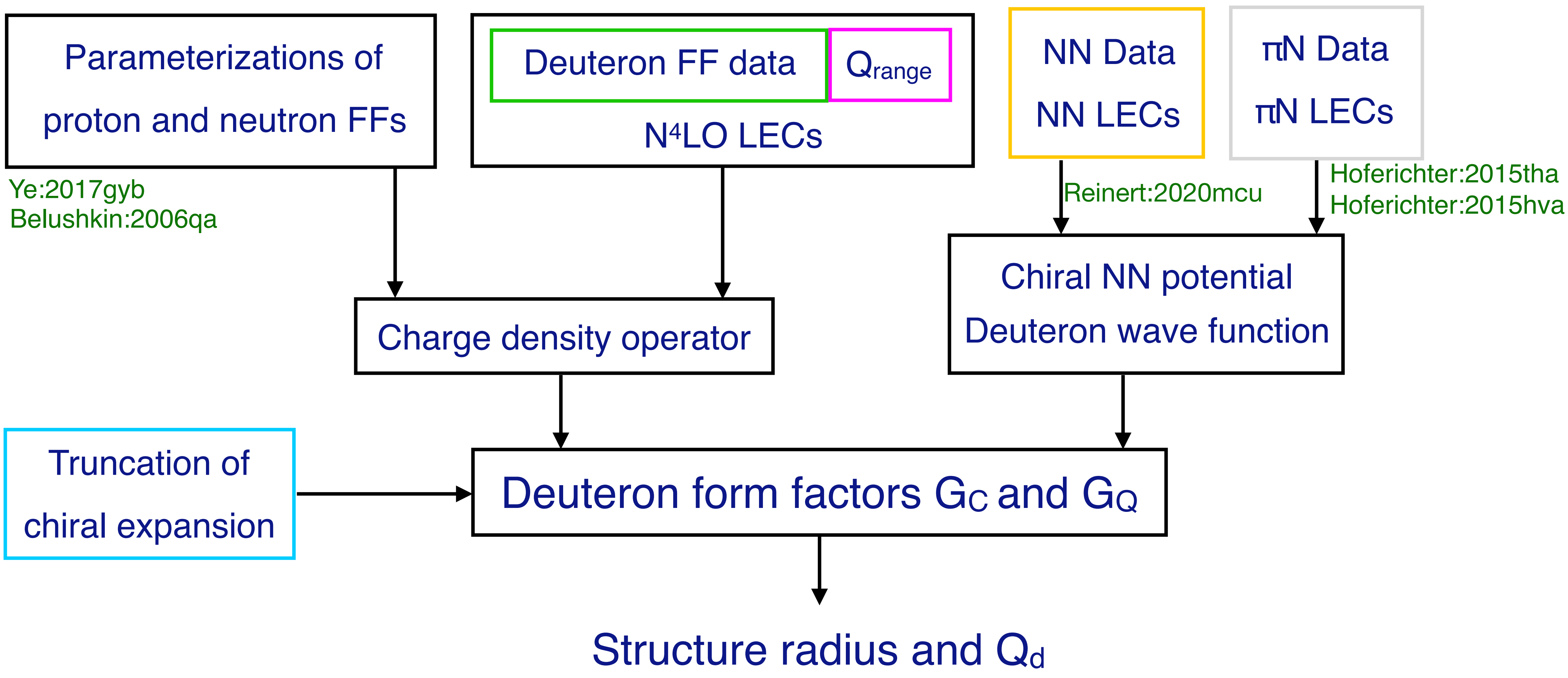
Our predictions for r_d are not sensitive to the specific form of (realistic) nucleon FF parametrization

parametrization must reasonably describe proton and neutron data for Q between 1 and 2.5 fm^{-1}

low-Q (< 1 fm^{-1}) behavior of parametrization is not important for our fits

Extensive uncertainty analysis

We propagate uncertainties from multiple sources



	Central	Truncation	$\rho^{\text{reg}}_{\text{Cont}}$	πN LECs RSA	2N LECs and f_i^2	Q range	Total
r^2_{str} (fm^2)	3.8925	± 0.0030	± 0.0024	± 0.0003	± 0.0025	$^{+0.0035}_{-0.0005}$	$^{+0.0058}_{-0.0046}$
Q_d (fm^2)	0.2854	± 0.0005	± 0.0007	± 0.0003	± 0.0016	$^{+0.0035}_{-0.0005}$	$^{+0.0038}_{-0.0017}$

Results

Predictions for deuteron structure radius and quadrupole moment including all uncertainties:

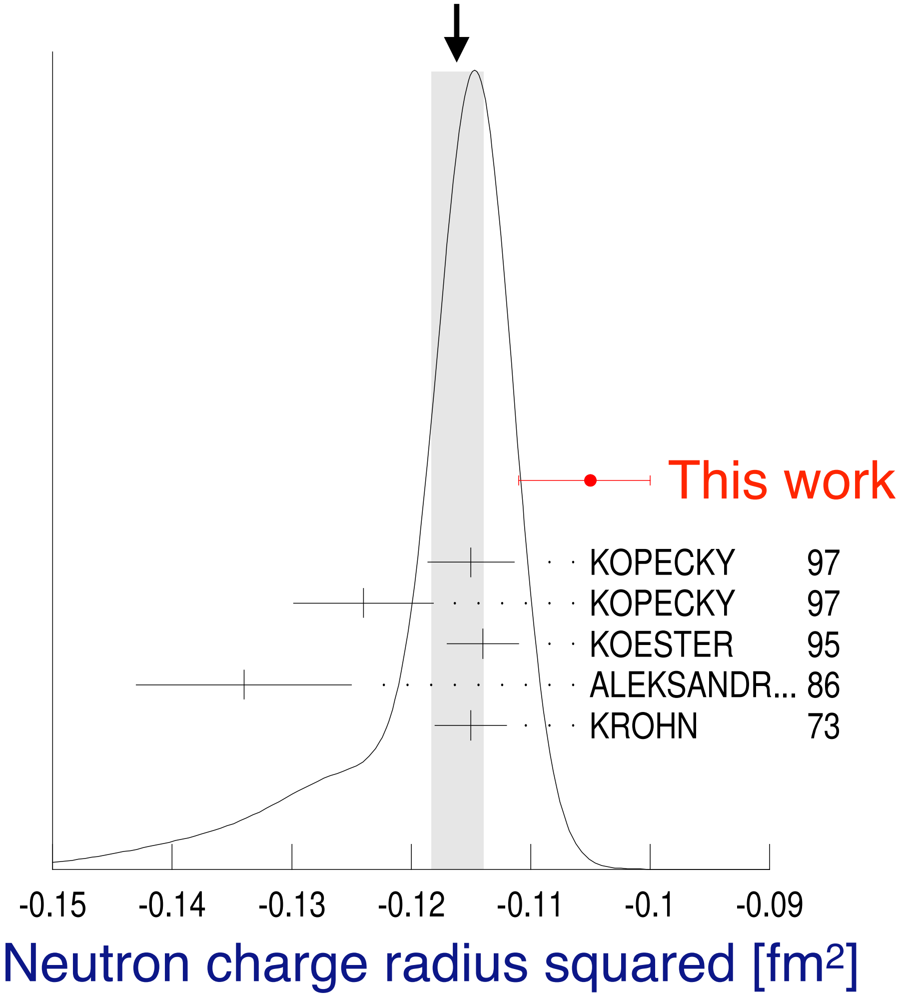
$$r_{str} = 1.9729^{+0.0015}_{-0.0012} fm^2 \qquad Q_d = 0.2854^{+0.0038}_{-0.0017} fm^2$$

Extraction of neutron radius from $(r_d^2 - r_p^2) = 3.82070(31) fm^2$ (atomic spectroscopy + QED corrections)

$$r_n^2 = -0.105^{+0.005}_{-0.006} fm^2$$

$$r_n^2 = (r_d^2 - r_p^2) - \frac{3}{4m_p^2} - r_{str}^2$$

1.9σ deviation from the PDG2020 weighted average $r_n^2 = -0.1161(22) fm^2$

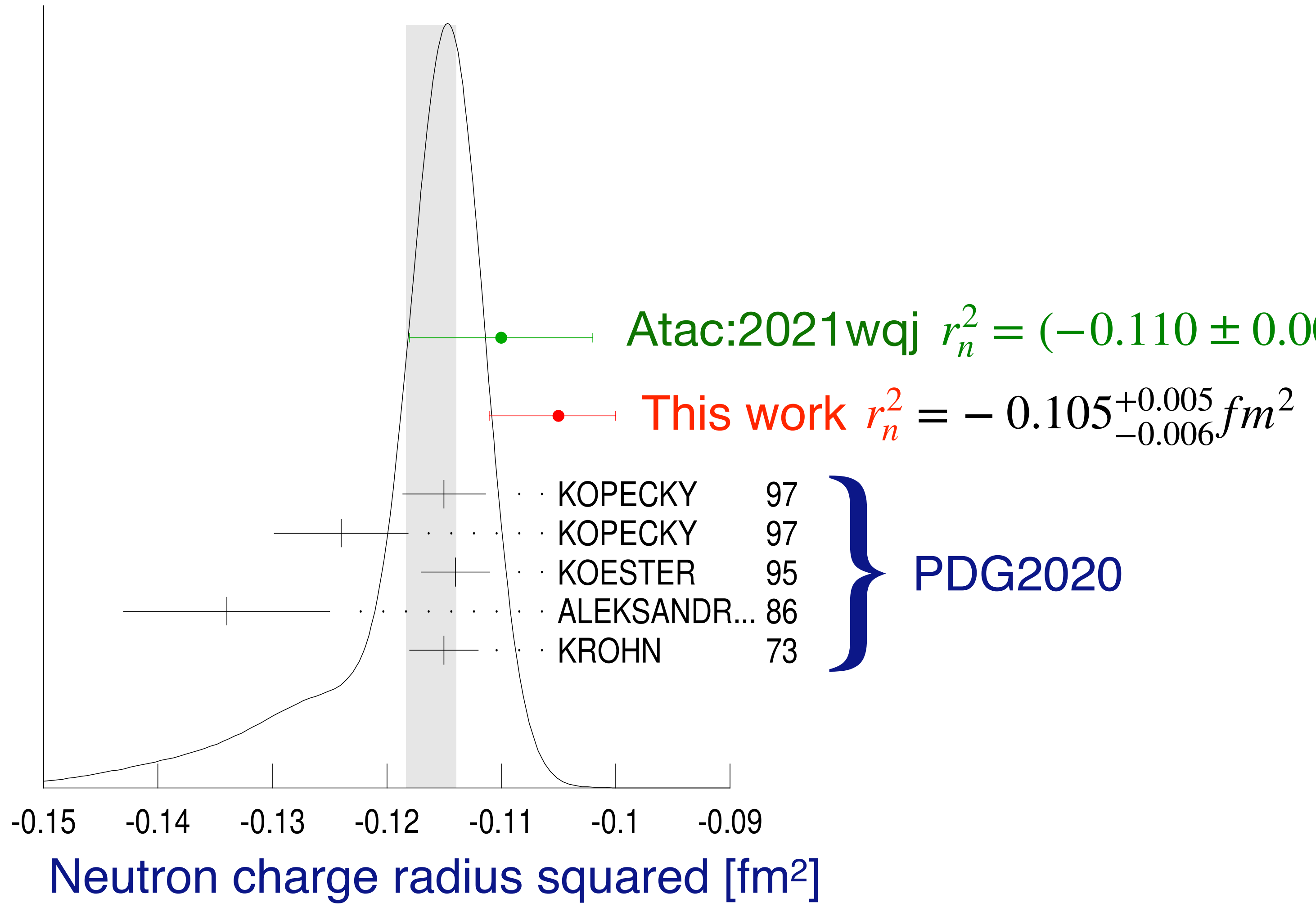


Recently published results

New extraction of neutron charge radius

Atac:2021wqj (*Nat Commun* 12, 1759 (2021))

(extraction from quadrupole transition form factors of proton)



confirms our result

New determination of deuteron quadrupole moment

Puchalski:2020jkt *PRL*125 253001 (2020)

Value: $Q_d = 0.285699(15)(18) fm^2$

Very good agreement with our result ($Q_d = 0.2854^{+0.0038}_{-0.0017} fm^2$)

Summary

Comprehensive analysis of deuteron charge and quadrupole form factors in χEFT

- Pushed to N^4LO for the first time — PRL124 082501 (2020), Phys.Rev.C 103, 024313 (2021)
- Two unknown LECs are determined from FF data
- Prediction for deuteron structure radius and quadrupole moment
- Extended analysis of various sources of uncertainty

Extraction of neutron charge radius

- New method using hydrogen-deuterium 1S-2S isotope shift data
- 1.9σ-deviation from the value in PDG2020
- Confirmed by very recent determination Atac:2021wqj (*Nat Commun* 12, 1759 (2021))

Outlook

- Analysis of the deuteron magnetic form factor
- Analysis of ^3H, ^3He and ^4He electromagnetic form factors