The pion LCDA
from the pion electromagnetic form factor

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Introduction

- Pion light-cone distribution amplitudes (LCDAs)
  - ∗ are used to describe hard exclusive processes with pions, e.g.:

$\gamma^*(Q^2)\pi^\pm \to \pi^\pm$ $\gamma\gamma^*(Q^2) \to \pi^0$ $B \to \pi\ell\nu_\ell$ $B \to \pi\pi$

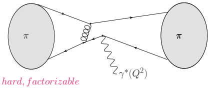
  - ∗ represent nonperturbative part of continuum-QCD methods:

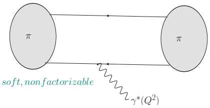
  factorization theorems, QCDF, light-cone sum rules (LCSRs)

- The leading twist-two LCDA: asymptotics of the pion e.m. form factor

$$\lim_{Q^2\to\infty} F_\pi(Q^2) = \frac{8\pi\alpha_s(Q^2)}{9Q^2}\left| f_\pi \int_0^1 \frac{\varphi_\pi(u)}{1-u}\right|^2$$

[V.Chernyak, A.Zhitnitsky ,V.Serbo (1977)], [G.Farrar, D.Jackson(1979)], [A.Efremov, A.Radyushkin (1979)], [S.Brodsky, G.Lepage(1979)]





- We suggest to use data on the timelike e.m. pion form factor, measured in $e^+e^- \to \pi^+\pi^-$ to determine/constrain the pion LCDA

  [Shan Cheng, AK, Aleksey Rusov, 2007.05550]

Structure of the pion LCDA

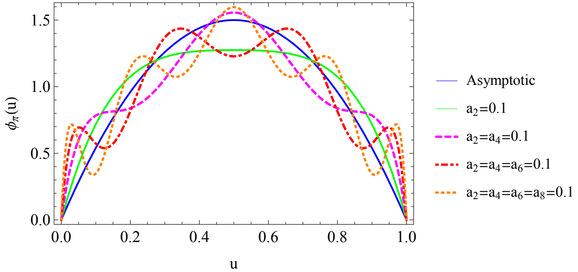
- Definition:

$$\langle \pi^+(p)|\bar{u}(x)[x,0]\gamma_\mu\gamma_5 d(0)|0\rangle\Big|_{x^2\to 0} = -i\, f_\pi p_\mu \int_0^1 du\, e^{iup\cdot x} \underbrace{\varphi_\pi(u)}_{twist-two} + \underbrace{\ldots}_{higher\ \ twists}$$

- Conformal expansion, Gegenbauer moments:

$$\varphi_\pi(u,\mu) = \underbrace{6u(1-u)}_{\varphi_\pi^{as}}\Big[1 + \sum_{n=2,4,\ldots} a_n(\mu)\ \underbrace{C_n^{(3/2)}(2u-1)}_{Gegenbauer\ \ polynomials}\Big]$$

  - ∗ ERBL evolution: $a_n(\mu) \to 0$ at large μ
  - ∗ $a_n(\mu \sim 1\text{GeV})$ to be determined by nonperturbative methods
  - ∗ models with $\{a_2, a_4, \ldots, a_{n_{max}}\}$



Pion electromagnetic form factor

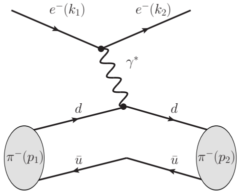
- The standard definition:

$$\langle \pi(p_2)|\, j_\mu^{\rm em}\, |\pi(p_1)\rangle = F_\pi(q^2)\,(p_1+p_2)_\mu, \qquad q = p_2 - p_1,$$

- spacelike form factor: $q^2 = -Q^2$, $Q^2 \geq 0$, $\boxed{F_\pi(0) = 1}$

  * $e^-\pi^\pm \to e^-\pi^\pm$,
  measured only at small Q^2 [NA7 Collab. (1986)]

  * accessible indirectly in $eN \to e\pi N$
  measured at $Q^2 \lesssim 2.0$ GeV^2
  [Jefferson Lab, G.Huber et al. (2008)]



- timelike form factor: $q^2 = s \geq 4m_\pi^2$

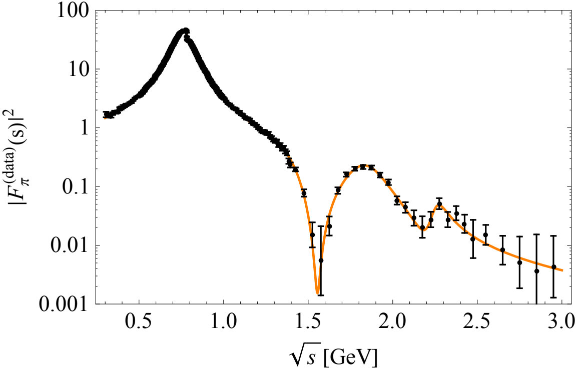
- $F_\pi(q^2)$ is real valued at $q^2 < 4m_\pi^2$
  and develops imaginary part at $q^2 > 4m_\pi^2$
  (ρ-resonance poles and branch points of thresholds)

The timelike form factor: data

- Measured in $e^+e^- \to \pi^+\pi^-$

$$\sigma^{(e^+e^-\to\pi^+\pi^-)}(s) \sim |F_\pi(s)|^2, \qquad s = q^2 > 4m_\pi^2$$

- BaBar collaboration (using 232 fb^{-1} data, ISR) [J.Lees et al. (2012)]



The timelike form factor: parametrization

- The BaBar data at $\sqrt{s} < 3.0$ GeV - fitted using a ρ-resonance ansatz:

[Lees et al. (2012)]

$$F_\pi^{\rm data}(s) = \frac{\sum\limits_{\rho_n} c_{\rho_n}^\pi \, {\rm BW}_{\rho_n}^{\rm GS}(s)}{1 + \sum\limits_{\rho_n} c_{\rho_n}^\pi}, \qquad \rho_n = \{\rho, \omega, \rho', \rho'', \rho'''\}, \quad F_\pi(0) = 1$$

  * the Gounaris-Sakurai resonance formula [Gounaris, Sakurai (1968)]

$${\rm BW}_{\rho_n}^{\rm GS}(s) = \frac{m_{\rho_n}^2 + m_{\rho_n}\Gamma_{\rho_n} d(m_{\rho_n})}{m_{\rho_n}^2 - s + f(s, m_{\rho_n}, \Gamma_{\rho_n}) - i m_{\rho_n}\Gamma(s, m_{\rho_n}, \Gamma_{\rho_n})}$$

- we extrapolate above $\sqrt{s} > 3.0\,{\rm GeV}$, employing the dual-QCD model

[C.Dominguez, (2001)] [C.Bruch, AK, J.Kühn (2005)]

$$F_\pi^{\rm dQCD}(s) = \sum_n^\infty c_{\rho_n}^\pi \, {\rm BW}_{\rho_n}^{\rm dQCD}(s)$$

$${\rm BW}_{\rho_n}^{\rm dQCD}(s) = \frac{m_{\rho_n}^2}{m_{\rho_n}^2 - s - i m_{\rho_n}\Gamma_{\rho_n}}, \quad c_{\rho_n}^\pi = \frac{(-1)^n \Gamma(\beta - 1/2)}{\alpha' m_{\rho_n}^2 \sqrt{\pi}\,\Gamma(n+1)\Gamma(\beta - 1 - n)}$$

Pion form factor: dispersion relations

- "Standard" dispersion relation (unsubtracted, due to QCD asymptotics)

$$F_\pi(q^2) = \frac{1}{\pi}\int_{s_0}^{\infty} ds \frac{\mathrm{Im} F_\pi(s)}{s - q^2}, \qquad s_0 = 4m_\pi^2$$

  * Depends on a model for imaginary part at $s > 4m_\pi^2$

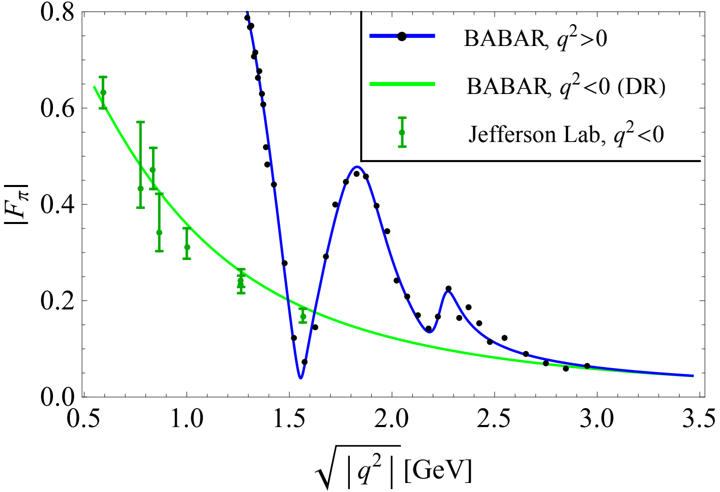
- A different form: modulus representation [B.Geshkenbein (2000)]

$$F_\pi(q^2) = \exp\left[\frac{q^2\sqrt{s_0 - q^2}}{2\pi}\int_{s_0}^{\infty} ds \frac{\ln |F_\pi(s)|^2}{\sqrt{s - s_0}\, s\,(s - q^2)}\right], \qquad q^2 < s_0$$

valid if $F_\pi(q^2)$ has no zeros in the q^2 plane [H.Leutwyler (2002)]

  * Depends on the directly measured $|F_\pi(s)|^2$

Pion form factor: timelike vs spacelike



- ∗ spacelike form factor measurement in agreement with dispersion relation
- ∗ onset of "duality": asymptotics of $F(Q^2)$ and $F_\pi(s)$ coincide above few first ρ resonances

LCSR for the spacelike form factor

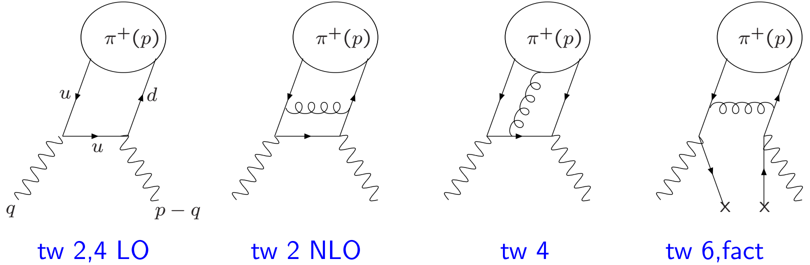
[V.Braun, I.Halperin (1994)], [V.Braun, AK, M.Maul (2000)], [J.Bijnens, AK (2002)]

- The correlation function: vacuum-pion transition amplitude

$$F_{\mu\nu}(p,q) = i\int d^4x e^{iqx}\langle 0|T\{j_{\mu 5}(0)j_\nu^{em}(x)\}|\pi^+(p)\rangle = F((p-q)^2, Q^2)p_\mu p_\nu + ...$$

$j_{\mu 5} = \overline{d}\gamma_\mu\gamma_5 u$ - the pion interpolating current,

∗ Operator-product expansion near $x^2 \sim 0$ in terms of pion DAs:



- factorization of the correlation function at $|(p-q)^2|, Q^2 \gg \Lambda^2_{QCD}$:

$$F^{OPE}((p-q)^2, Q^2) = \sum_{t=2,4,..}\int \mathcal{D}u_i T_t((p-q)^2, Q^2; u_i, \mu)\otimes \phi_t(u_i,\mu),$$

$F_\pi(Q^2)$ from LCSR

- Matching the OPE to the hadronic dispersion relation in $(p-q)^2$:

$$F^{OPE}((p-q)^2, Q^2) = 2i \underbrace{\frac{f_\pi F_\pi(Q^2)}{m_\pi^2 - (p-q)^2}}_{\text{pion pole}} + \int\limits_{(3m_\pi)^2}^{\infty} ds \underbrace{\frac{\rho^h(Q^2, s)}{s-(p-q)^2}}_{\text{heavier states}}$$

  * quark-hadron duality: $\rho^h(Q^2, s) \to \rho^{OPE}(Q^2, s)\theta(s - s_0^\pi)$
  * $s_0^\pi = 0.7 \pm 0.1\ \mathrm{GeV}^2$ (QCD sum rule for f_π) [M.Shifman,A.Vainshtein, V.Zakharov (1979)]

- the LCSR after Borel transform: $u_0 = Q^2/(s_0^\pi + Q^2)$ [V.Braun, AK, M.Maul (2000)]

$$F_\pi(Q^2) = \int\limits_{u_0}^{1} du\, \varphi_\pi(u, \mu) \exp\left(-\frac{\bar{u}Q^2}{uM^2}\right) + \{tw2\ NLO\} + \{tw\ 4, 6\}$$

  * contains both hard-scattering (factorizable) and soft end-point (nonfactorizable) contributions to $F_\pi(Q^2)$
  * the tw2 NLO term $\sim \alpha_s/Q^2$ at $Q^2 \to \infty$, and reproduces the QCD asymptotics
  * subleading OPE terms determined by δ_π^2 (twist 4), and $\langle 0|\bar{q}q|0\rangle$ (fact. twist 6)
    $\delta_\pi^2(2\mathrm{GeV}) = (0.18 \pm 0.06)\,\mathrm{GeV}^2$ (QCD SR) in agreement with lattice QCD [G.Bali et al (2018)]

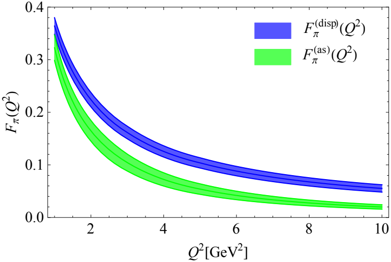
Probing Gegenbauer moments with $F_\pi(Q^2)$

- Isolating the dependence on Gegenbauer coefficients at $\mu_0 \sim 1$ GeV:

$$F_\pi^{\rm LCSR}(Q^2) = F_\pi^{(tw2,as)}(Q^2) + \sum_{n=2,4,\dots} a_n^\pi(\mu_0)\, f_n(Q^2,\mu,\mu_0) + F_\pi^{tw4,LO}(Q^2) + F_\pi^{(tw6,fact)}(Q^2)$$

$$f_n = 6\left[\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right]^{\gamma_{2n}/\beta} \int\limits_{u_0}^{1} du\, u(1-u) e^{-\bar{u}Q^2/(uM^2)} C_{2n}^{(3/2)}(2u-1) + f_n^{NLO}$$

- assuming purely asymptotic $\varphi_\pi(u)$, comparing with dispersion relation:



- nonasymptotic terms are important!

Fit of the Gegenbauer moments

Model	$a_2(1\,\mathrm{GeV})$	$a_4(1\,\mathrm{GeV})$	$a_6(1\,\mathrm{GeV})$	$a_8(1\,\mathrm{GeV})$	$\chi^2_{\mathrm{min}}/\mathrm{ndf}$
$\{a_2\}$	0.302 ± 0.046				4.08
$\{a_2, a_4\}$	0.279 ± 0.047	0.189 ± 0.060			0.75
$\{a_2, a_4, a_6\}$	0.270 ± 0.047	0.179 ± 0.060	0.123 ± 0.086		0.073
$\{a_2, a_4, a_6, a_8\}$	0.269 ± 0.047	0.185 ± 0.062	0.141 ± 0.096	0.049 ± 0.116	0.013

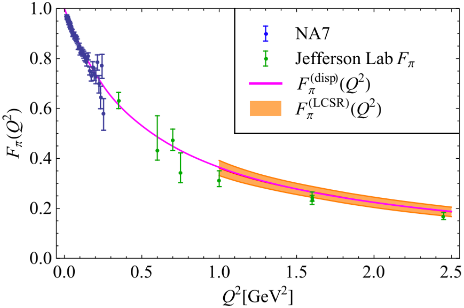


FIG. 6. The pion spacelike form factor calculated from the dispersion relation (39) (magenta curve; central input) and from the LCSR with the fitted Gegenbauer moments (orange band; the model $\{a_2, a_4, a_6, a_8\}$) compared with the measurements of NA7

Comparison

- Comparison of the second and fourth Gegenbauer moments obtained with various methods

Method	$a_2(1\,\mathrm{GeV})$	$a_4(1\,\mathrm{GeV})$	Ref.
Lattice QCD	0.135 ± 0.032	–	[1]
QCD sum rule	0.28 ± 0.08	–	[2]
QCD sum rule with nonlocal condensate	$0.203^{+0.069}_{-0.057}$	$-0.143^{+0.094}_{-0.087}$	[3]
LCSR fitted to Jlab data	0.17 ± 0.08	0.06 ± 0.10	[4]
LCSR fitted to dispersion relation	0.22 - 0.33	0.12 - 0.25	this work

[1] G.Bali et al. (2019) see the talk by Gunnar Bali

[2] P.Ball, V.Braun, A.Lenz (2006)

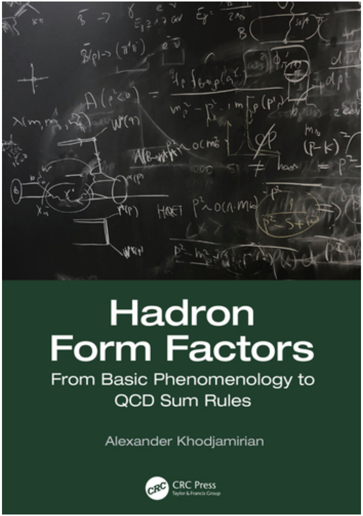
[3] S.Mikhailov, A.Pimikov, N.Stefanis (2016), Stefanis (2020)

[4] AK, T.Mannel, N.Offen, Y-M.Wang (2011)

Outlook

- New method to assess the pion twist-2 LCDA:
  combining LCSR, dispersion relation and data on the timelike F_π
- Global fit involvong other methods:
  - LCSR and data on the $F_{\pi\gamma\gamma^*}$ form factor
  - lattice QCD
- Future applications:
  - the pion vector/isovector form factor in $\tau \to \pi^-\pi^0\nu_\tau$ decays,
  - kaon DA (with $a_1^K, a_3^K \neq 0$) from the kaon timelike e.m. form factor,
- Comparing the modulus representation with $F_\pi(Q^2)$ on the lattice
  [HPQCD, J.Koponen et al (2017)]

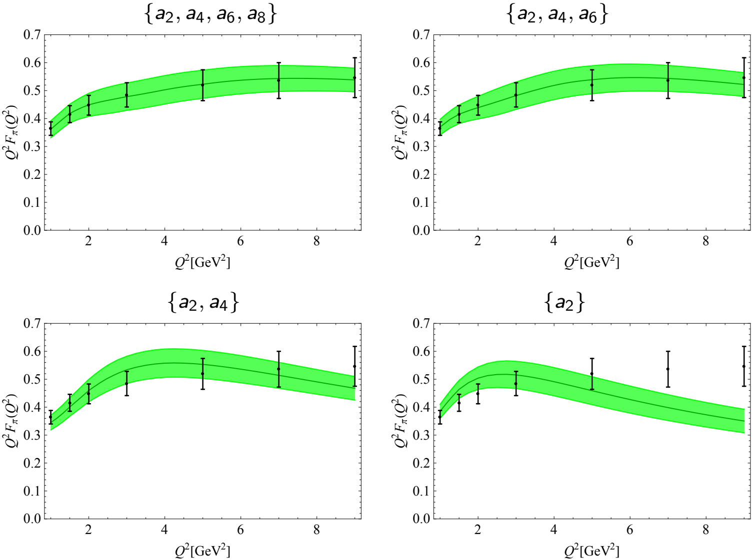
In conclusion, let me mention
the recently published book,
containing an introduction
to the subjects of this talk:



see: https://www.routledge.com/Hadron-Form-Factors-From-Basic-Phenomenology-to-QCD-Sum-Rules/Khodjamirian/p/book/9781138306752

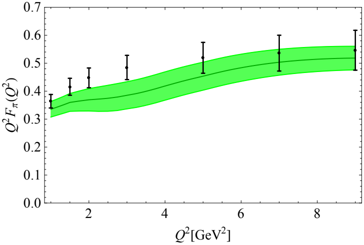
Backup

Fit in detail

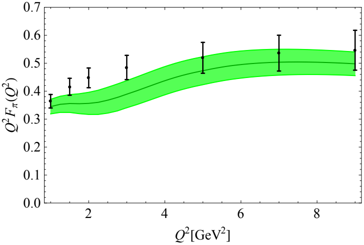


Different version of the fit

$\{a_2, a_4, a_6, a_8\}$

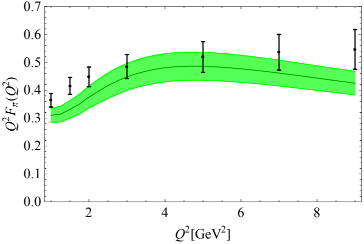


$\{a_2, a_4, a_6\}$



Fixing $a_2^\pi(1\,\mathrm{GeV}) = 0.135 \pm 0.032$

$\{a_2, a_4\}$



- Using a value $a_2^\pi(2\,\mathrm{GeV}) = 0.101 \pm 0.024$ from LQCD [Bali et al. (2019)]

$$\Rightarrow a_2^\pi(1\,\mathrm{GeV}) = 0.135 \pm 0.032$$

Model	$a_4(1\,\mathrm{GeV})$	$a_6(1\,\mathrm{GeV})$	$a_8(1\,\mathrm{GeV})$	$\chi^2_{\mathrm{min}}/\mathrm{ndf}$
$\{a_2, a_4\}$	0.218 ± 0.059			3.93
$\{a_2, a_4, a_6\}$	0.203 ± 0.060	0.157 ± 0.086		2.81
$\{a_2, a_4, a_6, a_8\}$	0.210 ± 0.061	0.179 ± 0.095	0.062 ± 0.116	2.71