Recent progress in hadronic light-by-light scattering

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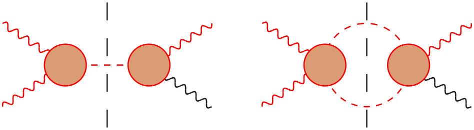
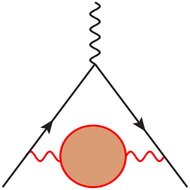
August 5, 2021

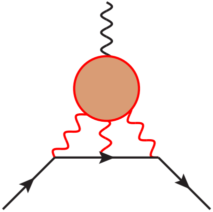
A Virtual Tribute to Quark Confinement and the Hadron Spectrum 2021

Hadronic light-by-light scattering

- In the past: hadronic models, inspired by various QCD limits, but error estimates difficult
- Dispersive approach: use again **analyticity**, **unitarity**, **crossing**, and **gauge invariance** for data-driven approach Colangelo, MH, Procura, Stoffer 2014, . . .
- For simplest intermediate states: relation to $\pi^0 \to \gamma^*\gamma^*$ **transition form factor** and $\gamma^*\gamma^* \to \pi\pi$ **partial waves**







HLbL scattering: white paper strategy

- Reference points:

$$a_\mu^{\text{HLbL}}\Big|_{\text{“Glasgow consensus” 2009}} = 105(26) \times 10^{-11}$$

$$a_\mu^{\text{HLbL}}\Big|_{\text{Jegerlehner, Nyffeler 2009}} = 116(39) \times 10^{-11}$$

- Strategy in the white paper
  - Take well-controlled results for the low-energy contributions
  - Combine errors in quadrature
  - Take best guesses for medium-range and short-distance matching
  - Add these errors linearly, since errors hard to disentangle at the moment
- **Recommended value**

$$a_\mu^{\text{HLbL}}\,(\text{phenomenology}) = 92(19) \times 10^{-11}$$

- **Lattice QCD**: first complete calculation RBC/UKQCD 2019 (Mainz 2021 after WP, see below)

$$a_\mu^{\text{HLbL}}\,(\text{lattice}, uds) = 79(35) \times 10^{-11}$$

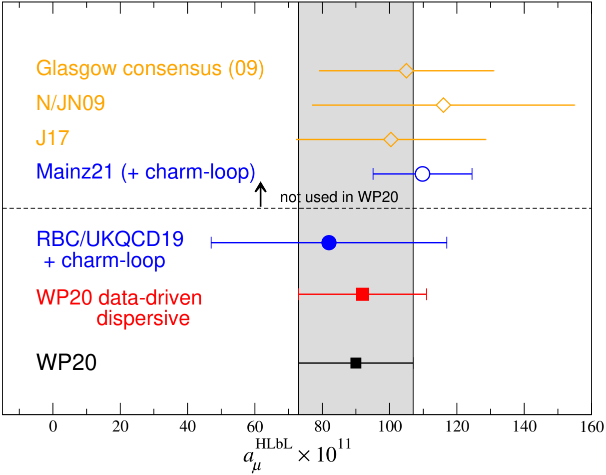
$\hookrightarrow$ can combine with phenomenological value

HLbL scattering: white paper details

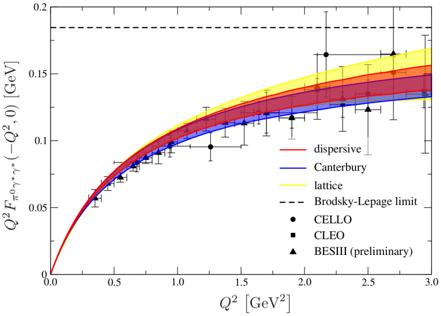
Contribution	PdRV(09)	N/JN(09)	J(17)	Our estimate
π^0, η, η'-poles	114(13)	99(16)	95.45(12.40)	93.8(4.0)
π, K-loops/boxes	$-19(19)$	$-19(13)$	$-20(5)$	$-16.4(2)$
S-wave $\pi\pi$ rescattering	$-7(7)$	$-7(2)$	$-5.98(1.20)$	$-8(1)$
subtotal	88(24)	73(21)	69.5(13.4)	69.4(4.1)
scalars	–	–	–	} $-1(3)$
tensors	–	–	1.1(1)	
axial vectors	15(10)	22(5)	7.55(2.71)	6(6)
u, d, s-loops / short-distance	–	21(3)	20(4)	15(10)
c-loop	2.3	–	2.3(2)	3(1)
total	105(26)	116(39)	100.4(28.2)	92(19)

All to be compared to projected **final E989 precision**: $\Delta a_\mu^{\text{E989}} = 16 \times 10^{-11}$

Status of HLbL scattering



HLbL scattering: pion pole



- Pion pole from data MH et al. 2018, Masjuan, Sánchez-Puerto 2017 and lattice QCD Gérardin et al. 2019

$$a_\mu^{\pi^0\text{-pole}}\Big|_{\text{dispersive}} = 63.0^{+2.7}_{-2.1} \times 10^{-11} \qquad a_\mu^{\pi^0\text{-pole}}\Big|_{\text{Canterbury}} = 63.6(2.7) \times 10^{-11}$$

$$a_\mu^{\pi^0\text{-pole}}\Big|_{\text{lattice+PrimEx}} = 62.3(2.3) \times 10^{-11} \qquad a_\mu^{\pi^0\text{-pole}}\Big|_{\text{lattice}} = 59.7(3.6) \times 10^{-11}$$

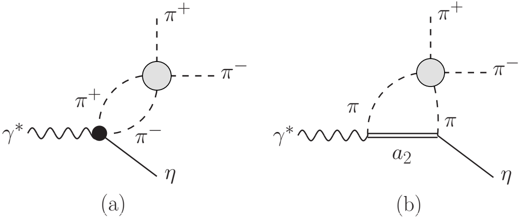
$\hookrightarrow$ agree within uncertainties well below Fermilab goal

- Singly-virtual results agree well with BESIII measurement

- **Subleading contributions**
  1. η, η' poles
  2. Subleading two-pion and multi-hadron intermediate states

     $\hookrightarrow$ narrow-resonance description
  3. Short-distance constraints talk by A. Rodríguez-Sánchez and their implementation
- In the following: brief review of status and prospects
- For more details: see talks by J. Bijnens, G. Colangelo, B. Kubis, A. Rebhan, P. Stoffer at recent meeting Muon $g-2$ Theory Initiative meeting (virtual at KEK) `https://www-conf.kek.jp/muong-2theory/`

HLbL scattering: η, η' poles



Holz et al. 2021

- So far only based on **Canterbury approximants** Masjuan, Sánchez-Puerto 2017

$$a_\mu^{\eta\text{-pole}}\big|_{\text{Canterbury}} = 16.3(1.4) \times 10^{-11} \qquad a_\mu^{\eta'\text{-pole}}\big|_{\text{Canterbury}} = 14.5(1.9) \times 10^{-11}$$

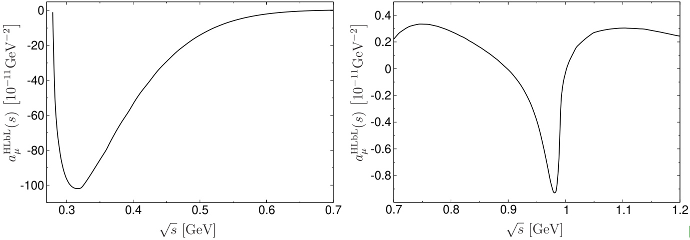
- Impact of **factorization-breaking terms** not well understood: in general

  $F_{\eta\gamma^*\gamma^*}(q_1^2, q_2^2) \neq F(q_1^2)F(q_2^2)$

- Can be cross checked with data on $e^+e^- \to \eta\pi\pi$ Holz et al. 2021

  $\hookrightarrow$ need more differential data to ascertain role of **left-hand cut from a_2 diagram**

HLbL scattering: scalar contributions



Danilkin, MH, Stoffer 2021

- Single-particle poles in general depend on the **choice of tensor basis**

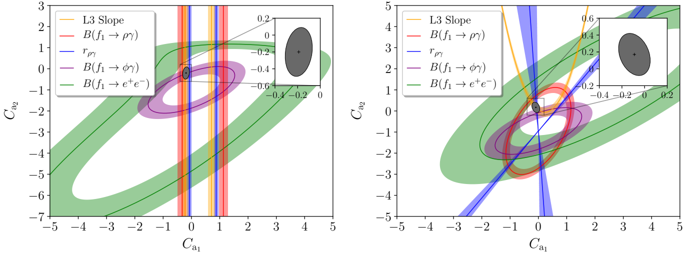
  $\hookrightarrow$ basis independence only ensured by **sum rules** for entire HLbL tensor

- Exception: pseudoscalar poles
- Scalar contributions first non-trivial test case
- For $f_0(500)$ and $f_0(980)$ implementation in terms of $\gamma^*\gamma^* \to \pi\pi/\bar{K}K$

  $\hookrightarrow$ can **compare full and narrow-resonance description** for $f_0(980)$

$$a_\mu^{\rm HLbL}[f_0(980)]\big|_{\rm rescattering} = -0.2(1)\times 10^{-11} \qquad a_\mu^{\rm HLbL}[f_0(980)]\big|_{\rm NWA} = -0.37(6)\times 10^{-11}$$

HLbL scattering: axial-vector contributions



Zanke, MH, Kubis 2021

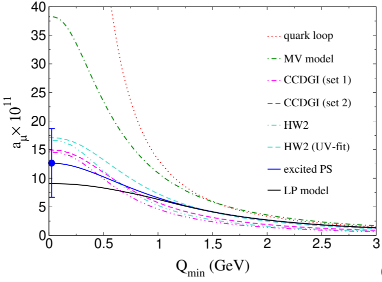
- Challenges regarding axial-vector states
    - Require multi-hadron channels: $a_1 \to 3\pi$, $f_1 \to \eta\pi\pi, \ldots$

      $\hookrightarrow$ **narrow-resonance approximation**
    - Limited information on transition form factors

      $\hookrightarrow$ global analysis of f_1 decays Zanke, MH, Kubis 2021, asymptotic constraints MH, Stoffer 2020

      $\hookrightarrow$ **improved measurement of $f_1 \to e^+e^-$ would be valuable**
    - Need tensor basis in which kinematic singularities are manifestly absent

HLbL scattering: short-distance constraints



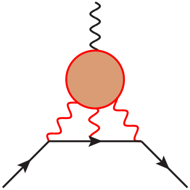
Colangelo et al. 2021

- Open issue how to best **implement the short-distance constraints**
  1. Melnikov–Vainshtein model: anomaly exact in chiral limit, low-energy 2π and 3π cuts missing
  2. Holographic QCD Leutgeb–Rebhan, Cappiello et al. 2019: model for QCD, implementation in terms of axial-vector states
  3. Regge model for excited pseudoscalars Colangelo et al. 2019: individual pseudoscalar contributions not affected by sum rules, but works only away from chiral limit
  4. Interpolation between low- and high-energy constraints Lüdtke, Procura 2020

$\hookrightarrow$ good agreement among 2.–4. for the effect on HLbL

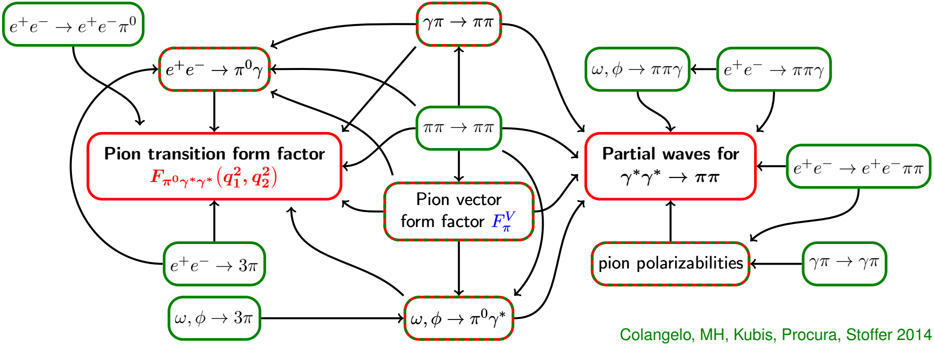
Conclusions

- **Hadronic light-by-light scattering**
  - Use dispersion relations to remove model dependence as far as possible (π^0 and leading $\pi\pi$ effects done)
  - Evaluation of subleading terms and comparison to lattice-QCD calculations in progress
- **Subleading terms**
  - η, η' poles
  - Short-distance constraints talk by A. Rodríguez-Sánchez and their implementation
  - Subleading two-pion and multi-hadron intermediate states $\hookrightarrow$ resonance description



- Current theory matches expected experimental precision after first E989 release, **but need to go further**!

Hadronic light-by-light scattering: data input



Colangelo, MH, Kubis, Procura, Stoffer 2014

- Reconstruction of $\gamma^*\gamma^* \to \pi\pi, \pi^0$: combine experiment and theory constraints
- Need input on $\gamma^*\gamma^*$ matrix elements for as many states as possible