A universal holographic wavefunction for light hadrons

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A virtual tribute to Quark Confinement and the Hadron Spectrum

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Based on work done with M. Ahmady, D. Chakraborti, S. Kaur, and C. Mondal
Light-front QCD

Ordinary time

Light-front time

\[ x^+ = x^0 + x^3 \]

LF Schrodinger-like Equation in the conformal limit

\[ \left( -\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + U_{\text{eff}}(\zeta) \right) \phi(\zeta) = M^2 \phi(\zeta) \]

\[ \zeta = \sqrt{\frac{x}{1-x}} \sum_{j=1}^{N-1} x_j b_{\perp,j} \]

\[ x = \frac{k^+}{p^+} \]
Holographic dictionary

Light front transverse distance maps onto 5th dimension of AdS

$$\zeta \leftrightarrow z_5$$

(Orbital angular momentum)$^2$ maps onto (AdS mass parameter x radius)$^2$ and spin

$$L^2 = (\mu R)^2 + (2 - J)^2$$
Unique confinement potential

Confinement in physical spacetime $\iff$ dilaton field in AdS

$$U(\zeta) = \frac{1}{2} \varphi''(\zeta) + \frac{1}{4} \varphi'(\zeta)^2 + \frac{2J - 3}{2\zeta} \varphi'(\zeta)$$

$$\varphi = \kappa^2 z_5^2$$

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (J - 1)$$

$\kappa$ : emerging mass scale !

Quadratic dilaton/potential is required by underlying conformal symmetry
Supersymmetric light-front holography

Dosch, de Teramond, Brodsky, Phys. Rev. D 95 034016 (2017)

\[ H \ket{\phi} = M^2_{\perp} \ket{\phi} \]

\[ H = \begin{pmatrix}
-\frac{d^2}{d\zeta^2} + \frac{4L^2 - 1}{4\zeta^2} & U_M(\zeta) \\
0 & -\frac{d^2}{d\zeta^2} + \frac{4L^2 - 1}{4\zeta^2} + U_B(\zeta)
\end{pmatrix} \]

\[ |\phi\rangle = \begin{pmatrix}
\phi_M(L_M = L_B + 1) & \psi^-(L_B + 1) \\
\psi^+(L_B) & \phi_T(L_T = L_B)
\end{pmatrix} \]

\[ U_M(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L_M + S_M - 1) \]
\[ U_B(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L_B + S_D) \]

\[ M^2_{\perp,M} = 4\kappa^2 \left( n_i + L_M + \frac{S_M}{2} \right) \]
\[ M^2_{\perp,B} = 4\kappa^2 \left( n_i + L_B + \frac{S_D}{2} + 1 \right) \]
\[ M^2_{\perp,T} = 4\kappa^2 \left( n_i + L_T + \frac{S_T}{2} + 1 \right) \]

Each baryon has two supersymmetric partners: a meson and a tetraquark
A universal holographic mass scale

\[ \kappa = 523 \pm 24 \text{ MeV} \]

Brodsky, de Teramond, Dosch, Lorce (2013)

Universal holographic wavefunction for ground state

\[ \Psi(x, k_\perp^2) \propto \frac{1}{\sqrt{x\bar{x}}} \exp \left( -\frac{M^2}{2k^2} \right) \quad M^2 = \frac{k^2_\perp}{x\bar{x}} \]

Fourier conjugate to \( \zeta \)

\[ k^2_\perp \rightarrow k^2_\perp + m^2 \quad \text{For massive quarks} \]
For a successful phenomenology, we need to account for dynamical effects of quark masses and spins.

\[ \Psi_{h,h}^{P,V}(x, k) = S_{h,h}^{P,V}(x, k) \Psi(x, k_\perp), \]

Mesons (quark-antiquark)

\[
S_{h_q h_{\bar{q}}}^{V(\lambda)}(x, k) \propto \frac{\bar{u}_{h_q}(1 - x) P^+, -k)}{\sqrt{x}} \frac{\epsilon_\lambda^\ast \cdot \gamma}{\sqrt{x}} u_{h_{\bar{q}}}(x P^+, k) \]

Nucleons (quark-diquark)

\[
S_{h_N h_{\bar{q}}}^{N(\lambda)}(x, k) \propto \frac{\bar{u}_{h_q}(x P^+, k)}{\sqrt{x}} \frac{1}{1} u_{h_N}(P^+, 0) \]

\[
B \gg 1
\]
EM transition form factors

Light-front holographic radiative transition form factors for light mesons

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\[ V \rightarrow P + \gamma^* \]
Predictions for radiative decay widths

TABLE I. Our predictions for the \((\rho, \omega, \phi) \rightarrow \pi \gamma\) decay widths, compared to the PDG averages [2].

<table>
<thead>
<tr>
<th>Decay widths</th>
<th>Spin-improved LFH [keV]</th>
<th></th>
<th></th>
<th>PDG (2018) [keV]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>B = 0</td>
<td>B = 1</td>
<td>B \gg 1</td>
<td></td>
</tr>
<tr>
<td>(\Gamma(\rho^\pm \rightarrow \pi^\pm \gamma))</td>
<td>23.46 ± 3.12</td>
<td>64.52 ± 6.94</td>
<td>66.37 ± 7.00</td>
<td>67.10 ± 7.82</td>
</tr>
<tr>
<td>(\Gamma(\rho^0 \rightarrow \pi^0 \gamma))</td>
<td>23.46 ± 3.12</td>
<td>64.52 ± 6.94</td>
<td>66.37 ± 7.00</td>
<td>70.08 ± 9.32</td>
</tr>
<tr>
<td>(\Gamma(\omega \rightarrow \pi^0 \gamma))</td>
<td>221.03 ± 29.90</td>
<td>607.96 ± 65.44</td>
<td>625.38 ± 66.03</td>
<td>713.16 ± 25.40</td>
</tr>
<tr>
<td>(\Gamma(\phi \rightarrow \pi^0 \gamma))</td>
<td>1.84 ± 0.33</td>
<td>5.06 ± 0.80</td>
<td>5.21 ± 0.82</td>
<td>5.52 ± 0.22</td>
</tr>
</tbody>
</table>

Pure \(\gamma^5\)
Predictions for the transition form factors

\[ \omega \rightarrow \pi + \gamma^* \]

\[ \varphi \rightarrow \pi + \gamma^* \]
Excellent agreement at low momentum transfer

Large uncertainties for neutron where LO contributions tend to cancel out
### Predictions for the EM radii of nucleons

<table>
<thead>
<tr>
<th>Radius</th>
<th>Our prediction</th>
<th>Experimental data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\langle r_E \rangle_p$ fm</td>
<td>0.833 ± 0.010</td>
<td>0.833 ± 0.010 [48]; 0.831 ± 0.019 [50]; 0.841 ± 0.084 [49]</td>
</tr>
<tr>
<td>$\langle r_M \rangle_p$ fm</td>
<td>0.7985 ± 0.0313</td>
<td>0.851 ± 0.026 [52]</td>
</tr>
<tr>
<td>$\langle r_E^2 \rangle_n$ fm$^2$</td>
<td>−0.0704 ± 0.0434</td>
<td>−0.1161 ± 0.0022 [52]; −0.110 ± 0.008 [53]</td>
</tr>
<tr>
<td>$\langle r_M \rangle_n$ fm</td>
<td>0.8388 ± 0.0288</td>
<td>0.864$^{+0.009}_{-0.008}$ [52]</td>
</tr>
</tbody>
</table>
Conclusions & Acknowledgements

- Light hadrons share a universal holographic wavefunction which is modified differently by their spin structures

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