Three-pion scattering from lattice QCD

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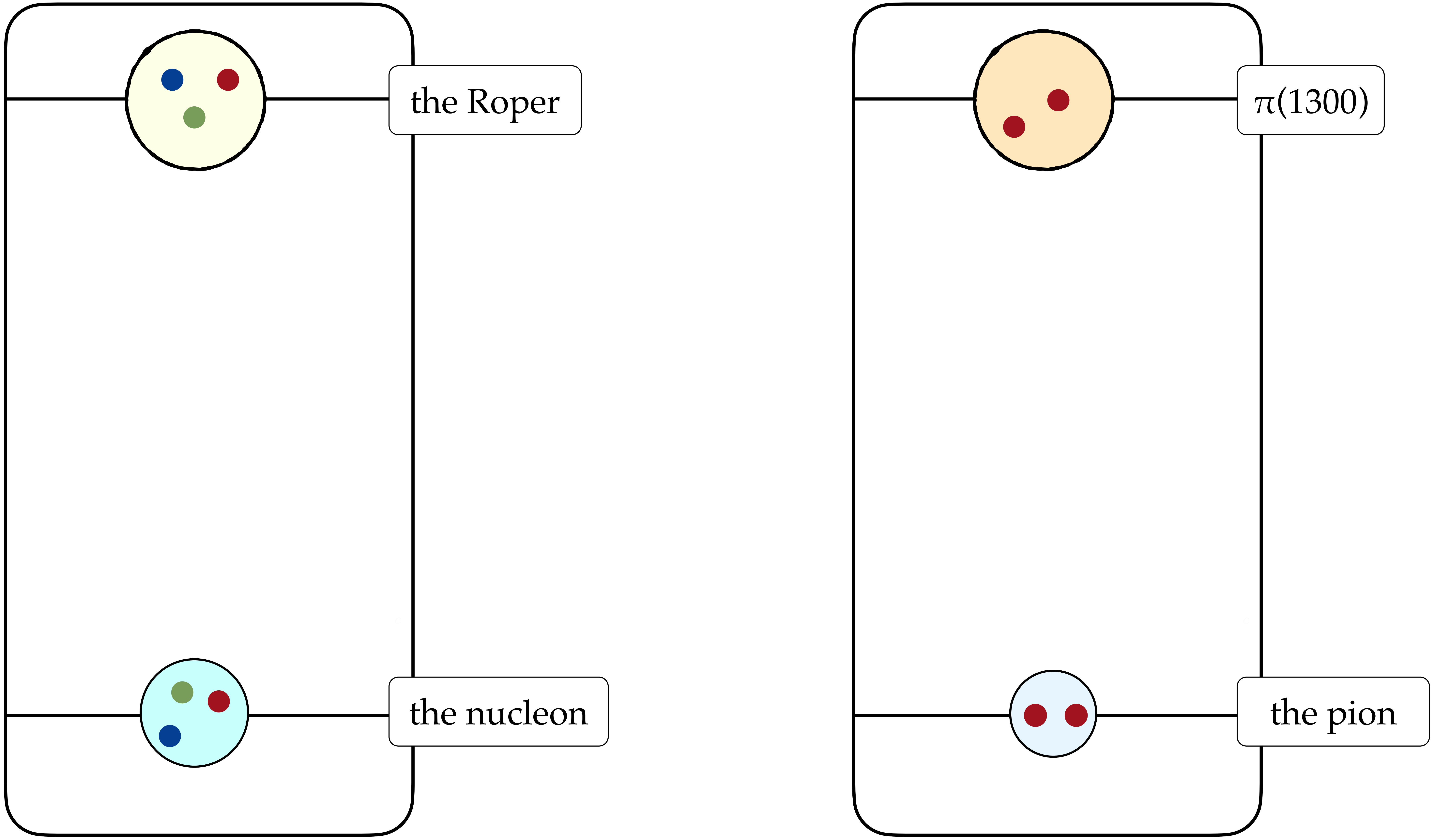
hadspec

the three-body problem in hadron spectroscopy

Most excited lie above three-particle thresholds and couple strongly onto these states…

…just look at the first excited states of the simplest QCD states



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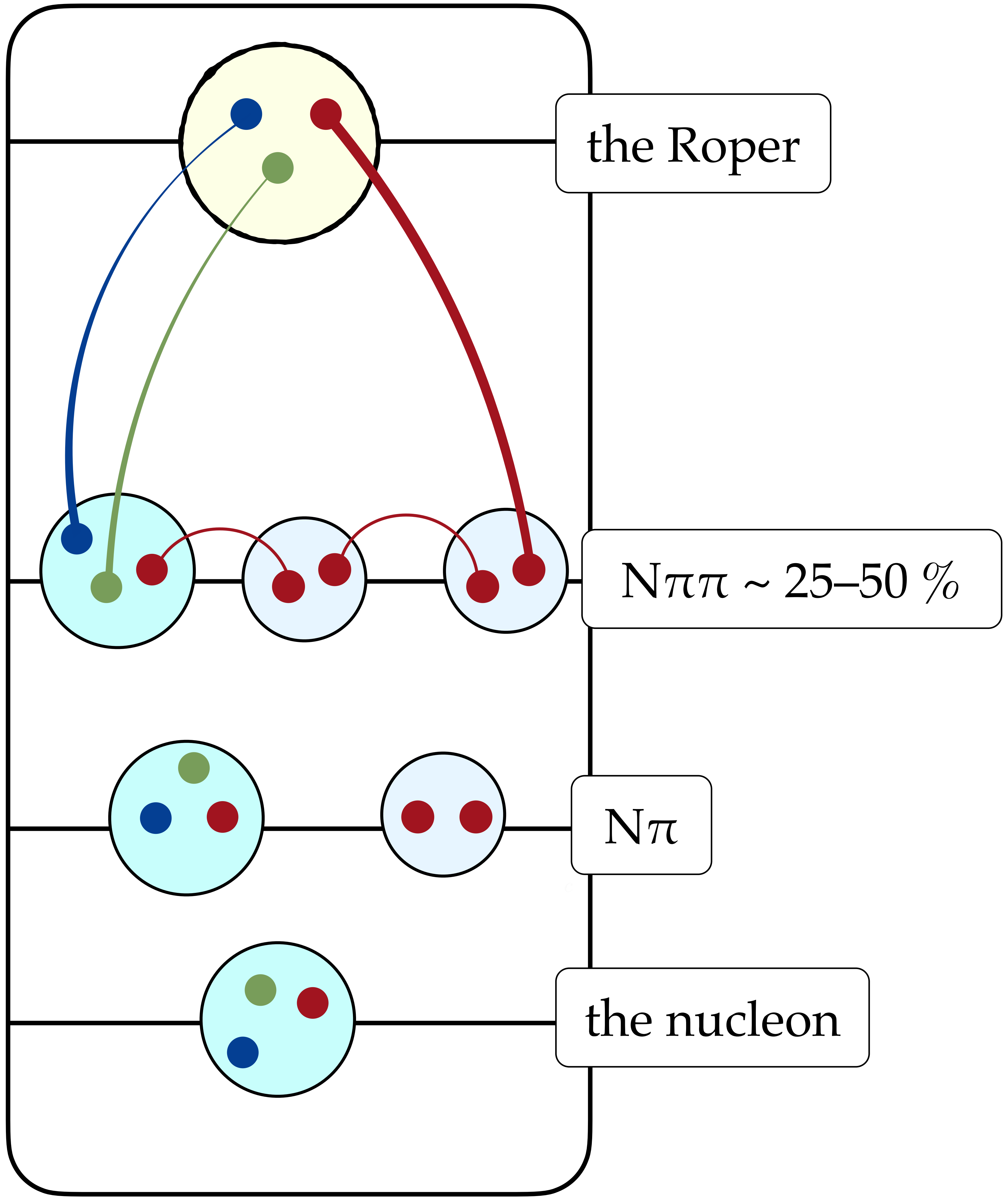
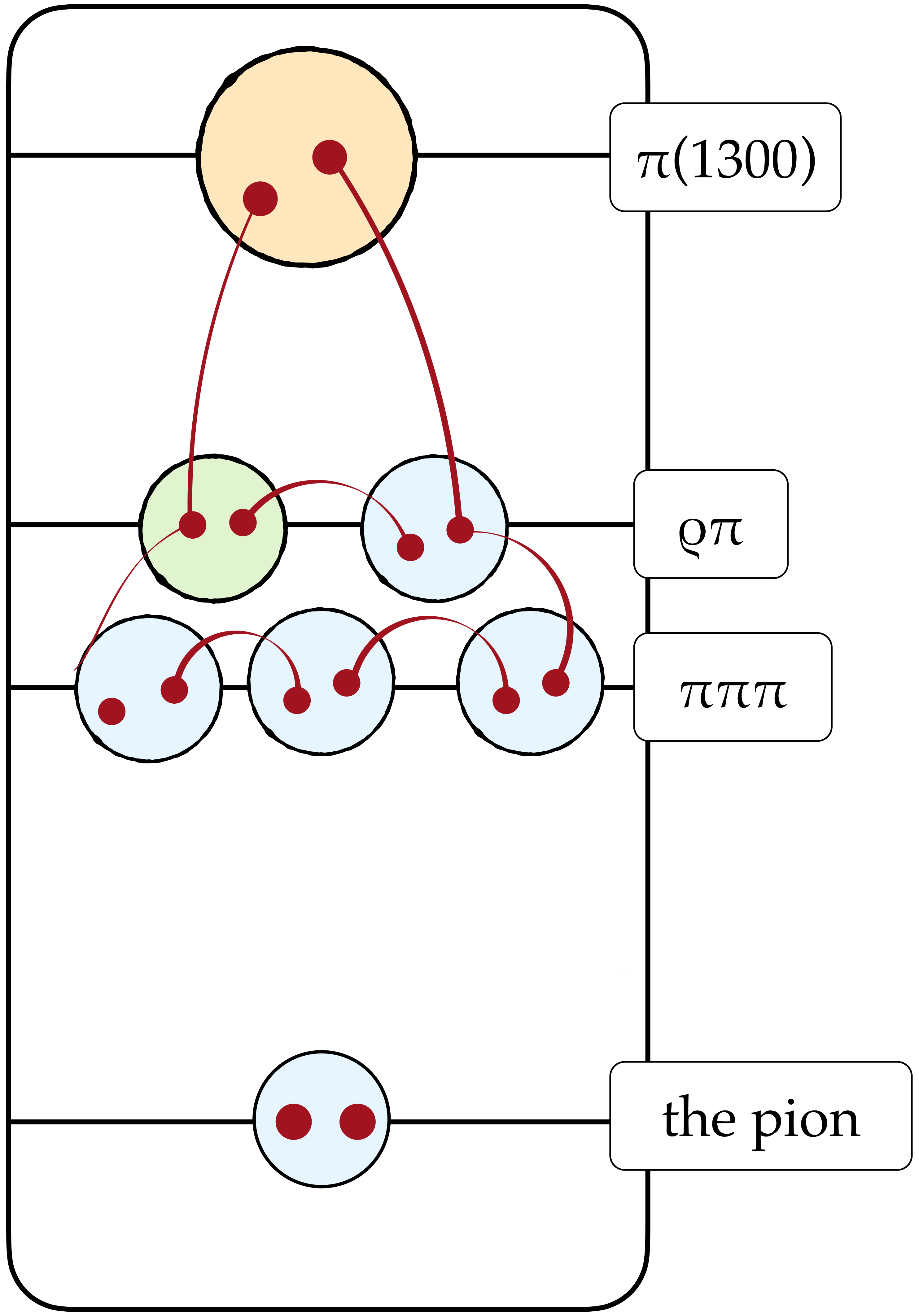
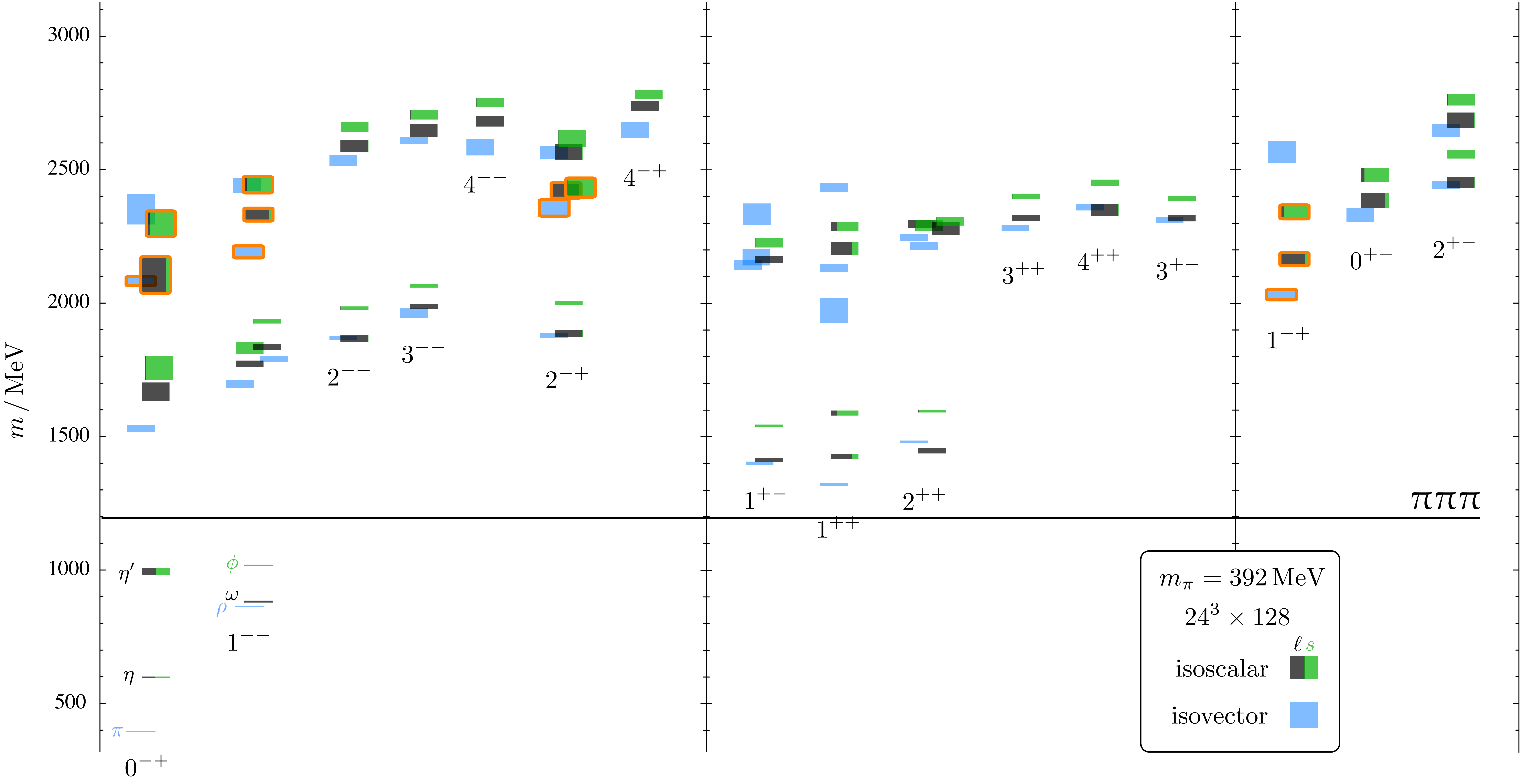




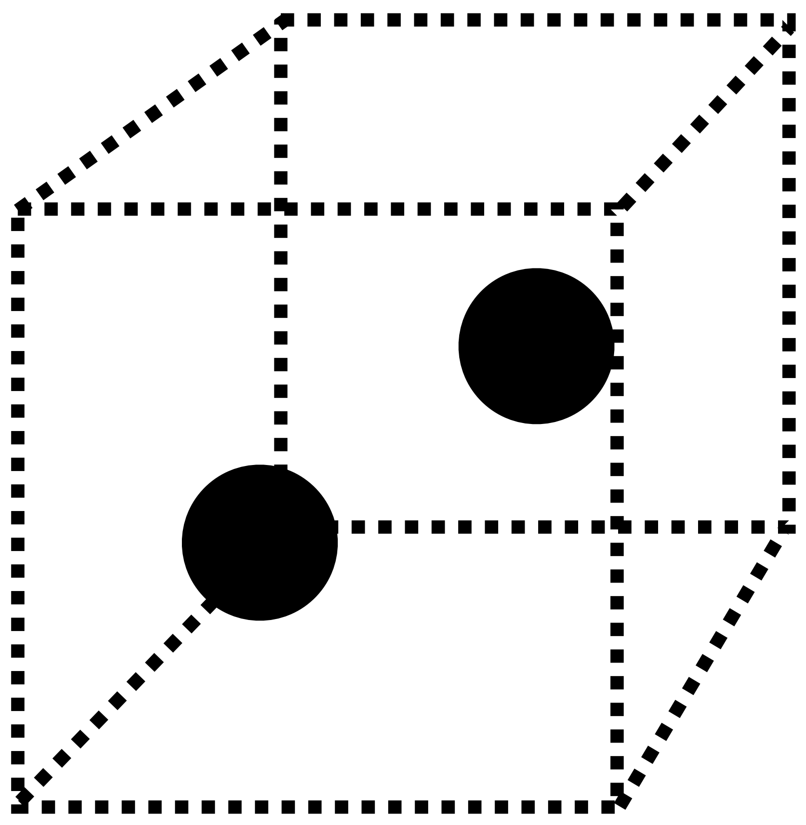
the three-body problem in lattice QCD

How does the three-body problem manifest itself in lattice QCD?



Dudek, Edwards, Guo, & Thomas (2013)

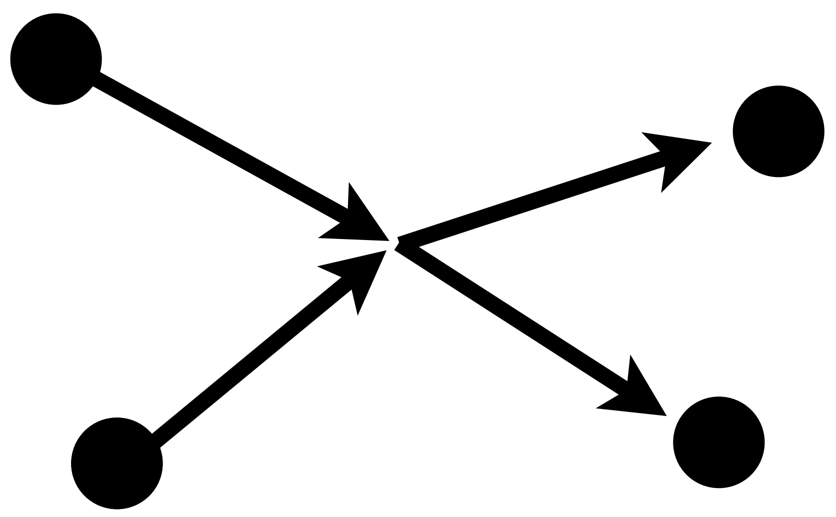
Two-hadron systems



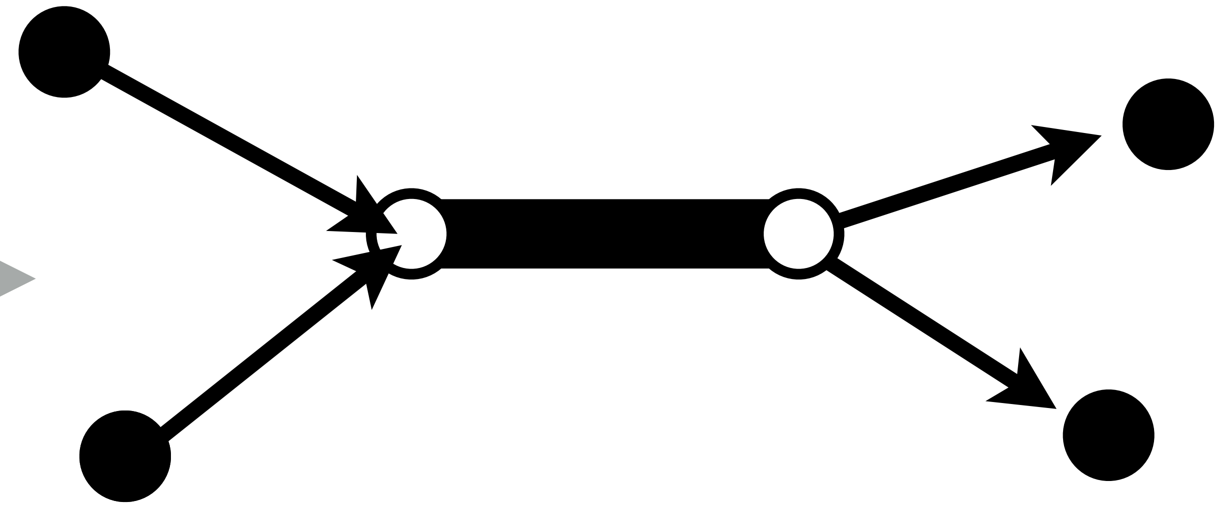
finite-volume
spectroscopy

$$\det\left[F(E_L,L)+\mathcal{M}^{-1}(E_L)\right]=0$$

Lüscher (1986, 1991)
Rummukainen & Gottlieb (1995)
Kim, Sachrajda, & Sharpe (2005)
Christ, Kim & Yamazaki (2005)
Feng, Li, & Liu (2004)
Hansen & Sharpe (2021)
RB & Davoudi (2012)
RB (2014)



infinite-volume
scattering amplitudes



bound state and
resonance poles

2+1 minimum requirements

Two "musts" for few-body systems:

☑ Generalized eigenvalue problem (GEVP),

☑ large basis of ops,

$\mathcal{O}_b \sim \bar{q}\,\Gamma_b\, q, \pi\pi, K\overline{K}, \ldots, 3\pi, \ldots$

☑ diagonalization,

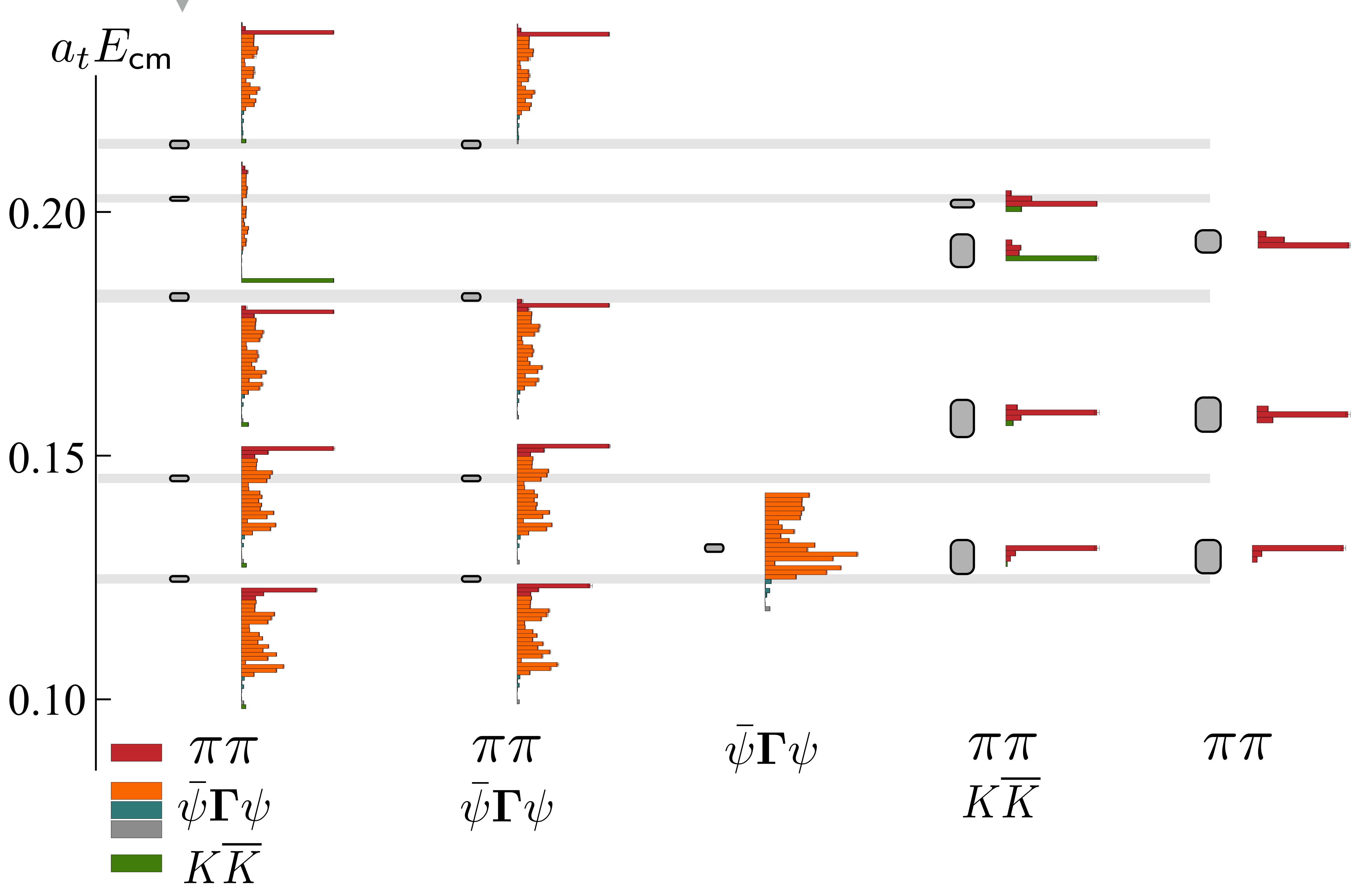
$$C^{2pt.}_{ab}(t, \mathbf{P}) \equiv \langle 0|\mathcal{O}_b(t,\mathbf{P})\mathcal{O}_a^\dagger(0,\mathbf{P})|0\rangle = \sum_n Z_{b,n} Z^*_{a,n} e^{-E_n t}$$

☑ Finite-volume formalisms.

One powerful tool to make GEVP practical:

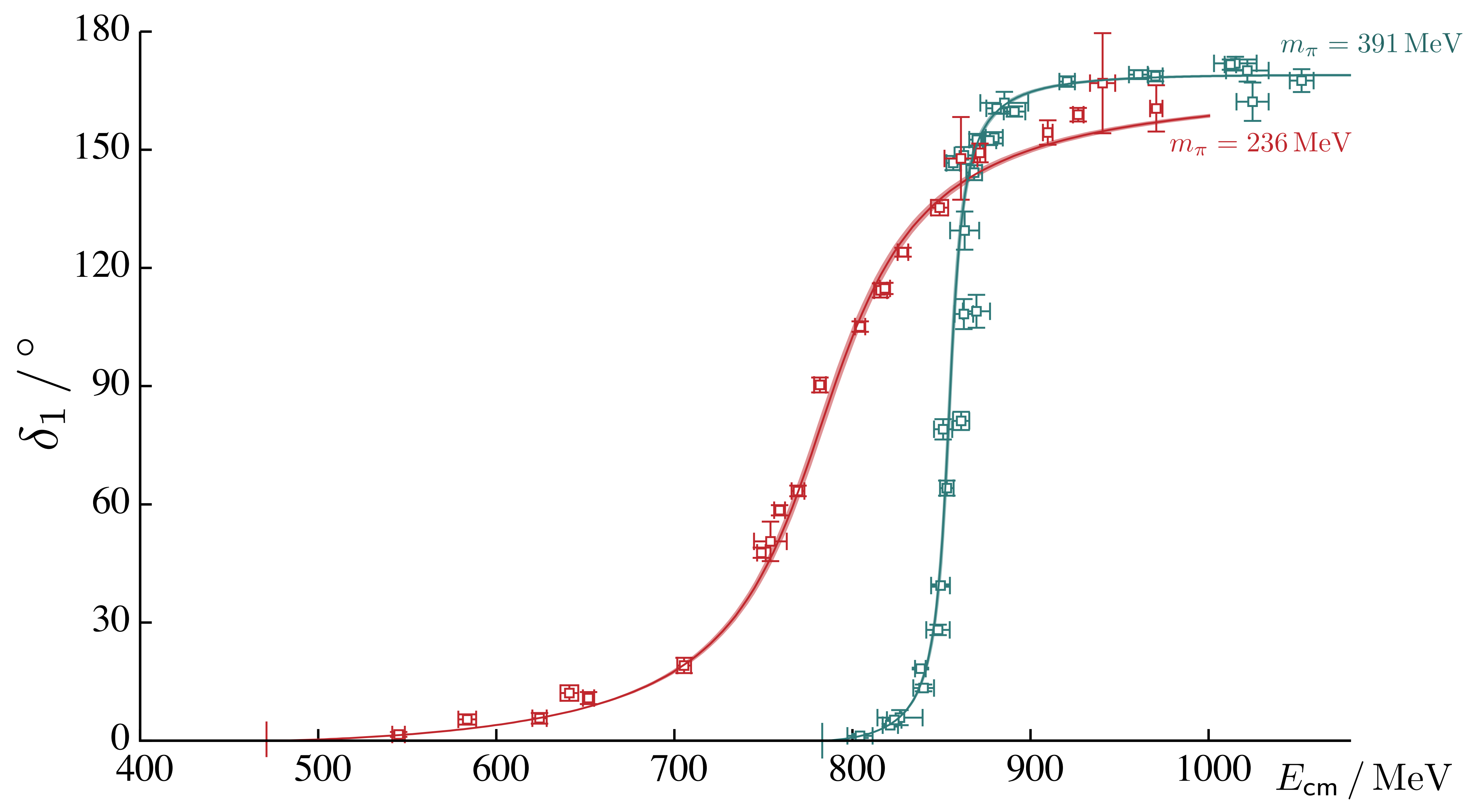
☑ Distillation [Peardon, *et al.* (HadSpec, 2009)].

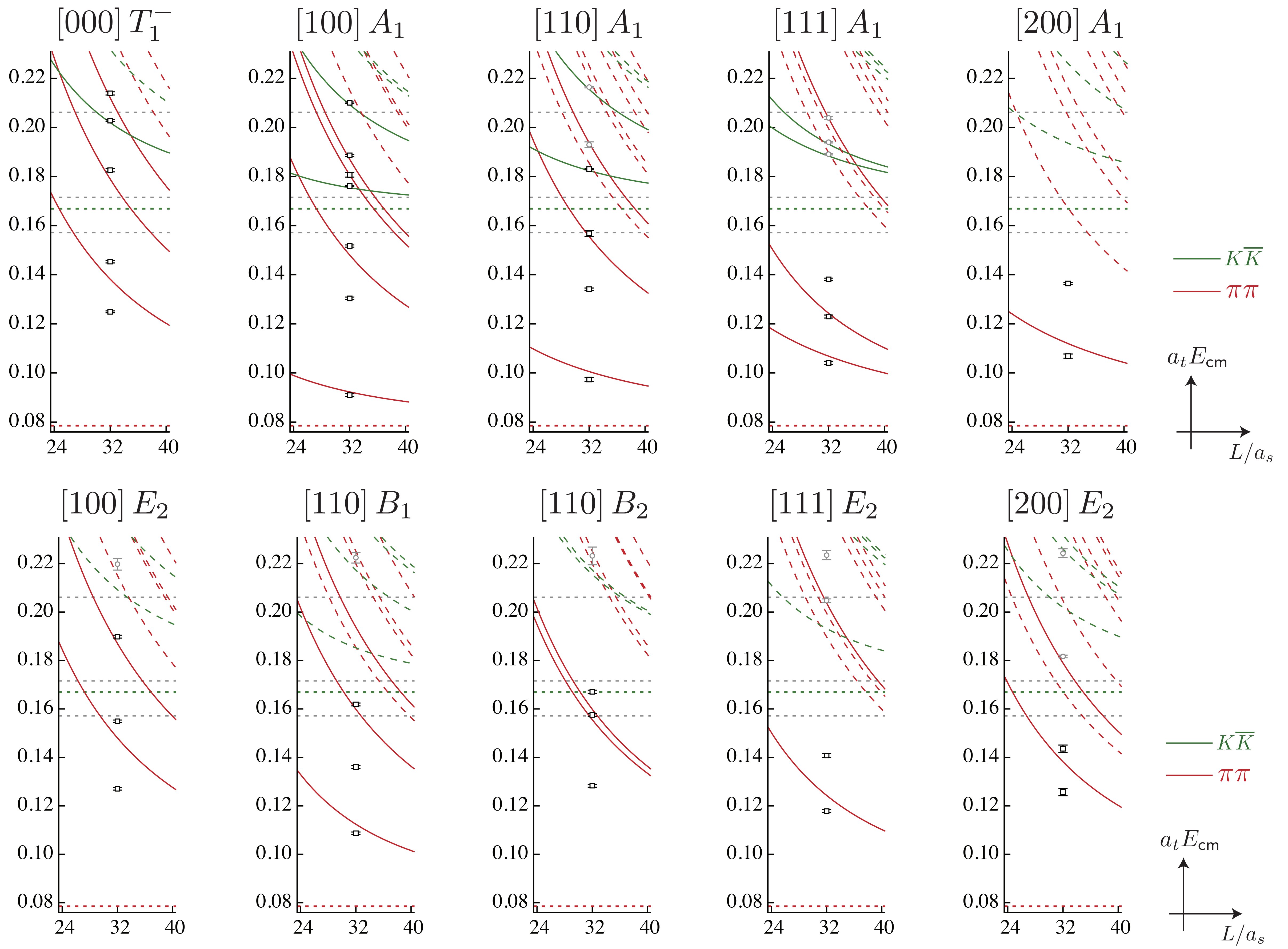
GEVP with large basic is **necessary** but **not sufficient** to trust spectrum!



Wilson, RB, Dudek, Edwards, & Thomas (2015)

$\pi\pi$ scattering
(I=1 channel)





$$\mathcal{M} \sim \frac{1}{p\cot\delta - ip}$$

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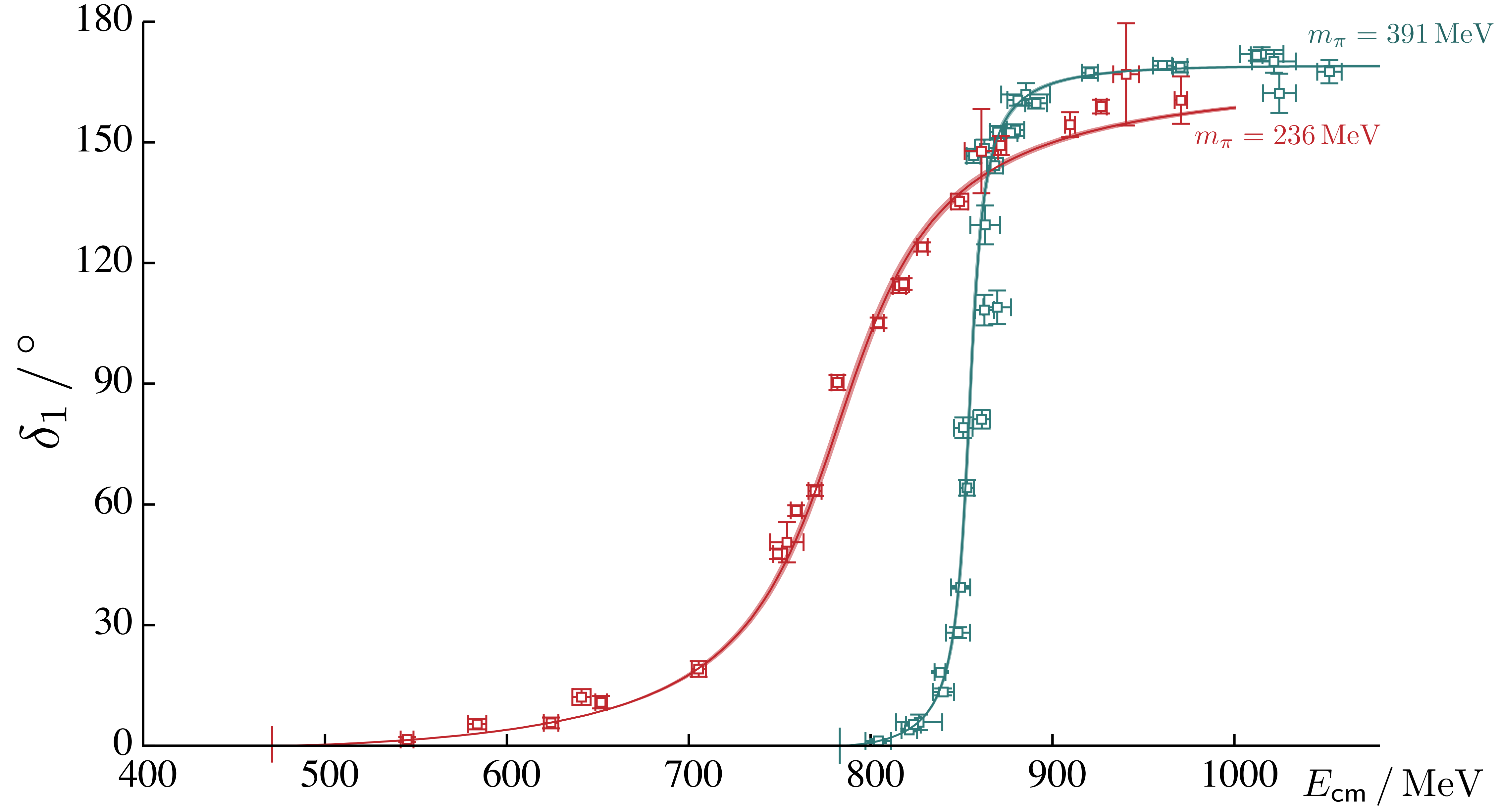
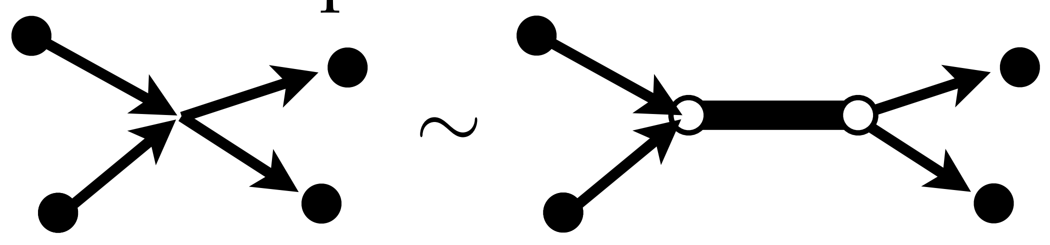


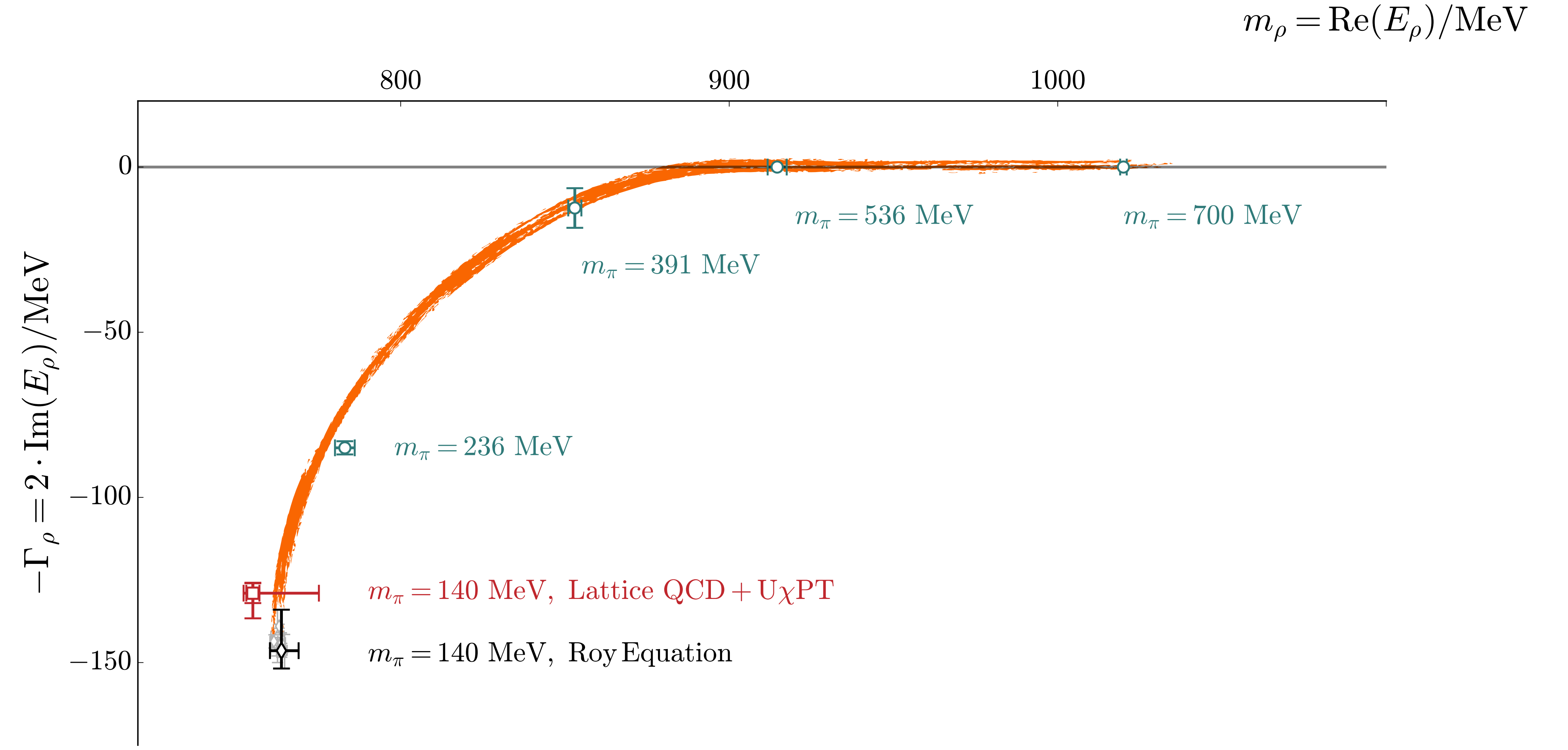
$$\mathcal{M} \sim \frac{1}{p\cot\delta - ip}$$

resonance pole



$$\sim \frac{1}{s - s_0}$$

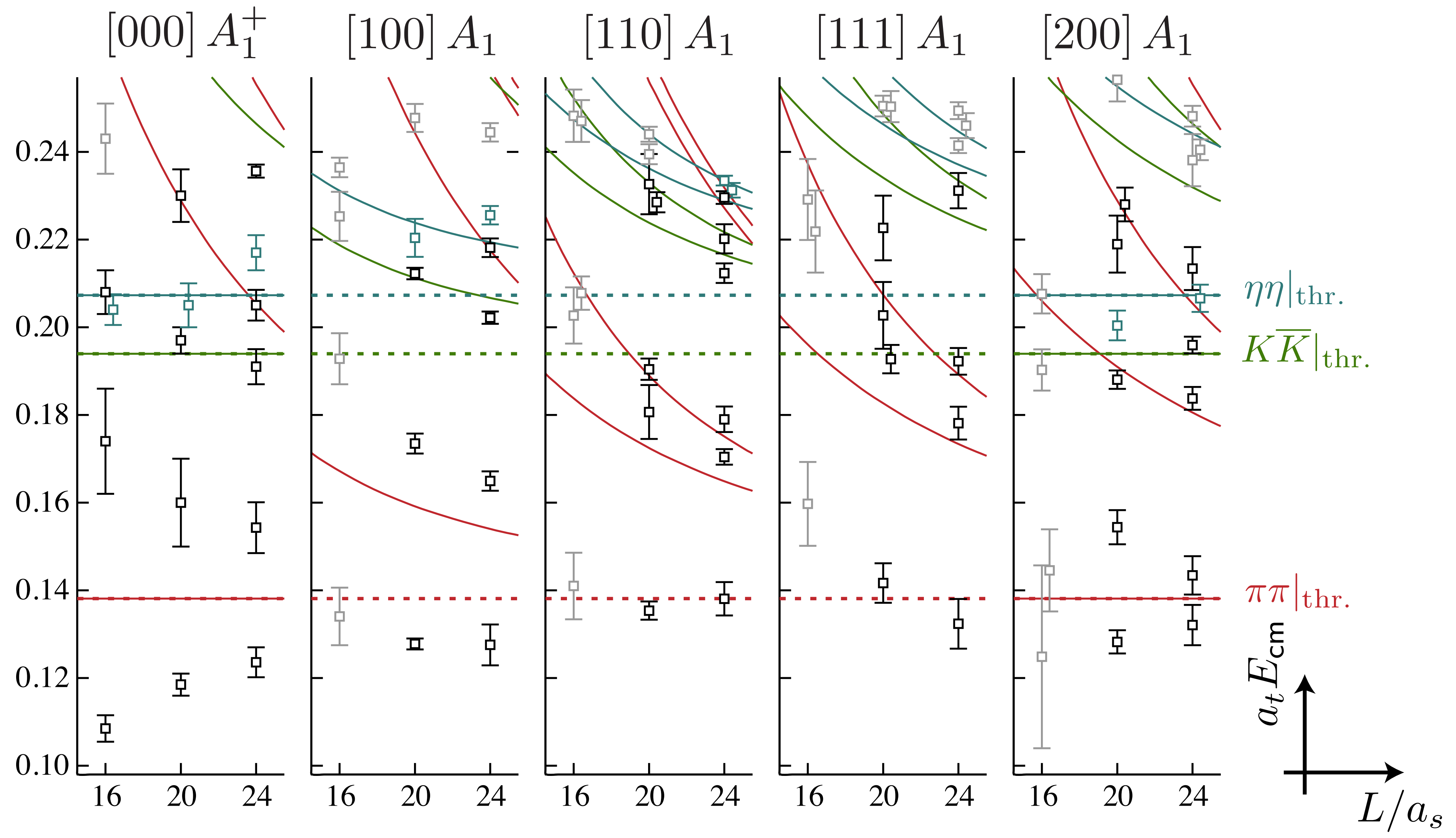
$$s_0 = (m_R - i\Gamma_R/2)^2$$

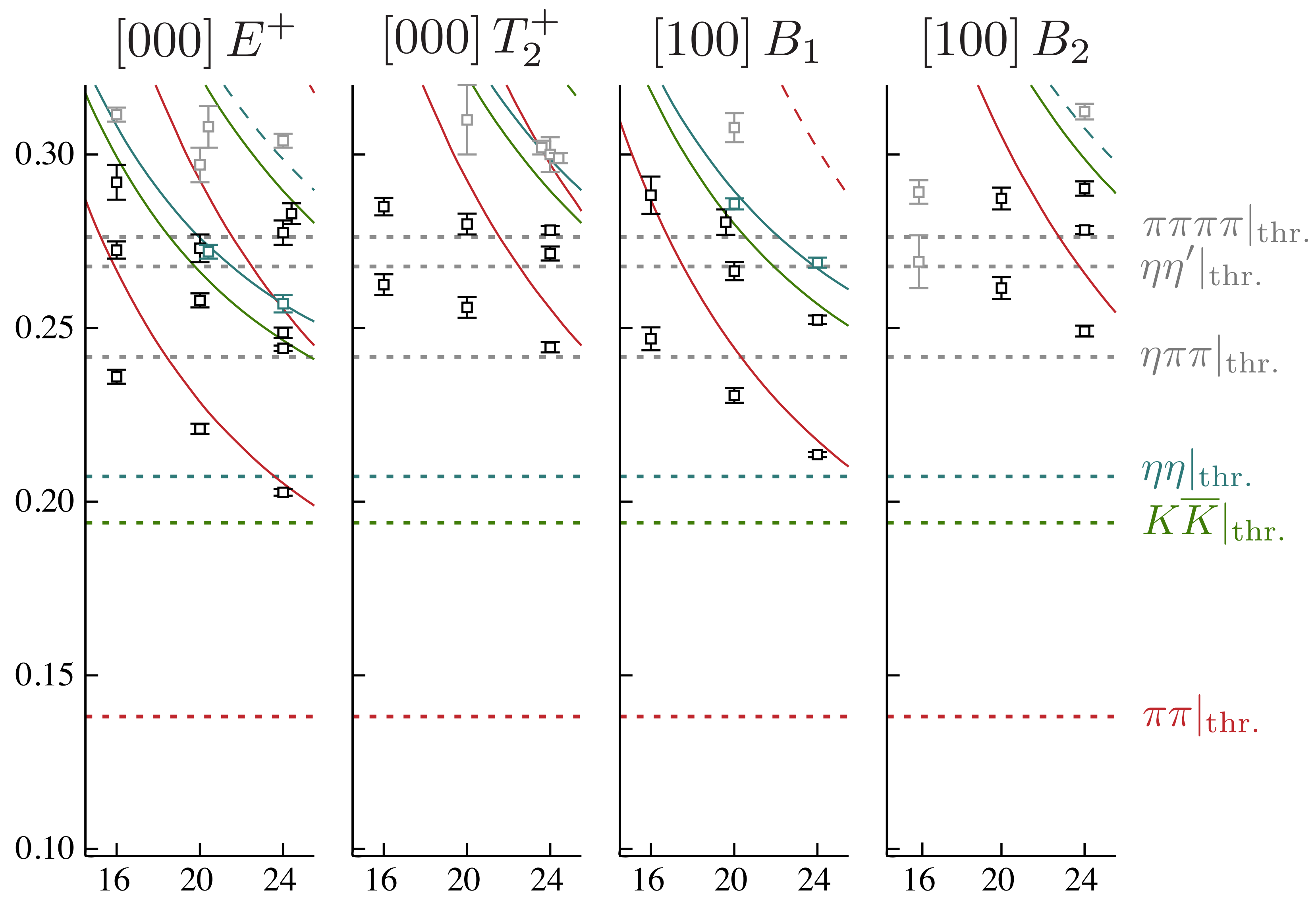


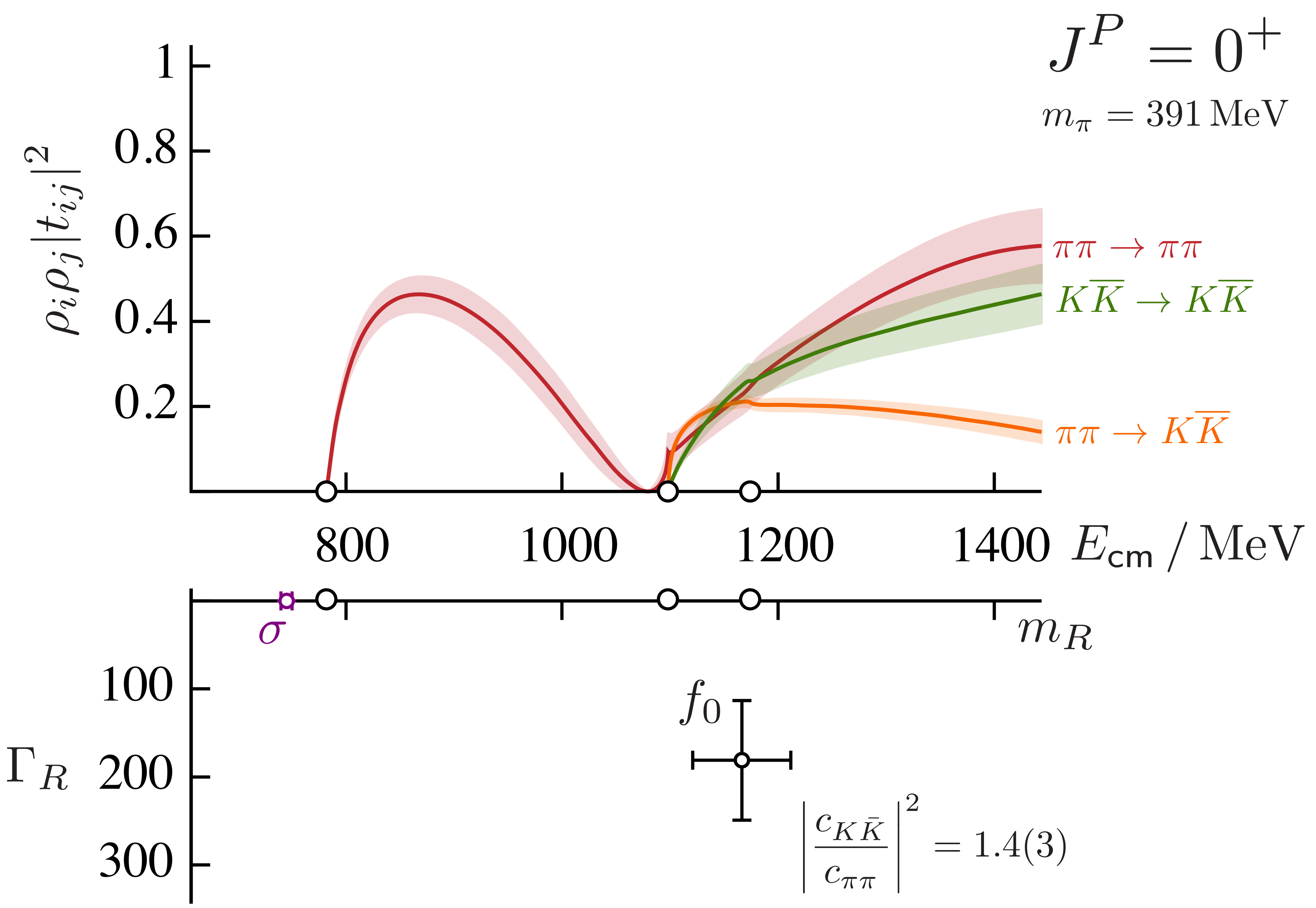
Dudek, Edwards, & Thomas (2012)

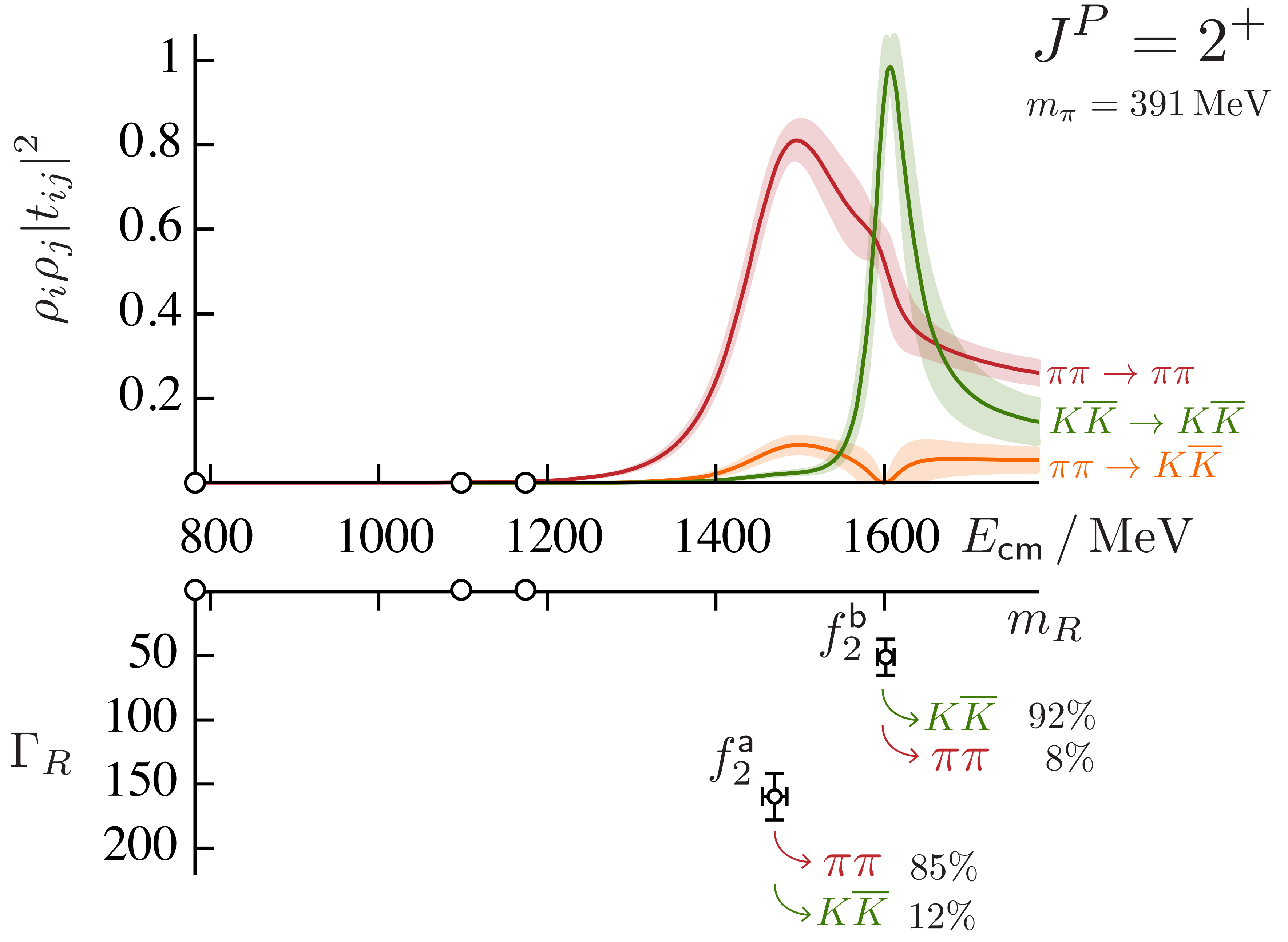
Wilson, RB, Dudek, Edwards, & Thomas (2015)

$\pi\pi$-KK
(I=0 channel)







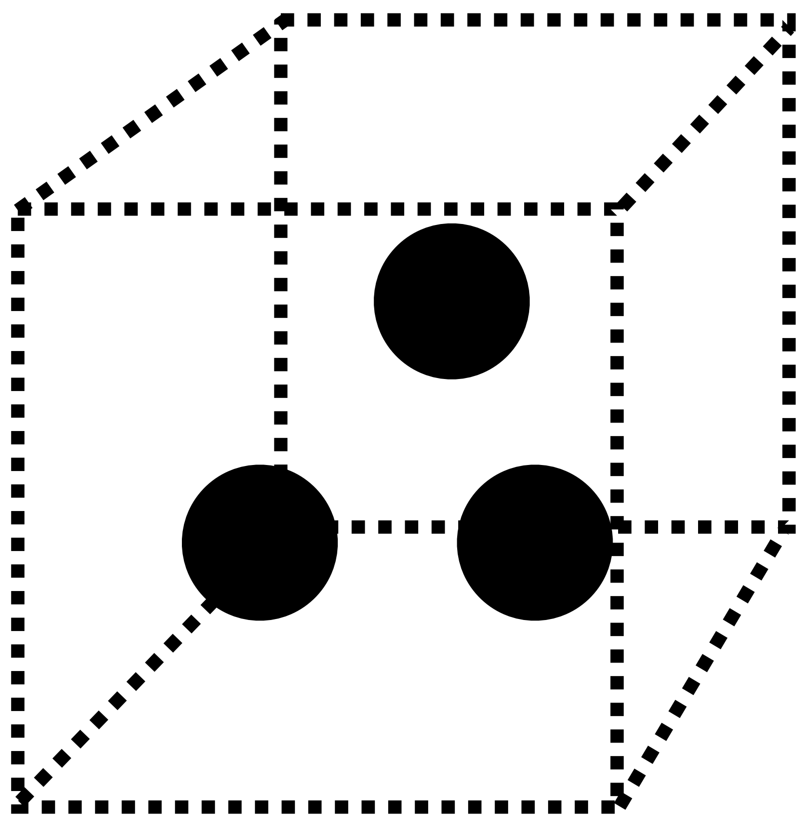


$m_\pi \sim 390$ MeV

RB, Dudek, Edwards, & Wilson (2016)

RB, Dudek, Edwards, & Wilson (2017)

Three-hadron systems



$$\det\left[F_3(E_L, L) + \mathcal{K}^{-1}_{\mathrm{df},3}(E_L)\right] = 0$$

Hansen & Sharpe ('14, '15)

Mai & Döring ('17)

RB, Hansen & Sharpe ('18)

Hansen, Romero-Lopez & Sharpe ('20)

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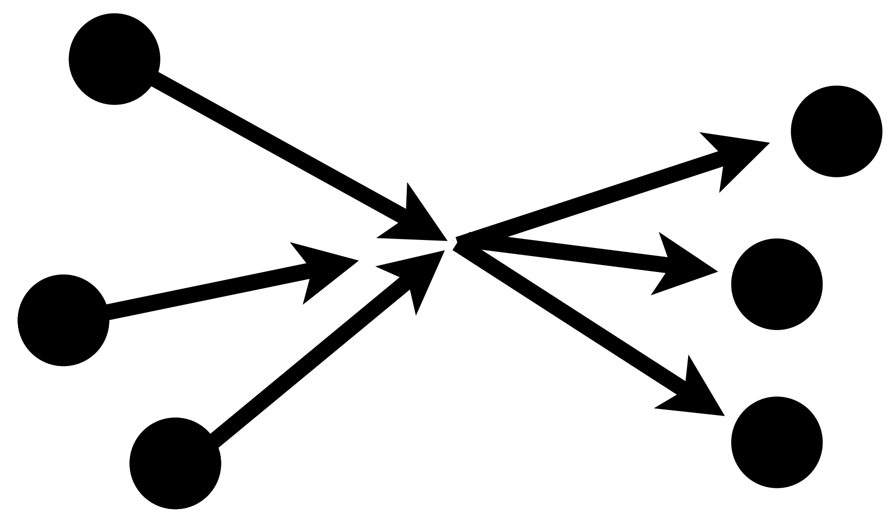
Jackura et al. ('20)

finite-volume
spectroscopy



infinite-volume
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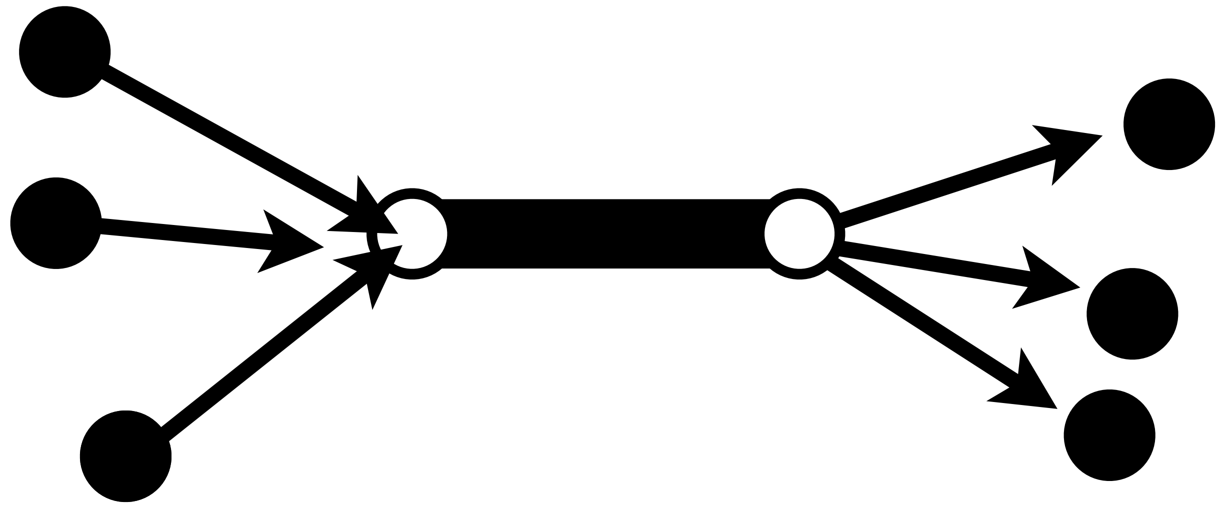
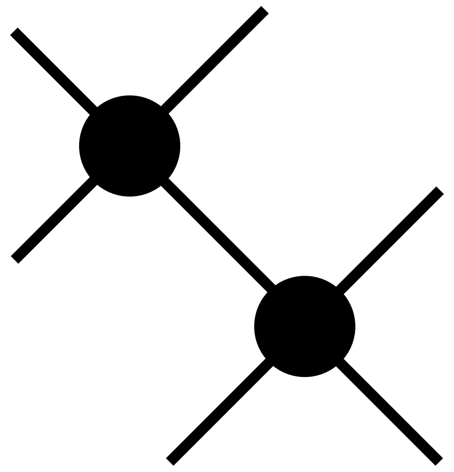
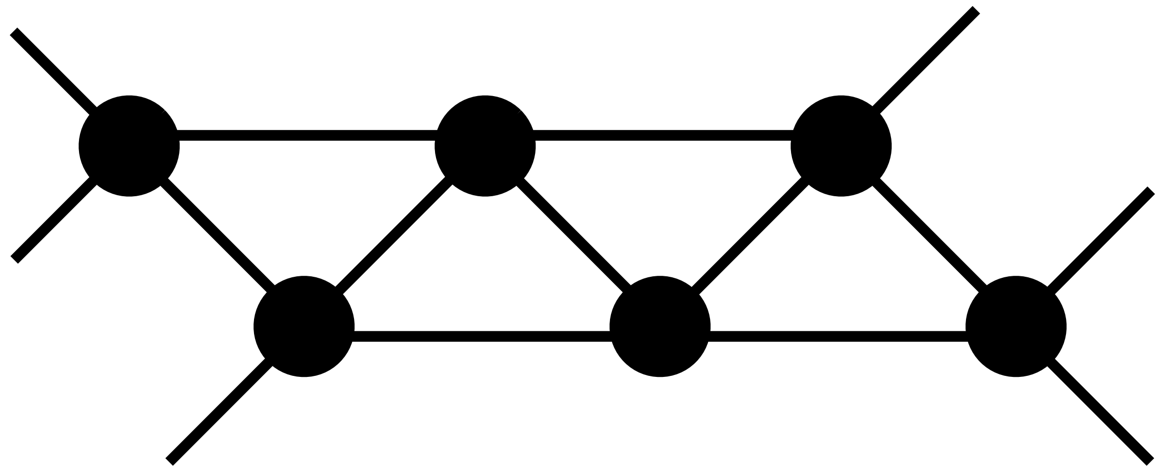


bound state and
resonance poles

Three hadrons in an infinite volume

The three-body scattering amplitude using all orders perturbation theory.

Sum over all connected 3-to-3 diagrams…

$i\mathcal{M}_{3,\text{con.}} =$  $+ \cdots +$  $+ \cdots$

satisfies an integral equations

$$i\mathcal{D} = i\mathcal{M}_2 iG i\mathcal{M}_2 + \int i\mathcal{M}_2 iG \, i\mathcal{D}$$

Three hadrons in an infinite volume

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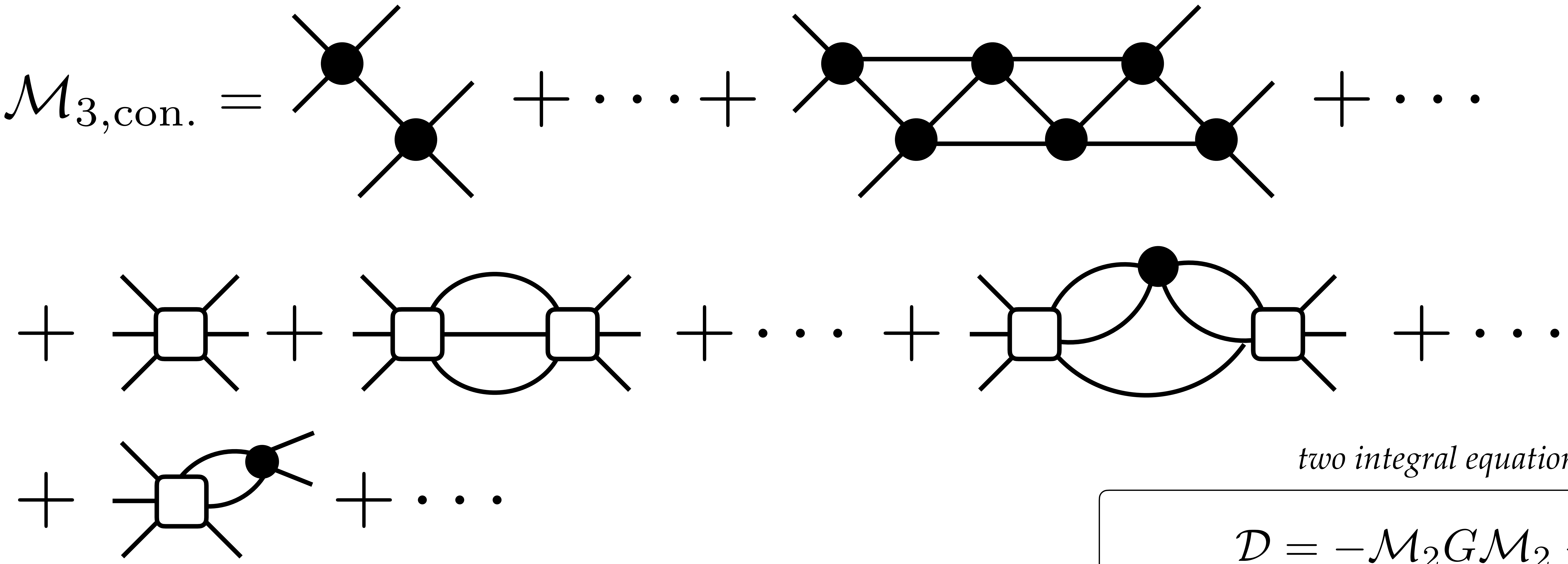


two integral equations and you're done!

$$\mathcal{D} = -\mathcal{M}_2 G \mathcal{M}_2 - \int \mathcal{M}_2 G\, \mathcal{D}$$

$$\mathcal{L} = \frac{1}{3} + \mathcal{M}_2 \rho - \mathcal{D}\rho$$

$$\mathcal{T} = \mathcal{K}_{\text{df},3} - \int \mathcal{K}_{\text{df},3}\, \rho\, \mathcal{L}\, \mathcal{T}$$

$$\mathcal{M}_{3,\text{con.}} = \mathcal{S}\left\{\mathcal{D} + \mathcal{L}\,\mathcal{T}\,\mathcal{L}^T\right\}$$

Solving integral equations

Assuming $\mathscr{K}_{\text{df}} = 0$ and partial wave-projecting onto $\ell = 0$ sector,

$$\mathcal{D}_{\mathsf{s}}^{(u,u)}(p,k) = -\mathcal{M}_2(E^\star_{2,p}) G_{\mathsf{s}}(p,k,\epsilon) \mathcal{M}_2(E^\star_{2,k}) - \mathcal{M}_2(E^\star_{2,p}) \int_0^{k_{\mathsf{max}}} \frac{k'^2\, dk'}{(2\pi)^2 \omega_{k'}} G_{\mathsf{s}}(p,k',\epsilon) \mathcal{D}_{\mathsf{s}}^{(u,u)}(k',k),$$

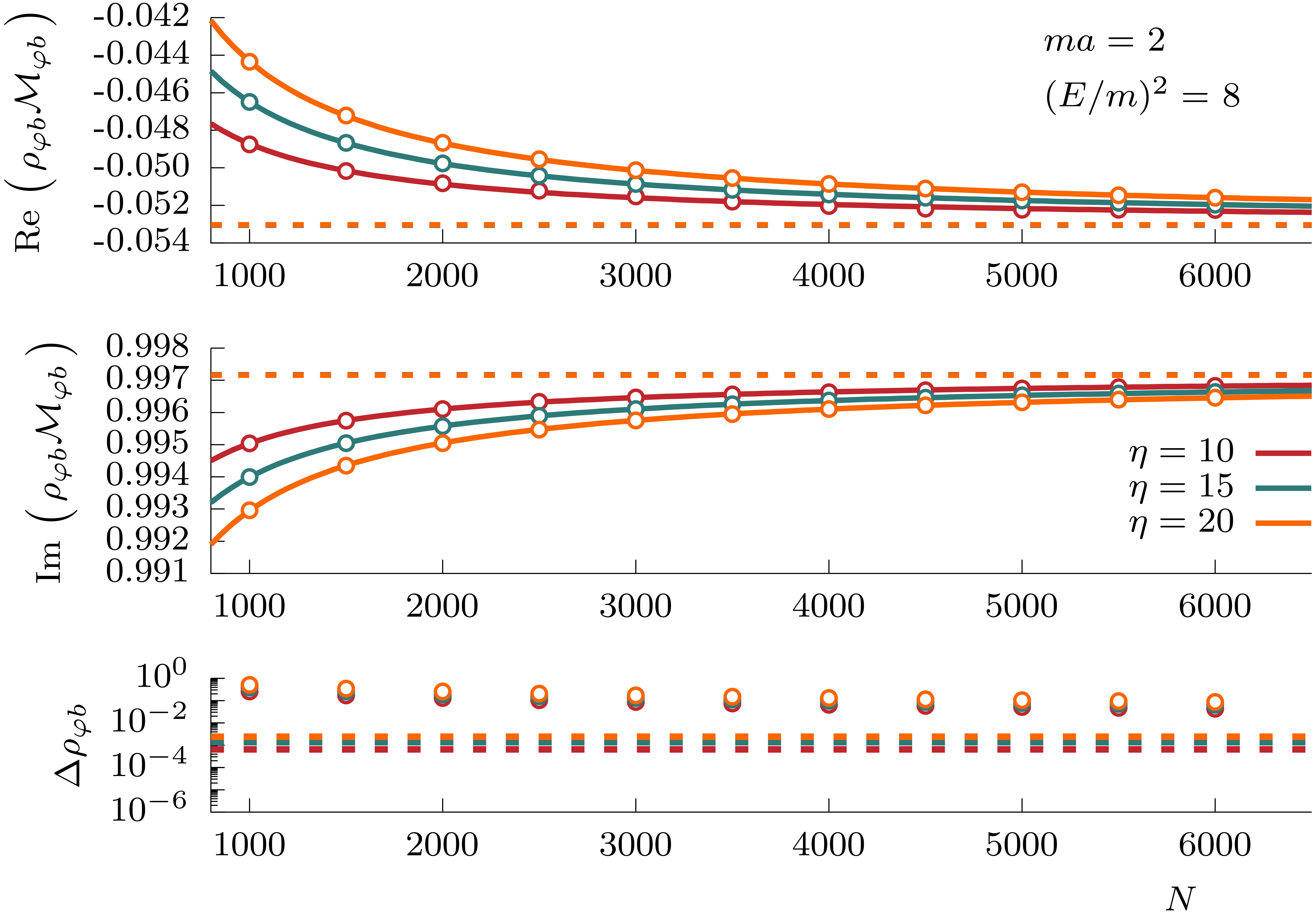
- Discretized momenta: meshes, splines,…
- Soften or isolate possible singularities: non-zero epsilon, integrate out poles analytically, …
- Write as a matrix form:

$$\boldsymbol{D}(N,\epsilon) = -\boldsymbol{\mathcal{M}} \cdot \boldsymbol{G}(\epsilon) \cdot \boldsymbol{\mathcal{M}} - \boldsymbol{\mathcal{M}} \cdot \boldsymbol{G}(\epsilon) \cdot \boldsymbol{P} \cdot \boldsymbol{D}(N,\epsilon)$$

- Recover the exact results when taking $\epsilon = \eta/N \to 0$.

Jackura, RB, Dawid, Islam, & McCarty (2020)

Convergence tests

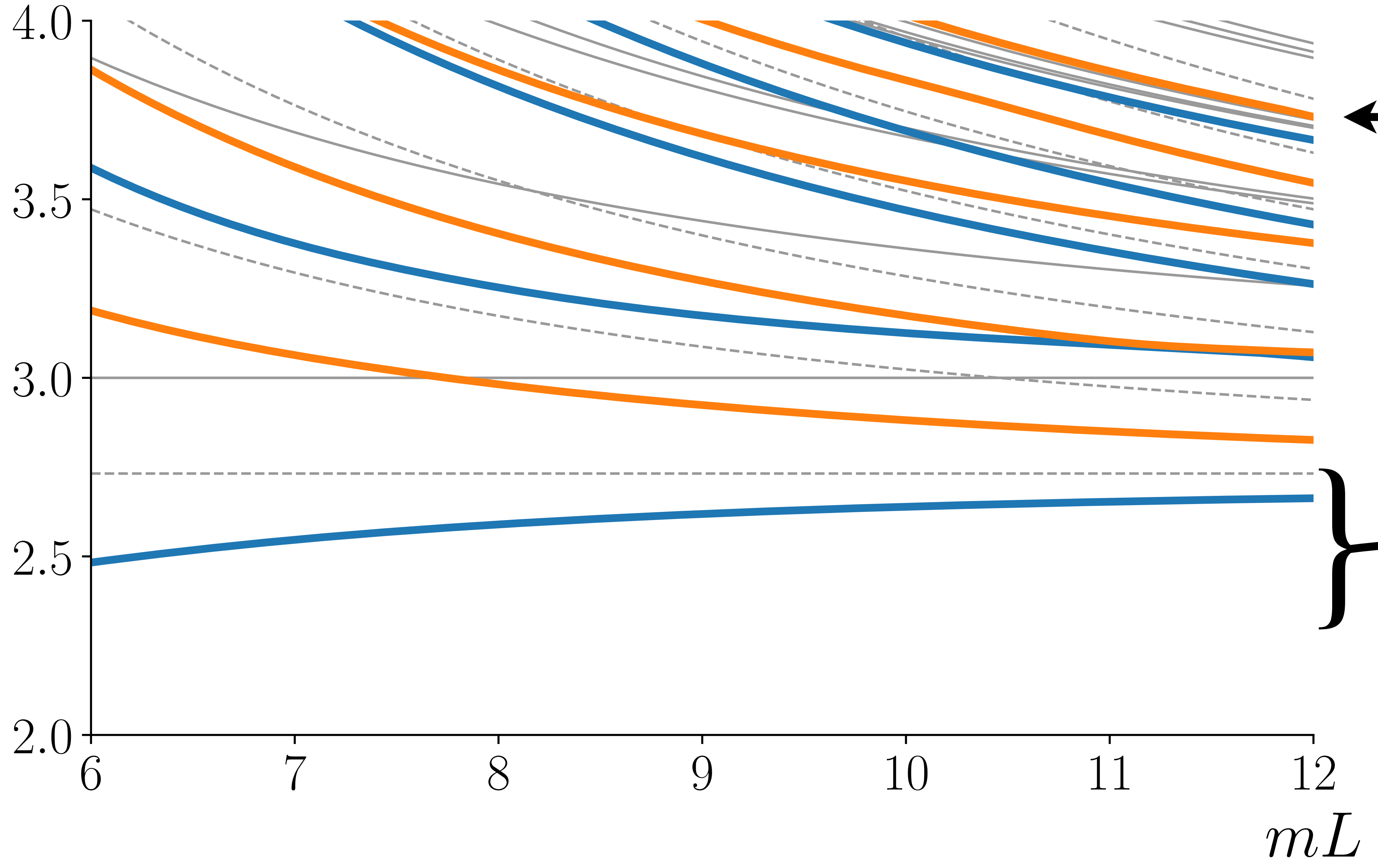


Jackura, RB, Dawid, Islam, & McCarty (2020)

Consistency checks for toy model

Alternatively to solving the integral equations, one can instead:

- ☐ consider theories with two-body bound states (),
- ☐ obtain E_L using 3-body quantization by Hansen & Sharpe (2014),
- ☐ obtain $q \cot \delta_{\varphi b}$ using the two-body Lüscher formalism.



$$\det\left[F_3(E_L, L) + \mathcal{K}_{\mathrm{df},3}^{-1}(E_L)\right] = 0$$

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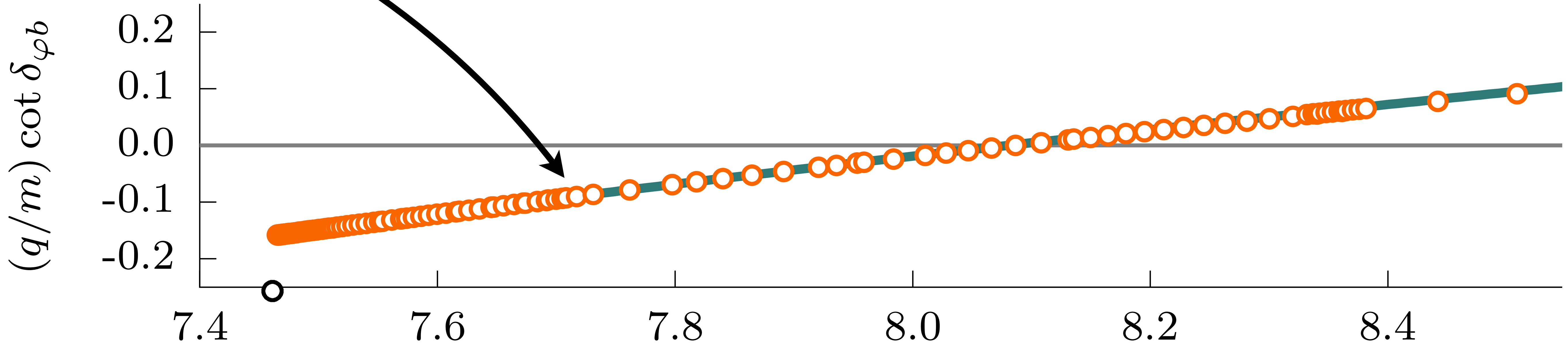
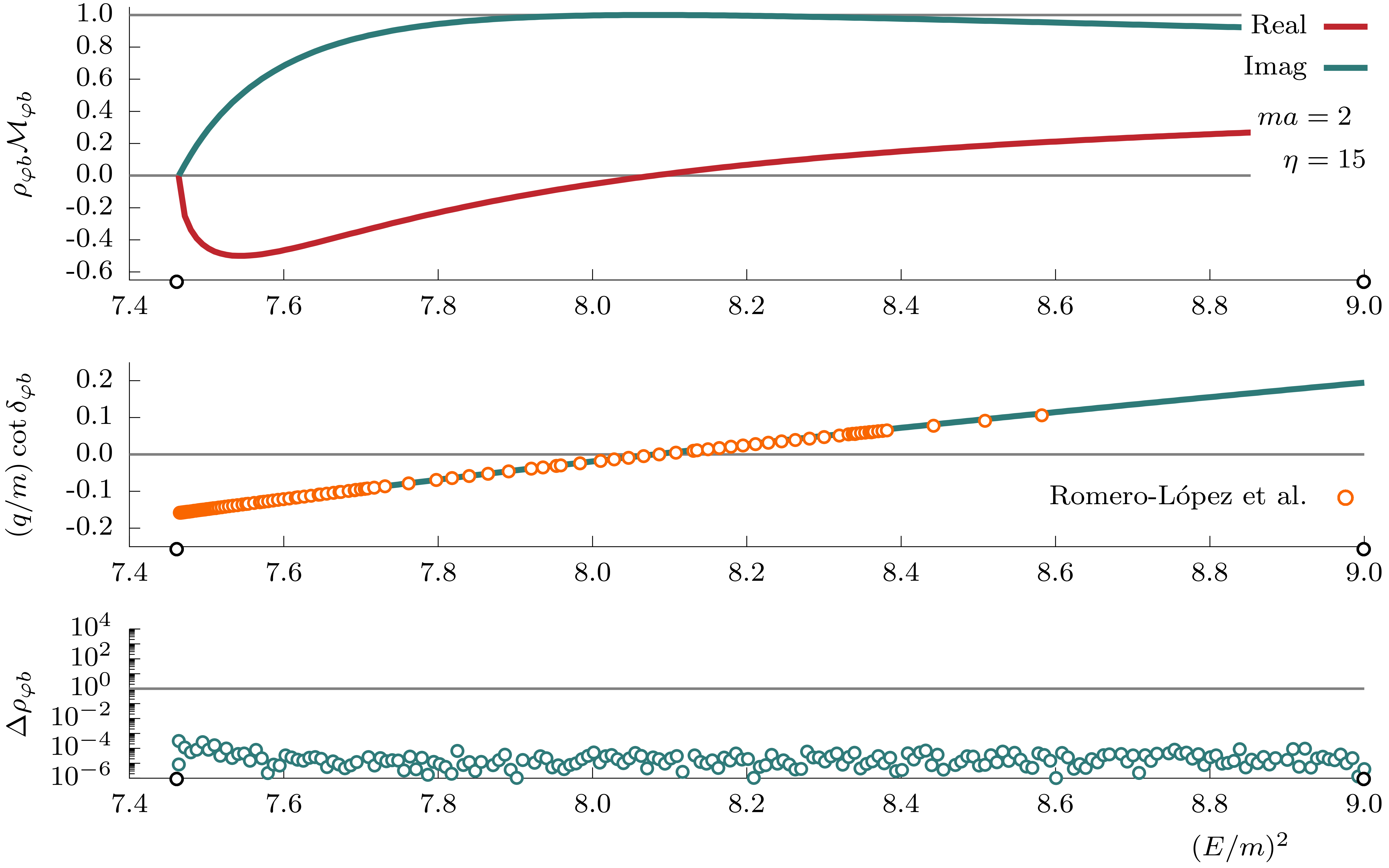


Romero-López, Sharpe, Blanton, RB, & Hansen (2019)

Consistency checks for toy model

percentage unitarity deviation

$$\Delta\rho_{\varphi b}(E;N) \equiv \left| \frac{\mathrm{Im}\left[\mathcal{M}_{\varphi b}^{-1}(E;N)\right] + \rho_{\varphi b}(E)}{\rho_{\varphi b}(E)} \right| \times 100$$



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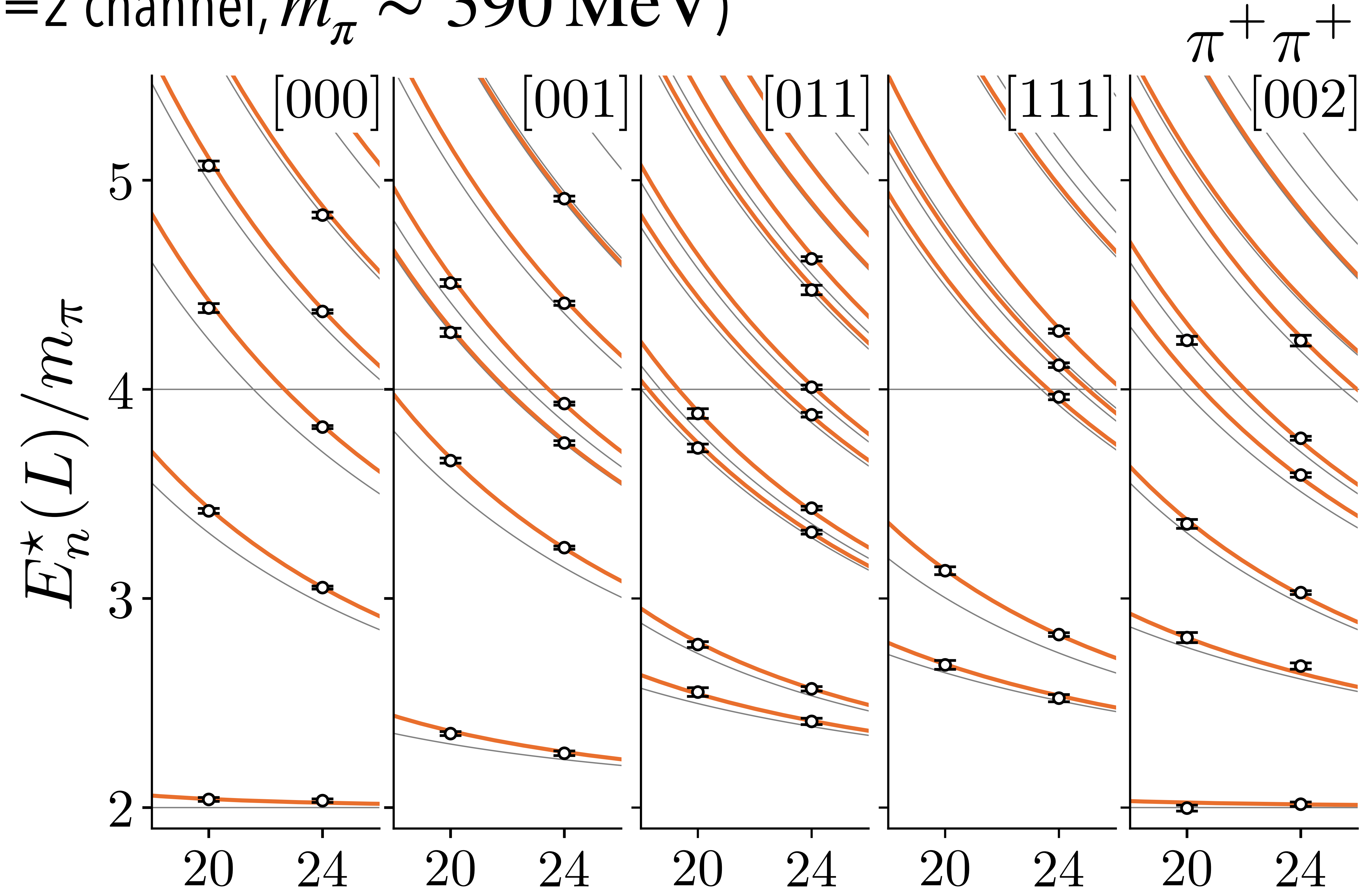
$\pi\pi$

(I=2 channel, $m_\pi \sim 390\,\text{MeV}$)



Hansen Edwards Thomas Wilson

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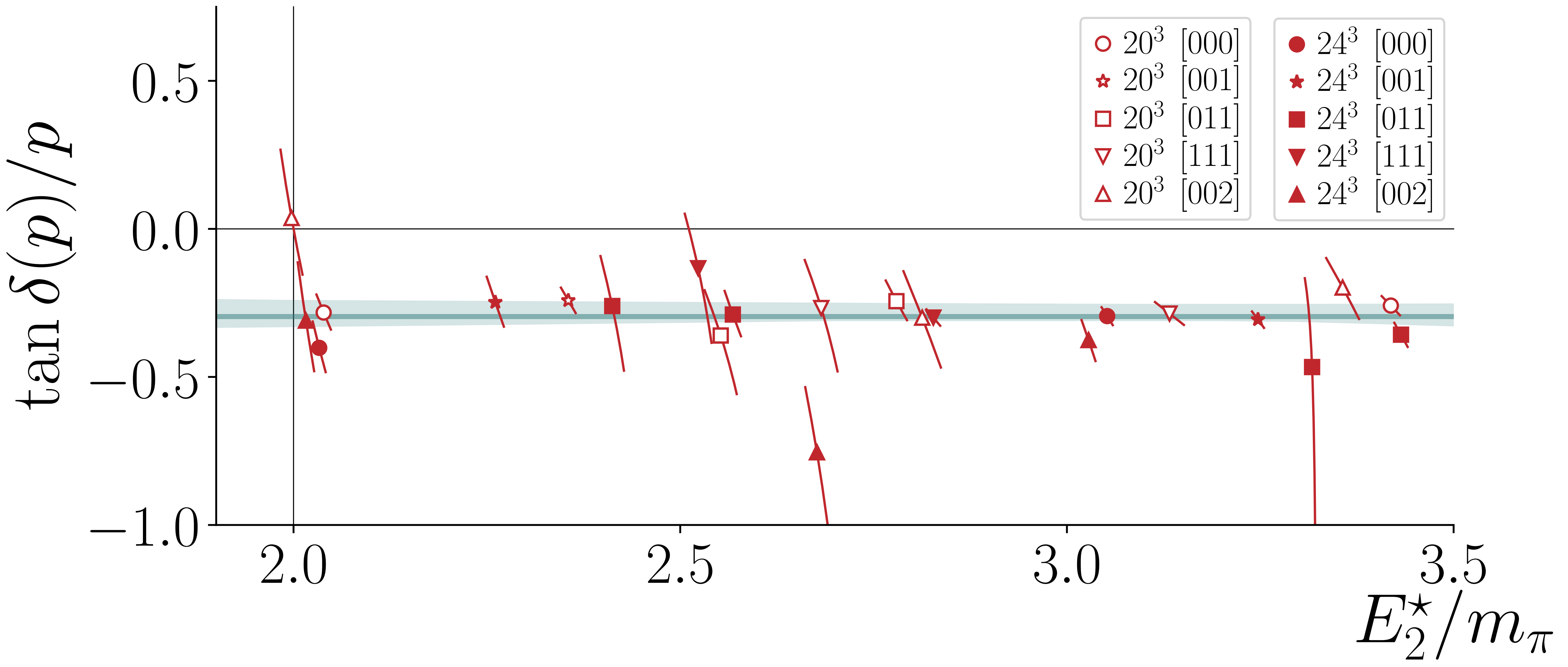
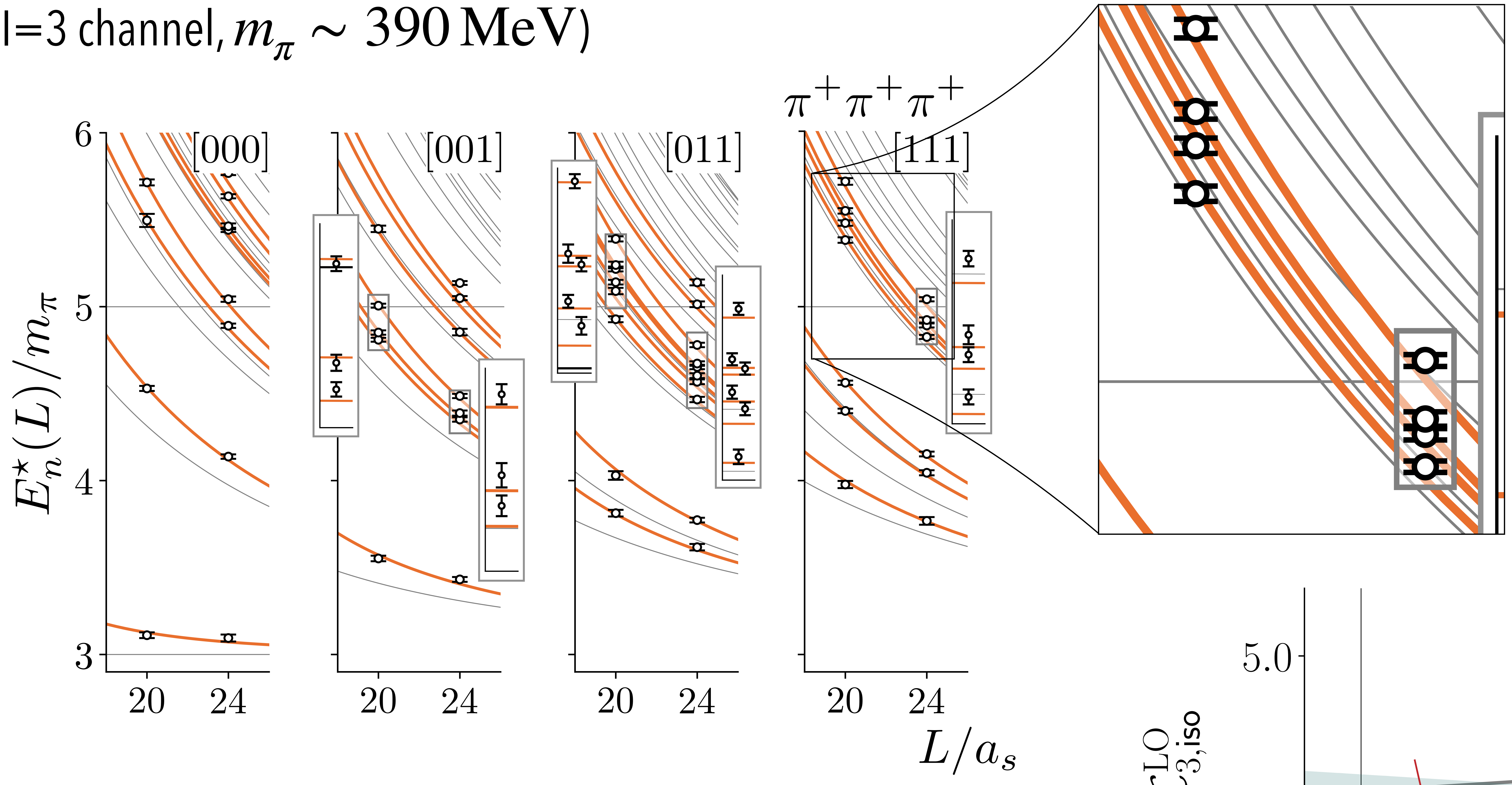


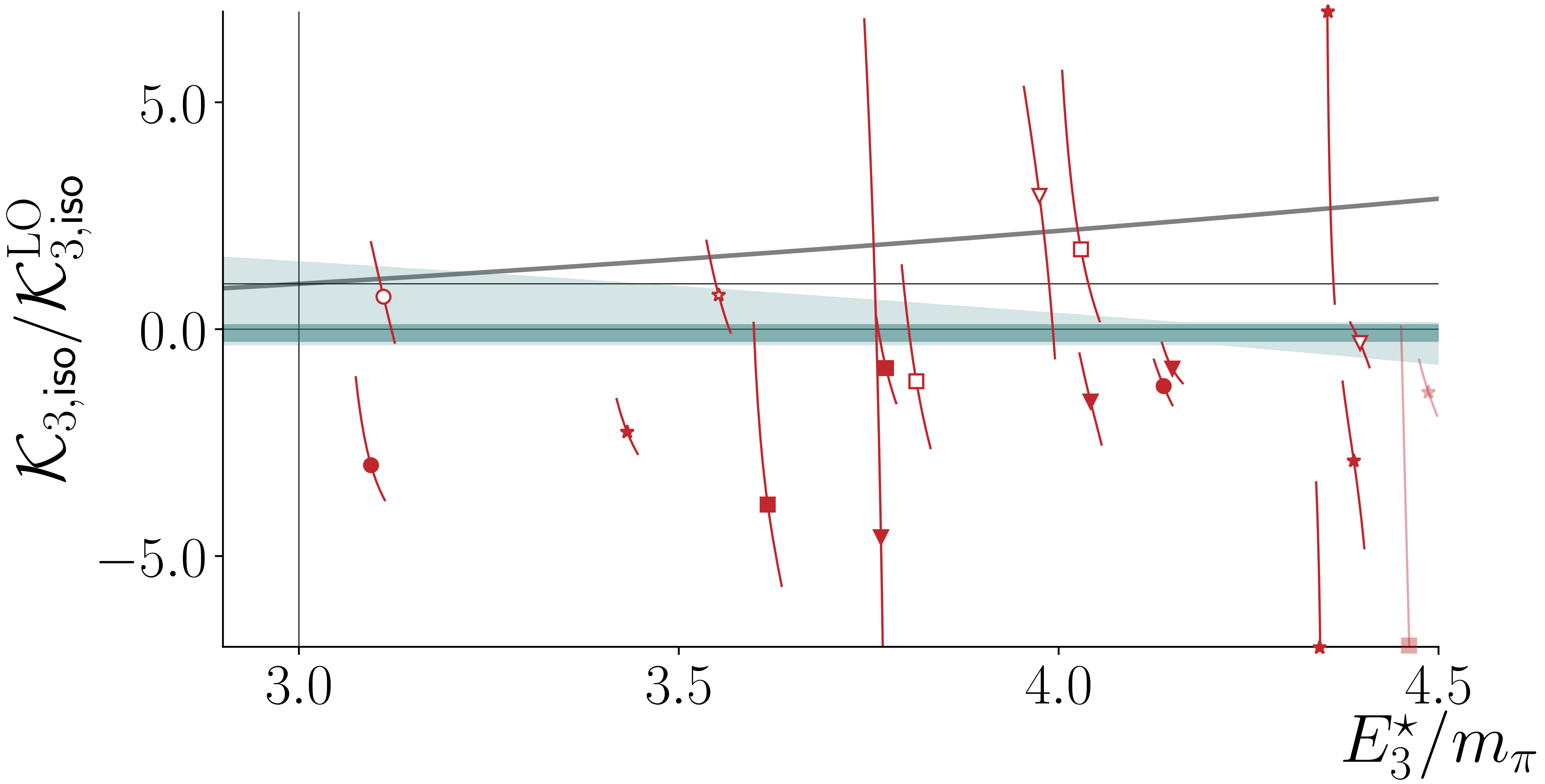
$m_\pi \sim 390\,\text{MeV}$

Hansen, RB, Edwards, Thomas, & Wilson (2020)

$\pi\pi\pi$

(I=3 channel, $m_\pi \sim 390\,\mathrm{MeV}$)





$m_\pi \sim 390\,\mathrm{MeV}$

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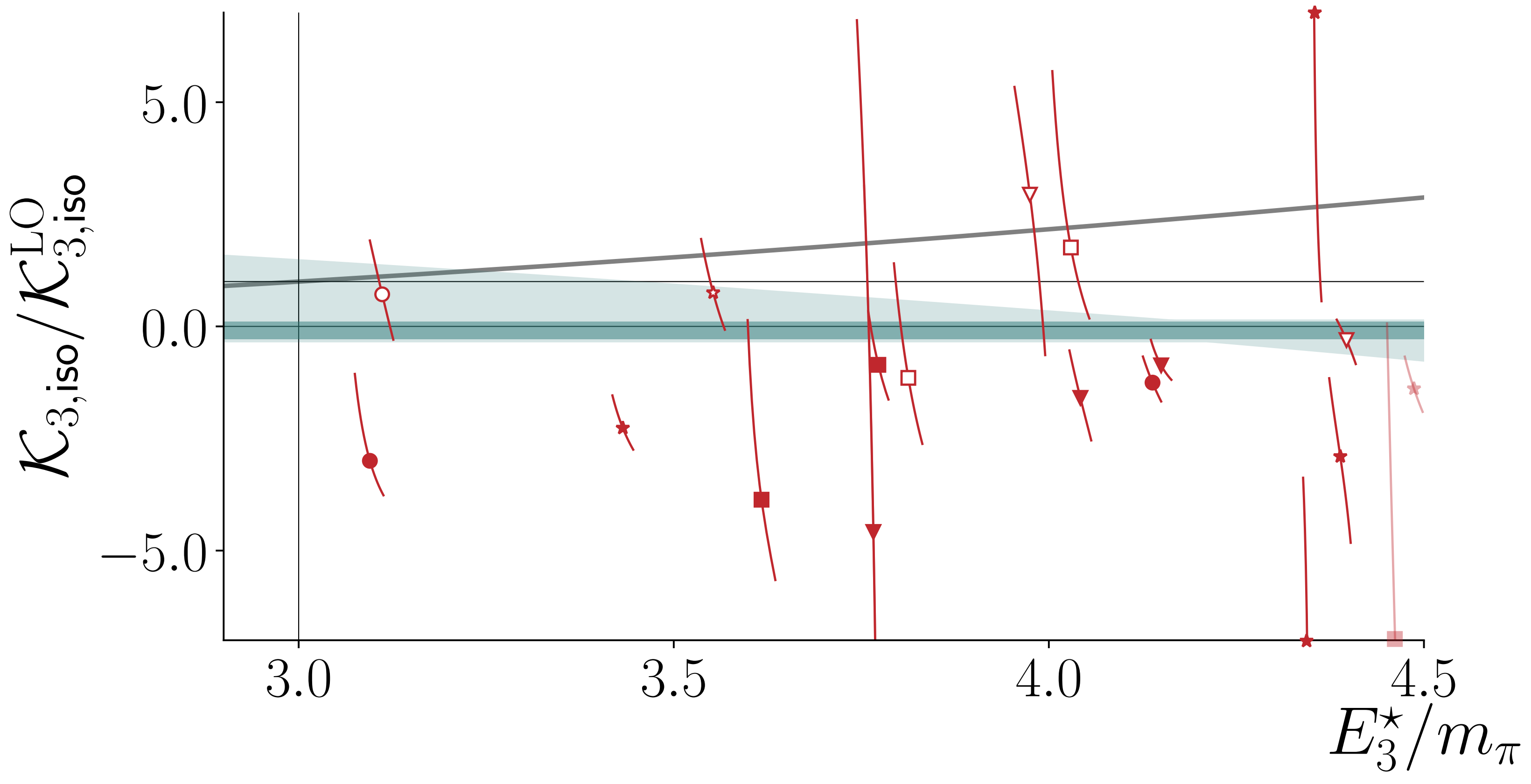
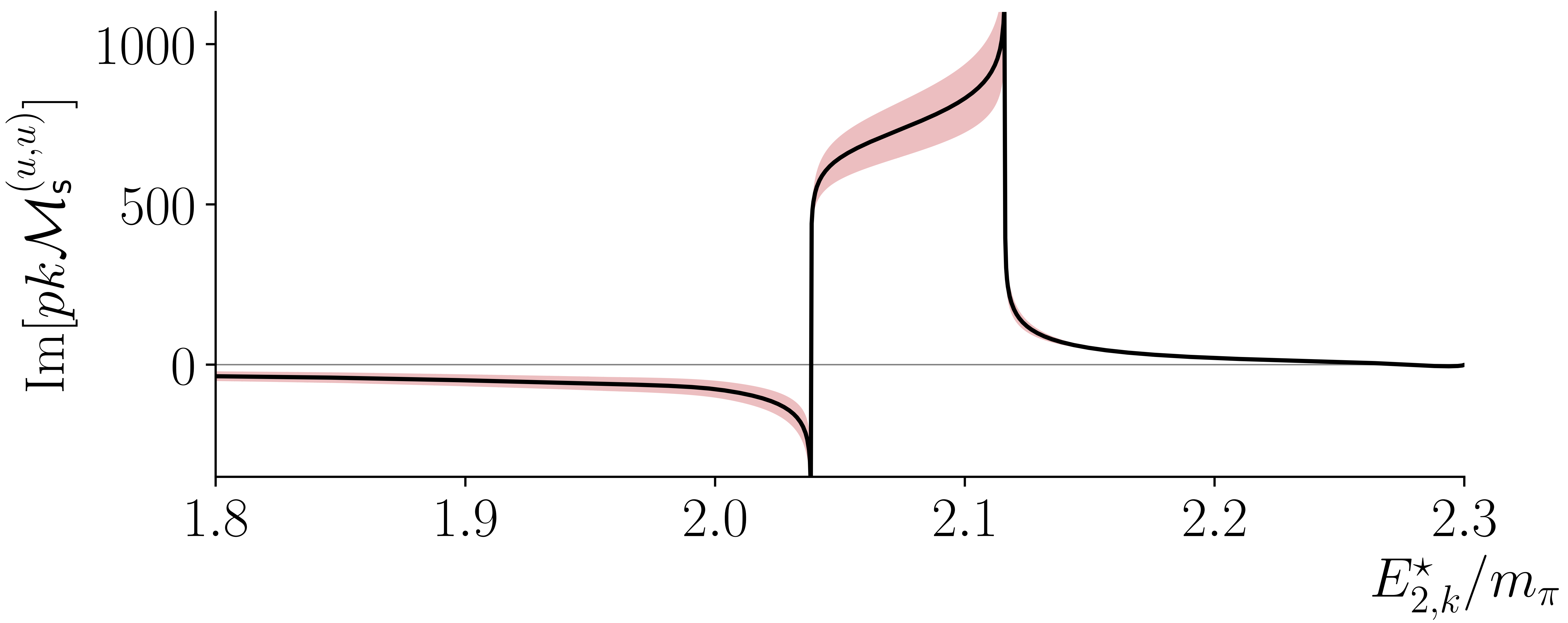
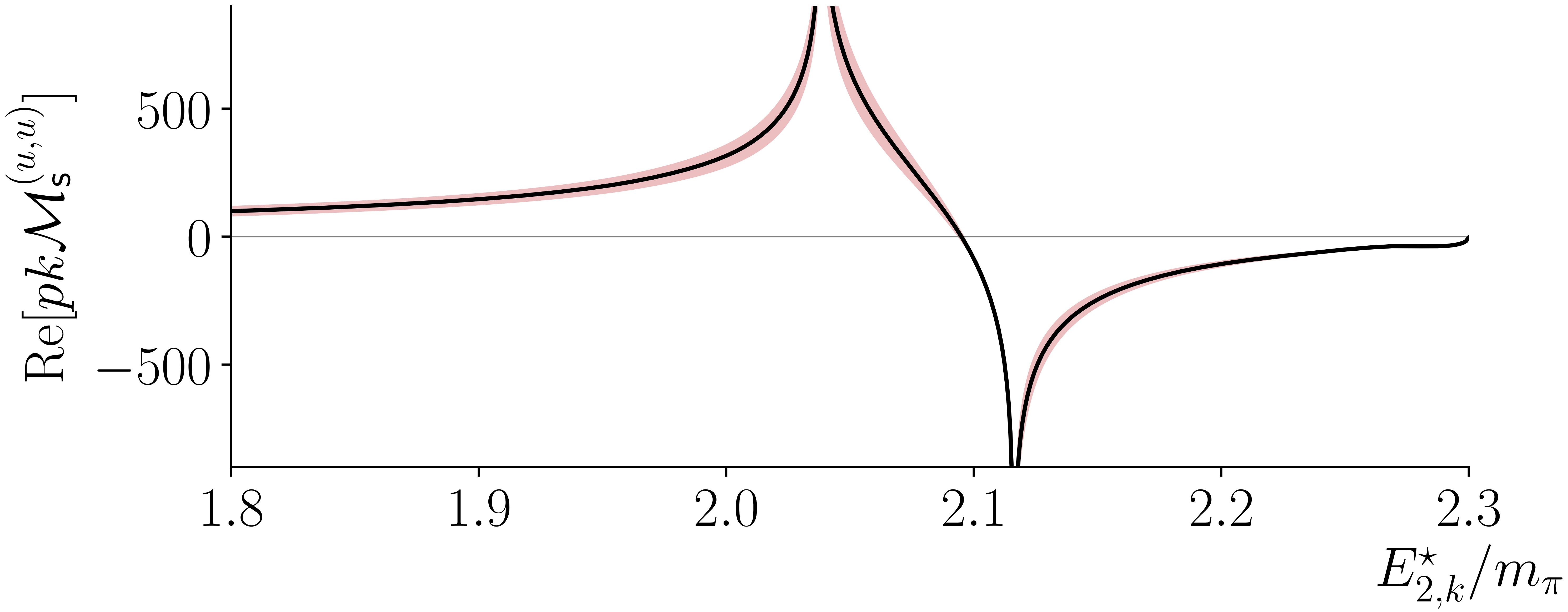
πππ scattering
(I=3 channel, $m_\pi \sim 390\,\mathrm{MeV}$)

first 3body scattering amplitude from the lattice QCD!







$m_\pi \sim 390\,\mathrm{MeV}$

Hansen, RB, Edwards, Thomas, & Wilson (2020)

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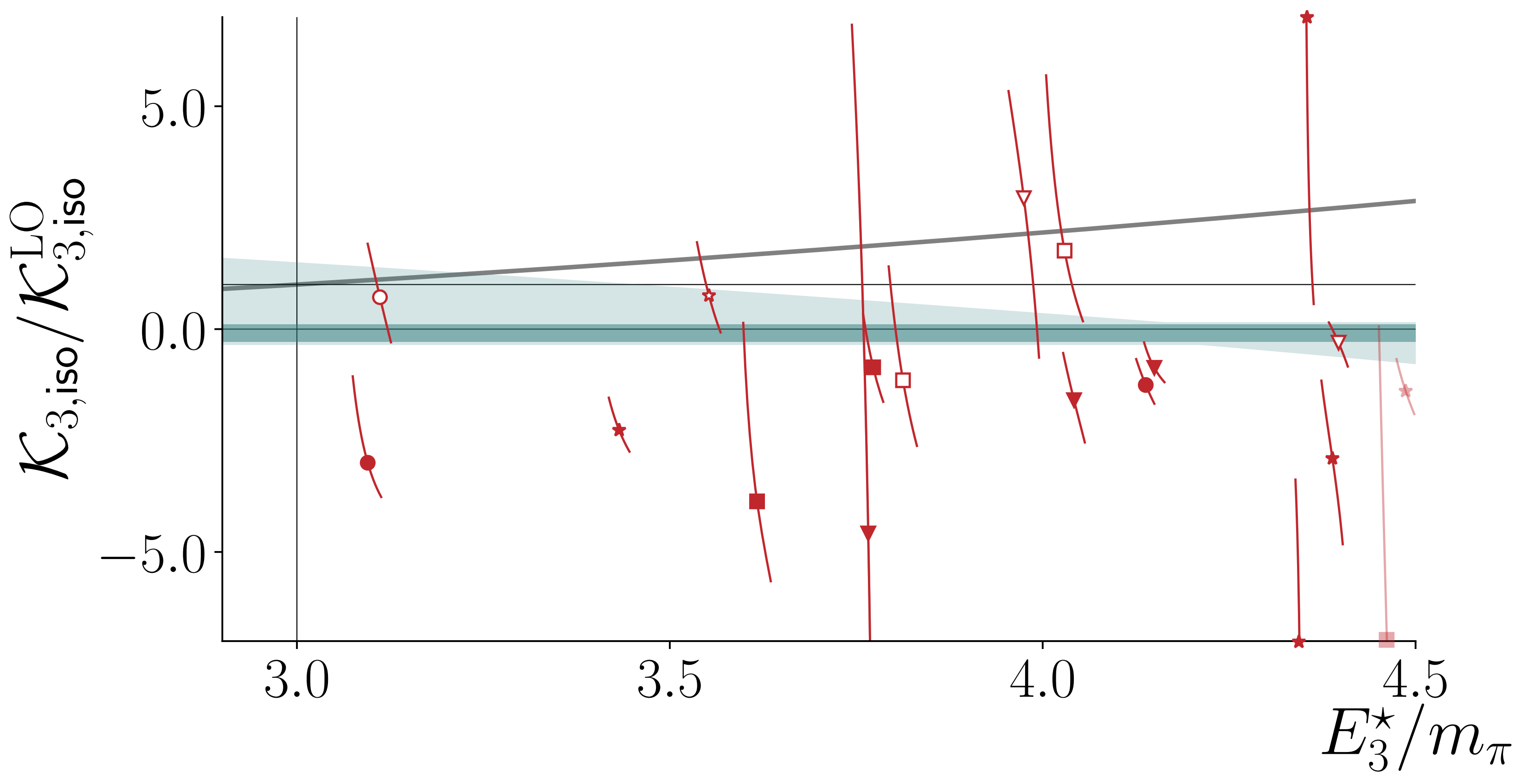

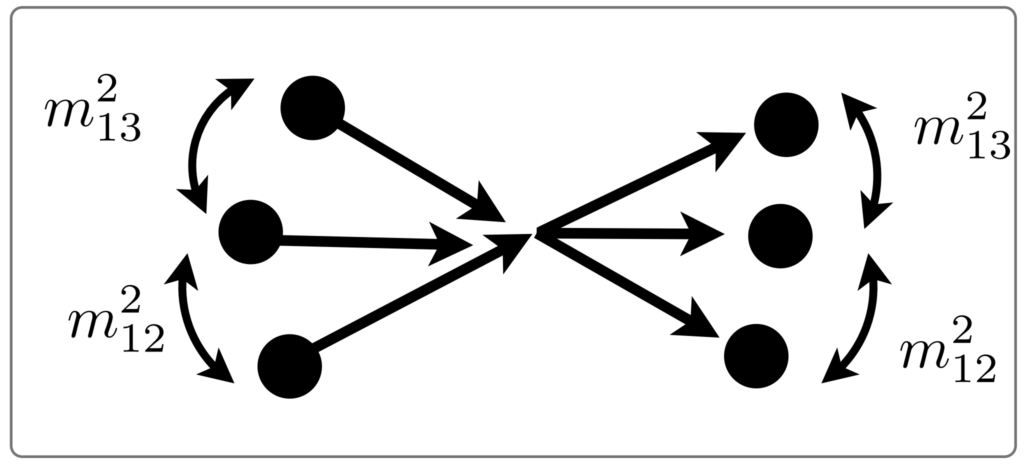

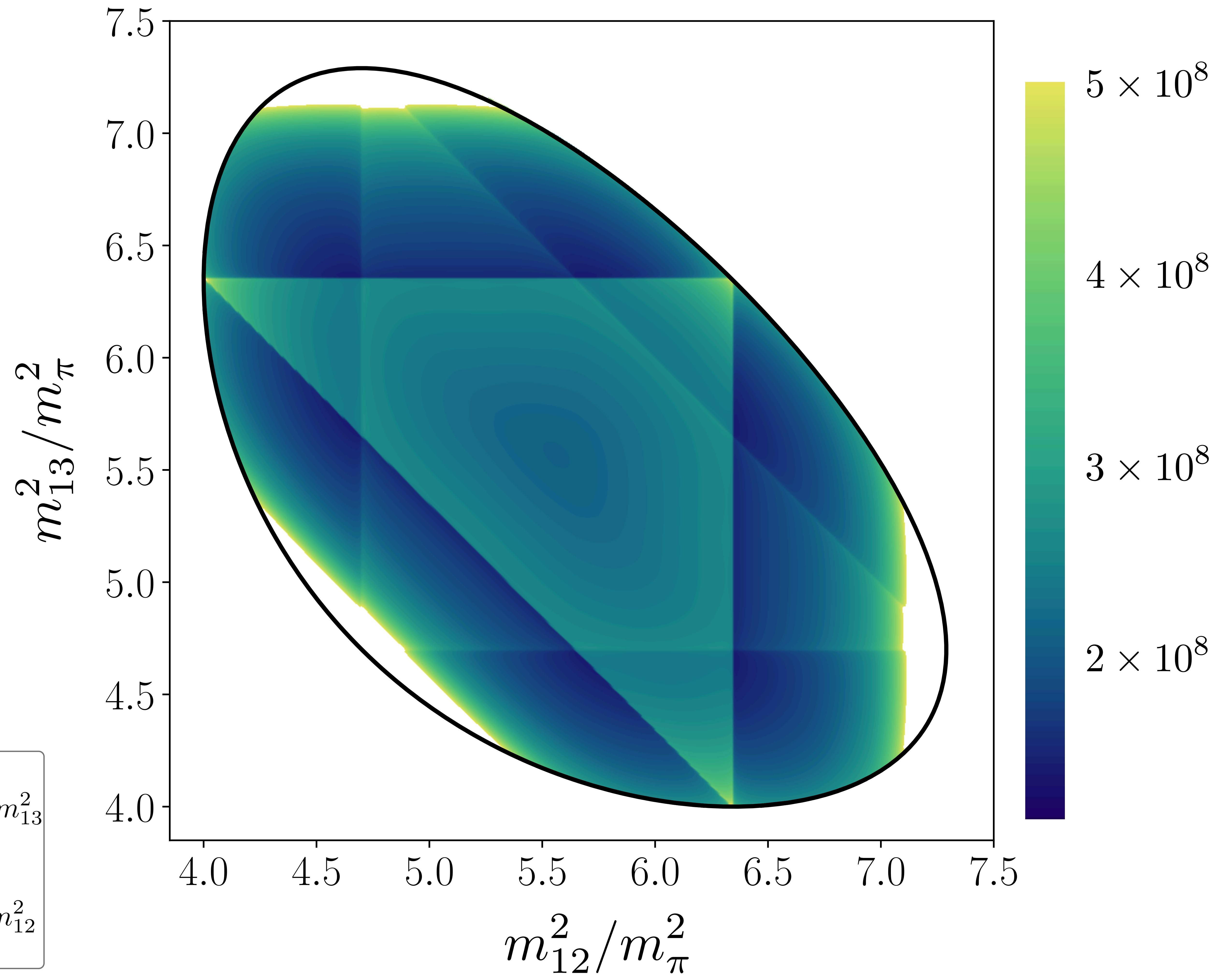

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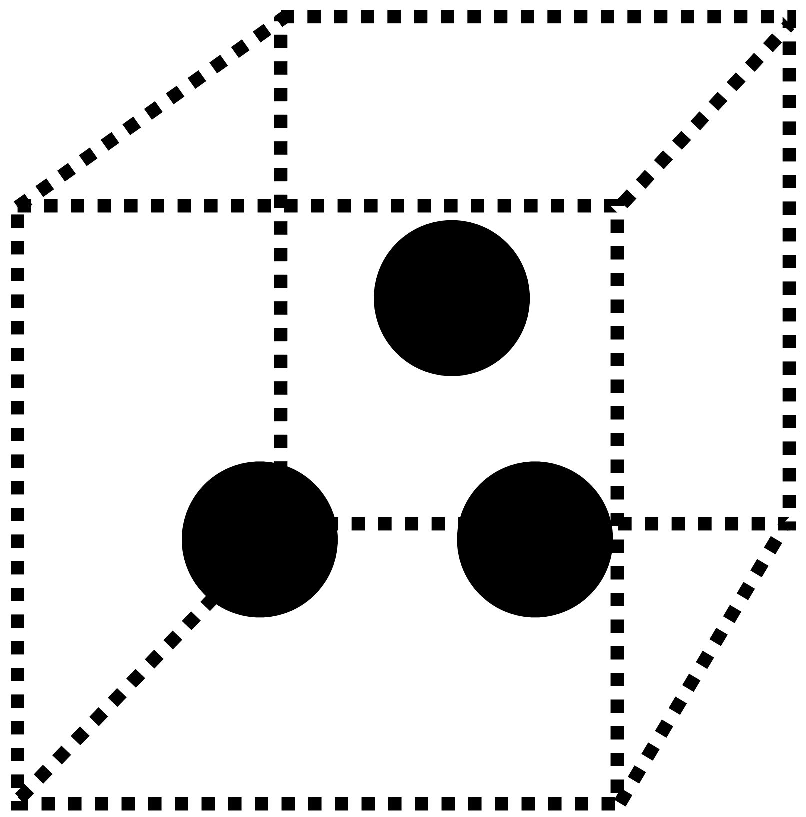
Hansen, RB, Edwards, Thomas, & Wilson (2020)

Future of spectroscopy



Hansen & Sharpe ('14, '15)

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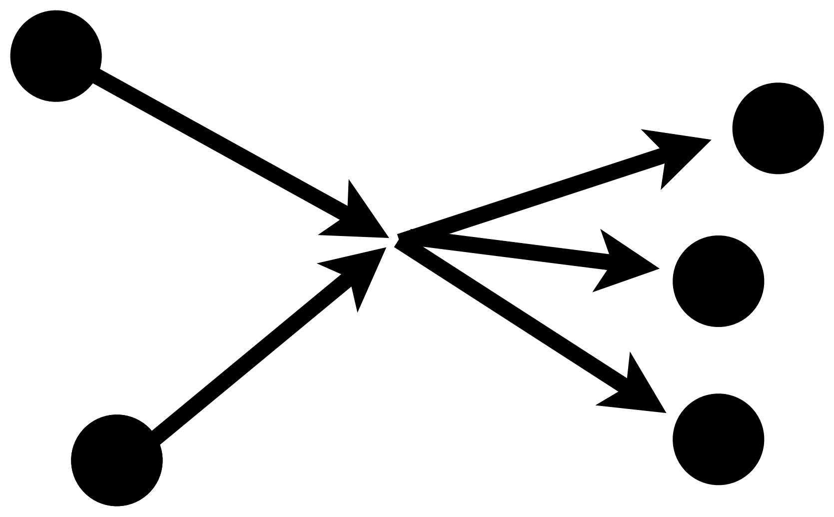
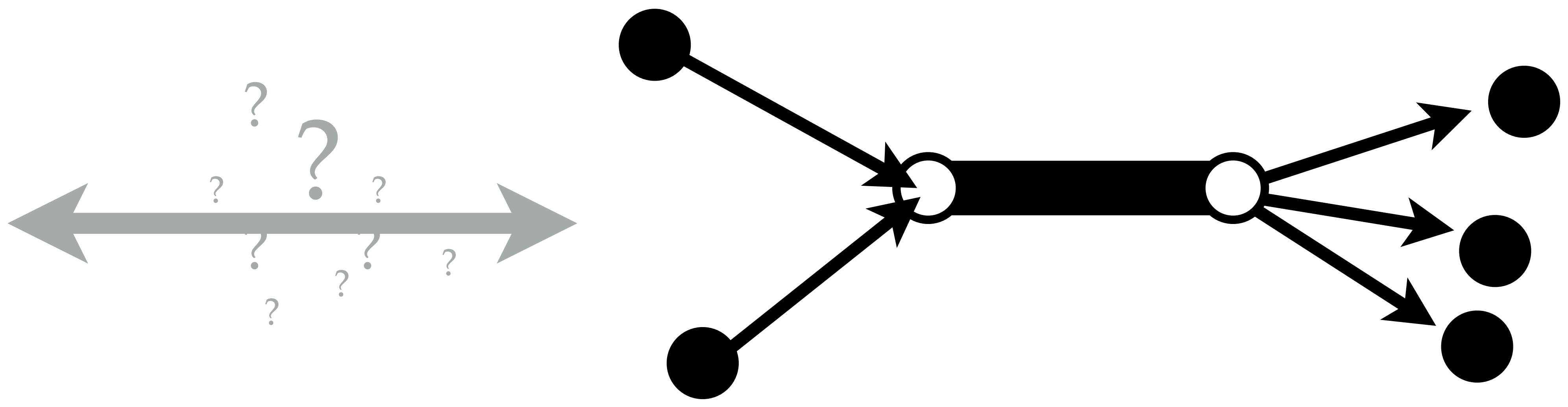
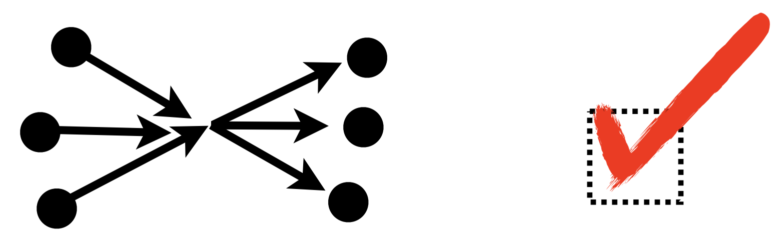
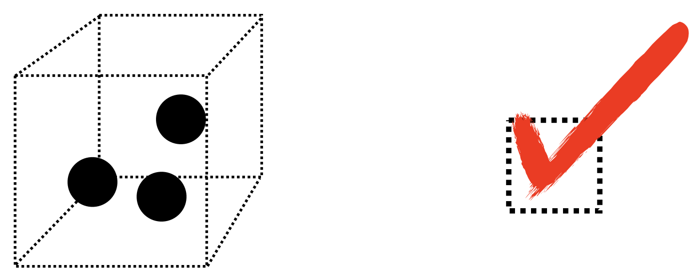




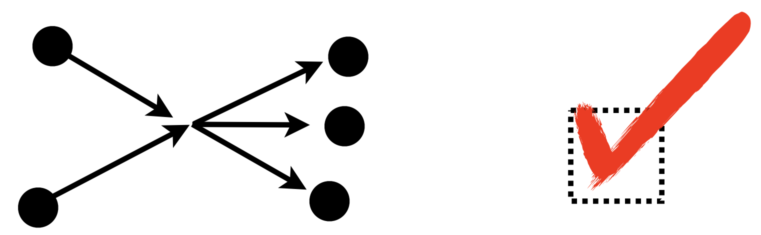
Unitarity in 3Body systems



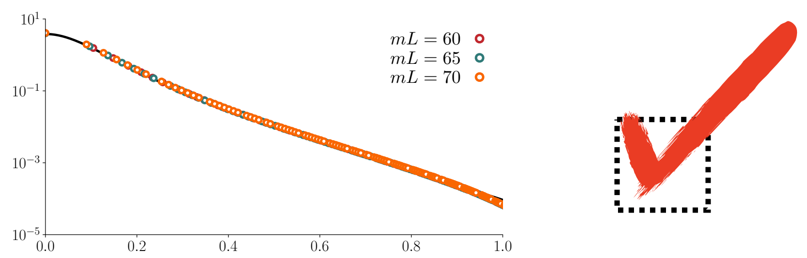
FV formalism for spinless states



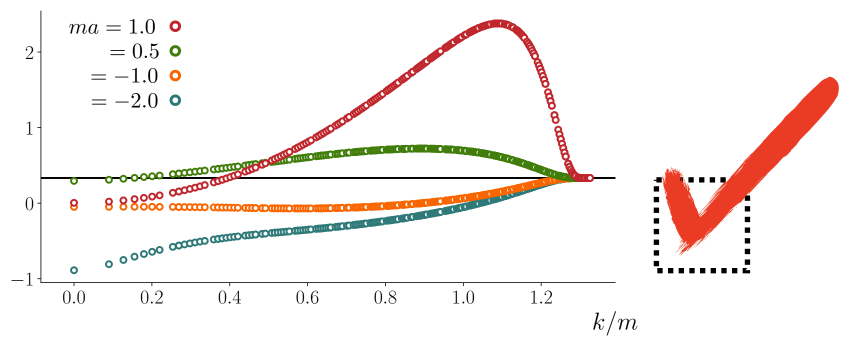
coupled 2/3B systems



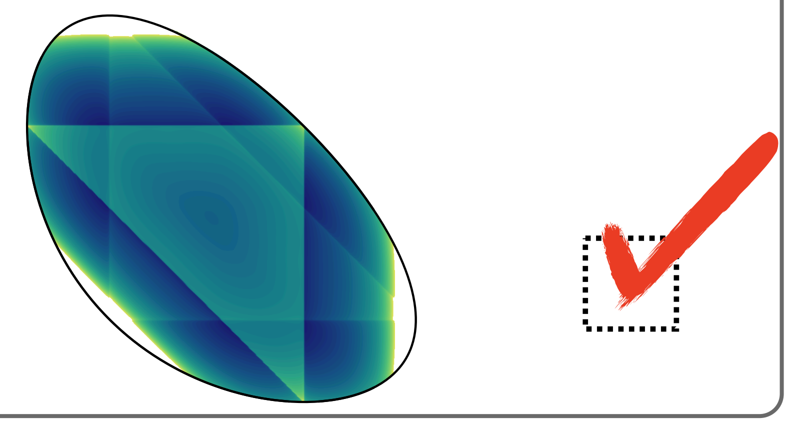
analytic and numerical checks



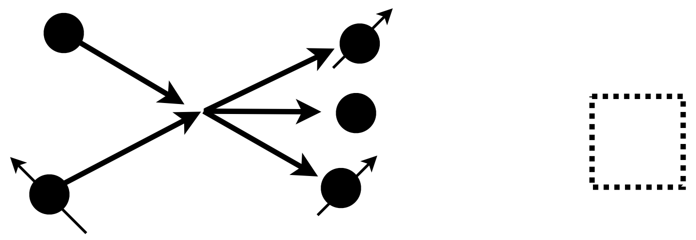
integral equations



Actual calculations



Spin, multichannels



Analytic continuation