The $\sin(2\phi - \phi_S)$ azimuthal asymmetry in the pion induced Drell-Yan process within TMD factorization

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OUTLINE

1. Introduction

The Boer-Mulders function $h_1^\perp$:

- transversely polarization asymmetry of quarks inside an unpolarized hadron
- T-odd distribution function (model calculations)
- chiral-odd distribution function (single transversely polarized Drell-Yan, $h_1^\perp \otimes h_1$, $\sin(2\phi - \phi_S)$ asymmetry)

Experimental measurement

- the first measurement on the $\sin(2\phi - \phi_S)$ asymmetry has been performed by the COMPASS ($\pi p^\uparrow$)
- indicates negative sign and substantial size

TMD factorization:

- It allows TMDs to dependent on the intrinsic transverse momentum
- It is applicable in the region the transverse momentum of the dilepton $q_\perp$ is much smaller than the hard scale Q
- The physical observables can be expressed as the convolution of the factors related to hard scattering and the well-defined TMDs

TMD evolutions:

- It provides a systematic approach to deal with the evolution of the TMDs

2. The TMD evolution of distribution functions $\tilde{F}(x, b; \mu, \zeta_F)$

- $\tilde{F}(x, b; \mu, \zeta_F)$: f_1, $h_1^\perp, h_1$.
- For the ζ_F dependence of the TMD distributions, it is determined by the Collins-Soper-Sterman (CSS) equation ($b = |\mathbf{b}_\perp|$):

$$\frac{\partial \ \ln\tilde{F}(x, b; \mu, \zeta_F)}{\partial \ \sqrt{\zeta_F}} = \tilde{K}(b; \mu), \tag{2.1}$$

- while the μ dependence is derived from the renormalization group equation as

$$\frac{d \ \tilde{K}}{d \ \ln\mu} = -\gamma_K(\alpha_s(\mu)), \tag{2.2}$$

$$\frac{d \ \ln\tilde{F}(x, b; \mu, \zeta_F)}{d \ \ln\mu} = \gamma_F(\alpha_s(\mu); \frac{\zeta_F^2}{\mu^2}), \tag{2.3}$$

- Solving those equations, one can obtain the general solution for the energy dependence of $\tilde{F}$:

$$\tilde{F}(x, b, Q) = \mathcal{F} \times e^{-S(Q,b)} \times \tilde{F}(x, b, \mu_i), \tag{2.4}$$

- Combining the perturbative part and the nonperturbative part, one has the complete result for the Sudakov form factor:

$$S(Q, b) = S_{\rm P}(Q, b_*) + S_{\rm NP}(Q, b). \tag{2.5}$$

- One set a parameter b_{max} to be the boundary between the two different regions to guarantee that b_* is always in the perturbative region.

- The perturbative part $S_{\rm P}(Q,b)$ has the following form:

$$S_{\rm P}(Q,b_*) = \int_{\mu_b^2}^{Q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2} \left[A(\alpha_s(\bar{\mu}))\ln\frac{Q^2}{\bar{\mu}^2} + B(\alpha_s(\bar{\mu})) \right]. \qquad (2.6)$$

- The coefficients A and B can be expanded as the series of α_s/π:

$$A = \sum_{n=1}^{\infty} A^{(n)} (\frac{\alpha_s}{\pi})^n, \qquad (2.7)$$

$$B = \sum_{n=1}^{\infty} B^{(n)} (\frac{\alpha_s}{\pi})^n. \qquad (2.8)$$

- We will take $A^{(n)}$ up to $A^{(2)}$ and $B^{(n)}$ up to $B^{(1)}$ in the accuracy of next-to-leading-logarithmic (NLL) order:

$$A^{(1)} = C_F, \tag{2.9}$$

$$A^{(2)} = \frac{C_F}{2}\left[C_A\left(\frac{67}{18} - \frac{\pi^2}{6}\right) - \frac{10}{9}T_R n_f\right], \tag{2.10}$$

$$B^{(1)} = -\frac{3}{2}C_F. \tag{2.11}$$

- In the small b region $1/Q \ll b \ll 1/\Lambda$, the TMD distributions at a fixed scale μ can be expressed as:

$$\tilde{F}_{q/H}(x, b; \mu) = \sum_i C_{q \leftarrow i} \otimes F_{i/H}(x, \mu). \tag{2.12}$$

The convolution $\otimes$ regarding the momentum fraction of x is given by

$$C_{q \leftarrow i} \otimes F_{i/H}(x, \mu) \equiv \int_x^1 \frac{d\xi}{\xi} C_{q \leftarrow i}(x/\xi, b; \mu) F_{i/H}(\xi, \mu), \tag{2.13}$$

where $\mu = c_0/b_*$, with $c_0 = 2e^{-\gamma_E}$ and the Euler Constant $\gamma_E \approx 0.577$.

- It is straightforward to rewrite the scale-dependent TMD distribution function $\tilde{F}$ of the proton and the pion in b space:

$$\tilde{F}_{q/H}(x, b; Q) = e^{-\frac{1}{2}S_{\rm P}(Q,b_*)-S_{\rm NP}^{F_{q/H}}(Q,b)}\mathcal{F}(\alpha_s(Q)) \sum_i C^F_{q\leftarrow i} \otimes F_{i/H}(x, \mu_b), \qquad (2.14)$$

The factor of $\frac{1}{2}$ in front of $S_{\rm P}$ comes from the fact that $S_{\rm P}$ of quarks and antiquarks satisfies the relation:

$$S^q_{\rm P}(Q, b_*) = S^{\bar{q}}_{\rm P}(Q, b_*) = S_{\rm P}(Q, b_*)/2. \qquad (2.15)$$

Hereafter, we take $\mathcal{F} = 1$ and $C^F_{q\leftarrow i} = \delta_{qi}\delta(1 - x)$ for $F = f_1, h_1$ and $h_1^\perp$.

- **SIYY** parametrization:

(1) $S_{\rm NP}$ associated with a single TMD distribution function of the proton can be expressed as [**arXiv:1406.3073**]:

$$S_{\rm NP}^{f_{1,q/p}}(Q,b) = \frac{g_1}{2}b^2 + \frac{g_2}{2}\ln\frac{b}{b_*}\ln\frac{Q}{Q_0}. \tag{2.16}$$

$$Q_0^2 = 2.4\ {\rm GeV}^2, \quad g_1 = 0.212^{+0.006}_{-0.007}, \quad g_2 = 0.84^{+0.040}_{-0.035}.$$

(2) For the nonperturbative form factors of the pion distribution function [**arXiv:1707.05207**]:

$$S_{\rm NP}^{f_{1,q/\pi}} = g_1^\pi\, b^2 + g_2^\pi \ln\frac{b}{b_*}\ln\frac{Q}{Q_0}, \tag{2.17}$$

$$Q_0^2 = 2.4\ {\rm GeV}^2, \quad g_1^\pi = 0.082 \pm 0.022, \quad g_2^\pi = 0.394 \pm 0.103.$$

- **SIYY** parametrization:

The original CSS approach propose the so-called b_* prescription as:

$$b_* = b/\sqrt{1 + b^2/b_{\max}^2}\ ,\ b_{\max} < 1/\Lambda_{\rm QCD}, \tag{2.18}$$

where $b_{\max} \approx 1\ \mathrm{GeV}^{-1}$ and $b_* \approx b$ at small b value and $b_* \approx b_{\max}$ at large b value.

- **BDPRS** parametrization [**arXiv:1703.10157**]:

The evolution formalism for the distribution function was suggested:

$$\tilde{f}_1^a(x, b^2; Q^2) = f_1^a(x; \mu^2) e^{-S(\mu^2, Q^2)} e^{\frac{1}{2} g_K(b) ln(Q^2/Q_0^2)} \tilde{f}_{1\text{NP}}^a(x, b^2), \tag{2.19}$$

with $g_K = -g_2 b^2/2$.

$$\tilde{f}_{1\text{NP}}^a(x, b^2) = \frac{1}{2\pi} e^{-g_1 \frac{b^2}{4}} (1 - \frac{\lambda g_1^2}{1 + \lambda g_1} \frac{b^2}{4}), \tag{2.20}$$

$$g_1(x) = N_1 \frac{(1-x)^\alpha x^\sigma}{(1-\hat{x})^\alpha \hat{x}^\sigma} \tag{2.21}$$

$$\hat{x} = 0.1 \text{ GeV}^2, \quad \alpha = 2.95 \pm 0.05, \quad \sigma = 0.17 \pm 0.02,$$
$$\lambda = 0.86 \pm 0.78 \text{ GeV}^{-2}, \quad N_1 = 0.28 \pm 0.06 \text{ GeV}^2.$$

- **BDPRS** parametrization [**arXiv:1703.10157**]:

The new b_* prescription different from **SIYY** parametrization was proposed as

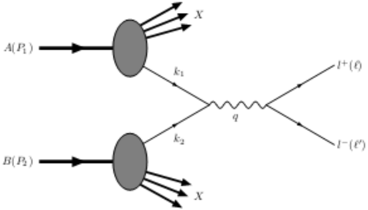
$$b_* = b_{\max}\left(\frac{1 - e^{-b^4/b_{\max}^4}}{1 - e^{-b^4/b_{\min}^4}}\right)^{1/4} \quad (2.22)$$

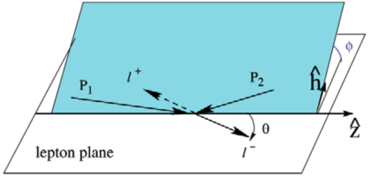
with $b_{\max} = 2e^{-\gamma_E}\ \mathrm{GeV}^{-1} \approx 1.123\ \mathrm{GeV}^{-1}$.

3. Formalism of the $\sin(2\phi - \phi_S)$ asymmetry in Drell-Yan process

- The pion-induced Drell-Yan process

$$\pi^-(P_\pi) + p^\uparrow(P_p) \longrightarrow \gamma^*(q) + X \longrightarrow l^+(\ell) + l^-(\ell') + X \tag{3.1}$$





The Collins-Soper frame

- The kinematical variables:

$$s = (P_\pi + P_p)^2, \quad x_\pi = \frac{Q^2}{2P_\pi \cdot q}, \quad x_p = \frac{Q^2}{2P_p \cdot q},$$
$$x_F = 2q_L/s = x_\pi - x_p, \quad \tau = Q^2/s = x_\pi x_p, \quad y = \frac{1}{2}\ln\frac{q^+}{q^-} = \frac{1}{2}\ln\frac{x_\pi}{x_p}. \tag{3.2}$$

- x_π and x_p can be expressed as functions of x_F, τ and of y, τ

$$x_{\pi/p} = \frac{\pm x_F + \sqrt{x_F^2 + 4\tau}}{2}, \quad x_{\pi/p} = \sqrt{\tau}e^{\pm y}. \tag{3.3}$$

- In leading twist, the differential cross section in πp Drell-Yan for a transversely polarized target has the following general form:

$$\begin{aligned}\frac{d\sigma}{d^4q d\Omega} =& \frac{\alpha_{em}^2}{Fq^2}\hat{\sigma}_U \Big\{ \Big(1 + D_{[\sin^2\theta]} A_U^{\cos 2\phi} \cos 2\phi\Big) \\ & + |\mathbf{S}_T| \Big[A_T^{\sin\phi_S} \sin\phi_S \\ & + D_{[\sin^2\theta]} \Big(A_T^{\sin(2\phi+\phi_S)} \sin(2\phi+\phi_S) \\ & + A_T^{\sin(2\phi-\phi_S)} \sin(2\phi-\phi_S)\Big)\Big]\Big\}.\end{aligned} \tag{3.4}$$

- The $\sin(2\phi - \phi_S)$ asymmetry:

$$A_T^{\sin(2\phi-\phi_S)}(x_1, x_2, Q) = \frac{F_T^{\sin(2\phi-\phi_S)}(x_1, x_2, Q)}{F_U^1(x_1, x_2, Q)}. \quad (3.5)$$

- The denominator:

$$F_U^1 = \mathcal{C}[f_{1,q/\pi} f_{1,\bar{q}/p}], \quad (3.6)$$

- The numerator $(\mathbf{h} = \hat{\mathbf{q}} \equiv \mathbf{q}_\perp / |\mathbf{q}_\perp|)$:

$$F_T^{\sin(2\phi-\phi_S)} = -\mathcal{C}[\frac{\mathbf{h} \cdot \mathbf{k}_{a\perp}}{M_\pi} h_{1,q/\pi}^\perp h_{1,\bar{q}/p}]. \quad (3.7)$$

- The convolution of TMDs in the transverse momentum space is defined through the following notation:

$$\begin{aligned}
&\mathcal{C}[\omega(\mathbf{k}_{a\perp}, \mathbf{k}_{b\perp}) f_1 \bar{f}_2] \\
&= \frac{1}{N_c} \sum_q e_q^2 \int d^2\mathbf{k}_{a\perp} d^2\mathbf{k}_{b\perp} \delta^2(\mathbf{k}_{a\perp} + \mathbf{k}_{b\perp} - \mathbf{q}_\perp) \omega(\mathbf{k}_{a\perp}, \mathbf{k}_{b\perp}) \\
&\quad \times \Big[f_1^q(x_a, \mathbf{k}_{a\perp}^2) f_2^{\bar{q}}(x_b, \mathbf{k}_{b\perp}^2) + f_1^{\bar{q}}(x_a, \mathbf{k}_{a\perp}^2) f_2^q(x_b, \mathbf{k}_{b\perp}^2) \Big]. \qquad (3.8)
\end{aligned}$$

- The Fourier transformation:

$$\delta^2(\mathbf{k}_{a\perp} + \mathbf{k}_{b\perp} - \mathbf{q}_\perp) = \frac{1}{(2\pi)^2} \int d^2\mathbf{b}_\perp e^{-i\mathbf{b}_\perp \cdot (\mathbf{k}_{a\perp} + \mathbf{k}_{b\perp} - \mathbf{q}_\perp)}. \qquad (3.9)$$

- The spin-dependent structure function $F_T^{\sin(2\phi-\phi_S)}$:

$$
\begin{aligned}
& F_T^{\sin(2\phi-\phi_S)} \\
&= -\frac{1}{N_c}\sum_q e_q^2 \int d^2\mathbf{k}_{a\perp} d^2\mathbf{k}_{b\perp} \int \frac{d^2\mathbf{b}_\perp}{(2\pi)^2} e^{-i\mathbf{b}_\perp\cdot(\mathbf{k}_{a\perp}+\mathbf{k}_{b\perp}-\mathbf{q}_\perp)} \\
& \quad \times \frac{\mathbf{h}\cdot\mathbf{k}_{a\perp}}{M_\pi} h_{1,q/\pi}^\perp(x_\pi, \mathbf{k}_{a\perp}^2) h_{1,\bar{q}/p}(x_p, \mathbf{k}_{b\perp}^2) + (q \leftrightarrow \bar{q}) \\
&= -\frac{1}{N_c}\sum_q e_q^2 \int_0^\infty \frac{db}{4\pi} b^2 J_1(q_\perp b) h_{1,q/p}(x_p, \mu_b) T^{(\sigma)}_{\bar{q}/\pi,F}(x_\pi, x_\pi, \mu_b) \\
& \quad \times e^{-\left(S_{\rm NP}^{f_{1,q/p}} + S_{\rm NP}^{f_{1,q/\pi}} + S_{\rm P}\right)} + (q \leftrightarrow \bar{q}). \qquad (3.10)
\end{aligned}
$$

- The unpolarized structure function F_U^1:

$$\begin{aligned} F_U^1 =& \frac{1}{N_c} \sum_q e_q^2 \int d^2\mathbf{k}_{a\perp} d^2\mathbf{k}_{b\perp} \int \frac{d^2\mathbf{b}_\perp}{(2\pi)^2} e^{-i(\mathbf{k}_{a\perp}+\mathbf{k}_{b\perp}-\mathbf{q}_\perp)\cdot\mathbf{b}_\perp} \\ & \times f_{1,q/\pi}(x_\pi, \mathbf{k}_{a\perp}^2) f_{1,\bar{q}/p}(x_p, \mathbf{k}_{b\perp}^2) \\ =& \frac{1}{N_c} \sum_q e_q^2 \int_0^\infty \frac{bdb}{2\pi} J_0(q_\perp b) f_{1,q/\pi}(x_\pi, \mu_b) f_{1,\bar{q}/p}(x_p, \mu_b) \\ & \times e^{-\left(S_{\mathrm{NP}}^{f_{1,q/p}} + S_{\mathrm{NP}}^{f_{1,q/\pi}} + S_{\mathrm{P}}\right)} + (q \leftrightarrow \bar{q}). \end{aligned} \tag{3.11}$$

4. Numerical calculation

- We apply two different model calculations $h_{1\pi}^{\perp}$:
  (1) The result based on **the light-cone wave function** of the pion meson from Ref. [arXiv:1702.03637].
  (2) The result from **the light-front constituent quark model** in Ref.
  [arXiv:1406.2056].
- For consistency, in each calculation of the asymmetry, we apply the unpolarized distribution function of the pion meson $f_{1\pi}(x)$ from the same model.
- The unpolarized distribution function $f_1(x)$ of the proton:
  we adopt the NLO set of the CT10 parametrization (central PDF set).
- The proton transversity distribution h_1 extracted from SIDIS data via the same TMD evolution formalism [**arXiv:1505.05589**].

- **Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equations:**

(1) We apply the QCDNUM evolution package to perform the evolution of $f_{1,q/\pi}$ from the model scale μ_0 to another energy.
(2) As for the energy evolution of the twist-3 collinear correlation function $T_{q,F}^{(\sigma)}$, we only consider the homogenous term in the evolution kernel:

$$P_{qq}^{T_{q,F}^{(\sigma)}}(x) \approx \Delta_T P_{qq}(x) - N_C \delta(1-x), \tag{4.1}$$

$$\Delta_T P_{qq}(x) = C_F \left[\frac{2z}{(1-z)_+} + \frac{3}{2}\delta(1-x) \right]. \tag{4.2}$$

(3)$\Delta_T P_{qq}$ being the evolution kernel for the transversity distribution function $h_1(x)$.

- In COMPASS Collaboration, 190 GeV π^- beam colliding on a polarized NH_3 target with the following kinematical ranges:

$$\begin{aligned} &0.05 < x_N < 0.4, \quad 0.05 < x_\pi < 0.9, \quad s = 357\,\mathrm{GeV}^2, \\ &4.3\ \mathrm{GeV} < Q < 8.5\ \mathrm{GeV}, \quad -0.3 < x_F < 1. \end{aligned} \tag{4.3}$$

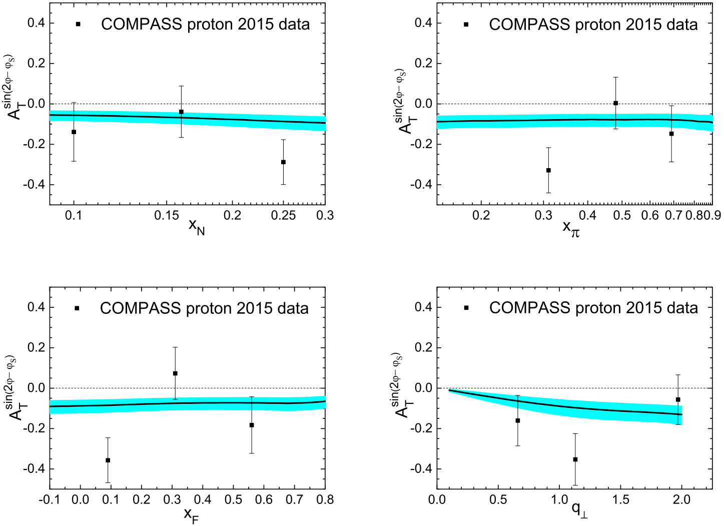


FIG.1. The $\sin(2\phi - \phi_S)$ azimuthal asymmetry in the $\pi^- N^\uparrow$ Drell-Yan process calculated from the **SIYY** parametrization on the nonperturbative form factor and the pion Boer-Mulders function from **light-cone model**.

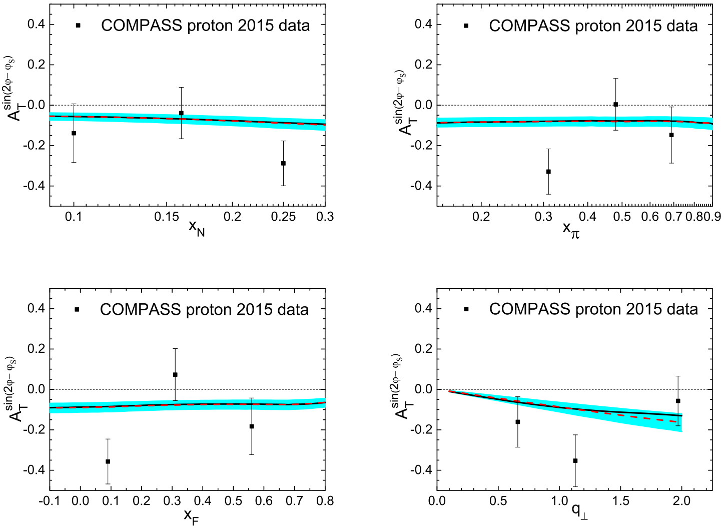


FIG.2. Similar to FIG.1, but the asymmetry calculated from the **BDPRS** parametrization on the nonperturbative form factor.

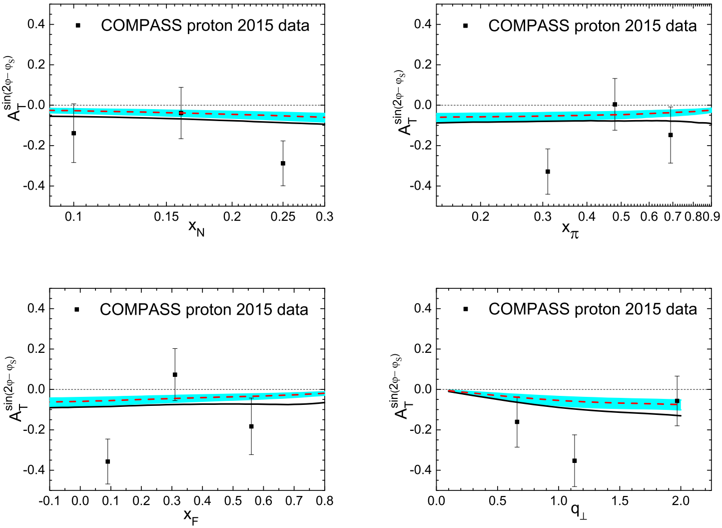


FIG.3. Similar to FIG.1, but the asymmetry calculated from the Boer-Mulders function of the pion in **the light-front constituent model**.

5. Conclusion

- The $A_T^{\sin(2\phi-\phi_S)}$ is sizable at the kinematics of COMPASS and is qualitatively consistent with the COMPASS measurement.

- The asymmetry is sensitive to the choice of the pion distribution function, while different choice on the nonperturbative part of the TMD evolution formalism will only affect the shape of the $q_\perp$-dependent of the asymmetry.

- Our study may provide a framework to access the Boer-Mulders function of the pion and the corresponding nonperturbative Sudakov form factor through transversely polarized πp data.

THANK YOU