A theoretical analysis of the semileptonic decays $\eta^{(\prime)} \to \pi^0 \ell^+ \ell^-$ and $\eta' \to \eta \ell^+ \ell^-$

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Emilio Royo (eroyo@ifae.es)
UAB & IFAE-BIST

Ongoing collaboration with Rafel Escribano & Pablo Sánchez-Puertas

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Universitat Autònoma de Barcelona

EXCELENCIA SEVERO OCHOA

BIST Barcelona Institute of Science and Technology

Outline

- Motivation
- SM Calculations
- SM Results
- BSM – *CP* Violation

Motivation

- Standard Model of particle physics *painfully* successful

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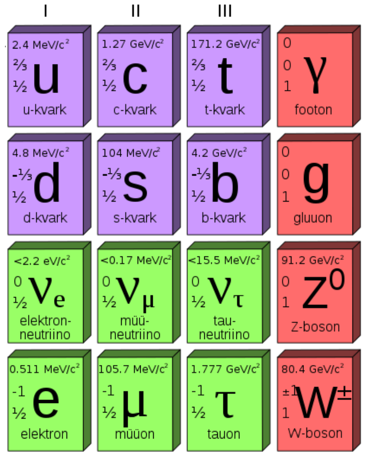

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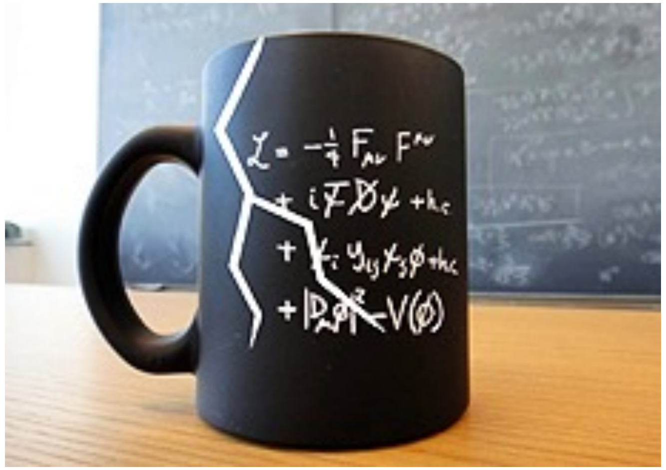

Motivation

- A number of low-energy precision measurements are sensitive to BSM Physics*
  - SM prediction for the measured quantity is precisely known
  - SM background is small
- Experiments currently underway
  - Muon g – 2
  - EDMs
  - Neutron decays
  - Etc.

* *PoS* ConfinementX (2012) 023

Motivation

- The η and η' mesons are special*:
  - The η is a pseudo-Goldstone boson
  - The η' is largely influenced by the $U(1)_A$ anomaly
  - The η and η' are eigenstates of the *C*, *P*, *CP* and *G* operators: $I^G J^{PC} = 0^+0^{-+}$
  - All their additive quantum numbers are zero: flavour conserving decays
  - All their strong and EM decays are forbidden at lowest order
  - Decays are mostly free from SM background

➡ **Perfect laboratory to stress-test the SM in search for physics BSM**

* https://redtop.fnal.gov/the-physics/

Motivation

- The semileptonic decays $\eta^{(\prime)} \to \pi^0 \ell^+ \ell^-$ and $\eta' \to \eta \ell^+ \ell^-$ (ℓ = e or μ) can be used as fine probes to assess physics BSM.
  - SM contributes through the *C*-conserving exchange of two photons that is highly suppressed (no contribution at tree-level, only corrections at one-loop and higher orders)
- Latest theoretical estimations for $\eta \to \pi^0 \ell^+ \ell^-$ date back to the 90s
- No theoretical studies for $\eta' \to \pi^0 \ell^+ \ell^-$ or $\eta' \to \eta \ell^+ \ell^-$ to the best of our knowledge

Calculations

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- Analysis of the *C*-conserving semileptonic decays $\eta^{(\prime)} \to \pi^0 \ell^+ \ell^-$ and $\eta' \to \eta \ell^+ \ell^-$
  - Assess accurately SM background
- Calculations performed within the Vector Meson Dominance (VMD) framework
  - Decay processes dominated by the exchange of vector resonances
- VMD coupling constants parametrised using an existing phenomenological model
  - Numerical values obtained from an optimisation fit to $V \to P\gamma$ and $P \to V\gamma$ radiative decays ($V = \rho^0, \omega, \phi$ and $P = \pi^0, \eta, \eta'$)
  - See *Phys. Lett. B* 807 (2020) 135534, arXiv:2003.08379 for details

Calculations

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- Contributing Feynman diagrams

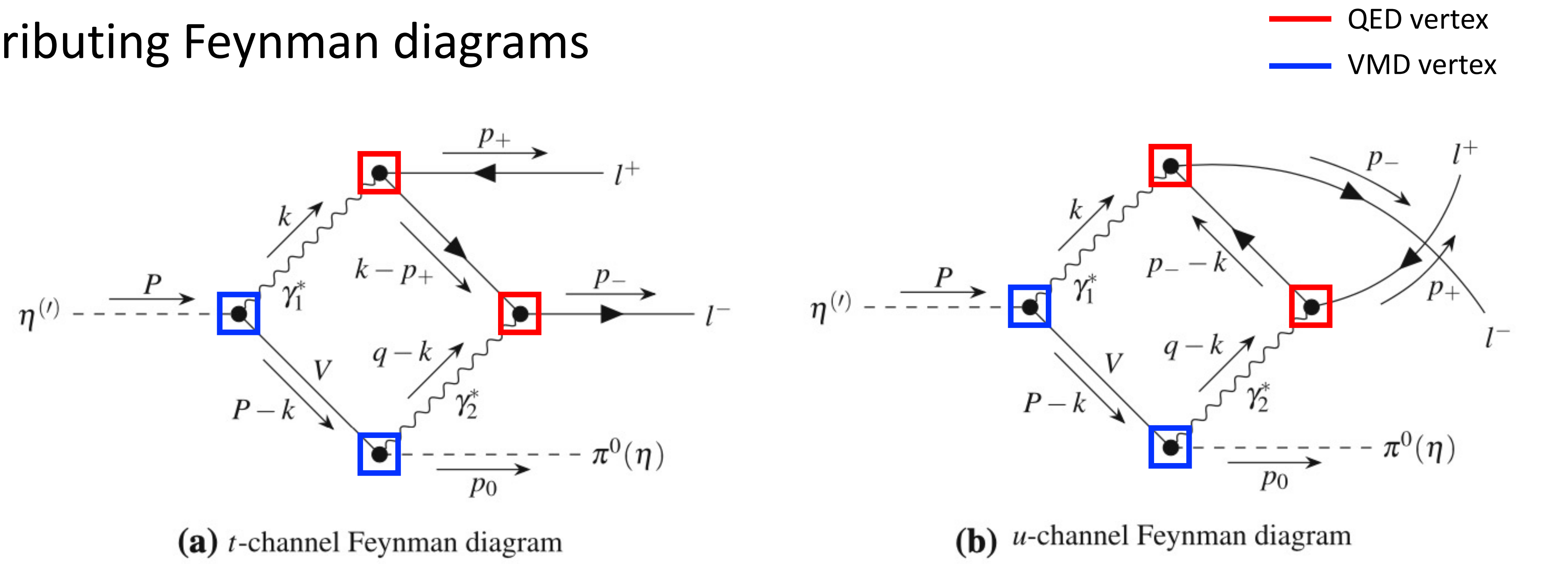

Fig. 1 Feynman diagrams contributing to the C-conserving semileptonic decays $\eta^{(\prime)} \to \pi^0 l^+ l^-$ and $\eta' \to \eta l^+ l^-$ ($l = e$ or μ). Note that $q = p_+ + p_-$ and $V = \rho^0, \omega, \phi$

Calculations

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- *VP*γ interaction amplitude consistent with Lorentz, *P*, *C* and EM gauge invariance can be written as

$$\mathcal{M}(V \to P\gamma) = g_{VP\gamma}\, \epsilon_{\mu\nu\alpha\beta} \epsilon^{\mu}_{(V)} p^{\nu}_{V} \epsilon^{*\alpha}_{(\gamma)} q^{\beta} \boxed{\hat{F}_{VP\gamma}(q^2)}$$

Normalised form factor to account for off-shell photons mediating the transition

Calculations

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- Unpolarised squared amplitude

$$\overline{|\mathscr{M}|^2} = \overbrace{4\left\{2(P\cdot p_+)(P\cdot p_-) - m^2_{\eta^{(\prime)}}\left[(p_+\cdot p_-)+m_l^2\right]\right\}\times \mathrm{Abs}(\Omega)^2}^{\text{Vector current}} + \overbrace{8m_l^2\left[(P\cdot p_+)-(P\cdot p_-)\right]\mathrm{Re}(\Omega\Sigma^*)}^{\text{Interference}}$$
$$+\underbrace{4m_l^2\left[(p_+\cdot p_-)-m_l^2\right]\mathrm{Abs}(\Sigma)^2}_{\text{Scalar current}}$$

Calculations

Eur. Phys. J. C (2020) 80: 1190, [arXiv:2007.12467]

with

$$\Omega = \sum_{V=\rho^0,\omega,\phi} \alpha_V + \sigma_V \,,$$

$$\Sigma = \sum_{V=\rho^0,\omega,\phi} \beta_V + \tau_V \,,$$

$$\alpha_V = e^2 \frac{g_{V\eta^{(\prime)}\gamma}\, g_{V\pi^0(\eta)\gamma}}{16\pi^2} \int dxdydz \left[\frac{2A_1}{\Delta_{1V}+i\epsilon} - \frac{B_1}{(\Delta_{1V}+i\epsilon)^2} \right],$$

$$\beta_V = e^2 \frac{g_{V\eta^{(\prime)}\gamma}\, g_{V\pi^0(\eta)\gamma}}{16\pi^2} \int dxdydz \left[\frac{2C_1}{\Delta_{1V}+i\epsilon} - \frac{D_1}{(\Delta_{1V}+i\epsilon)^2} \right],$$

$$\sigma_V = e^2 \frac{g_{V\eta^{(\prime)}\gamma}\, g_{V\pi^0(\eta)\gamma}}{16\pi^2} \int dxdydz \left[\frac{2A_2}{\Delta_{2V}+i\epsilon} - \frac{B_2}{(\Delta_{2V}+i\epsilon)^2} \right],$$

$$\tau_V = e^2 \frac{g_{V\eta^{(\prime)}\gamma}\, g_{V\pi^0(\eta)\gamma}}{16\pi^2} \int dxdydz \left[\frac{2C_2}{\Delta_{2V}+i\epsilon} - \frac{D_2}{(\Delta_{2V}+i\epsilon)^2} \right],$$

Calculations

Phys. Lett. B 807 (2020) 135534, arXiv:2003.08379

- VMD couplings parametrisation

$$g_{\rho^0\pi^0\gamma} = \frac{1}{3} g \,,$$

$$g_{\rho^0\eta\gamma} = g z_{\rm NS} \cos\phi_P \,,$$

$$g_{\rho^0\eta'\gamma} = g z_{\rm NS} \sin\phi_P \,,$$

$$g_{\omega\pi^0\gamma} = g \cos\phi_V \,,$$

$$g_{\omega\eta\gamma} = \frac{1}{3} g \left(z_{\rm NS} \cos\phi_P \cos\phi_V - 2\frac{\overline{m}}{m_s} z_{\rm S} \sin\phi_P \sin\phi_V \right) ,$$

$$g_{\omega\eta'\gamma} = \frac{1}{3} g \left(z_{\rm NS} \sin\phi_P \cos\phi_V + 2\frac{\overline{m}}{m_s} z_{\rm S} \cos\phi_P \sin\phi_V \right) ,$$

$$g_{\phi\pi^0\gamma} = g \sin\phi_V \,,$$

$$g_{\phi\eta\gamma} = \frac{1}{3} g \left(z_{\rm NS} \cos\phi_P \sin\phi_V + 2\frac{\overline{m}}{m_s} z_{\rm S} \sin\phi_P \cos\phi_V \right) ,$$

$$g_{\phi\eta'\gamma} = \frac{1}{3} g \left(z_{\rm NS} \sin\phi_P \sin\phi_V - 2\frac{\overline{m}}{m_s} z_{\rm S} \cos\phi_P \cos\phi_V \right) ,$$

Calculations

Phys. Lett. B 807 (2020) 135534, arXiv:2003.08379

- Numerical values from optimisation fit to $VP\gamma$ radiative decays

$$
\begin{aligned}
&g = 0.70 \pm 0.01\ \mathrm{GeV}^{-1}, \quad z_{\mathrm{S}}\overline{m}/m_s = 0.65 \pm 0.01\ , \\
&\phi_P = (41.4 \pm 0.5)^\circ, \quad \phi_V = (3.3 \pm 0.1)^\circ\ , \\
&z_{\mathrm{NS}} = 0.83 \pm 0.02\ .
\end{aligned}
$$

Results

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- Decay widths and branching ratios

Table 1 Decay widths and branching ratios for the six C-conserving decays $\eta^{(\prime)} \to \pi^0 l^+ l^-$ and $\eta' \to \eta l^+ l^-$ ($l = e$ or μ). First error is experimental, second is down to numerical integration and third is due to model dependency

Decay	Γ_{th}	BR_{th}	BR_{exp}	
$\eta \to \pi^0 e^+ e^-$	$2.8(2)(3)(5) \times 10^{-6}$ eV	$2.1(1)(2)(4) \times 10^{-9}$	$< 7.5 \times 10^{-6}$ (CL=90%)	WASA-at-COSY
$\eta \to \pi^0 \mu^+ \mu^-$	$1.6(1)(2)(2) \times 10^{-6}$ eV	$1.2(1)(1)(2) \times 10^{-9}$	$< 5 \times 10^{-6}$ (CL=90%)	PDG
$\eta' \to \pi^0 e^+ e^-$	$8.7(0.5)(0.9)(1.0) \times 10^{-4}$ eV	$4.5(3)(5)(6) \times 10^{-9}$	$< 1.4 \times 10^{-3}$ (CL=90%)	PDG
$\eta' \to \pi^0 \mu^+ \mu^-$	$3.5(2)(4)(5) \times 10^{-4}$ eV	$1.8(1)(2)(3) \times 10^{-9}$	$< 6.0 \times 10^{-5}$ (CL=90%)	PDG
$\eta' \to \eta^0 e^+ e^-$	$7.6(0.4)(0.8)(1.3) \times 10^{-5}$ eV	$3.9(3)(4)(7) \times 10^{-10}$	$< 2.4 \times 10^{-3}$ (CL=90%)	PDG
$\eta' \to \eta^0 \mu^+ \mu^-$	$3.1(2)(3)(2) \times 10^{-5}$ eV	$1.6(1)(2)(1) \times 10^{-10}$	$< 1.5 \times 10^{-5}$ (CL=90%)	PDG

Results

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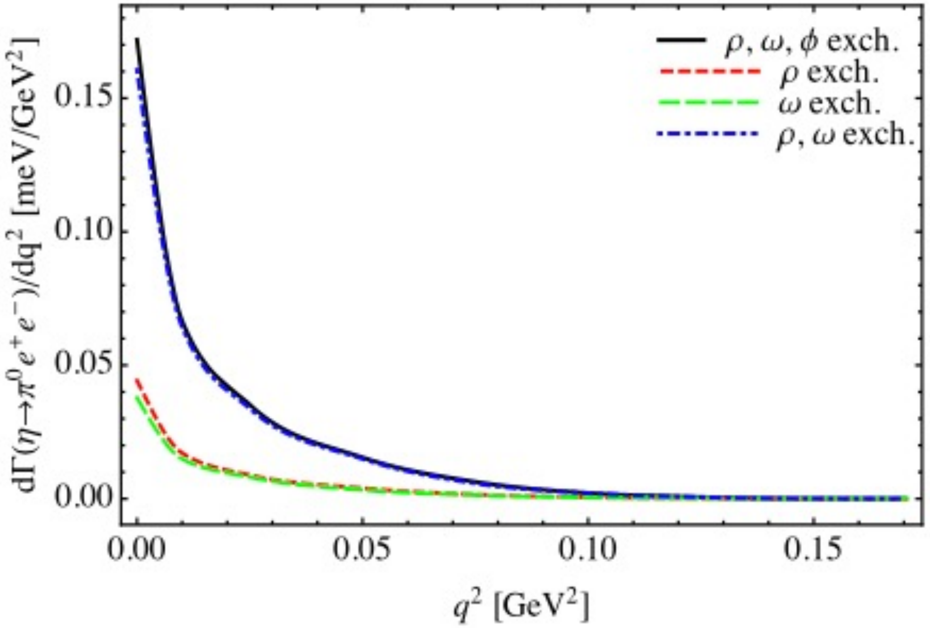
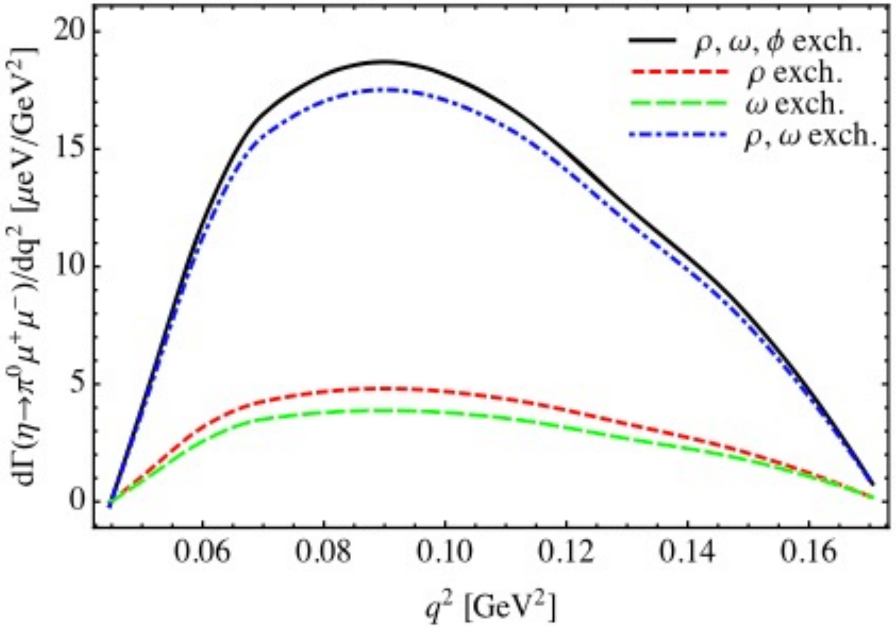
- $\eta \to \pi^0 \ell^+ \ell^-$ dilepton spectrum



(a) $\eta \to \pi^0 e^+ e^-$



(b) $\eta \to \pi^0 \mu^+ \mu^-$

Results

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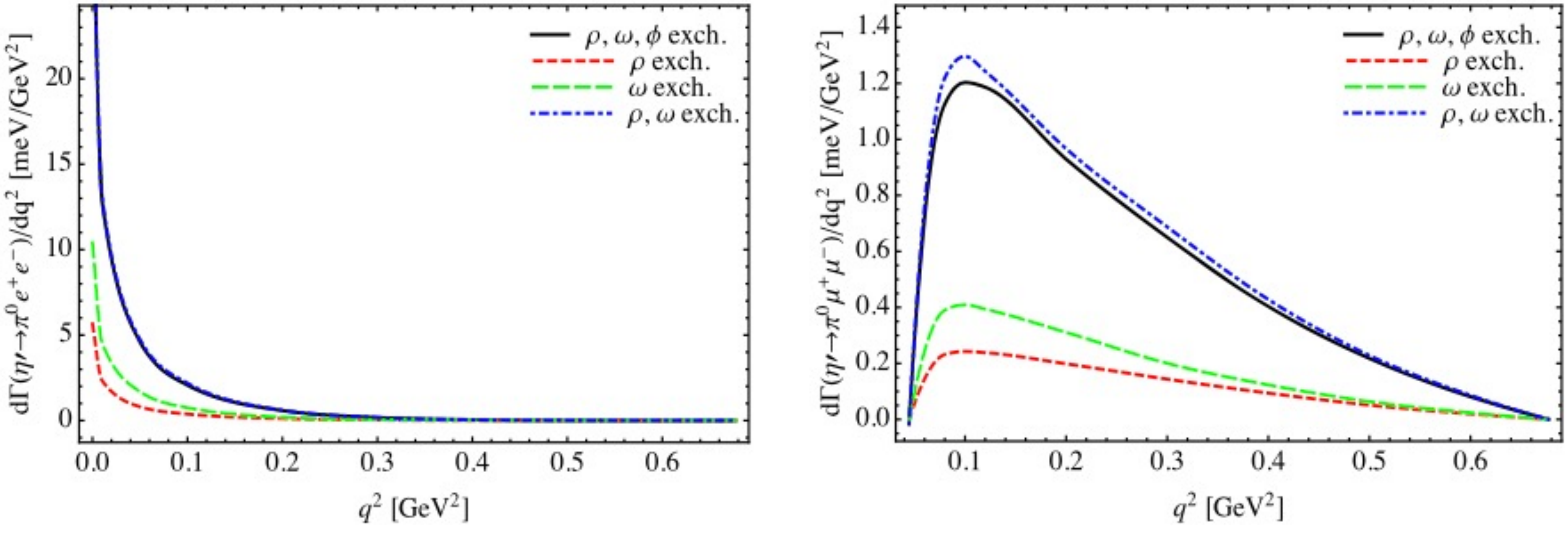
- $\eta' \to \pi^0 \ell^+ \ell^-$ dilepton spectrum



(c) $\eta' \to \pi^0 e^+ e^-$ **(d)** $\eta' \to \pi^0 \mu^+ \mu^-$

Results

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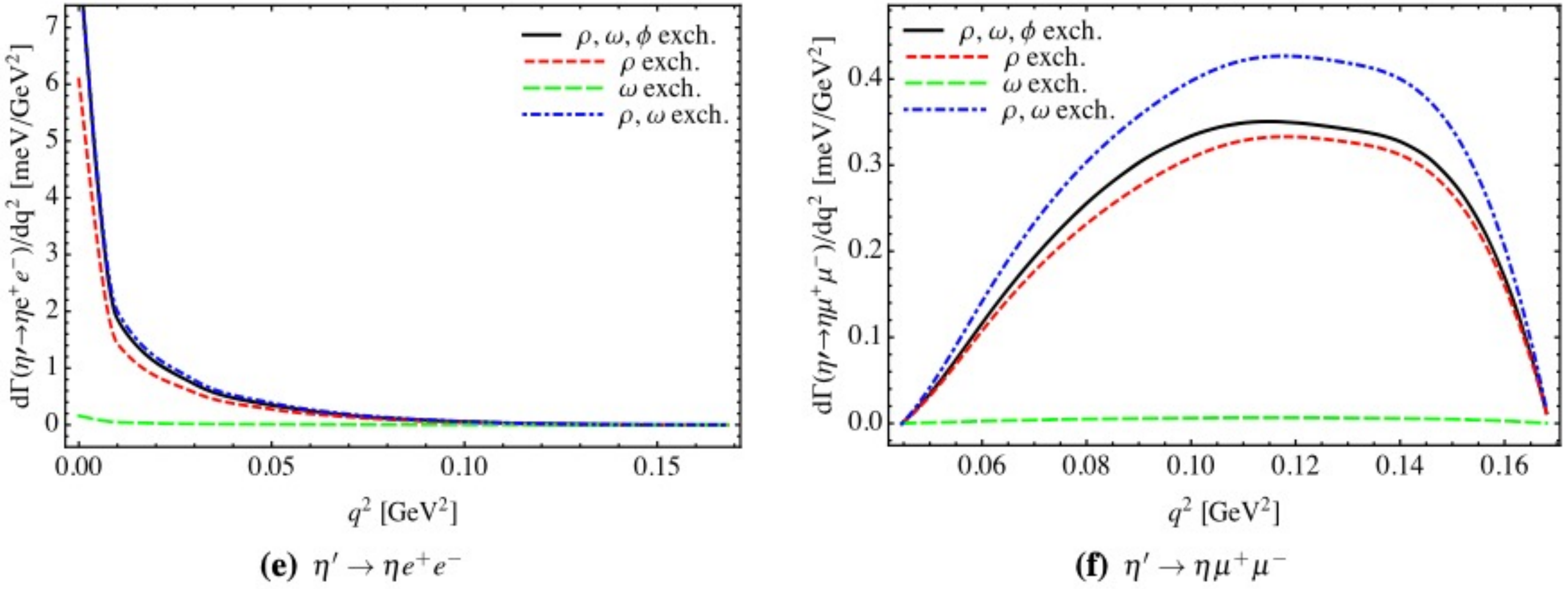
- $\eta' \to \eta \ell^+ \ell^-$ dilepton spectrum



Results

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- REDTOP is a new Fermilab project that belongs to the high intensity class of experiments
  - It aims at detecting small variations from the SM by looking at a large number of events produced with very intense beams
- 1.8 GeV continuous proton beam impinging on a target made with 10 foils of beryllium to produce about $2.5 \times 10^{13}\,\frac{\eta}{year}$ and $2.5 \times 10^{11}\,\frac{\eta'}{year}$
- More information about REDTOP can be found in https://redtop.fnal.gov
- REDTOP may be able measure these BRs with significantly improved accuracy!

CP violation

- Joined forces with Pablo Sánchez-Puertas for this study.
- Use results from previous section to fix SM background
- Some of Pablo's previous work on *CP* violation:
  - JHEP 01, 031 (2019), arXiv:1810.13228
  - arXiv:1909.07491

CP violation

- The SMEFT assumes that new physics states are heavy
- The SMEFT is a consistent EFT generalization of the SM constructed out of a series of $SU_c(3) \times SU_L(2) \times U_Y(1)$ invariant higher dimensional operators, built out of SM fields*

$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM} + \underbrace{\sum_i \frac{c_i}{\Lambda} \mathcal{O}_i^{d=5} + \boxed{\sum_i \frac{c_i}{\Lambda^2} \mathcal{O}_i^{d=6}} + \mathcal{O}(\Lambda^{-3})}_{\text{BSM effects}}$$

- Relevant operators for our study

$$\mathcal{O}_{\ell edq}^{prst} = (\bar{\ell}_p^i e_r)(\bar{d}_s q_t^i), \qquad \mathcal{O}_{\ell equ}^{(1)prst} = (\bar{\ell}_p^i e_r)(\bar{q}_s^j u_t)\epsilon_{ij}$$

* Phys. Rept. 793 (2019) 1-98, arXiv:1706.08945

CP violation

- The most general form factor decomposition for the $\eta^{(\prime)} \to \pi^0(\eta)\mu^+\mu^-$ processes

$$\langle \ell^+ \ell^- | \eta \pi^0 \rangle = i\mathcal{M}(2\pi)^4 \delta(p_{\ell^+} + p_{\ell^-} - p_\eta - p_\pi)$$

is

$$\mathcal{M} = m_\ell(\bar{u}v)F_1 + (\bar{u}i\gamma^5 v)F_2 + (\bar{u}\not{k}v)F_3 + i(\bar{u}\not{k}\gamma^5 v)F_4,$$

where

$$k = p_\eta + p_\pi$$

CP violation

- Noting that
  - Within the SM, only $F_{1,3}$ terms contribute if one neglects electroweak effects
  - Possible effects to F_4 within the SM arise via electroweak loops
  - Within the SMEFT, the Fermi operators involving vector currents are irrelevant

one arrives at

$$F_1 = \Sigma \qquad \text{(cf. slide 15)}$$

$$F_2 = \left[\mathrm{Im}\, c_{\ell edq}^{2211} \langle 0| \bar{d}d |\eta\pi^0\rangle + \mathrm{Im}\, c_{\ell edq}^{2222} \langle 0| \bar{s}s |\eta\pi^0\rangle - \mathrm{Im}\, c_{\ell equ}^{(1)2211} \langle 0| \bar{u}u |\eta\pi^0\rangle\right] v^{-2}$$

$$F_3 = \tfrac{1}{2}\Omega \qquad \text{(cf. slide 15)}$$

$$F_4 = 0$$

CP violation

- Strong upper bounds for the Wilson coefficients come from nEDMs and charm decays, such as $D_s^- \to \mu^- \bar{\nu}_\mu$

$$Im\ c_{lequ}^{(1)2211} < 0.001$$

$$Im\ c_{ledq}^{2211} < 0.002$$

$$Im\ c_{ledq}^{2222} < 0.02$$

CP violation

- At LO in $LN_c\ \chi$PT, the above matrix elements can be expressed as

$$\langle \pi^0 | \bar{u}u | \eta \rangle = B_0 (c\phi_{23} + \epsilon_{13} s\phi_{23}) \,, \qquad \langle \pi^0 | \bar{u}u | \eta' \rangle = -B_0 (\epsilon_{13} c\phi_{23} - s\phi_{23})$$

$$\langle \pi^0 | \bar{d}d | \eta \rangle = -B_0 (c\phi_{23} - \epsilon_{13} s\phi_{23}) \,, \qquad \langle \pi^0 | \bar{d}d | \eta' \rangle = -B_0 (\epsilon_{13} c\phi_{23} + s\phi_{23})$$

$$\langle \pi^0 | \bar{s}s | \eta \rangle = -2B_0 \epsilon_{13} s\phi_{23} \,, \qquad \langle \pi^0 | \bar{s}s | \eta' \rangle = 2B_0 \epsilon_{13} c\phi_{23}$$

$$\langle \eta | \bar{u}u | \eta' \rangle = -B_0 [\epsilon_{13} (c\phi_{23}^2 - s\phi_{23}^2) - (1 - 2\epsilon_{12}) c\phi_{23} s\phi_{23}]$$

$$\langle \eta | \bar{d}d | \eta' \rangle = B_0 [\epsilon_{13} (c\phi_{23}^2 - s\phi_{23}^2) + (1 + 2\epsilon_{12}) c\phi_{23} s\phi_{23}]$$

$$\langle \eta | \bar{s}s | \eta' \rangle = -2B_0 c\phi_{23} s\phi_{23}$$

CP violation

- Differential decays widths for $\eta^{(\prime)} \to \pi^0(\eta)\mu^+\mu^- \to \pi^0(\eta)e^+\nu_e\bar{\nu}_\mu e^-\bar{\nu}_e\nu_\mu$

$$d\Gamma = \sum_{\lambda\bar{\lambda}} \underbrace{\frac{ds\,dc_\theta}{64(2\pi)^3 m_\eta}\frac{\lambda_K^{1/2}\beta_\mu}{m_\eta^2}|\mathcal{M}(\lambda\boldsymbol{n},\bar{\lambda}\bar{\boldsymbol{n}})|^2}_{\eta^{(\prime)} \to \pi^0(\eta)\mu^+\mu^-} \underbrace{\left[\frac{d\Omega}{4\pi}dx\; n(x)\left(1-\lambda b(x)\boldsymbol{\beta}\cdot\boldsymbol{n}\right)\right]}_{\mu^+ \to e^+\nu_e\bar{\nu}_\mu} \underbrace{\left[\frac{d\bar{\Omega}}{4\pi}d\bar{x}\; n(\bar{x})\left(1+\bar{\lambda}b(\bar{x})\bar{\boldsymbol{\beta}}\cdot\bar{\boldsymbol{n}}\right)\right]}_{\mu^- \to e^-\bar{\nu}_e\nu_\mu}$$

$$d\Gamma = \frac{ds\,dc_\theta}{64(2\pi)^3 m_\eta}\frac{\lambda_K^{1/2}\beta_\mu}{m_\eta^2}\left[\frac{d\Omega}{4\pi}dx\; n(x)\right]\left[\frac{d\bar{\Omega}}{4\pi}d\bar{x}\; n(\bar{x})\right]\Big[\tilde{c}_1|F_1|^2+\tilde{c}_3|F_3|^2+\tilde{c}_{13}^R\,\mathrm{Re}\,F_1F_3^*+\tilde{c}_{13}^I\,\mathrm{Im}\,F_1F_3^*$$
$$+\tilde{c}_2|F_2|^2+\tilde{c}_{12}^R\,\mathrm{Re}\,F_1F_2^*+\tilde{c}_{12}^I\,\mathrm{Im}\,F_1F_2^*+\tilde{c}_{23}^R\,\mathrm{Re}\,F_2F_3^*+\tilde{c}_{23}^I\,\mathrm{Im}\,F_2F_3^*\Big],$$

CP violation

- Longitudinal and transverse asymmetries:

$$A_L = \frac{N(c_{\theta_{e^+}} > 0) - N(c_{\theta_{e^+}} < 0)}{N} = -\frac{2}{3}\frac{\int ds dc_\theta \lambda_K^{1/2} \beta_\mu m_\mu \left[\beta_\mu s \,\mathrm{Im}\, F_1 F_2^* + 2\lambda_K^{1/2} c_\theta \,\mathrm{Im}\, F_3 F_2^*\right]}{64(2\pi)^3 m_\eta^3 \int d\Gamma}$$

$$A_T = \frac{N(s_{\bar{\phi}-\phi} > 0) - N(s_{\bar{\phi}-\phi} < 0)}{N} = \frac{\pi}{18}\frac{\int ds dc_\theta \lambda_K^{1/2} \beta_\mu m_\mu \left[\beta_\mu s \,\mathrm{Re}\, F_1 F_2^* + 2\lambda_K^{1/2} c_\theta \,\mathrm{Re}\, F_3 F_2^*\right]}{64(2\pi)^3 m_\eta^3 \int d\Gamma}$$

CP violation

- **<u>Preliminary</u>** results @LO in χPT for $\eta \to \pi^0\mu^+\mu^-$

$$A_L = -0.179(9)\ \overbrace{Im\ c_{lequ}^{(1)2211}}^{<\,0.001} - 0.176(10)\ \overbrace{Im\ c_{ledq}^{2211}}^{<\,0.002} - 7.92(32)\cdot 10^{-3}\ \overbrace{Im\ c_{ledq}^{2222}}^{<\,0.02} < -6.9(4)\cdot 10^{-4}$$

$$A_T = 0.0800(34)\ Im\ c_{lequ}^{(1)2211} + 0.0786(26)\ Im\ c_{ledq}^{2211} + 3.52(15)\cdot 10^{-3}\ Im\ c_{ledq}^{2222} < 3.0(1)\cdot 10^{-4}$$

CP violation

- **<u>Preliminary</u>** results @LO in χPT for $\eta' \to \pi^0\mu^+\mu^-$

$$A_L = -0.0757(23)\,\overbrace{Im\, c_{lequ}^{(1)2211}}^{<\,0.001} - 0.0798(23)\,\overbrace{Im\, c_{ledq}^{2211}}^{<\,0.002} + 4.38(18)\cdot 10^{-3}\,\overbrace{Im\, c_{ledq}^{2222}}^{<\,0.02} < -1.5(1)\cdot 10^{-4}$$

$$A_T = 6.39(47)\cdot 10^{-3}\,Im\, c_{lequ}^{(1)2211} + 6.84(36)\cdot 10^{-3}\,Im\, c_{ledq}^{2211} - 3.76(17)\cdot 10^{-4}\,Im\, c_{ledq}^{2222} < 1.3(1)\cdot 10^{-5}$$

CP violation

- **<u>Preliminary</u>** results @LO in χPT for $\eta' \to \eta\mu^+\mu^-$

$$A_L = -0.0375(9)\ \overbrace{Im\ c_{lequ}^{(1)2211}}^{<\,0.001} + 0.0422(12)\ \overbrace{Im\ c_{ledq}^{2211}}^{<\,0.002} - 0.0794(20)\ \overbrace{Im\ c_{ledq}^{2222}}^{<\,0.02} < -1.55(4)\cdot 10^{-3}$$

$$A_T = 6.47(1.34)\cdot 10^{-4}\ Im\ c_{lequ}^{(1)2211} - 6.25(1.91)\cdot 10^{-4}\ Im\ c_{ledq}^{2211} + 1.21(9)\cdot 10^{-3}\ Im\ c_{ledq}^{2222} < 2.3(6)\cdot 10^{-5}$$

CP violation

- REDTOP can perform muon polarimetry
- The expected asymmetry noise, for example, for the $\eta \to \pi^0\mu^+\mu^-$ is

$$\frac{1}{\sqrt{N}} = \frac{1}{\sqrt{2.5\times10^{13} \cdot 1.2\times10^{-9}}} \approx 6\times10^{-3}$$

- On the other hand, the asymmetries calculated at LO are of the order $\sim 1 \times 10^{-3}$
- Thus, it appears that the statistics of REDTOP may fall short
- Results using matrix elements at NLO with q^2 dependence still pending

<u>STAY TUNNED</u>

Conclusions

- The study of the η and η' phenomenology can provide a very interesting way to find physics BSM
- Theoretical predictions have been presented for the BRs of the six $\eta^{(\prime)} \to \pi^0 \ell^+ \ell^-$ and $\eta' \to \eta \ell^+ \ell^-$ semileptonic processes
- Theoretical estimations for the longitudinal and transverse asymmetries of the three $\eta^{(\prime)} \to \pi^0(\eta)\mu^+\mu^-$ processes have been presented, which will enable the assessment of *CP* violating effects from physics BSM in these processes. Work ongoing.