A brief review of the anomalous magnetic moment of the muon (theory)

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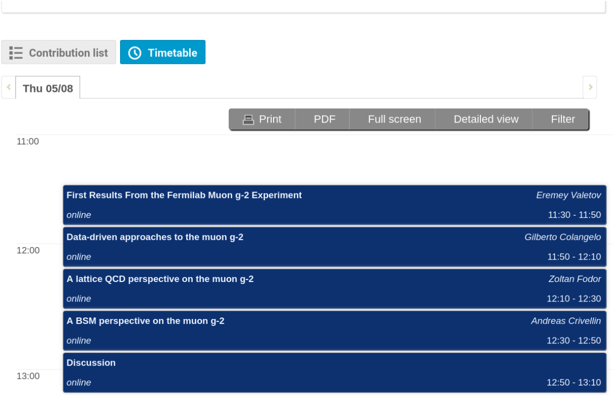
A Virtual Tribute to Quark Confinement
and the Hadron Spectrum 2021 (2-6/August)

Apetizers for next plenary panel session



Basic reference: arXiv 2006:04822. White paper from "g-2 Theory Initiative" https://muon-gm2-theory.illinois.edu/

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The anomalous magnetic moment of the muon in the Standard Model

T. Aoyama [1,2,3], N. Asmussen [4], M. Benayoun [5], J. Bijnens [6], T. Blum [7,8], M. Bruno [9], I. Caprini [10], C.M. Carloni Calame [11], M. Cè [9,12,13], G. Colangelo [14,*], F. Curciarello [15,16], H. Czyż [17], I. Danilkin [12], M. Davier [18,*], C.T.H. Davies [19], M. Della Morte [20], S.I. Eidelman [21,22,*], A.X. El-Khadra [23,24,*], A. Gérardin [25], D. Giusti [26,27], M. Golterman [28], Steven Gottlieb [29], V. Gülpers [30], F. Hagelstein [14], M. Hayakawa [31,2], G. Herdoíza [32], D.W. Hertzog [33], A. Hoecker [34], M. Hoferichter [14,35,*], B.-L. Hoid [36], R.J. Hudspith [12,13], F. Ignatov [21], T. Izubuchi [37,8], F. Jegerlehner [38], L. Jin [7,8], A. Keshavarzi [39], T. Kinoshita [40,41], B. Kubis [36], A. Kupich [21], A. Kupść [42,43], L. Laub [14], C. Lehner [26,37,*], L. Lellouch [25], I. Logashenko [21], B. Malaescu [5], K. Maltman [44,45], M.K. Marinković [46,47], P. Masjuan [48,49], A.S. Meyer [37], H.B. Meyer [12,13], T. Mibe [1,*], K. Miura [12,13,3], S.E. Müller [50], M. Nio [2,51], D. Nomura [52,53], A. Nyffeler [12,*], V. Pascalutsa [12], M. Passera [54], E. Perez del Rio [55], S. Peris [48,49], A. Portelli [30], M. Procura [56], C.F. Redmer [12], B.L. Roberts [57,*], P. Sánchez-Puertas [49], S. Serednyakov [21], B. Shwartz [21], S. Simula [27], D. Stöckinger [58], H. Stöckinger-Kim [58], P. Stoffer [59], T. Teubner [60,*], R. Van de Water [24], M. Vanderhaeghen [12,13], G. Venanzoni [61], G. von Hippel [12], H. Wittig [12,13], Z. Zhang [18], M.N. Achasov [21], A. Bashir [62], N. Cardoso [47], B. Chakraborty [63], E.-H. Chao [12], J. Charles [25], A. Crivellin [64,65], O. Deineka [12], A. Denig [12,13], C. DeTar [66], C.A. Dominguez [67], A.E. Dorokhov [68], V.P. Druzhinin [21], G. Eichmann [69,47], M. Fael [70], C.S. Fischer [71], E. Gámiz [72], Z. Gelzer [23], J.R. Green [9], S. Guellati-Khelifa [73], D. Hatton [19], N. Hermansson-Truedsson [14], S. Holz [36], B. Hörz [74], M. Knecht [25], J. Koponen [1], A.S. Kronfeld [24], J. Laiho [75], S. Leupold [42], P.B. Mackenzie [24], W.J. Marciano [37], C. McNeile [76], D. Mohler [12,13], J. Monnard [14], E.T. Neil [77], A.V. Nesterenko [68], K. Ottnad [12], V. Pauk [12], A.E. Radzhabov [78], E. de Rafael [25], K. Raya [79], A. Risch [12], A. Rodríguez-Sánchez [6], P. Roig [80], T. San José [12,13], E.P. Solodov [21], R. Sugar [81], K. Yu. Todyshev [21], A. Vainshtein [82], A. Vaquero Avilés-Casco [66], E. Weil [71], J. Wilhelm [12], R. Williams [71], A.S. Zhevlakov [78]

What is g-2 ?

An orbiting particle of mass m and electric charge e has a magnetic dipole moment

$$\vec{\mu} = \frac{e}{2m}\vec{L} = \mu_B \vec{L} \qquad \text{(if the particle is an electron)}$$

and interacts with a magnetic filed through a term

$$H = -\vec{\mu} \cdot \vec{B}$$

The spin produces an intrinsic magnetic moment

$$\vec{\mu} = g \frac{e}{2m}\vec{S}$$

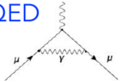
g is the gyromagnetic ratio. The deviation from the tree-level value 2

$$a_\ell = \frac{g_\ell - 2}{2} \, , \qquad \ell = \text{lepton index}$$

is the anomalous magnetic moment.

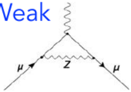
$$a_\mu = a_\mu(QED) + a_\mu(Weak) + a_\mu(Hadronic)$$

QED



$+\ldots$ $\qquad 116\,584\,718.9\,(1) \times 10^{-11}$ $\qquad$ 0.001 ppm

Weak

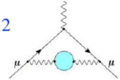


$+\ldots$ $\qquad 153.6\,(1.0) \times 10^{-11}$ $\qquad$ 0.01 ppm

Hadronic…

…Vacuum Polarization (HVP) $\qquad 6845\,(40) \times 10^{-11}$ [0.6%] $\qquad$ 0.37 ppm

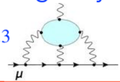
α^2



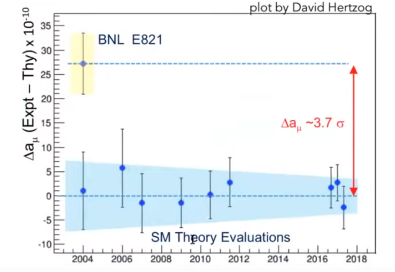
$+\ldots$

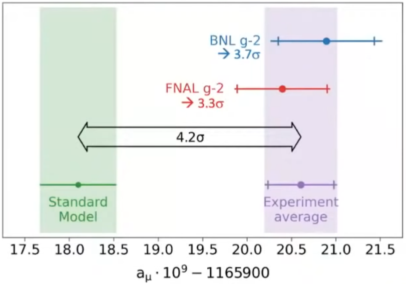
…Light-by-Light (HLbL) $\qquad 92\,(18) \times 10^{-11}$ [20%] $\qquad$ 0.15 ppm

α^3



$+\ldots$



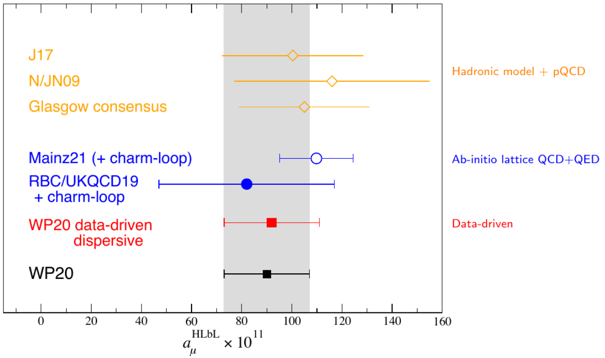


Theo-Exp tension $\simeq 250 \cdot 10^{-11}$

- size of EW contribution (subpercent error)
- 2-3 times light-by-light contribution
- 3% of HVP contribution

Let us have a closer look at estimates of hadronic contributions

Hadronic Light-by-Light



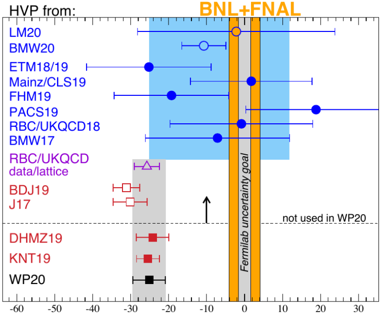
It seems under control

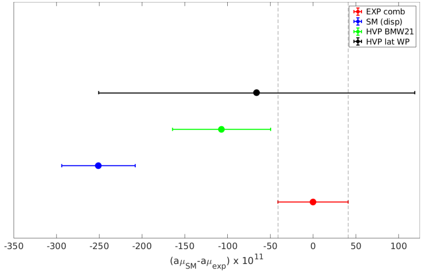
Dispersive, data driven, systematically improvable approach: progress in assessing uncertainties [G. Colangelo et al., JHEP09(2014)091]
Lattice: [Mainz, arXiv:2104.02632], [RBC/UKQCD, Phys.Rev.Lett. 124 (2020) 13]

Hadronic Vacuum Polarization

$(a_\mu^{SM} - a_\mu^{exp}) \times 10^{10}$ with a_μ^{HVP} from:

after April 7 [Phys.Rev.Lett. 126 (2021) 14, 141801]





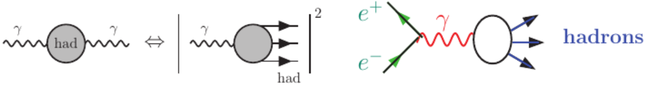
more accurate disperisive estimate used in WP (producing the 4.2σ)

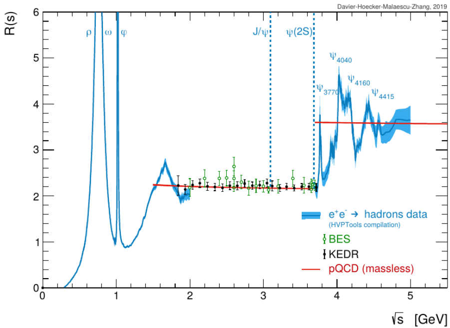
now lattice BMW [Nature 593 (2021) 7857, 51-55] at the same level but at 2σ from dispersive

The *theory* number uses the experimentally measured(KLOE, BABAR, Belle, BESIII, Novosibirsk) hadronic e^+e^- annihilation cross-section

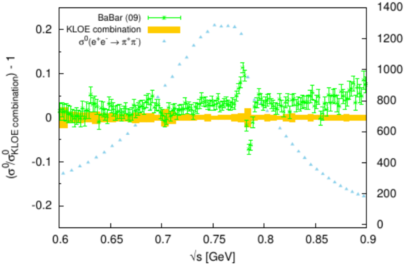
Disperive Relation : $$\Pi(k^2) - \Pi(0) = \frac{k^2}{\pi}\int_0^\infty ds \frac{\mathrm{Im}\Pi(s)}{s(s-k^2-i\varepsilon)}$$

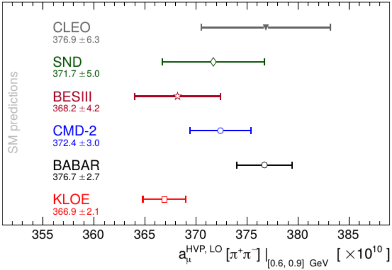
+ optical theorem : $$\mathrm{Im}\Pi(s) \propto s\sigma_{tot}(e^+ e^- \to \text{anything})$$





50 exclusive channels. E.g.: $\pi^+\pi^-$





The difference KLOE-BABAR reduces to 5.5×10^{-10} when integrating up to 1.8 GeV.

Half of that is included in systematic error by DHMZ and WP

On the lattice

The Euclidean hadronic vacuum polarisation tensor is defined as

$$\Pi_{\mu\nu}^{(N_f)}(q) = i \int d^4x e^{iqx} \langle J_\mu^{(N_f)}(x) J_\nu^{(N_f)}(0) \rangle$$

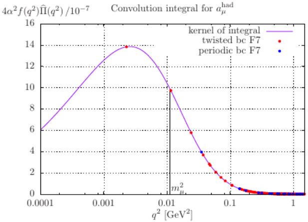
Euclidean invariance and current conservation imply

$$\Pi_{\mu\nu}^{(N_f)}(q) = (q_\mu q_\nu - g_{\mu\nu} q^2) \Pi^{(N_f)}(q^2)$$

The relation between $\Pi_{\mu\nu}^{(N_f)}(q^2)$ and a_μ^{HLO} is [E. De Rafael, 1994 and T. Blum, 2002]

$$a_\mu^{HLO} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dq^2\, f(q^2) \hat{\Pi}(q^2)$$

The kernel $f(q^2)$ is dominated by momenta $\simeq m_\mu^2$. Smallest lattice momentum is $\frac{2\pi}{L}$. One needs $L \simeq 10$ fm and in general expects large finite size effects.



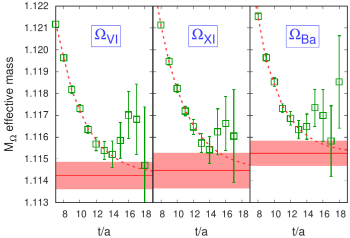
Looking at BMW21 [Nature 593 (2021) 7857, 51-55] defining the state of the art

- physical quark masses and QED
- Lattice extents from 6 fm to 11 fm
- lattice spacings from 0.13 to 0.064 fm (largest volume $96^3 \times 144$)

Final result is systematic dominated and moved from $\underline{712.4(4.5) \times 10^{-10}}$ (arXiv, Feb 2020) to $\underline{707.5(5.5) \times 10^{-10}}$ (Nature, Apr 2021 ,revised analysis after input from the community)

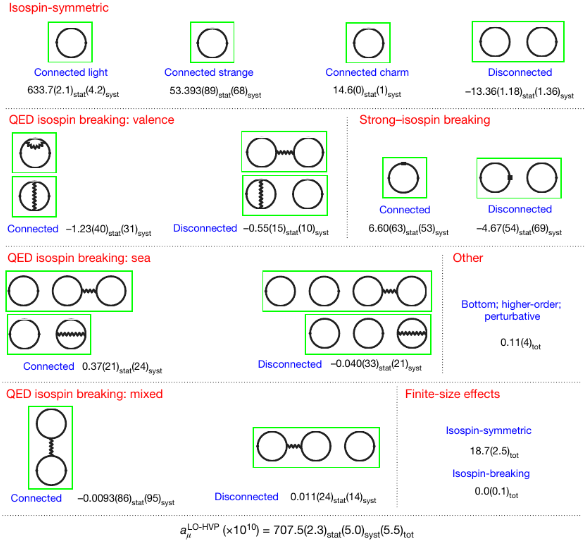
Main systematic: continuum extrapolation (lattice spacing $\to 0$).

Uncertainty on the value of the lattice spacing is also an important effect. It enters quadratically in a_μ [Della Morte et al., JHEP 10 (2017) 020]. Few permille on a needed for subpercent calculations. In BMW21 a is fixed through m_Ω.



It requires to have multi(4)-state fits under control (e.g. using priors or GEVP).

Still from BMW21:



Summing Isospin symmetric only: $a_\mu = 707(2)(5) \times 10^{-10}$

Internal, accurate, checks of lattice computations

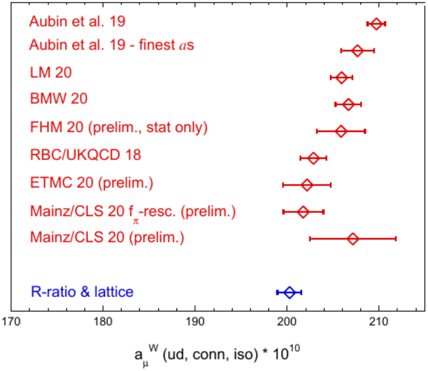
Window observables: Integrating the 2-pt function in a region safe from both cutoff and finite-size effects *D. Giusti@LATTICE21*

$$a_\mu^{SD}(f;t_0,\Delta) \equiv 4\alpha_{em}^2 \int_0^\infty dt\, K_\mu(t) V^f(t) \left[1 - \Theta\left(t, t_0, \Delta\right)\right]$$

$$a_\mu^{W}(f;t_0,t_1,\Delta) \equiv 4\alpha_{em}^2 \int_0^\infty dt\, K_\mu(t) V^f(t) \left[\Theta\left(t, t_0, \Delta\right) - \Theta\left(t, t_1, \Delta\right)\right]$$

$$a_\mu^{LD}(f;t_1,\Delta) \equiv 4\alpha_{em}^2 \int_0^\infty dt\, K_\mu(t) V^f(t)\, \Theta\left(t, t_1, \Delta\right)$$

$(t_0, t_1, \Delta) = (0.4, 1.0, 0.15)$ fm



Perspectives

Everything can move by 1σ

- Exp. (a_μ): stat. dominated
- Theory (a_μ^{HVP}): *lattice;* BMW21 moved by 1σ changing analysis
  dispersive; BABAR - KLOE tension amounts $\simeq 1\sigma$

Ongoing efforts reviewed at last Theory Initiative workshop (virtual, KEK, end of June): https://www-conf.kek.jp/muong-2theory/

- Lattice: RBC/UKQCD, Fermilab and Mainz working on subpercent determinations of a_μ^{HVP}. Timescale: 1 year. ETMC also improving their result.
- Dispersive:
  - Belle II: phase 3, 0.5% on $a_\mu^{\pi^+\pi^-}$ in 2021
  - BABAR: New analysis ($\pi^+\pi^-$ channel) using full-statistics and new particle ID; 7 times previous sample
  - CMD-3 and SND exploiting increased luminosity at VEPP-2000
  - BES III: Possibly extending data sample

New version of WP by next year

Many possible scenarios. In [A. Crivellin et al., Phys.Rev.Lett. 125 (2020) 9]:
disp $\to$ lattice (BMW21) $\simeq$ exp. No NP in a_μ, however

$$a_\mu^{HVP} \propto \int_{s_{thr}}^{\infty} ds \frac{\hat{K}(s)}{s^2} R_{had}(s), \quad \Delta\alpha_{had}(M_Z^2) \propto \int_{s_{thr}}^{\infty} ds \frac{R_{had}(s)}{s(M_Z^2 - s)}$$

Ref. value: $\Delta\alpha_{had}|_{e^+e^-} = 276.1(1.1) \times 10^{-4}$.
Modifying $R_{had}(s)$ to get agreement with exp value of a_μ

$$\begin{aligned}
\Delta\alpha^{(5)}_{\rm had}\big|_{\rm proj,\,\infty} &= 283.8(1.3) \times 10^{-4}, \\
\Delta\alpha^{(5)}_{\rm had}\big|_{\rm proj,\,\leq 11.2\,GeV} &= 280.3(1.3) \times 10^{-4}, \\
\Delta\alpha^{(5)}_{\rm had}\big|_{\rm proj,\,\leq 1.94\,GeV} &= 277.9(1.1) \times 10^{-4},
\end{aligned}$$

producing (correlated) tensions at (4.5,2.5,4.5)σ level with ref. value.

Specific observables such as $A_\ell = \frac{2Re[g_V^\ell/g_A^\ell]}{1+(Re[g_V^\ell/g_A^\ell])^2}$ also are in tensions up to 3σ with EW fits.

Another (extreme) scenario: in 1-2 years: Disp. stays and lattice $\simeq$ exp. confirmed:

- Lattice, by construction, gives SM contribution to HVP $\Rightarrow$ New Physics in HVP ? That is low energy.
- On the other hand if the lattice value for HVP gives results consistent with exp. for a_μ it means that there must be NP contributions to a_μ compensating the one in HVP.

Many other scenarios 'in between'.

Sensitivity of a_μ to NP $\propto \frac{m_\mu^2}{M_{NP}^2}$.
The exp-dispersive discrepancy sets $M_{NP} \approx 2\ TeV$

Conclusions

- 'Short blanket' situation in $(g-2)_\mu$, with many different possible NP scenarios. Model builders already correlate the tension with flavor anomalies ($R_K^{(*)}$ and$R_D^{(*)}$).
- Need to corroborate theory results (lattice and data driven) $\approx$ 2 years.
- A lot going on also on the Experimental side
  - Fermilab: current result is 6% of planned stats. Factor 4 (in stats) by next year.
  - Approved exp at J-PARC: ultracold muons $\rightarrow$ different systematics wrt Fermilab.
  - MuonE at CERN (currently testing): very timely. It will provide results for HVP at space-like momenta (direct comparison with lattice). If approved, results in 3-4 years (goal 0.3% stat. on a_μ^{HLO}).