Present status of radiative and rare kaon decays

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Radiative and rare kaon decay programs

- Rare kaon decays: extremely useful probes of

  (a) new physics (when SM-suppressed and short-distance dominated).

  (b) the SM itself in the nonperturbative regime (when long-distance dominated).

- Ongoing programs of NA62, LHCb and KOTO
- In this talk:
  - $K^+ \to \pi^+ \bar{\nu}\nu$
  - $K_L \to \pi^0 \bar{\nu}\nu$
  - $K^+ \to \pi^0 \ell^+ \nu\gamma$
  - $K \to \pi\gamma^{(*)}$ and $K \to \pi\pi\gamma^{(*)}$
- Not in this talk: exotic or forbidden channels (see talks by E. Minucci and C. Parkinson at EPS-HEP 2021)

$K^+ \to \pi^+\bar{\nu}\nu$

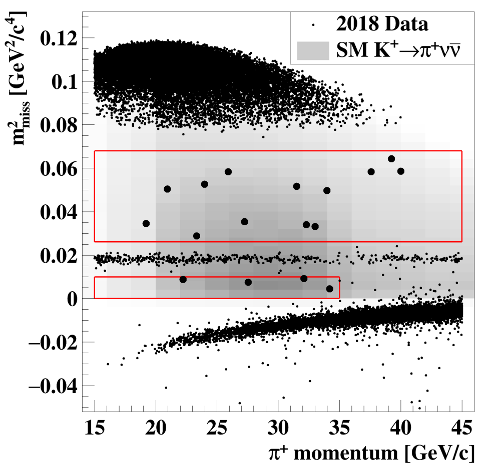
- Extremely clean theoretically, but FCNC and GIM-suppressed: [1503.02693 [hep-ph]]

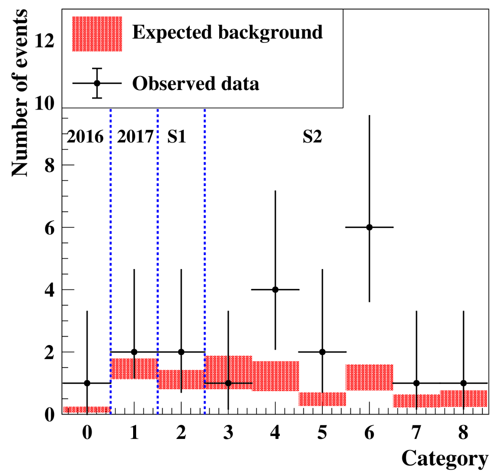
$$\mathcal{B}(K^+ \to \pi^+\bar{\nu}\nu) = (8.4 \pm 1.0)10^{-11}$$

- Computed to NLO (EW+QCD) for the top contribution and NLO (EW) and NNLO (QCD) for the charm contribution.
- Flagship of the NA62 experiment. Currently, [2103.15389 [hep-ex]]

$$\mathcal{B}(K^+ \to \pi^+\bar{\nu}\nu) = (10.6^{+4.0}_{-3.4} \pm 0.9)10^{-11}$$

  based on 20 signal events (3.4σ).
- Sizeable deviations from the SM excluded. Precision to increase to 10% with Run 2 data (2021-2023).





$K_L \to \pi^0\bar{\nu}\nu$

- Extremely clean theoretically but also extremely tiny

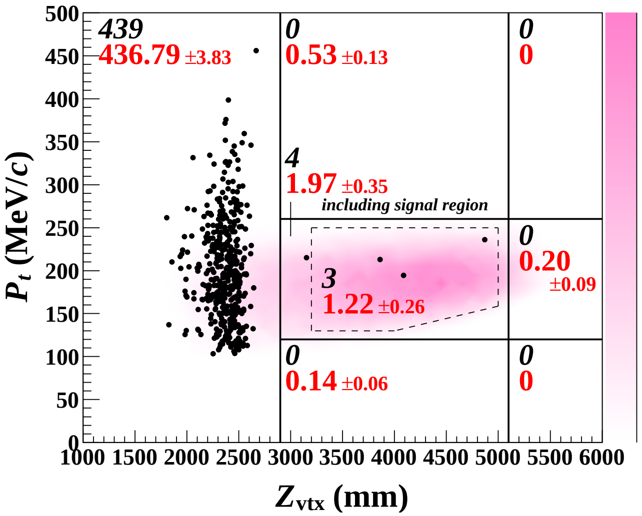
[1503.02693 [hep-ph]]

$$\mathcal{B}(K_L \to \pi^0\bar{\nu}\nu) = (3.4 \pm 0.6)10^{-11}$$

- Computed to NLO accuracy in both QCD and EW.
- Experimentally very challenging. Current bounds:

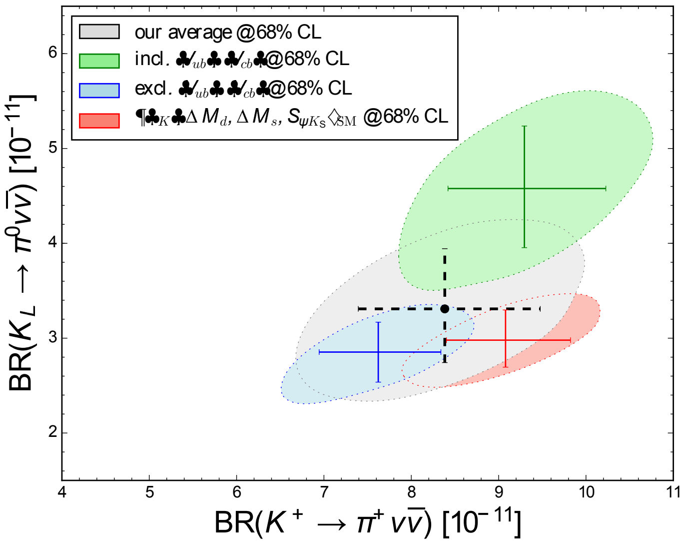
$$\mathcal{B}(K_L \to \pi^0\bar{\nu}\nu) < 3.0 \times 10^{-9} \qquad [2015 \text{ data}]$$
$$\mathcal{B}(K_L \to \pi^0\bar{\nu}\nu) < 4.9 \times 10^{-9} \qquad [2016 - 2018 \text{ data}]$$

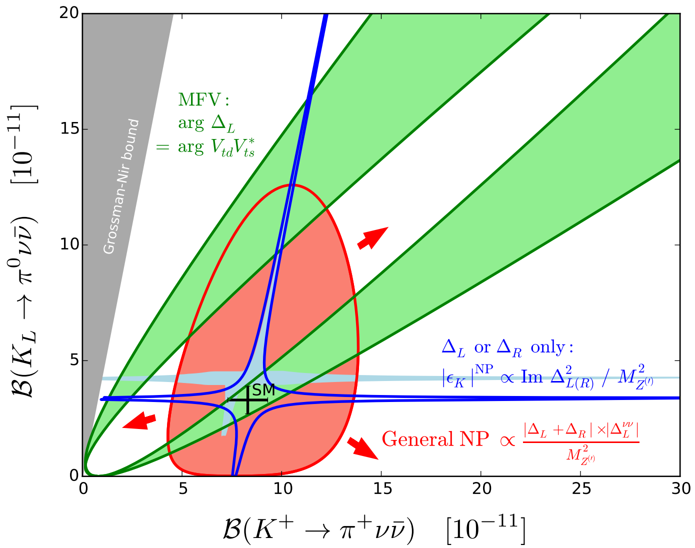
- Signal events in the 2016-2018 data deemed as statistically consistent with backgrounds. [2012.07571 [hep-ex]]



$K_L \to \pi^0\bar{\nu}\nu$ and $K^+ \to \pi^+\bar{\nu}\nu$

- Correlation between the modes within and beyond the SM:





- Isospin (Grossman-Nir bound):

$$\Gamma(K^+ \to \pi^+\bar{\nu}\nu) > \Gamma(K_L \to \pi^0\bar{\nu}\nu)$$

- CKM unitarity.

$K^+ \to \pi^0 \ell^+ \nu \gamma$

- Dominated by Bremsstrahlung, but structure-dependent pieces estimated to $\mathcal{O}(p^6)$

[0611366, 1012.0147 [hep-ph]]

- Measurements of ratios, with angular and energy cuts:

$$R_j = \frac{\mathcal{B}(Ke3\gamma_j)}{\mathcal{B}(Ke3)}$$

- With 2016-2017 data (preliminary!) [Bizioli, EPS-HEP 2021]

	ChPT (0611366)	NA62 preliminary
$R_1(\times 10^2)$	$1.804(21)$	$1.684(5)(10)$
$R_2(\times 10^2)$	$0.640(8)$	$0.599(3)(5)$
$R_3(\times 10^2)$	$0.559(6)$	$0.523(3)(3)$

- Precision at the percent level.
- Systematic deviation of $\mathcal{O}(6\%)$ with respect to theory.

$K \to \pi\gamma^{(*)}$ *and* $K \to \pi\pi\gamma^{(*)}$

- Theoretical main tool: ChPT in the $\Delta S = 1$ sector.

$$\mathcal{L}_{\Delta S=1} = G_8 f_\pi^4 \, \mathrm{tr}\left[\lambda_6 D_\mu U^\dagger D^\mu U\right] + G_8 f_\pi^2 \sum_j N_j \, W_j(U, D_\mu U, \lambda_6) + \mathcal{O}(p^6)$$

with

$$U = \exp\left[i\frac{\phi^a \tau^a}{f_\pi}\right]; \qquad D_\mu U = \partial_\mu U + ieA_\mu[Q, U]$$

- The LO is universal, NLO order contains nonperturbative (hadronic) information inside N_i.
- Radiative kaon decays: out of the 37 NLO operators, sensitive to combinations of $W_{14}, ..., W_{18}$ (CP-even) and $W_{28}, ..., W_{31}$ (CP-odd).
- General structure of the amplitudes:

$$\mathcal{M}(K \to X\gamma^{(*)}) = \underbrace{\mathcal{M}_B(\mathcal{O}(p^2))}_{\mathrm{Brems.}} + \underbrace{\mathcal{M}_E(\mathcal{O}(p^4))}_{\mathrm{electric,\,CP-even}} + \underbrace{\mathcal{M}_M(\mathcal{O}(p^4))}_{\mathrm{magnetic,\,CP-odd}}$$

Experimental status and future prospects

- Main strategy: determine/overconstrain $\Delta S = 1$ ChPT up to NLO.

- Measured modes:

$$
\begin{array}{llllllll}
K^\pm \to \pi^\pm \gamma^* & [10^{-7}]_{3\%}\,; & K_S \to \pi^0 \gamma^* & [10^{-9}]_{50\%}\,; & K_L \to \pi^0 \gamma^* & [< 10^{-10}] \\
K^\pm \to \pi^\pm \pi^0 \gamma & [10^{-6}]_{7\%}\,; & K_S \to \pi^+ \pi^- \gamma & [10^{-3}]_{3\%}\,; & K_L \to \pi^+ \pi^- \gamma & [10^{-5}]_{4\%} \\
K^\pm \to \pi^\pm \gamma\gamma & [10^{-6}]_{6\%}\,; & K_S \to \pi^0 \gamma\gamma & [10^{-8}]_{37\%}\,; & K_L \to \pi^0 \gamma\gamma & [10^{-6}]_{3\%} \\
K^\pm \to \pi^\pm \pi^0 \gamma^* & [10^{-6}]_{3\%}\,; & K_S \to \pi^+ \pi^- \gamma^* & [10^{-5}]_{3\%}\,; & K_L \to \pi^+ \pi^- \gamma^* & [10^{-7}]_{6\%}
\end{array}
$$

- Near-future upgrades:

$$
\begin{array}{ll}
K^\pm \to \pi^\pm \pi^0 \gamma^* & \mathrm{NA62}\,(?) \\
K_S \to \pi^+ \pi^- \gamma^* & \mathrm{LHCb}
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Status of chiral tests

- Weak chiral couplings are related to the slope of the differential decay rate, most easily accessed through the interference term between Bremsstrahlung and electric emission.

- Experimental effort for the last 15 years, and ongoing:

$K^\pm \to \pi^\pm \gamma^* :$	$a_+ = -0.578 \pm 0.016$ *	[NA48/2, 2009 – 11]
$K_S \to \pi^0 \gamma^* :$	$a_S = (1.06^{+0.26}_{-0.21} \pm 0.07)$ *	[NA48/1, 2003 – 04]
$K^\pm \to \pi^\pm \pi^0 \gamma :$	$X_E = (-24 \pm 4 \pm 4)\ \text{GeV}^{-4}$	[NA48/2, 2010]
$K^+ \to \pi^+ \gamma\gamma :$	$\hat{c} = 1.56 \pm 0.23 \pm 0.11$	[NA62, 2014]

- The slopes are linked to ChPT through [Ecker et al; D'Ambrosio et al]

$$\mathcal{N}_E^{(1)} \equiv N_{14}^r - N_{15}^r = \frac{3}{64\pi^2}\left(\frac{1}{3} - \frac{G_F}{G_8}a_+ - \frac{1}{3}\log\frac{\mu^2}{m_K m_\pi}\right) - 3L_9^r$$

$$\mathcal{N}_S \equiv 2N_{14}^r + N_{15}^r = \frac{3}{32\pi^2}\left(\frac{1}{3} + \frac{G_F}{G_8}a_S - \frac{1}{3}\log\frac{\mu^2}{m_K^2}\right)$$

$$\mathcal{N}_E^{(0)} \equiv N_{14}^r - N_{15}^r - N_{16}^r - N_{17} = -\frac{|\mathcal{M}_K| f_\pi}{2G_8} X_E$$

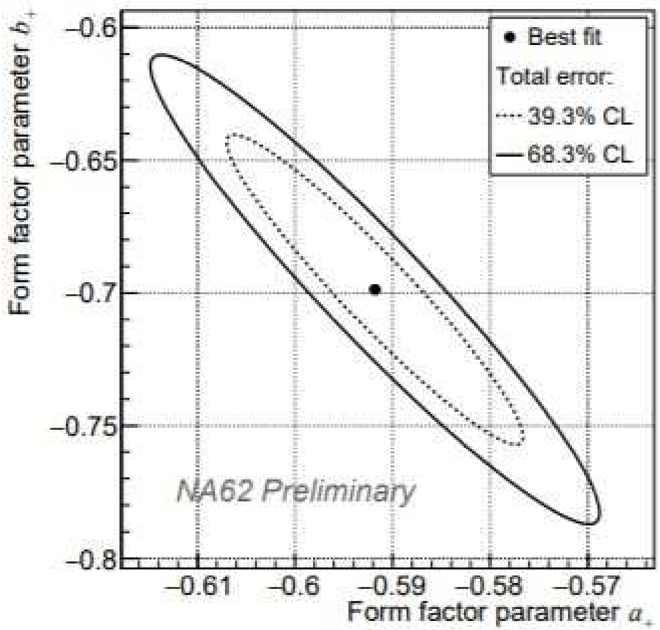
$$\mathcal{N}_0 \equiv N_{14}^r - N_{15}^r - 2N_{18}^r = \frac{3}{128\pi^2}\hat{c} - 3(L_9^r + L_{10}^r)$$

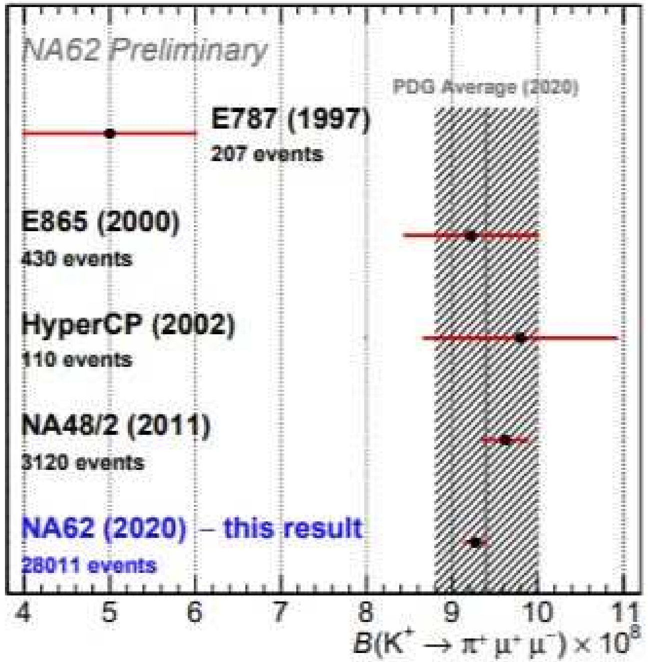
$K^+ \to \pi^+\ell^+\ell^-$

- Customary to parametrize it as

$$\frac{d^2\Gamma}{dzdx} = \frac{\alpha^2 m_K}{(4\pi)^5} f(x,z)|W(z)|^2, \qquad W(z) = G_F m_K^2 (a_+ + b_+ z) + W^{\pi\pi}(z)$$

- Recent improvement of the muon channel (preliminary!) [Bician, PoS ICHEP2020(2021)364]





- Preference for negative parameters,

$$a_+ = -0.592(13)(7)(1); \qquad b_+ = -0.699(46)(35)(3)$$

A positive solution at $a_+ = 0.368$, $b_+ = 2.045$ is disfavored. When compared to the electron channel, no indications of LFUV.

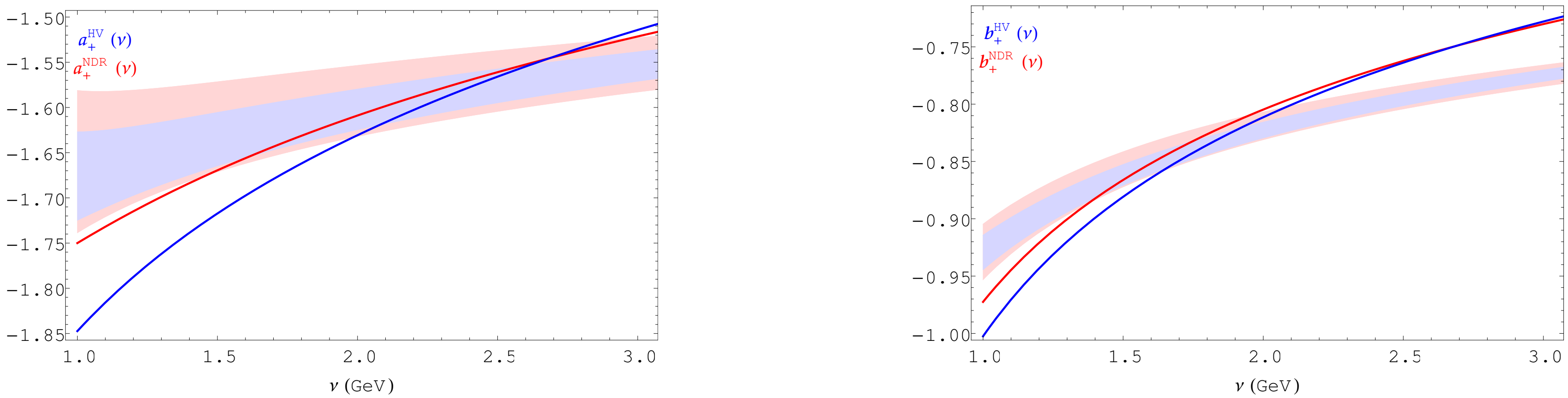
$K^+ \to \pi^+ \ell^+ \ell^-$

- The existence of two close solutions is a feature. [1812.00735 [hep-ph]],[1906.03046 [hep-ph]]
- Theoretical determination of a_+ and b_+ is feasible, but not straightforward. It requires to go beyond low energies...
- First approximation:

$$W(z) = W^{\pi\pi}(z) + W^{\rm res}(z;\nu) + W^{\rm SD}(z;\nu)$$

Strategy: model the form factor with an infinite tower of states and match long and short distances.

- Through loop-level matching one can determine the scale dependence of a_+ and b_+



- The value for a_+ is off, but also the model is too crude (e.g. no KK rescattering).

Status of NLO chiral counterterms

Decay mode	counterterm combination	expt. value *
$K^\pm \to \pi^\pm \gamma^*$	$N_{14} - N_{15}$	$-0.0167(13)$
$K_S \to \pi^0 \gamma^*$	$2N_{14} + N_{15}$	$+0.016(4)$
$K^\pm \to \pi^\pm \pi^0 \gamma$	$N_{14} - N_{15} - N_{16} - N_{17}$	$+0.0022(7)$
$K^\pm \to \pi^\pm \gamma\gamma$	$N_{14} - N_{15} - 2N_{18}$	$-0.0017(32)$

- From $K \to \pi\gamma^*$ decays,

$$N_{14} = (-2 \pm 18) \times 10^{-4}; \qquad N_{15} = (1.65 \pm 0.22) \times 10^{-2}$$

- Adding $K \to \pi\gamma\gamma$,

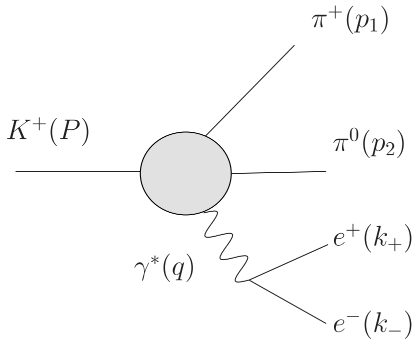
$$N_{18} = (-7.5 \pm 2.3) \times 10^{-3}$$

- So far, only the combination $N_{16} + N_{17}$ constrained. One extra combination needed:

$$K \to \pi\pi\ell^+\ell^-$$

- Most promising channel: $K^\pm \to \pi^\pm\pi^0 e^+ e^-$, recently analyzed by NA48/2 ([1809.02873 [hep-ex]]). Important alternatives: $K_{S,L} \to \pi^+\pi^- e^+ e^-$.

$K^+ \to \pi^+\pi^0 e^+ e^-$



- Relevant weak coupling combinations:

$$\mathcal{N}_E^{(0)} \equiv N_{14}^r - N_{15}^r - N_{16}^r - N_{17} = +0.0022(7)$$
$$\mathcal{N}_E^{(1)} \equiv N_{14}^r - N_{15}^r = -0.0167(13)$$
$$\mathcal{N}_E^{(2)} = N_{14}^r + 2N_{15}^r - 3(N_{16}^r - N_{17})$$

- Similar in size as $K^+ \to \pi^+\gamma^*$ due to Bremsstrahlung $\mathcal{O}(p^2)$ contribution. Challenge:

$$\Gamma_M \sim \frac{1}{70}\Gamma_B; \qquad \Gamma_{\rm INT} \sim 10^{-2}\Gamma_B$$

- Best strategy:

  (a) Low's theorem for Bremsstrahlung. Input: $\mathcal{M}(K^+ \to \pi^+\pi^0)$ (FSI included)

  (b) Bremsstrahlung peaked at low q^2. Use cuts in the dilepton invariant mass.

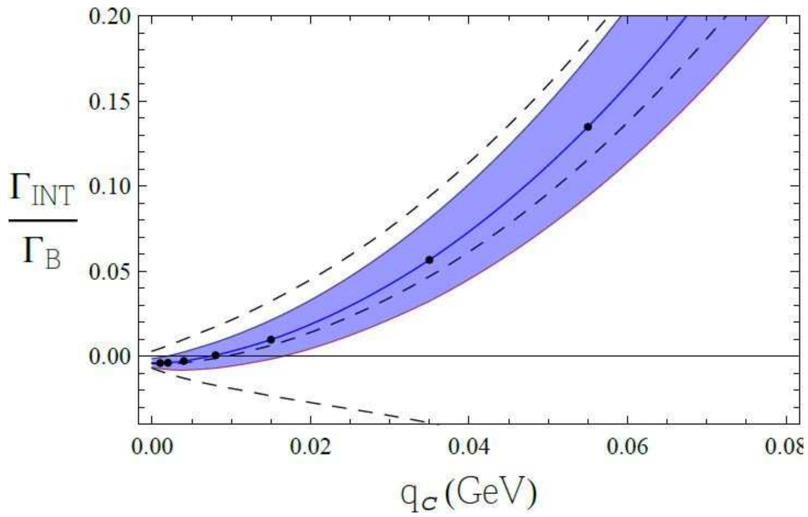
[0010284; 1112.5184 [hep-ph]; 1712.10270 [hep-ph]]

$K^+ \to \pi^+\pi^0 e^+ e^-$

- Using the already measured radiative decays:

$$\mathcal{N}_E^{(2)} = +0.089(11) + 6N_{17} \sim +\mathcal{O}(10^{-1})$$

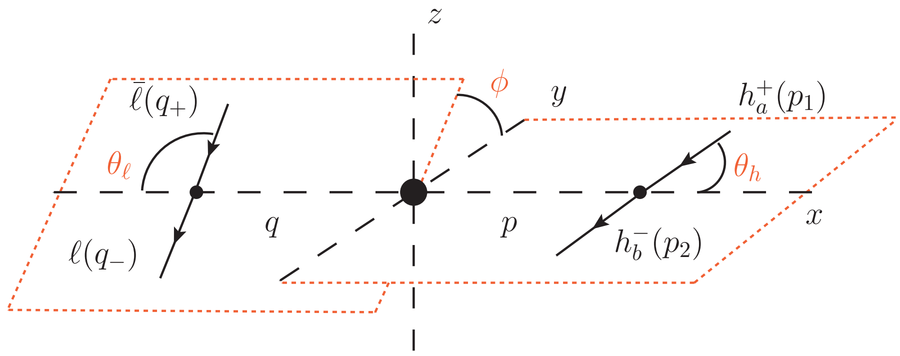
- Assumption: N_{17} small (theoretically favored).
- The interference has a characteristic pattern depending mostly on $\mathcal{N}_E^{(0)}$ and $\mathcal{N}_E^{(2)}$.



- NA48/2 has recently confirmed the Bremsstrahlung and magnetic contributions. [arXiv:1809.02873 [hep-ex]]
- Present status: more statistics needed to extract $\mathcal{N}_E^{(2)}$.

$K \to \pi\pi\ell^+\ell^-$: *reach onto new physics*

- 4-body decays have rich kinematical distributions.



$$\frac{d^5\Gamma}{dE_\gamma^* dT_c^* dq^2 d\cos\theta_\ell d\phi} = \mathcal{A}_1 + \mathcal{A}_2 \sin^2\theta_\ell + \mathcal{A}_3 \sin^2\theta_\ell \cos^2\phi + \mathcal{A}_4 \sin 2\theta_\ell \cos\phi$$
$$+ \mathcal{A}_5 \sin\theta_\ell \cos\phi + \mathcal{A}_6 \cos\theta_\ell + \mathcal{A}_7 \sin\theta_\ell \sin\phi$$
$$+ \mathcal{A}_8 \sin 2\theta_\ell \sin\phi + \mathcal{A}_9 \sin^2\theta_\ell \sin 2\phi$$

- P conserving.
- P violating (short-distance).
- P violating (long-distance).

$K_S \to \pi^+\pi^- e^+ e^-$

- A similar analysis can be performed for $K_S \to \pi^+\pi^- e^+ e^-$. Total branching ratio measured:

$$BR(K_S \to \pi^+\pi^- e^+ e^-)_{\text{exp}} = (4.79 \pm 0.15) \times 10^{-5}$$

  Access to interference term requires typically a percent precision. Challenging but in the LHCb agenda.

- On top of $\mathcal{N}_E^{(0)}$ (known), new counterterm, but not independent:

$$\mathcal{N}_E^{(3)} = N_{14} - N_{15} - 3(N_{16} + N_{17}) = -2\mathcal{N}_E^{(1)} + 3\mathcal{N}_E^{(0)} = +0.040(5)$$

- Prediction of the LO and NLO chiral contributions: [1712.10270 [hep-ph]]

$$BR(K_S \to \pi^+\pi^- e^+ e^-) = \underbrace{4.74 \cdot 10^{-5}}_{\text{Brems.}} + \underbrace{4.39 \cdot 10^{-8}}_{\text{Int.}} + \underbrace{1.33 \cdot 10^{-10}}_{\text{DE}}$$

- Extraction of N_{17} from neutral kaon decays possible with information on $K_L \to \pi^+\pi^- e^+ e^-$:

$$\mathcal{N}_E^{(4)} - \mathcal{N}_E^{(3)} = 6N_{17}$$

- Muon decay extremely suppressed by phase space:

$$BR(K_S \to \pi^+\pi^- \mu^+ \mu^-) = \underbrace{4.17 \cdot 10^{-14}}_{\text{Brems.}} + \underbrace{4.98 \cdot 10^{-15}}_{\text{Int.}} + \underbrace{2.17 \cdot 10^{-16}}_{\text{DE}}$$

Conclusions

- A number of ongoing programs on rare kaon decays (NA62, LHCb, KOTO).

- $K^+ \to \pi^+ \bar{\nu}\nu$ is currently compatible with the SM prediction. Run 2 just started, with a 10% target precision. $K_L \to \pi^0 \bar{\nu}\nu$ has not yet been detected, with an upper bound at 10^{-9}.

- $K \to \pi\pi\ell^+\ell^-$: very interesting probes of both long and short distances.

- A full determination of all the NLO weak chiral couplings relevant for radiative kaon decays is feasible but requires (a) better experimental data on $K \to \pi\pi\ell^+\ell^-$ or $K_{S,L} \to \pi^+\pi^- e^+ e^-$; (b) improved theoretical determinations.