Nucleon Charges from Lattice QCD

Boram Yoon (Los Alamos National Laboratory)

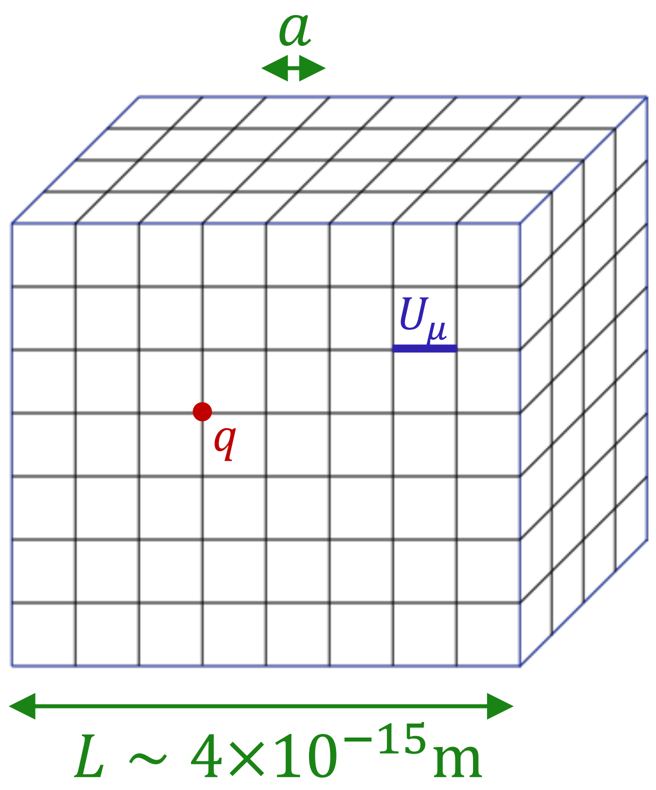
In collaboration with

Tanmoy Bhattacharya (LANL), Vincenzo Cirigliano (LANL), Rajan Gupta (LANL), Martin Hoferichter (Bern U.), Yong-Chull Jang (Columbia U.), Bálint Jóo (ORNL), Huey-Wen Lin (MSU), Emanuele Mereghetti (LANL), Santanu Mondal (LANL), Sungwoo Park (LANL), Frank Winter (JLab)

A Virtual Tribute to Quark Confinement and the Hadron Spectrum 2021, Aug 2-6 July 2021, online

Lattice QCD

- Non-perturbative approach to solving QCD on discretized Euclidean space-time
  - Hypercubic lattice with lattice spacing a
  - Quark fields placed on sites
  - Gluon fields on the links between sites; U_μ
- Numerical lattice QCD calculations using Monte Carlo methods
  - $t \to -i\tau$; $\mathrm{e}^{-\mathrm{iHt}} \to \mathrm{e}^{-\mathrm{E}\tau}$, $\int \mathrm{e}^{iS} \to \int \mathrm{e}^{-S_E}$
  - Computationally intensive
  - Use supercomputers
- Continuum results are obtained in $a \to 0$
- Has been successful for many QCD observables
  - Some results are with less than 1% error



Lattice QCD

- Correlation functions

$$\langle O \rangle = Z^{-1} \int dU dq d\bar{q}\, O(U, q, \bar{q}) e^{-S_g - \bar{q}(D+m_q)q}$$

$$= Z^{-1} \int dU \left[O\left(U, (D + m_q)^{-1}\right) e^{-S_g} \det(D + m_q) \right]$$

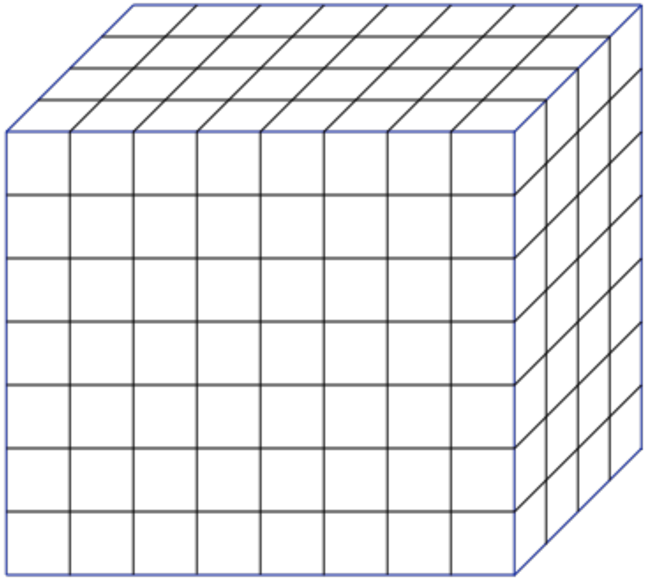
- Monte-Carlo integration
  - Integration variable U is huge

$$N_s^3 \times N_t \times 4 \times 8 \sim 10^9$$

  - Generate Markov chain of gauge configurations U
  - Calculate average as expectation value

$$\langle O \rangle \approx \frac{1}{N} \sum_i O_i \left(U, (D + m_q)^{-1}\right)$$

  - Calculation of $O_i\left(U, (D + m_q)^{-1}\right)$: measurement
  - $(D + m)^{-1}$ is computationally expensive



Physical Results from Unphysical Simulations

- **Finite Lattice Spacing**
  - Simulations at finite lattice spacings $0.6\ \mathrm{fm} \lesssim a \lesssim 0.15\ \mathrm{fm}$

  ⇒ Extrapolate to continuum limit, $a = 0$

- **Heavy Pion Mass**
  - Lattice simulation:

    Smaller quark mass → Larger computational cost and noisy results
  - Simulations at (heavy) pion masses $130\ \mathrm{MeV} \lesssim M_\pi \lesssim 310\ \mathrm{MeV}$

  ⇒ Extrapolate to physical pion mass, $M_\pi = M_\pi^{\mathrm{phys}}$

- **Finite Volume**
  - Simulations at finite lattice volume: $M_\pi L = 3 \sim 6$

  ⇒ Extrapolate to infinite volume, $M_\pi L = \infty$

Nucleon Charges

- Isovector charges: $g_{A,S,T}^{u-d}$

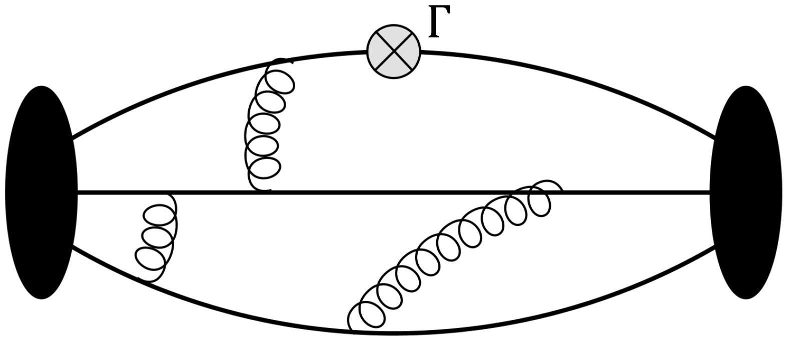
$$\langle p|\bar{u}\Gamma d|n\rangle = g_{\Gamma}^{u-d}\bar{\psi}_p\Gamma\psi_n$$

  - In the isospin limit ($m_u = m_d$),
    $\langle p|\bar{u}\Gamma d|n\rangle = \langle p|\bar{u}\Gamma u - \bar{d}\Gamma d|p\rangle = \langle n|\bar{d}\Gamma d - \bar{u}\Gamma u|n\rangle$
  - Cancel contributions from disconnected diagrams
  - Less systematics; better precision

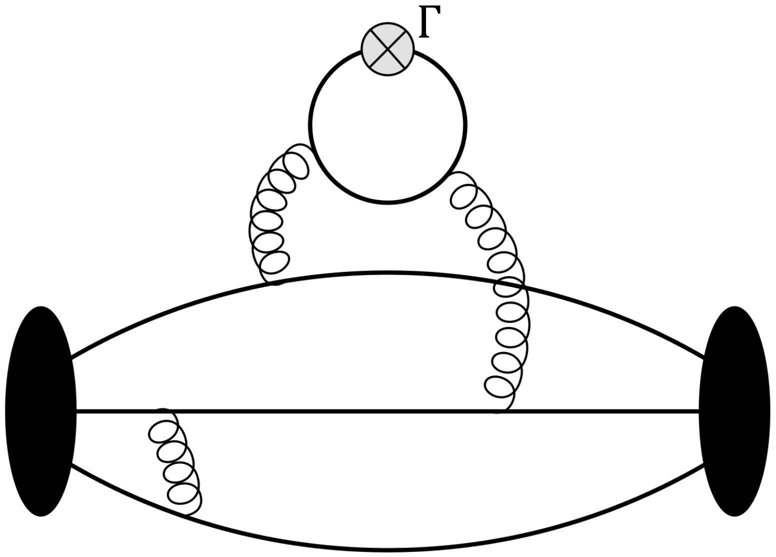
- Flavor-diagonal charges: $g_{A,S,T}^{u}, g_{A,S,T}^{d}, g_{A,S,T}^{s}, \ldots$

$$\langle p|\bar{q}\Gamma q|p\rangle = g_{\Gamma}^{q}\bar{\psi}_p\Gamma\psi_p \quad \text{for } q = u, d, s, \ldots$$

  - Need calculations of disconnected diagrams
  - More systematics; worse precision



Quark-line connected diagram



Quark-line disconnected diagram

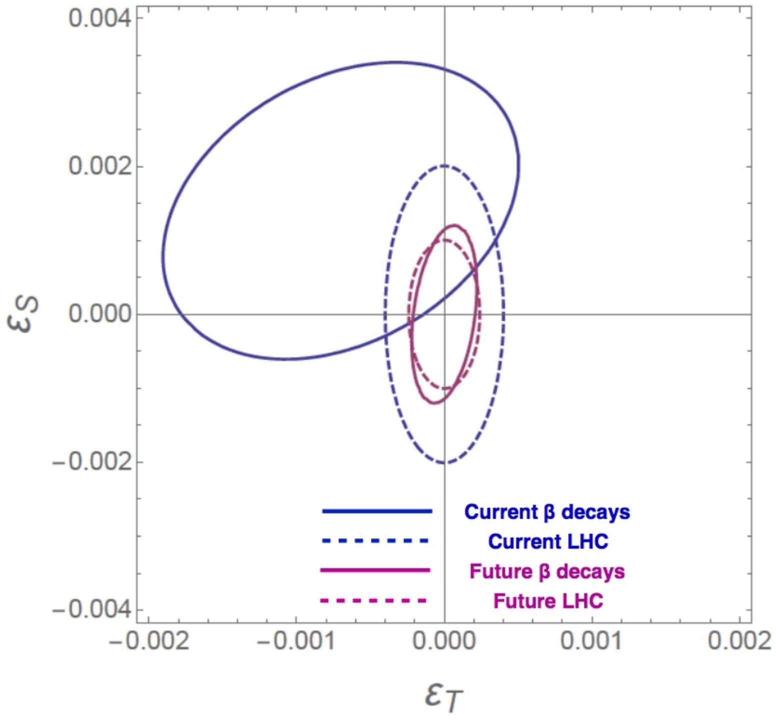
Nucleon Isovector Charges

$$\langle p|\bar{u}\Gamma d|n\rangle = g_\Gamma^{u-d}\bar{\psi}_p\Gamma\psi_n$$

Nucleon Isovector Charges

- **Axial charge g_A**
  - Weak interactions of nucleons
  - $g_A/g_V = 1.27641(45)(33)$ from cold neutron decay experiments (Märkisch, et al., PRL, 2019)
  - Nuclear beta decay, pion exchange between nucleons, nucleosynthesis, …

PRD98.034503(2018)



- **Scalar and tensor charges g_S, g_T**
  - Helicity-flip parameters b and b_ν in neutron decay
    $b = 0.34 g_S \epsilon_S - 5.22 g_T \epsilon_T$,
    $b_\nu = 0.44 g_S \epsilon_S - 4.85 g_T \epsilon_T$
  - When combined with (ultra) cold neutron decay experiments, g_S, g_T provides bounds on novel scalar and tensor interactions that can arise in BSM
    (Bhattacharya, et al., PRD 2012)

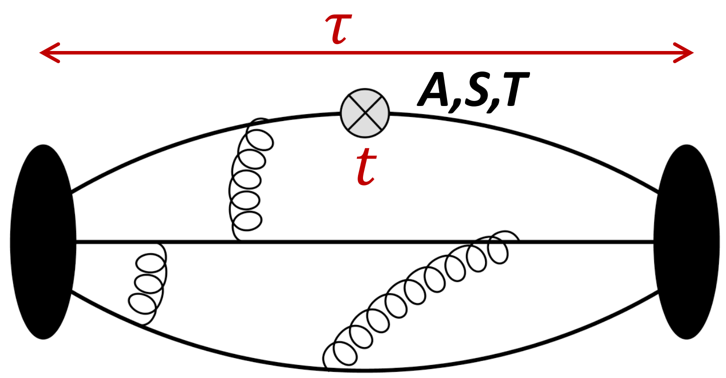
Extracting Nucleon Charges on the Lattice

$$\langle p|\bar{u}\Gamma d|n\rangle = g_\Gamma^{u-d}\bar{\psi}_p\Gamma\psi_n$$

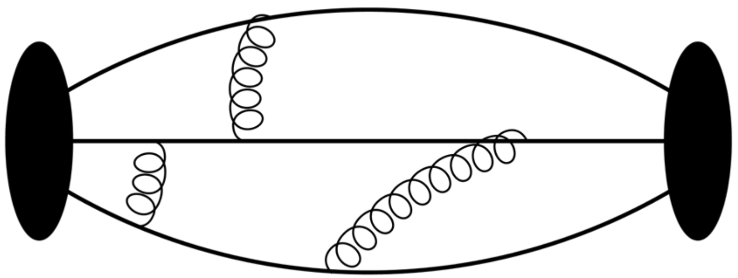
- g_Γ^{u-d} is extracted from the ratio of two- and three-point correlation functions

$$\frac{C_{3pt}^{\Gamma}(t,\tau)}{C_{2pt}(\tau)} \rightarrow g_\Gamma^{u-d}$$

  - $C_{2pt}(\tau) = \langle 0|\chi(\tau)\bar{\chi}(0)|0\rangle$
  - $C_{3pt}^{\Gamma}(t,\tau) = \langle 0|\chi(\tau)\mathrm{O}_\Gamma(\mathrm{t})\bar{\chi}(0)|0\rangle$
  - $\chi = \epsilon^{abc}[u^{aT}C\gamma_5(1+\gamma_4)d^b]u^c$
- χ introduces excited states of nucleons



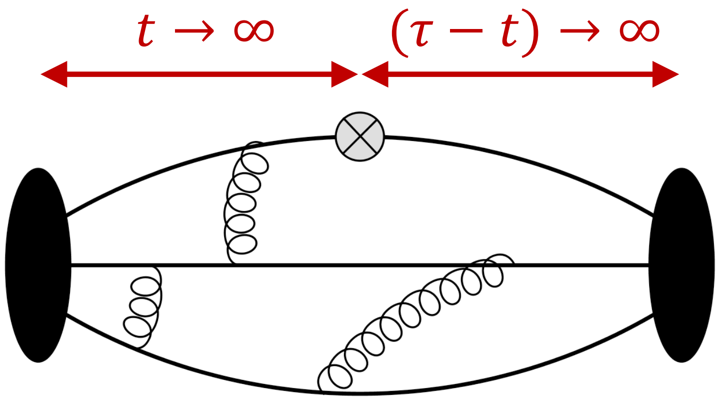
$C_{3pt}(\tau,t)$



$C_{2pt}(\tau)$

Removing Excited-state Contamination

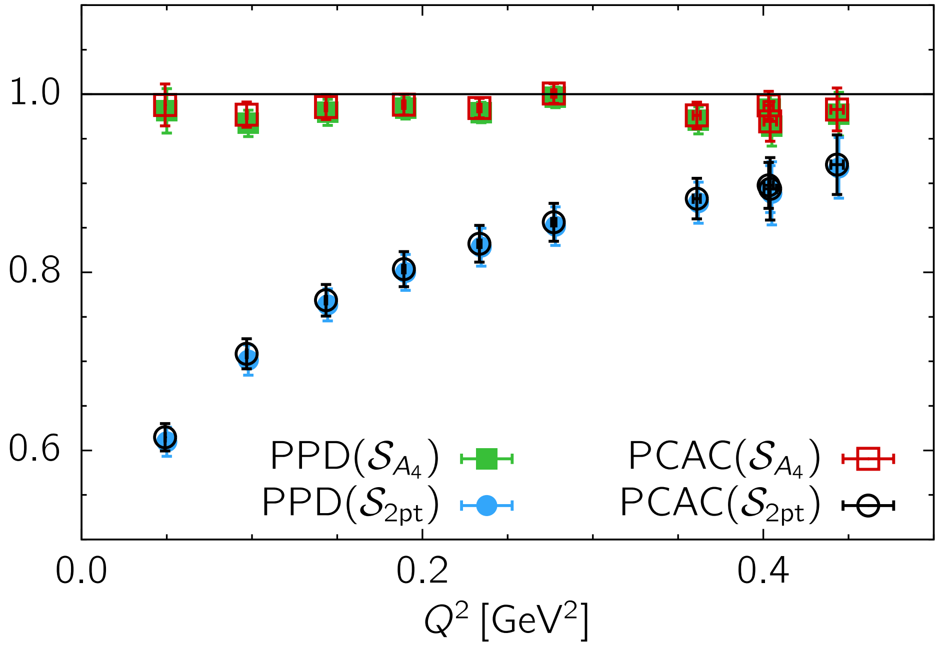
- Excited-states (ES) effect exponentially diminishes for large Euclidean time
  - Separating proton sources far from each other leaves only the ground-state
  - But signal-to-noise ratio drops exponentially in time for baryons ($R \sim e^{-\left(M_N - \frac{3}{2}m_\pi\right)\tau}$)
- For reasonably small t and τ, fit correlators to a function including ES
  - $C_{2pt}(\tau) = \sum_i |A_i|^2 e^{-M_i \tau}$
  - $C_{3pt}^{\Gamma}(\tau, t) = \sum_{i,j} A_i A_j^* \langle i | O_\Gamma | j \rangle e^{-M_i t - M_j (\tau - t)}$
  - $C_{3pt}^{\Gamma}(t, \tau)/C_{2pt}(\tau) \to \ g_\Gamma^{u-d}$ as $t \to \infty, (\tau - t) \to \infty$



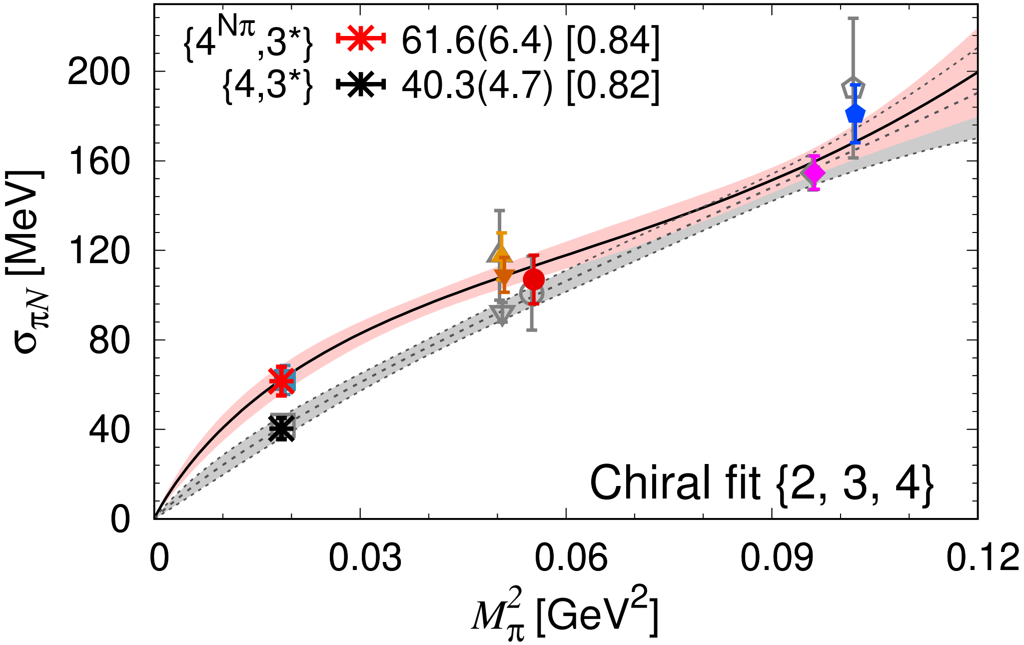
- Tower of possible excited states
  - Radial excitations: $N(1440), N(1710)$, …
  - Multi-hadron states: $N(\boldsymbol{p})\pi(-\boldsymbol{p}), N(\boldsymbol{0})\pi(\boldsymbol{0})\pi(\boldsymbol{0})$, …
  - But, which states contribute significantly?

Removing Excited-state Contamination

- Excited state fits to C_{2pt} gives large first ES mass $\mathrm{M}_1 \gtrsim M_{N(1440)}$
- But various evidences advocate $\mathrm{M}_1 \ll M_{N(1440)}$
  - PCAC relation between $G_A, \tilde{G}_P$, and G_P is much better satisfied with smaller M_1
  - Nucleon σ-term results are consistent with $\chi\mathrm{PT}$ when $\mathrm{M}_1 \sim M_{N\pi,\, N\pi\pi}$



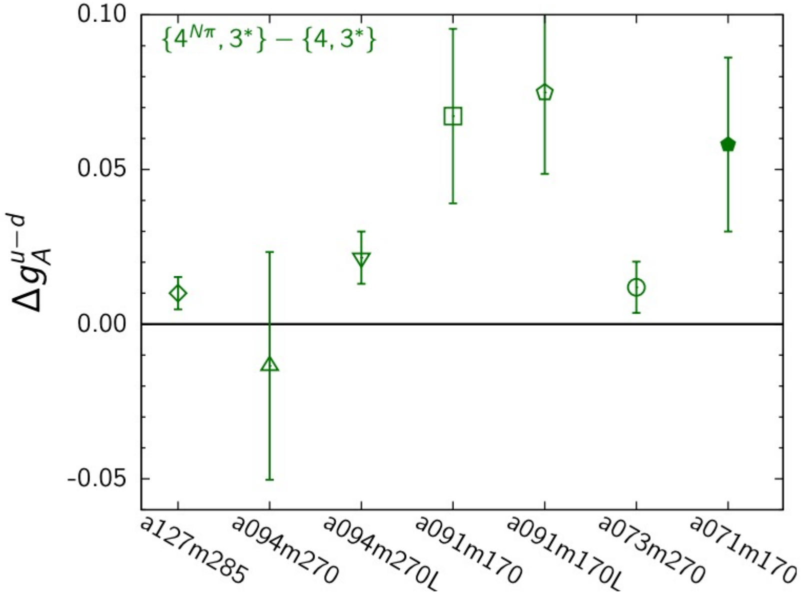
PRL 124, 072002 (2020)



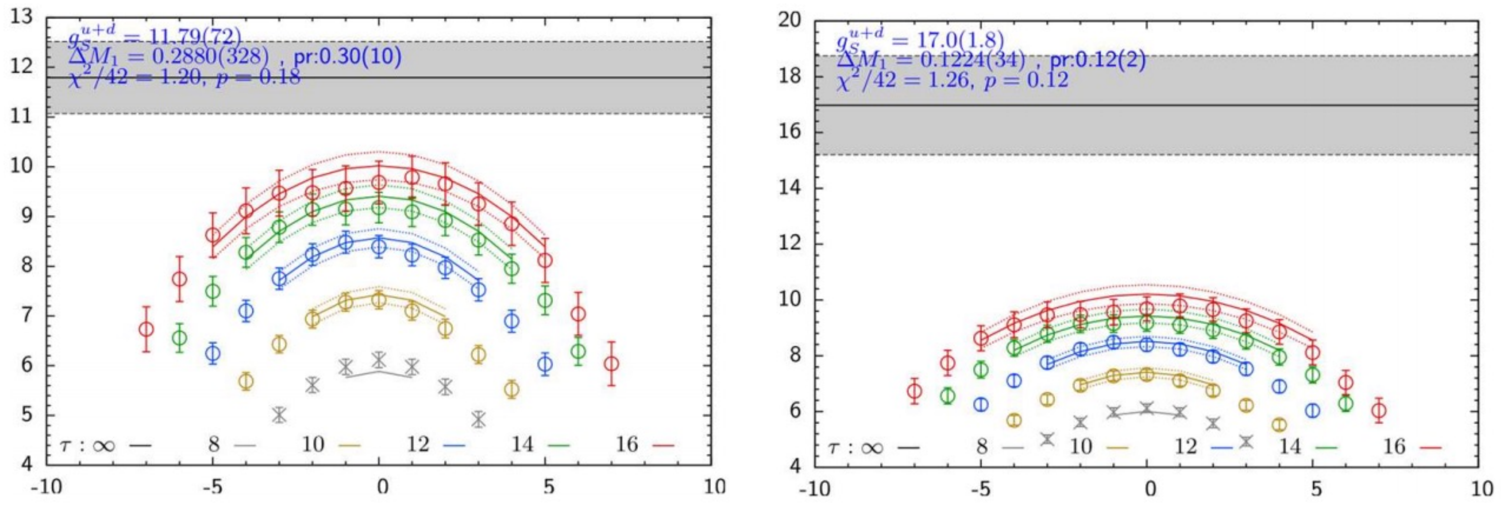
arXiv:2105.12095

Effect of $N\pi$ Excited States

- Excited state fit results with and without $N\pi$-state is huge:
  - Smaller M_1 $\rightarrow$ smaller mass gap $\Delta M_1 = M_1 - M_0$ $\rightarrow$ longer extrapolation in $t \rightarrow \infty, (\tau - t) \rightarrow \infty$ $\left(C_{3pt}^{\Gamma}(t,\tau)/C_{2pt}(\tau) = g_{\Gamma}^{u-d} + O\left(e^{-\Delta M_1 t}, e^{-\Delta M_1(\tau-t)}, e^{-\Delta M_1 \tau}\right)\right)$
- But χ^2 of 3pt fits are not sensitive to the low-lying excited-state mass
- Larger effect with smaller pion mass



arXiv:2103.05599



arXiv:2105.12095, Sungwoo Park's talk at Lattice 2021

Isovector Charges – Lattice Setup

$N_f = 2 + 1$ Clover-on-Clover

EnsID	a (fm)	M_π (MeV)	Volume	$M_\pi L$	Confs
a127m285	0.127(2)	285(5)	$32^3 \times 96$	5.87	2,002
a094m270	0.094(1)	269(3)	$32^3 \times 64$	4.09	2,469
a094m270L	0.094(1)	269(3)	$48^3 \times 128$	6.15	4,510
a091m170	0.091(1)	169(2)	$48^3 \times 96$	3.75	4,012
a091m170L	0.091(1)	170(2)	$64^3 \times 128$	5.03	2,002
a073m270	0.0728(8)	272(3)	$48^3 \times 128$	4.81	4,720
a071m170	0.0707(8)	166(2)	$72^3 \times 192$	4.28	2,500

arXiv:2103.05599

$N_f = 2 + 1$ Clover-on-HISQ

EnsID	a (fm)	M_π (MeV)	Volume	$M_\pi L$	Confs
a15m310	0.1510(20)	320.6(4.3)	$16^3 \times 48$	3.93	1,917
a12m310	0.1207(11)	310.2(2.8)	$24^3 \times 64$	4.55	1,013
a12m220S	0.1202(12)	225.0(2.3)	$24^3 \times 64$	3.29	946
a12m220	0.1184(10)	227.9(1.9)	$32^3 \times 64$	4.38	744
a12m220L	0.1189(09)	227.6(1.7)	$40^3 \times 64$	5.49	1,010
a09m310	0.0888(08)	313.0(2.8)	$32^3 \times 96$	4.51	2,263
a09m220	0.0872(07)	225.9(1.8)	$48^3 \times 96$	4.79	964
a09m130	0.0871(06)	138.1(1.0)	$64^3 \times 96$	3.9	1,290
a06m310	0.0582(04)	319.6(2.2)	$48^3 \times 144$	4.52	1,000
a06m220	0.0578(04)	235.2(1.7)	$64^3 \times 144$	4.41	650
a06m135	0.0570(01)	135.6(1.4)	$96^3 \times 192$	3.7	675

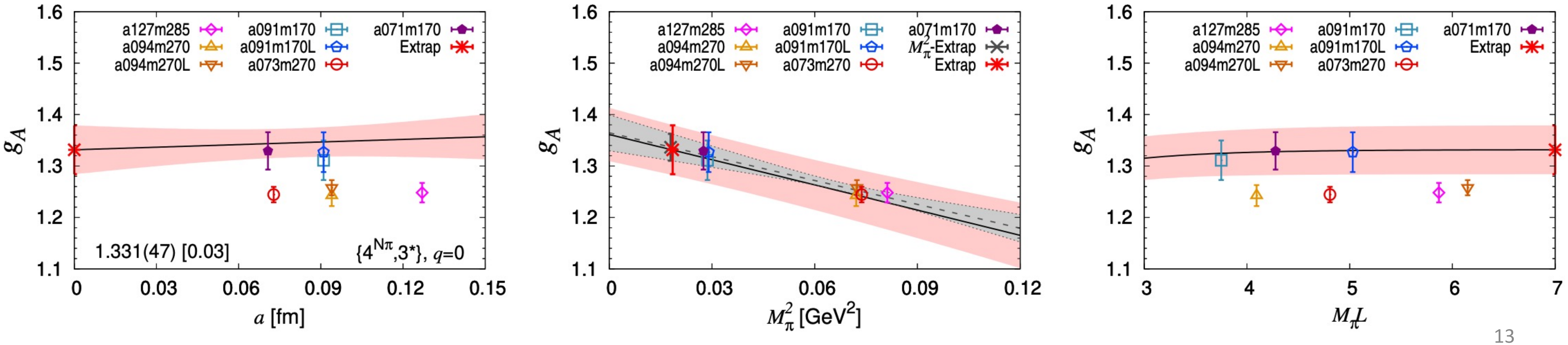
PRD 98, 034503 (2018)

Chiral, Continuum, Finite volume Extrapolation

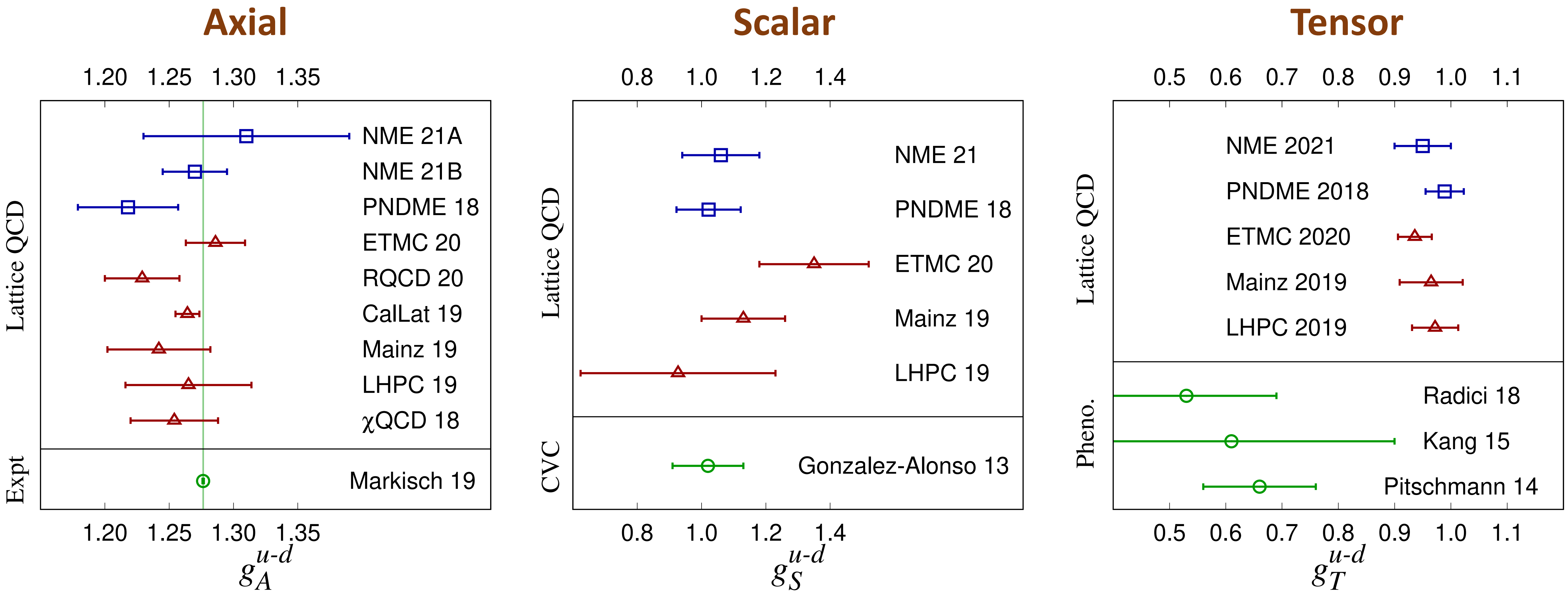
- **CCFV fit ansatz**

$$g(a, M_\pi, M_\pi L) = c_1 + c_2 a + c_3 M_\pi^2 + c_4 \frac{M_\pi^2}{\sqrt{M_\pi L}} e^{-M_\pi L}$$

- Only axial charge shows small FV effect
- Final results of mean and statistical/systematic errors are obtained by averaging/comparing various excited state fit ansatzes and CCFV (CC) fits



Isovector Charges - Results



- **CVC**: Conserved vector current relation for g_S; $g_S/g_V = (M_N - M_P)^{QCD}/(m_d - m_u)^{QCD}$
- **Pheno.**: Extraction of g_T from semi-inclusive deep-inelastic scattering (SIDIS) experimental data

Nucleon Flavor-diagonal Charges

$$\langle p|\bar{q}\Gamma q|p\rangle = g_{\Gamma}^{q}\bar{\psi}_p\Gamma\psi_p$$

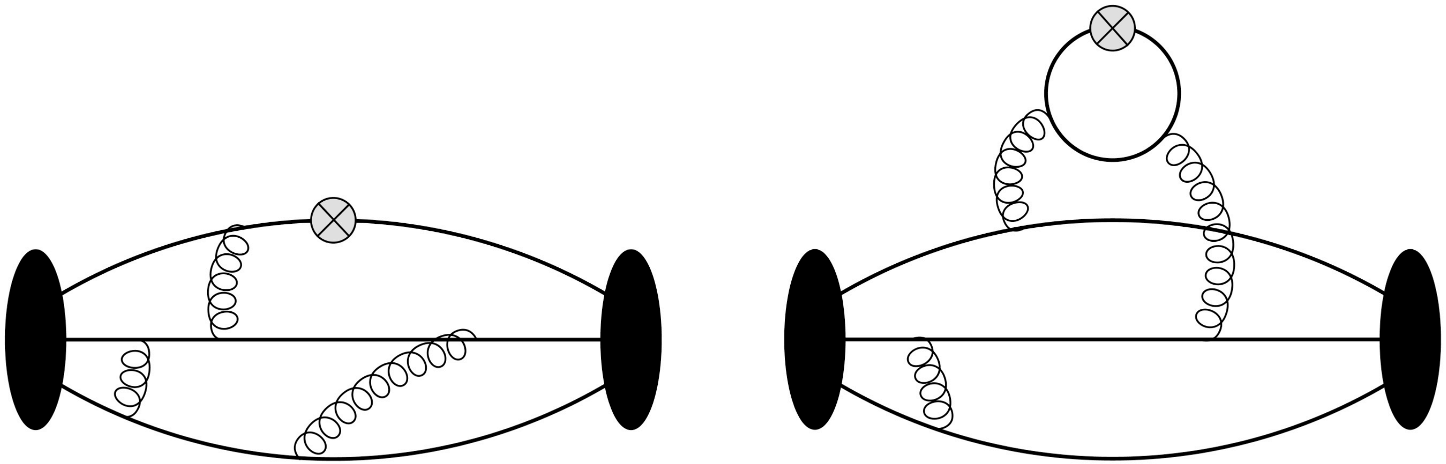
Flavor-diagonal Axial Charge: $g_A^{u,d,s}$

- Proton spin decomposition

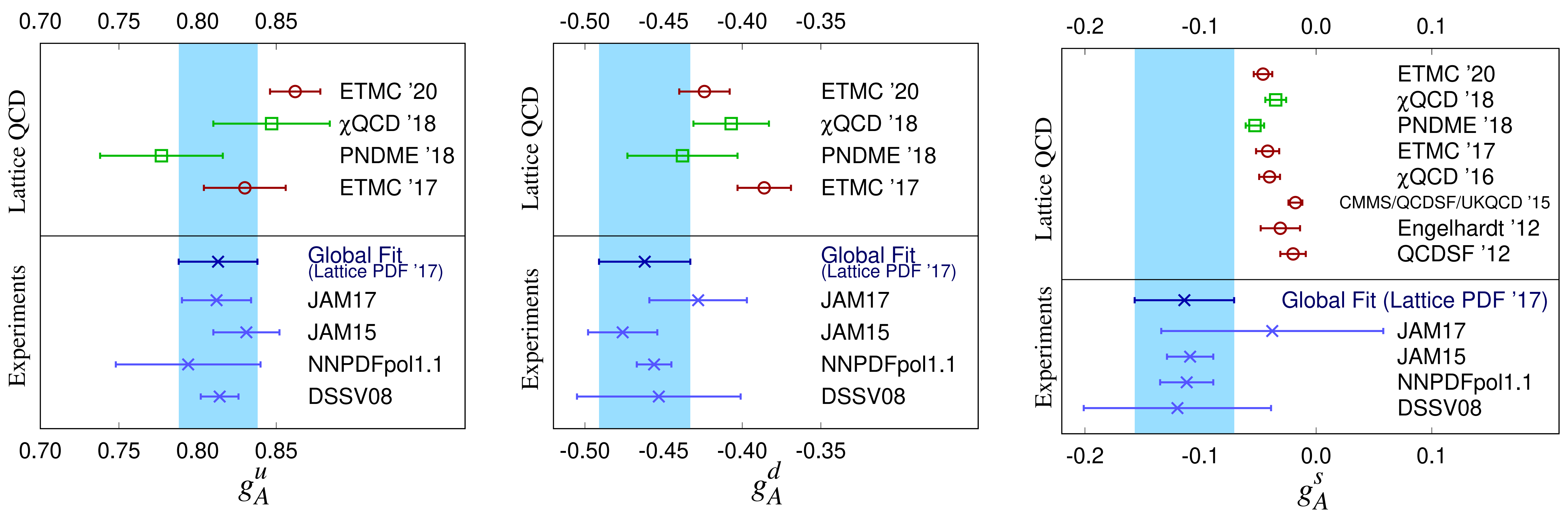
$$\frac{1}{2} = \sum_{q=u,d,s,\ldots} \left(\frac{1}{2}\Delta q + L_q\right) + J_g$$

where $g_A^q \equiv \Delta q = \int_0^1 dx\left(\Delta q(x) + \Delta\bar{q}(x)\right)$

- Need to calculate quark-line disconnected diagrams, in addition to connected ones, which is computationally expensive and noisy



Flavor-diagonal Axial charges - Current Status



- Only PNDME and χQCD'18 data are extrapolated to continuum limit
- Charm-quark results (g_A^c) can be found at
  - PNDME: Rui Zhang's talk at Lattice 2021;   ETMC: PRD 102, 054517 (2020)

Flavor-diagonal Tensor Charge: $g_T^{u,d,s}$

- Nonzero neutron electric dipole moment (nEDM) violates P and T, so CP
  → nEDM is a sensitive probe of new sources of CP violation in BSM

- No nonvanishing nEDM has been observed, but next-generation experiments will reach $d_N \sim 10^{-28} e \cdot cm$, which is the magnitude of nEDM predicted by many models in BSM

- To constrain BSM models using experimental value (or bound) of d_N, one needs corresponding QCD matrix elements

- One of the leading effective CPV Lagrangian term is

$$\mathcal{L}_{CPV}^{qEDM} = -\frac{i}{2} \sum_{q=u,d,s,\ldots} d_q \bar{q} \sigma_{\mu\nu} \gamma_5 q F^{\mu\nu}$$

- Neutron EDM from qEDM

$$d_N^{qEDM} = d_u g_T^u + d_d g_T^d + d_s g_T^s + \cdots$$

Flavor-diagonal Tensor charges - Current Status

	g_T^u	g_T^d	g_T^s
PNDME '18	0.784(28)(10)	-0.204(11)(10)	-0.0027(16)
ETMC '20	0.729(22)	-0.208(8)	-0.0027(6)

- PNDME '18 results are extrapolated to continuum limit from seven (six) ensembles for the strange (light) quarks: $M_\pi \approx 135, 220, 310$ MeV, $a \approx 0.06, 0.09, 0.12, 0.15$ fm
- ETMC '20 results are from a single ensemble at $M_\pi = 139$ MeV, $a = 0.08$ fm
- Charm-quark results (g_A^c) can be found at
  - PNDME: Rui Zhang's talk at Lattice 2021; ETMC: PRD 102, 054517 (2020)

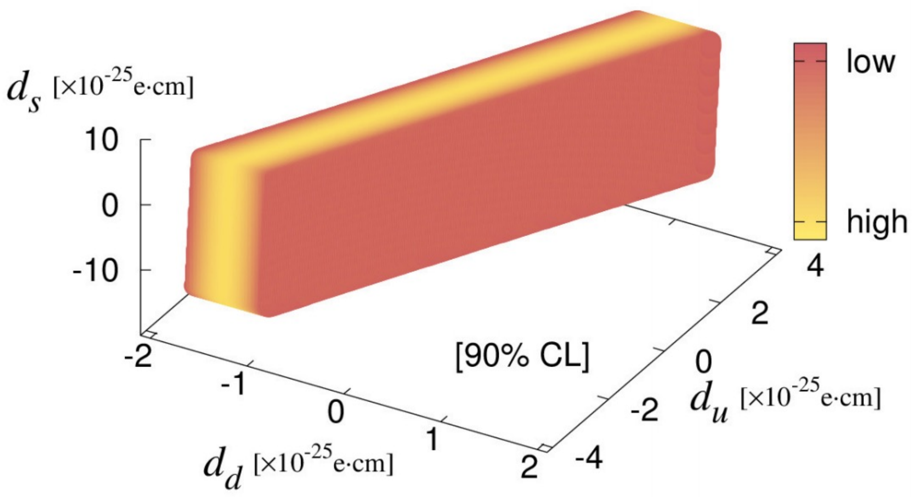
BSM Constraints from Tensor Charge

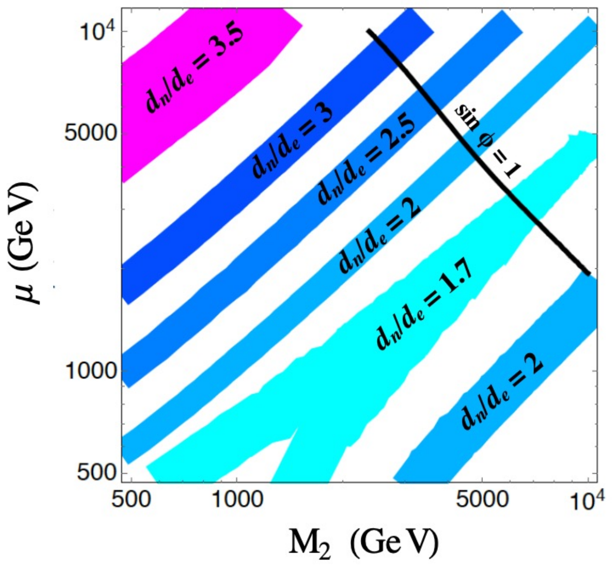
- For models in which qEDM is the dominant BSM source of CP violation

$$d_N = d_u g_T^u + d_d g_T^d + d_s g_T^s + \ \ldots$$

- Known parameters
  - $g_T^u = 0.784(28)(10), \ \ g_T^d = -0.204(11)(10), \ \ g_T^s = -0.0027(16)$
  - $|d_N| < 1.8{\times}10^{-26} e \cdot cm$ [nEDM experiment, PRL. 124, 081803 (2020)]

⇒ Constraints on BSM couplings d_u, d_d, d_s (left), and allowed $M_2 - \mu$ region for various values of d_n/d_e in split SUSY model (right)





PRL 115, 212002 (2015)
PRD 98, 091501 (2018)

Flavor-diagonal Scalar Charge: $g_S^{u,d,s}$

- Nucleon σ-terms

$$\sigma_{\pi N} = m_{ud}\langle N|\bar{u}u + \bar{d}d|N\rangle = m_{ud}(g_S^u + g_S^d \equiv g_S^{u+d})$$
$$\sigma_s = m_s\langle N|\bar{s}s|N\rangle = m_s g_S^s$$

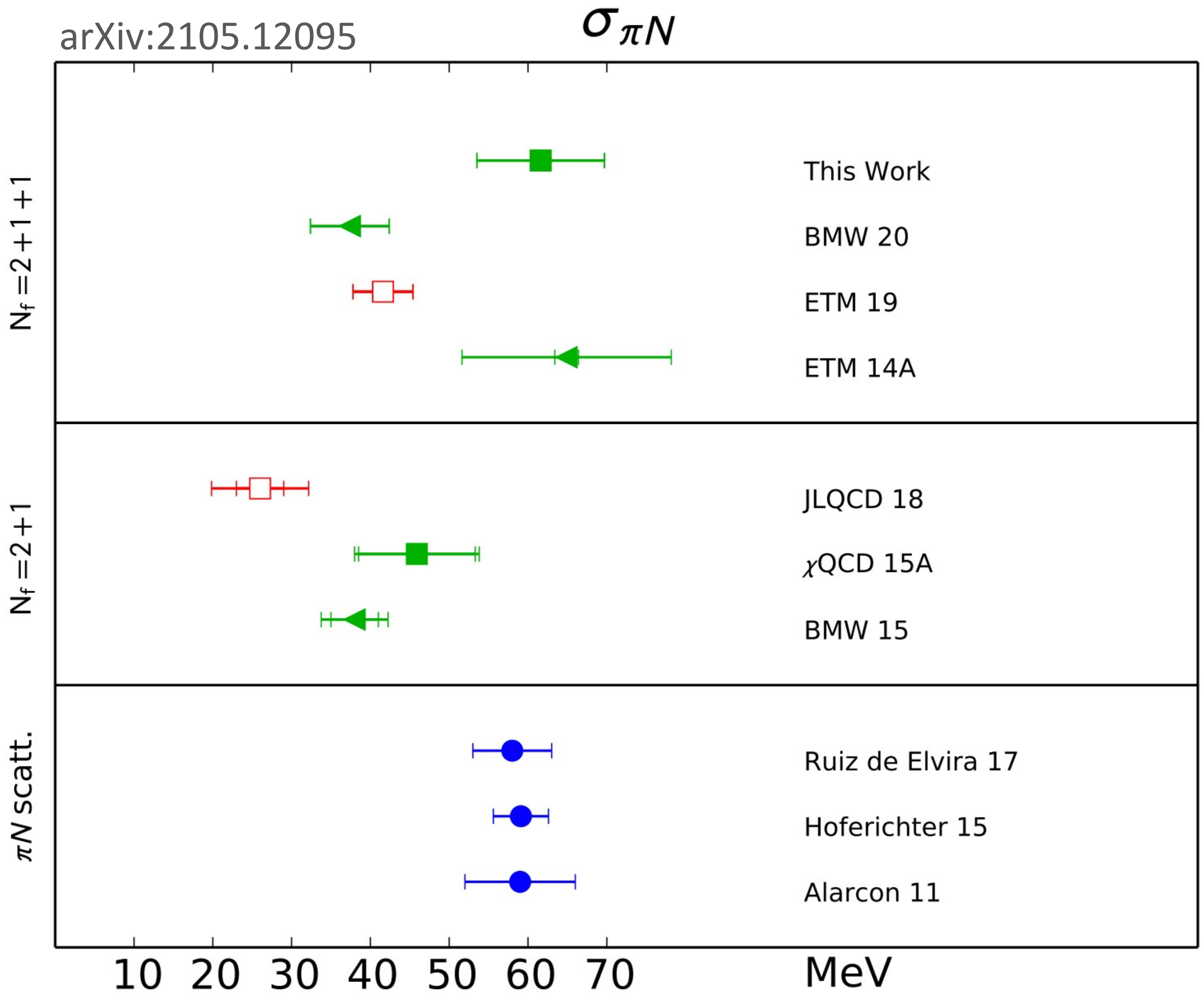
- Critical to include $N\pi$ excited-state (ES) terms to remove ES contamination

- Results with $N\pi$ ES-terms gives consistent chiral fit coefficients with χPT predictions

chiral fit	$\{4, 3^*\}$						$\{4^{N\pi}, 3^*\}$					
	d_2 (GeV^{-1})	d_3 (GeV^{-2})	d_4 (GeV^{-3})	d_{4L} (GeV^{-3})	$\frac{\chi^2}{\text{dof}}$	$\sigma_{\pi N}$ (MeV)	d_2 (GeV^{-1})	d_3 (GeV^{-2})	d_4 (GeV^{-3})	d_{4L} (GeV^{-3})	$\frac{\chi^2}{\text{dof}}$	$\sigma_{\pi N}$ (MeV)
χPT	4.44	−8.55	–	11.35	–	–	4.44	−8.55	–	11.35	–	–
$\{2, 3\}$	2.48(29)	−2.8(1.1)	–	–	0.68	38.3(2.8)	3.74(35)	−6.7(1.2)	–	–	1.48	51.6(3.5)
$\{2, 3, 4\}$	3.1(1.2)	−8(10)	10(20)	–	0.82	40.3(4.7)	6.7(1.6)	−31(13)	47(25)	–	0.84	61.6(6.4)
$\{2^\chi, 3^\chi, 4\}$	4.44	−8.55	−4.07(71)	–	12.2	58.50(24)	4.44	−8.55	−1.72(64)	–	2.30	59.28(21)
$\{2, 3^\chi, 4, 4L\}$	3.14(73)	−8.55	11(19)	−0.5(11.7)	0.82	40.4(4.7)	5.57(96)	−8.55	45(23)	26(15)	0.77	61.7(6.3)
$\{2, 3^\chi, 4, 4L^\chi\}$	3.86(17)	−8.55	29.6(2.2)	11.35	0.87	44.5(2.4)	4.65(21)	−8.55	21.7(2.4)	11.35	0.82	56.2(3.0)
$\{2^\chi, 3^\chi, 4, 4L\}$	4.44	−8.55	42.2(6.3)	19.8(2.7)	1.41	48.4(1.4)	4.44	−8.55	18.5(7.2)	8.9(3.2)	0.93	54.5(1.7)
$\{2^\chi, 3^\chi, 4, 4L^\chi\}$	4.44	−8.55	22.51(71)	11.35	3.13	52.72(24)	4.44	−8.55	23.97(64)	11.35	0.86	53.20(21)

arXiv:2105.12095

Current status of $\sigma_{\pi N}$ from Lattice QCD



- Phenomenological estimates using πN-scattering data gives
  $\sigma_{\pi N} \sim 60$ MeV
- Lattice results without $N\pi$ ES-terms gives
  $\sigma_{\pi N} \sim 40$ MeV
- Lattice result with $N\pi$ ES-terms gives
  $\sigma_{\pi N} = 61.6(6.4)$ MeV

Summary

- Nucleon charges play important role in analysis of experimental data and probing new physics in BSM
- Lattice QCD provides precise estimates of the nucleon charges
- Removing excited state contamination may need proper incorporation of $N\pi$ excited states
  - Current statistics does not provide a good determination of the first excited state, but indirect evidences (PCAC relation, $\chi\mathrm{PT}$ prediction) support strong effect of $N\pi$ excited states