Short-distance constraints for HLbL in $(g-2)_\mu$

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A Virtual Tribute to Quark Confinement and the Hadron Spectrum 2021
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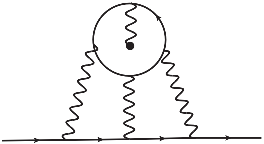
HLbL: a multiscale problem

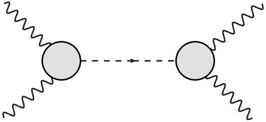


$$\Pi^q \sim \langle 0 | T\left(\prod_j^4 \int dx_j e^{-iq_j x_j} J^q(x_j)\right) e^{iS_{\rm int}} |0\rangle$$

$$J_q^\mu = Q_q \, \bar{q}\gamma^\mu q$$

$$a_\mu^{\rm HLbL} \sim \int_0^\infty dQ_{1,2,3} \sum_i T_i' \, \overline{\Pi}_i$$

- Π^q? A nonperturbative problem
- Weights T_i' enhance contributions near $Q_i \sim m_\mu$

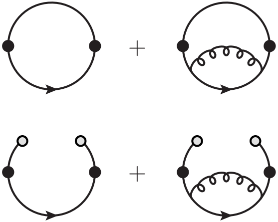


- From dispersion relations to resonance models using SD constraints Melnikov-Vainshtein, Brodsky-Lepage Talk by M. Hoferichter
- Basic question: asymptotic expansion of $(g-2)_\mu$ HLbL?

Operator Product Expansion (OPE)

Asymptotic behaviour of two-point correlation functions

$$\Pi(q) = \int dx\, e^{-iqx} \langle 0 | T(J_1(x) J_2(0) | 0 \rangle; \;\; J_i \sim \bar{q} \Gamma_i q$$



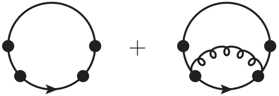
$$+ \ldots = c(\mathrm{pert})$$

$$+ \ldots = c_{qq} \langle m_q \bar{q} q \rangle$$

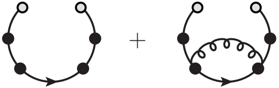
$$\Pi(Q) = \sum_{i,D} \frac{c_{i,D}(Q^2,\mu) \langle \mathcal{O}_{i,D}(\mu) \rangle}{Q^D}$$ Nucl.Phys.B 147 385-447

HLbL for $g-2$. Same procedure?

$$\Pi^{\mu_1\mu_2\mu_3\mu_4} = -i\int \frac{d^4q_3}{(2\pi)^4}\left(\prod_i^4 \int d^4x_i\, e^{-iq_ix_i}\right)\langle 0|\, T\left(\prod_j^4 J^{\mu_j}(x_j)\right)|0\rangle$$



$$+\ldots = c(\text{pert})$$

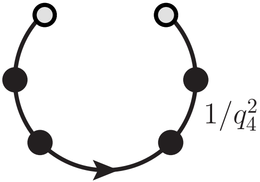


$$+\ldots = c_{qq}\langle m_q\bar{q}q\rangle$$

$$\Pi \sim \sum_{i,D} \frac{c_{i,D}(Q_i^2,\mu)\langle\mathcal{O}_{i,D}(\mu)\rangle}{Q_1^{d_1}\ldots Q_4^{d_4}} \qquad \sum_i d_i = D$$

HLbL for $g-2$. Same procedure?

- $\Pi \sim \sum_{i,D} \frac{c_{i,D}(Q_i^2,\mu)\langle \mathcal{O}_{i,D}(\mu)\rangle}{Q_1^{d_1}\ldots Q_4^{d_4}}$ $\qquad \sum_i d_i = D$
- Static limit: $\lim_{q_4\to 0} \frac{\partial \Pi^{\mu_1\mu_2\mu_3\nu_4}}{\partial q_{4,\,\mu_4}}$
- $\lim_{q_4\to 0} \Pi^{\mathrm{OPE}}$?



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- OPE only valid for large Euclidean Momenta!

Rethinking the problem: soft static photon

$$\langle 0|e^{iS}|\gamma_1\gamma_2\gamma_3\gamma_4\rangle \to \Pi^{\mu_1\mu_2\mu_3\mu_4}$$

One step backwards

$$\Pi^{\mu_1\mu_2\mu_3} \sim \int \frac{d^4q_3}{(2\pi)^4}\left(\prod_i^3 \int d^4x_i\, e^{-iq_ix_i}\right)\langle 0|\,T\left(\prod_j^3 J^{\mu_j}(x_j)\right)e^{iS_{\rm int}}|\gamma_E(q_4)\rangle$$

- $Q_{1,2,3} \gg \Lambda_{\rm QCD} \to$ OPE valid for the tensor
- We are looking for a static (soft) photon contribution: $F^{\mu\nu}$
- From the OPE keep those operator contributions with the same quantum numbers as the static photon, $F^{\mu\nu}$

  Nucl.Phys.B 232 109-142, Phys.Lett.B 129 328-334, Phys.Rev.D 67 073006

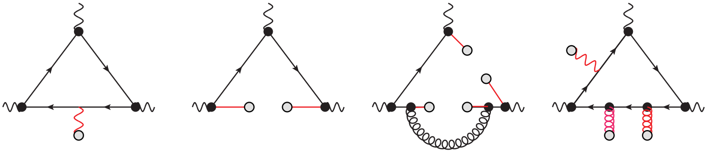
OPE with background photon

$$
\begin{aligned}
S_{1,\,\mu\nu} &\equiv e\, e_q F_{\mu\nu} \\
S_{2,\,\mu\nu} &\equiv \bar{q}\sigma_{\mu\nu} q \\
S_{3,\,\mu\nu} &\equiv i\ \bar{q}\, G_{\mu\nu} q \\
S_{4,\,\mu\nu} &\equiv i\, \bar{q}\, \bar{G}_{\mu\nu}\gamma_5 q \\
S_{5,\,\mu\nu} &\equiv \bar{q}q\ e\, e_q F_{\mu\nu} \\
S_{6,\,\mu\nu} &\equiv \frac{\alpha_s}{\pi}\, G_a^{\alpha\beta} G^a_{\alpha\beta}\ e\, e_q F_{\mu\nu} \\
S_{7,\,\mu\nu} &\equiv \bar{q}(G_{\mu\lambda} D_\nu + D_\nu G_{\mu\lambda})\gamma^\lambda q - (\mu \leftrightarrow \nu) \\
S_{\{8\},\,\mu\nu} &\equiv \alpha_s\, (\bar{q}\,\Gamma\, q\ \bar{q}\Gamma\, q)_{\mu\nu}
\end{aligned}
$$

$$\Pi^{\mu_1\mu_2\mu_3}(q_1, q_2) = \frac{1}{e}\vec{C}^{\,T,\mu_1\mu_2\mu_3\mu_4\nu_4}(q_1, q_2)\langle\vec{S}_{\mu_4\nu_4}\rangle; \quad \langle S_{i,\mu\nu}\rangle = e e_q X_S^i \langle F_{\mu\nu}\rangle$$

Renormalization program in JHEP 10 (2020) 203

Contributions from different operators



- Leading order: massless quark loop
- Magnetic susceptibility $\sim \frac{m_q \Lambda_{\mathrm{QCD}}}{Q^2}$
- First massless power corrections $\sim \frac{\Lambda^4_{\mathrm{QCD}}}{Q^4}$

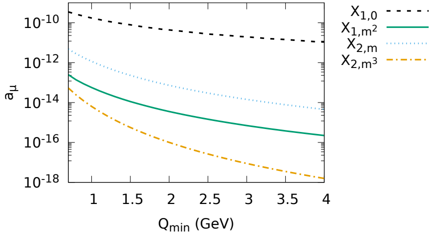
g-2 integral

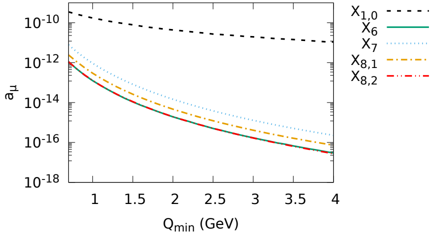
$$\Pi^{\mu_1\mu_2\mu_3}(q_1, q_2) = \vec{C}^{T,\mu_1\mu_2\mu_3\mu_4\nu_4}(q_1, q_2)\, \vec{X}\, \langle e_q F_{\mu_4\nu_4} \rangle$$

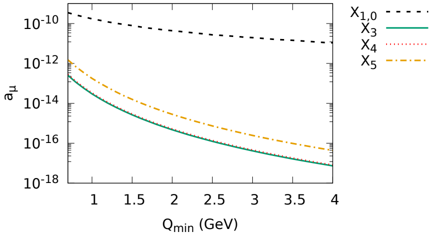
$a_\mu^{\mathrm{HLbL}} \sim \int_0^\infty dQ_{1,2} \int_{-1}^1 d\tau \sum_i T_i' \overline{\Pi}_i$ JHEP 09 (2015) 074, JHEP 04 (2017) 161

1. Build general projectors P: $P_{\mu_1\mu_2\mu_3\mu_4\nu_4} C^{\mu_1\mu_2\mu_3\mu_4\nu_4} \sim \overline{\Pi}$
2. Reduce scalar integrals KIRA, REDUZE
3. Perform the $g-2$ integral from some Q_{min} Rest Talk by M. Hoferichter

Numerical results: quark loop vs power corrections

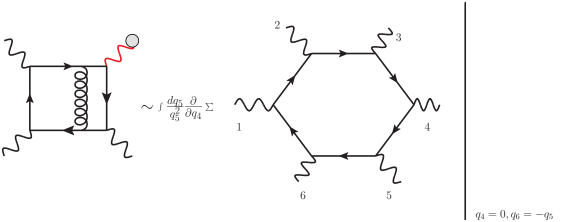




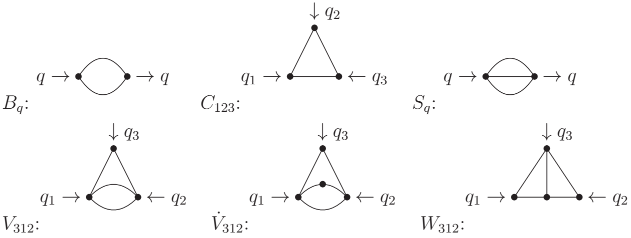


- Power corrections numerically very small above 1 GeV
- Gluonic corrections?...
- ...Let us compute them

The two loops: a symmetric sum of hexagons

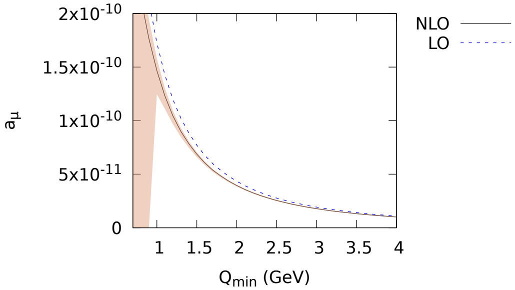


1. Build general projectors P: $P_{\mu_1\mu_2\mu_3\mu_4\nu_4} C^{\mu_1\mu_2\mu_3\mu_4\nu_4} = \overline{\Pi}$
2. Reduce $\sim \mathcal{O}(10^{3,4})$ scalar integrals (d dimensions) KIRA



Two loop: results

3. Master integrals known in terms of classical polylogs: analytic result for the HLbL tensor. Typically $\sim -\frac{\alpha_s}{\pi}$
4. Integrate (analytic expansions can improve precision) from Q_{min}



Above $\sim 1 - 2\,\text{GeV}$, gluonic corrections small and negative

Conclusions

- A systematic OPE with a background photon field can give a description of HLbL $g-2$ for large loop momenta
- The massless quark loop is the leading term
- Power corrections have been computed and found to be small
- Perturbative corrections are found small and negative
- Precise systematic expansion valid above $1-2\,\mathrm{GeV}$

Thank you