Controlling systematic uncertainties of IAM unitarization

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A VIRTUAL TRIBUTE TO
QUARK CONFINEMENT
AND THE HADRON SPECTRUM
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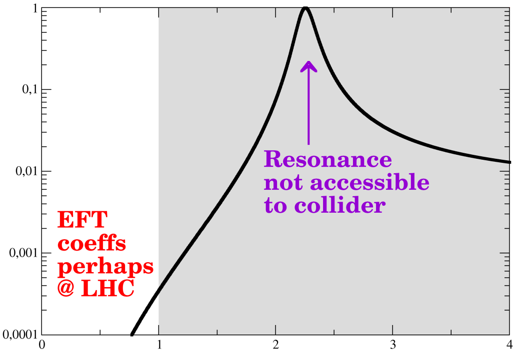
Is the LHC a high- or a low- energy machine?



Expand partial wave amplitudes

$$T_I(s,t,u) = 16\eta\pi \sum_{J=0}^{\infty} (2J+1) t_{IJ}(s) P_J(\cos\theta_s)$$

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$$t_{IJ}(s) \simeq \underbrace{t_0}_{O(s)} + \underbrace{t_1}_{O(s^2)} + \dots$$

(typical HEFT expansion)

Inverse Amplitude Method

$$\frac{1}{t} \simeq \frac{1}{t_0 + t_1} \simeq \frac{1}{t_0} - \frac{t_1}{t_0^2} \implies \boxed{t^{IAM} \simeq \frac{t_0^2}{t_0 - t_1}}$$

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Advantage: for $s > s_{\text{th}}$,

$$\text{Im}\frac{1}{t_{IJ}(s)} = -\sigma(s) \simeq -1$$

Perturbative vs exact (elastic) unitarity

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Perturbative vs exact (elastic) unitarity

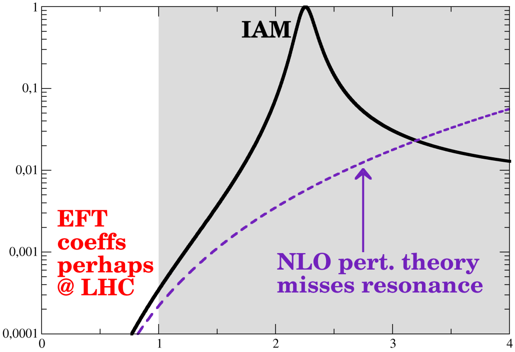
$$\text{Im}\, t_{IJ}(s) = \sigma(s)|t_{IJ}(s)|^2$$

- Exact in IAM
- Only order by order in EFT

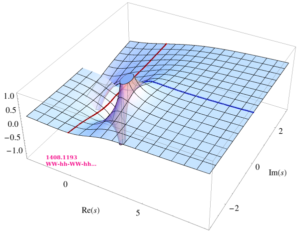
$$\text{Im}\, t_1(s) = \sigma(s)|t_0(s)|^2$$

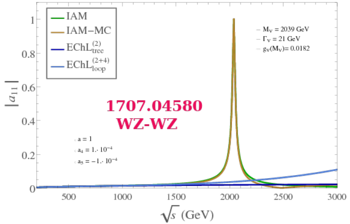
Why would anyone care?

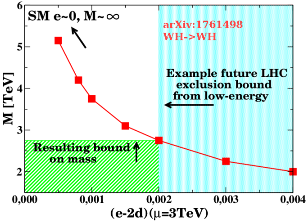
- EFT reliable only near threshold



Prediction of resonances from HEFT

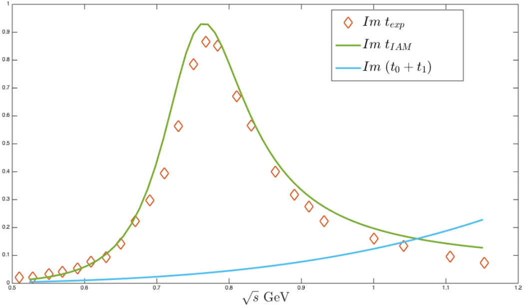






LHC bounds on HEFT coeffs $\Longrightarrow$ bounds on new physics scale

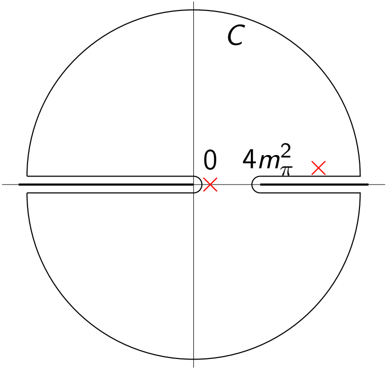
Much used in hadron physics to obtain resonances



(This is an IAM prediction from threshold data, not a fit)

Use its dispersive derivation: 2010.13709

- Causality $\implies$ analiticity
- Large circumference convergence $G \propto e^{-s}$ (1912.08747)
- Can apply Cauchy's theorem



to the function $t_0^2(s')/t(s')(s-s')s'^3$.

Master formula is a dispersion relation for $G(s) \equiv \frac{t_0^2(s)}{t(s)}$

$$
\begin{aligned}
G(s) = G(0) + G'(0)s + \frac{1}{2}G''(0)s^2 + PC(G) + \\
+ \frac{s^3}{\pi}\int_{RC} ds' \frac{\operatorname{Im} G(s')}{s'^3(s'-s)} + \\
+ \frac{s^3}{\pi}\int_{LC} ds' \frac{\operatorname{Im} G(s')}{s'^3(s'-s)}
\end{aligned}
$$

Dispersion relation: approximations

NLO subtraction constants

Neglected

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$$+ \frac{s^3}{\pi}\int_{LC} ds' \frac{\text{Im}\, G(s')}{s'^3(s'-s)}$$

NLO imaginary part Im G ⟶ -Im t1

Gives $\boxed{t \simeq t_0^2/(t_0 - t_1) = t_{IAM}}$.

Sources of uncertainty

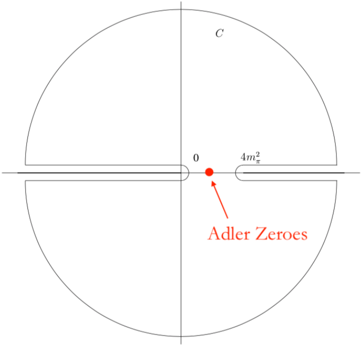
- Neglected pole contributions of t^{-1}:
  subthreshold Adler zeroes and CDD zeroes of t.

- Inelasticities due to KK (hh in HEFT), 4π, etc.

- $\mathcal{O}(p^4)$ truncation of subtraction constants.

- Left cut approximation $Im\ G \simeq -Im\ t_1$.

Adler zeroes of t near threshold



$$t_0 + t_1 = a + bs + cs^2$$

vanishes near $s = -a/b$

Adler zeroes of t near threshold

Tiny uncertainty in resonance region because at/below threshold these poles are nearly cancelled.

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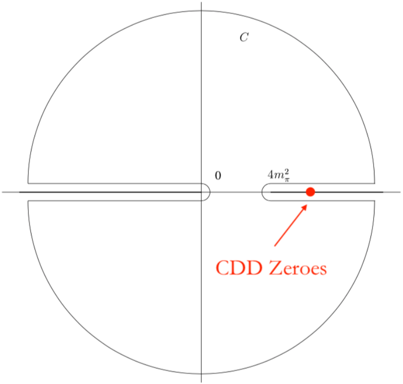
Uncertainty	Behavior	Displacement $\sqrt{s} = m_\rho$	improvable?
Adler zeroes of t	$(m_\pi/m_\rho)^4$	$10^{-3} - 10^{-4}$	Yes: mIAM

0712.2763

CDD poles (zeroes of t in resonance region)



("New" physics)

CDD poles (zeroes of t in resonance region)

Can affect a resonance calculation dramatically

Need to

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Can affect a resonance calculation dramatically

Need to

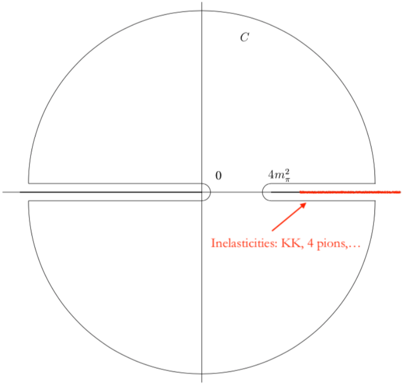
1. Check for CDD pole appearance: $t_0(s_C) + \mathrm{Re} t_1(s_C) = 0$
2. If present, modify

$$t_{\mathrm{IAM}} = \frac{t_0^2}{t_0 - t_1} \to \frac{t_0^2}{t_0 - t_1 + \frac{s}{s - s_c}\mathrm{Re}(t_1)} \; .$$

CDD poles (zeroes of t in resonance region)

Uncertainty	Behavior	Displacement m_ρ	improvable?
Adler zeroes of t	$(m_\pi/m_\rho)^4$	$10^{-3}-10^{-4}$	Yes: mIAM
CDD poles at M_0	M_R^2/M_0^2	0–$\mathcal{O}(1)$	Yes

Inelastic 2-body channels



Inelastic 2-body channels

- Hadrons: $\pi\pi \to \pi\pi$ couples to KK

$$\text{Im}\,\frac{1}{t_{\pi\pi}} \to -\sigma_{\pi\pi}\left(1 + \frac{\sigma_{K\bar{K}}}{\sigma_{\pi\pi}}\frac{|t_{\pi\pi\to K\bar{K}}|^2}{|t_{\pi\pi\to\pi\pi}|^2}\right)$$

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suppressed by phase-space $\frac{\sigma_{K\bar{K}}}{\sigma_{\pi\pi}}$ and low inelasticity in $t_{\pi\pi\to K\bar{K}}$

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- In HEFT only inelasticity in $\omega\omega - hh$ (actually zero in SM)
- We can use the coupled channel IAM directly or to estimate uncertainty in elastic IAM

Inelastic 2-body channels

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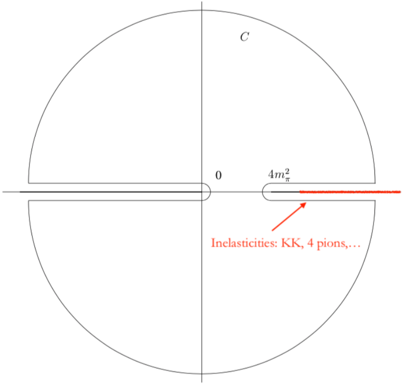
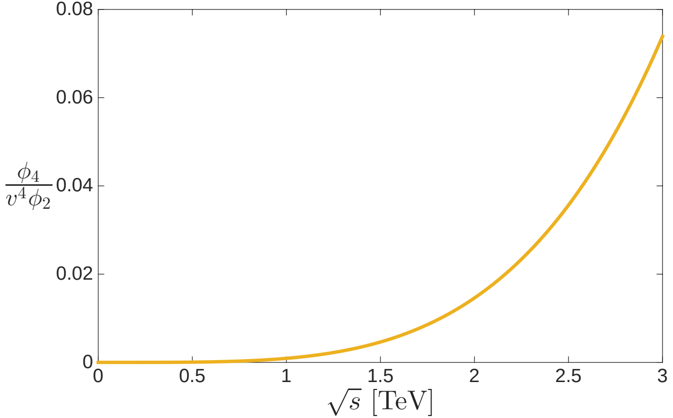
Inelastic 2-body	$(\sqrt{s}/(4\pi f_\pi))^4$	10^{-3}	Yes

Inelastic 4-body channels



- Difference with SMEFT: here, in ChPT or HEFT, additional particles *not* suppressed by the chiral counting.
  But phase space helps.

Inelastic 4-body channels



Inelastic 4-body channels

In hadron physics,
(with elastic and 4-π inelastic amplitudes taken as similar)

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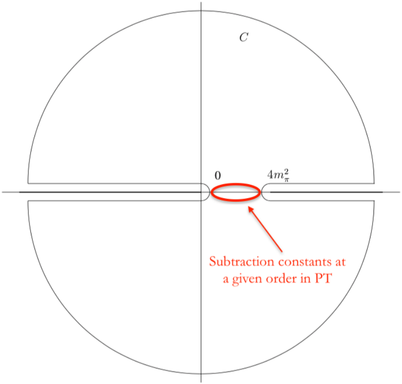
Adler zeroes of t	$(m_\pi/m_\rho)^4$	$10^{-3} - 10^{-4}$	Yes: mIAM
CDD poles at M_0	M_R^2/M_0^2	0–$\mathcal{O}(1)$	Yes
Inelastic 2-body	$(\sqrt{s}/(4\pi f_\pi))^4$	10^{-3}	Yes
Inelastic 4-body	$(\sqrt{s}/(4\pi f_\pi))^4$	10^{-4}	No

$O(p^4)$ truncation



Estimate based on size of NNLO counterterms ($\implies$ subtraction constants) from Resonance Effective Field Theory

$O(p^4)$ truncation

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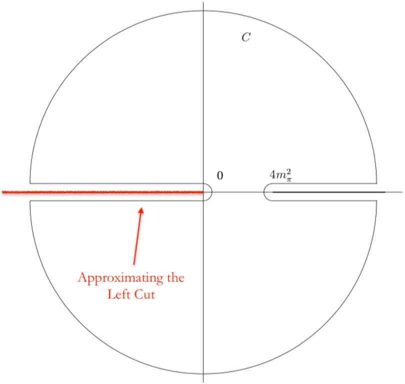
$O(p^4)$ truncation	$(\sqrt{s}/(4\pi f_\pi))^4$	10^{-2}	Yes

$$G(s) = \frac{t_0^2}{t} \simeq t_0 - t_1 - t_2 + \frac{t_1^2}{t_0}$$

Approximate left cut



Need to check

$$\int_{LC} ds' \frac{\text{Im}\, G + \text{Im}\, t_1}{s'^3(s'-s)} \; .$$

i.e., failure of IAM's

$$Im\ G = -Im\ t_1$$

over the left cut

Approximate left cut

Split interval in 3:

- Low-$|s|$ (ChPT/HEFT ✓) $|s|^{\frac{1}{2}} < 470\,\mathrm{MeV}$.
- Intermediate-$|s|$: Match to ChPT + natural-size counterterm. Currently studying LC parameterizations from GKPY eqns.
- High -$|s|$: Sugawara-Kanazawa relates it to right cut: Regge asymptotics there. Far from RC anyway.

Approximate left cut

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Inelastic 2-body	$(\sqrt{s}/(4\pi f_\pi))^4$	10^{-3}	Yes
Inelastic 4...-body	$(\sqrt{s}/(4\pi f_\pi))^4$	10^{-4}	No
$O(p^4)$ truncation	$(\sqrt{s}/(4\pi f_\pi))^4$	10^{-2}	Yes
Left Cut	$(\sqrt{s}/(4\pi f_\pi))^4$	0.17	Perhaps

Conclusion: if you know your EFT...

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- We have laid out (2010.13709 [hep-ph])
  its systematic theory uncertainties

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- Inverse Amplitude Method extends it to
  first resonance or $4\pi F$ or new: first zero (CDD-IAM)
- We have laid out (2010.13709 [hep-ph])
  its systematic theory uncertainties
- To make it more useful for BSM searches

Thank You!

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