Physics of warped dimensions and continuum spectra

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Other references: A. Falkowski, M. Pérez-Victoria PRD79 (2009); E.M., O.Pujolàs JHEP1408(2014) 081; E.M., G.Panico, O.Pujolàs, M.Quirós, JHEP1609(2016) 118; E.M., O.Pujolàs, M.Quirós, JHEP1605(2016) 137.

Issues

1. The extradimensional model
   - General formalism
   - The soft-wall model
2. Gapped continuum spectra
   - Green's functions for gauge bosons
   - Resonances
   - Spectral functions
   - Regularized continuum model
3. Phenomenological aspects
4. Conclusions

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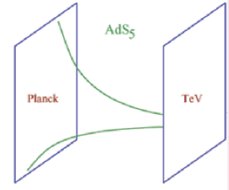
Introduction

Randall-Sundrum model in warped extra-dimension

- Proposed in 1999 by Randall and Sundrum (RS) **[PRL83, 3370 '99]**
- It was based on a 5D space-time with line element

$$ds^2 = e^{-2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2\,, \qquad A(y) = ky\,,$$

- and two branes:



$$\mathrm{TeV} = e^{-ky_1}M_{\mathrm{Planck}}\,, \qquad ky_1 \sim 35\,.$$

Higgs boson profile: $h(y) \propto e^{aky}$, $a > 2$.

AdS ⇔ CFT correspondence

- Heavy (light) fermions are mainly localized at the IR (UV) brane: composite (elementary).
- KK modes: $m_{\mathrm{KK}} \sim \mathrm{TeV} \ll M_{\mathrm{Planck}}$ ➞ Solve the hierarchy problem.
- Brane distance stabilized by a bulk scalar field ϕ with bulk/brane potentials fixing its VEVs **[W. Goldberger, M. Wise, PRD60, 107505 '99]**.

Issues

The Model

[Cabrer, Gersdorff, Quirós, (2010)]

- Scalar-gravity system with UV and IR branes:

$$S = \int d^5x \sqrt{|\det g_{MN}|} \left[-\frac{1}{2\kappa^2} R + \frac{1}{2} g^{MN} (\partial_M \phi)(\partial_N \phi) - V(\phi) \right] - \sum_\alpha \int_{B_\alpha} d^4x \sqrt{|\det \bar{g}_{\mu\nu}|} \lambda_\alpha(\phi) + S_{\mathrm{GHY}}$$

- Metric: $ds^2 = g_{MN} dx^M dx^N \equiv \underbrace{e^{-2A(y)} \eta_{\mu\nu}}_{\bar{g}_{\mu\nu}} dx^\mu dx^\nu - dy^2$.
- $V(\phi)$ bulk potential.
- λ_α $(\alpha = 0, 1) \equiv$ UV, IR 4-dim brane potentials at $(y(\phi_0), y(\phi_1))$.
- $S_{\mathrm{GHY}} :=$ Gibbons-Hawking-York boundary term.
- Solve the hierarchy problem ⟶ Brane dynamics should fix (ϕ_0, ϕ_1) to get $A(\phi_1) - A(\phi_0) \approx \mathcal{O}(35) \Longrightarrow M_{\mathrm{Planck}} \simeq 10^{15} M_{\mathrm{TeV}}$.

Issues

The soft-wall model

[Cabrer et al. NJP '10], [C. Csaki et al. '19], [E.M., M. Quirós, JHEP '19 & 2106.09598]

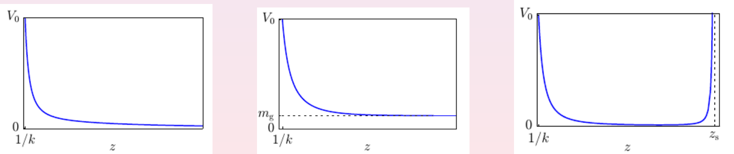
Superpotential: $W(\phi) = \frac{6k}{\kappa^2}\left(1 + e^{\nu\phi}\right)$.

- Critical case $\nu = \nu_c \equiv \kappa/\sqrt{3}$ ➞ Gapped continuum KK spectra.

$$A(z) \simeq \begin{cases} \log(kz) & 1/k \leq z \leq z_1 \\ \log(kz_1) + \rho(z - z_1) & z_1 < z < \infty \end{cases}.$$

- For $\nu > \nu_c$ ➞ discrete KK spectra with TeV spacing.
- For $\nu < \nu_c$ ➞ ungapped continuum KK spectra.

Effective potential for gauge bosons:



$\nu < \nu_c$ $\nu = \nu_c$ $\nu > \nu_c$

- $\rho \equiv k\, e^{-ky_1} \sim \mathrm{TeV}$ ➞ $A_1 \simeq 35$.
- Conformally flat coordinates: $ds^2 = e^{-2A(z)}\left(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2\right)$.

Green's functions: gapped continuum spectra

- Up to now, searches of new physics at the LHC ⟶ detection of bumps in the invariant mass of final states.
- An exploring possibility to cope with the elusiveness of signals is a continuum of Kaluza-Klein (KK) states beyond a mass gap m_g.
- ⟶ New physics associated with an excess in the measured cross section with respect to the SM prediction.
- The Green's functions generalize the particle propagators

$$\frac{1}{p^2 - m^2 + i\epsilon} = \mathcal{P}\frac{1}{p^2 - m^2} - i\pi\delta(p^2 - m^2)$$

- ... to Green's functions with an isolated pole (the zero mode) and a continuum of states with a mass gap m_g

$$G(p^2, m_g^2) = \mathrm{Re}\, G(p^2, m_g^2) + i\,\left[c_0\delta(p^2) + \eta(p^2, m_g^2)\Theta(p^2 - m_g^2)\right] .$$

- Same behavior as gapped unparticles **[M. Pérez-Victoria et al, PRD '09 & JHEP '09]**. Here $m_g \sim$TeV is linked to the solution of the *hierarchy problem*.

Issues

Massless gauge bosons

- Lagrangian for massless gauge bosons (in the gauge $A_5 = 0$):

$$\mathcal{L} = \int_0^{y_s} dy \left[-\frac{1}{4} \mathrm{tr} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} e^{-2A} \mathrm{tr} A'_\mu A'_\mu \right],$$

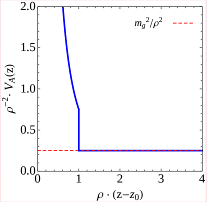
$$A_\mu(p,y) = f_A(y) A_\mu(p)/\sqrt{y_s}\,.$$

- Schrödinger like form of the EoM of the fluctuations

$$-\tilde{f}''_A(z) + V_A(z)\tilde{f}_A(z) = p^2 \tilde{f}_A(z)\,, \qquad [f_A(z) = e^{A(z)/2}\tilde{f}_A(z)]\,,$$

with potential

$$V_A(z) = \tfrac{1}{4}A'(z)^2 - \tfrac{1}{2}A''(z)\,, \qquad V_A(z) \underset{z>z_1}{\longrightarrow} m_g^2 = \left(\tfrac{\rho}{2}\right)^2.$$



➡ Existence of a mass gap.
➡ Continuum of states above the mass gap.

Massless gauge bosons

- The *Green's function* is given by

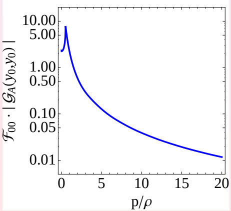
$$G_A^{\mu\nu}(y,y';p) = [\eta^{\mu\nu} - (1-\xi)p^\mu p^\nu/p^2]G_A(y,y';p)\,.$$

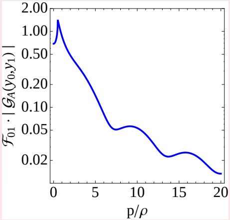
- EoM for the Green's function:

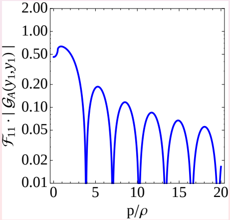
$$p^2 G_A(y,y';p) + \partial_y\left(e^{-2A(y)}\partial_y G_A(y,y';p)\right) = \delta(y-y')\,.$$

- All Green's functions include the zero-mode contribution:

$$G_A^0 = \frac{1}{y_s p^2} = \lim_{p\to 0} G_A(y,y';p) \quad \longrightarrow \quad \underline{\text{Define:}} \quad \mathcal{G}_A(y,y';p) = G_A(y,y';p) - G_A^0\,.$$







Issues

Green's functions in the complex plane: resonances

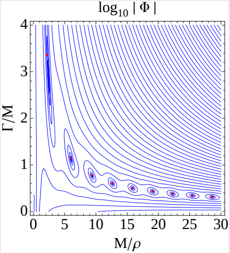
- Complex plane ($s \equiv p^2$):

$$s = M^2 - iM\Gamma = M^2(1 - ir), \qquad (r \equiv \Gamma/M)\,.$$

- Riemann sheets:

$$\begin{cases} \text{1st Riemann sheet} & \delta_A^{\mathrm{I}} = +\sqrt{1 - s/m_g^2} \\ \text{2nd Riemann sheet} & \delta_A^{\mathrm{II}} = -\sqrt{1 - s/m_g^2} \end{cases}.$$

- $G_A(y, y'; p) \propto \frac{1}{\Phi(p)}$ and $\Phi(p)|_{\text{2nd Riemann}} \propto \left[e^{i2p/\rho} - 8i(p/\rho)^2\right]$



→ Zeros of $\Phi(p) \equiv$ Poles of $G_A(y, y'; p)$:

$$\frac{s}{\rho^2} \simeq -\mathcal{W}_n\left[\pm\frac{1}{4}(1+i)\right]^2, \quad n = -1, -2, \cdots$$

$$\mathcal{W}_n \equiv \text{Lambert function}$$

Issues

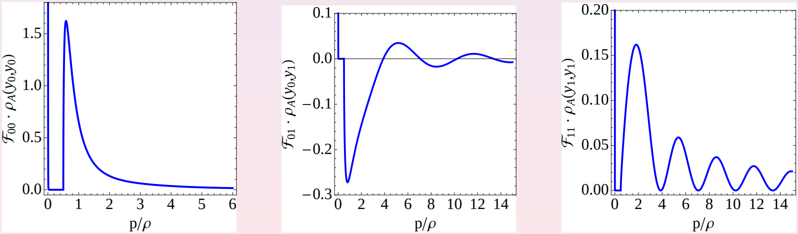
Spectral function (soft-wall model)

- Spectral function:

$$\rho_A(y,y';s) = -\frac{1}{\pi}\mathrm{Im}\, G_A(y,y';s+i\epsilon)\,, \qquad s \equiv p^2\,.$$

- Zero mode + continuum:

$$\rho_A(y,y';s) = \frac{1}{y_s}\,\delta(s) + \eta_A(s)\Theta(s-m_g^2)\,.$$



- Kramers-Kronig relations: $\mathrm{Re}G_A \Longleftrightarrow \mathrm{Im}G_A$ ➞
  ➞ All the information of G_A is in ρ_A.

Spectral function: positivity (RS model)

- The function $\rho_A(y, y')$ can be understood as a matrix element of a spectral operator **[L.L. Salcedo, Private communication]**

$$\hat{\rho}_A = -\frac{1}{\pi}\mathrm{Im}\ \hat{G}_A\,, \qquad \text{where} \qquad \mathrm{Im}\ \hat{G}_A = \frac{1}{2i}\left(\hat{G}_A - \hat{G}_A^\dagger\right),$$

$$\rightarrow \qquad \rho_A(y, y') = \langle y|\hat{\rho}_A|y'\rangle \equiv \rho_{yy'}\,.$$

- $\hat{\rho}_A$ is positive semidefinite.
- In the RS model

$$\lambda_{\mathrm{RS}}(p) \equiv \mathrm{tr}\,\hat{\rho}_{A,\mathrm{RS}} = \int_0^{y_1} dy\,\rho_{A,\mathrm{RS}}(y, y; p) = \sum_n \delta(p^2 - m_n^2) \geq 0\,,$$

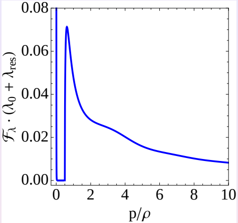
is interpreted as the *density of states*.

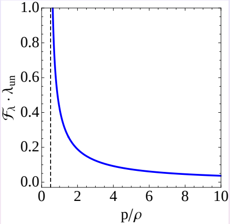
- The integral of $\lambda_{\mathrm{RS}}(s)$:

$$\int_0^\infty ds\,\lambda_{\mathrm{RS}}(s) = \int_0^\infty ds \sum_n \delta(s - m_n^2) = N_{\mathrm{states}} \to \infty\,,$$

is the *number of states*.

Spectral function: positivity (soft-wall model)





- $\lambda(p)$ has several contributions:

$$\lambda(p) = \lambda_0(p) + \lambda_{\rm resonant}(p) + \lambda_{\rm unparticles}(p)\,,$$

where

$$\begin{aligned}\lambda_0(p) &= \delta(p^2)\,,\\ \lambda_{\rm resonant}(p) &= \xi_A(p)\Theta(p^2 - m_g^2)\,,\\ \lambda_{\rm unparticles}(p) &= -\frac{\log(\rho\epsilon)}{2\pi\rho\sqrt{p^2 - m_g^2}}\Theta(p^2 - m_g^2)\,.\end{aligned}$$

See e.g. [A.Delgado, J.R. Espinosa, J.No, M.Quirós, PRD79 '09] for unparticles.

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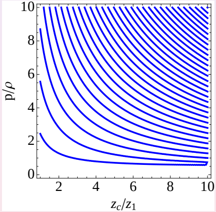
Regularized continuum model

[E.M., M. Pérez-Victoria, M. Quirós, to appear '21]

- Spectrum: $z \in [z_0, z_c]$ ➞ Discrete. $z \in [z_0, \infty]$ ➞ Continuum.
- *'Discrete'* $\longrightarrow$ *'Continuum'* by considering $z \in [z_0, z_c]$ with

$$z_0 < z_1 < z_c \quad \text{and} \quad z_c \to \infty\,.$$



$$\frac{p_n^2}{\rho^2} \simeq \frac{1}{4} + \left[\frac{z_1}{z_c}\pi\left(n - \frac{1}{4}\right)\right]^2, n = 1, 2, 3, \cdots$$

(Eigenvalues tighten as z_c increases).

- Green's function 'G_A' and spectral density 'σ':

$$G_A(z, z'; s) = \sum_n \frac{1}{||f_n||^2} \frac{f_n(z) f_n^*(z')}{s - p_n^2 + i\epsilon} \xrightarrow[z_c \to \infty]{} \int_0^\infty dp^2 \sigma(p^2) \frac{f_{p^2}(z) f_{p^2}^*(z')}{s - p^2 + i\epsilon},$$

$$\lambda(p^2) = ||f_{p^2}||^2 \sigma(p^2) \simeq \delta(p^2) + \frac{z_c}{2\pi\sqrt{p^2 - m_g^2}} \Theta(p^2 - m_g^2) \xrightarrow[z_c \to \infty]{} \infty\,.$$

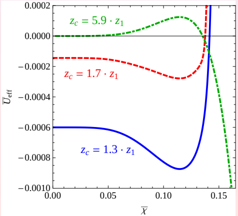
Radion effective potential

[E.M., M. Pérez-Victoria, M. Quirós, to appear '21] [Preliminary]

- Goldberger-Wise mechanism **[W. Goldberger, M. Wise PRD '99]** → Spontaneous breaking of conformal invariance.
- It then appears a *"light state"*: the radion/dilaton with interesting Higgs-like phenomenology **[C.Csaki et al., PRD63, 065002 '01]**.
- In the Linear Dilaton Model: $A(z) = \rho(z - z_0)\,;\ z \in [z_0, z_c]$. Action:

$$S_{\text{on-shell}} = S_{\text{bulk}} + S_{\text{brane}} + S_{\text{GHY}} = -\int d^4x\, U_{\text{eff}}\,.$$

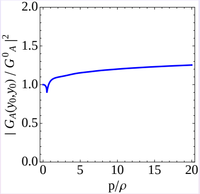
$$U_{\text{eff}}(\bar{\chi}) = \left[e^{-4A}(W + \Lambda_1)\right]_{z_c} + \left[e^{-4A}(-W + \Lambda_0)\right]_{z_0} \simeq \frac{1}{2} m_{\text{rad}}^2 (\bar{\chi} - \bar{\chi}_{\text{min}})^2$$



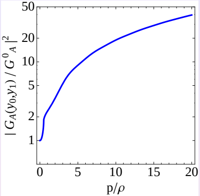
Light radion field: $\bar{\chi}(z_1) \simeq e^{-A(z_1)}$.

$U_{\text{eff}}(\bar{\chi})$ becomes unstable as $z_c \to \infty$ ($m_{\text{rad}}^2 < 0$).

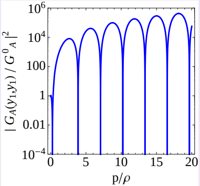
Brane-to-brane Green's functions



Little enhancement



Strong enhancement



Strong enhancement

- $G_A(y_0, y_{0,1}; p^2)$ ➞ production of fermions by Drell-Yan via gluon KK continuum ($p = \sqrt{\hat{s}} \equiv$ partonic c.o.m. energy):

$$\sigma(q\bar{q} \to g^* \to f_{UV}\bar{f}_{UV})/\sigma_{SM}(q\bar{q} \to g^{(0)} \to f_{UV}\bar{f}_{UV}) = |G_A(y_0, y_0; \hat{s})/G^0_A|^2 \,.$$

$$\sigma(q\bar{q} \to g^* \to f_{IR}\bar{f}_{IR})/\sigma_{SM}(q\bar{q} \to g^{(0)} \to f_{IR}\bar{f}_{IR}) = \underbrace{|G_A(y_0, y_1; \hat{s})/G^0_A|^2}_{\text{enhancement}} \,.$$

$$\sigma(f_{IR}\bar{f}_{IR} \to g^* \to f_{IR}\bar{f}_{IR}) \propto |G_A(y_1, y_1; \hat{s})|^2 \Longrightarrow R_{D^{(*)}}\,\text{anomalies}\,.$$

$g^* \equiv$ contribution from the gluon continuum. $g^{(0)} \equiv$ the SM gluon.
$q \equiv$ light fermions localized in the UV brane.
$f_{UV}/f_{IR} \equiv$ light/heavy fermions localized in the UV/IR brane.

Conclusions

- We have studied 5D warped models solving the hierarchy problem:
  - AdS_5 near the UV brane.
  - Linear dilaton theory near the IR singularity.
- The KK spectra of all particles (gauge bosons, fermions, graviton, radion, Higgs boson) are continua of states with a mass gap.
- We have computed the Green's functions $G(y, y'; p)$ and spectral functions $\rho(y, y'; p)$:
  - Gauge bosons $\Longrightarrow$ resonance effects at tree level.
  - Positivity of the spectral function.
  - Connection with unparticles.
- The existence of a continuum spectrum should modify the present searches of new physics.
  - Brane-to-brane Green's functions.
  - Increase in the cross section $\sigma(pp \to Q\bar{Q})$.
- Other phenomenological applications should be inspired on *unparticle phenomenology*.

Thank You!