Physics of warped dimensions and continuum spectra

Eugenio Megías

Manuel Pérez-Victoria, Mariano Quirós

1 Departamento Física Atómica, Molecular y Nuclear & Instituto Carlos I de Física Teórica y Computacional, Universidad de Granada, Spain.
2 Departamento Física Teórica y del Cosmos & CAFPE, Universidad de Granada, Spain.
3 Institut de Física d’Altes Energies (IFAE) and BIST, Barcelona, Spain.

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The extradimensional model
- General formalism
- The soft-wall model

Gapped continuum spectra
- Green’s functions for gauge bosons
- Resonances
- Spectral functions
- Regularized continuum model

Phenomenological aspects

Conclusions
**Issues**

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   - General formalism
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Introduction

Randall-Sundrum model in warped extra-dimension

- Proposed in 1999 by Randall and Sundrum (RS) [PRL83, 3370 ’99]
- It was based on a 5D space-time with line element
  \[ ds^2 = e^{-2A(y)} \eta_{\mu\nu} dx^\mu dx^\nu - dy^2, \quad A(y) = ky, \]

and two branes:

\[ \text{TeV} = e^{-ky_1} M_{\text{Planck}}, \quad ky_1 \sim 35. \]

Higgs boson profile: \( h(y) \propto e^{aky}, \quad a > 2. \)

AdS ⇔ CFT correspondence

- Heavy (light) fermions are mainly localized at the IR (UV) brane: composite (elementary).
- KK modes: \( m_{KK} \sim \text{TeV} \ll M_{\text{Planck}} \) Solve the hierarchy problem.
- Brane distance stabilized by a bulk scalar field \( \phi \) with bulk/brane potentials fixing its VEVs [W. Goldberger, M. Wise, PRD60, 107505 ’99].
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The Model

[Cabrera, Gersdorff, Quirós, (2010)]

- Scalar-gravity system with UV and IR branes:
  \[ S = \int d^5x \sqrt{|\det g_{MN}|} \left[ -\frac{1}{2\kappa^2} R + \frac{1}{2} g^{MN}(\partial_M \phi)(\partial_N \phi) - V(\phi) \right] 
  - \sum_{\alpha} \int_{B_\alpha} d^4x \sqrt{|\det \bar{g}_{\mu\nu}|} \lambda_\alpha(\phi) + S_{GHY} \]

- Metric: \( ds^2 = g_{MN} dx^M dx^N \equiv e^{-2A(y)} \eta_{\mu\nu} dx^\mu dx^\nu - dy^2 \).
- \( V(\phi) \) bulk potential.
- \( \lambda_\alpha (\alpha = 0, 1) \equiv \) UV, IR 4-dim brane potentials at \( (y(\phi_0), y(\phi_1)) \).
- \( S_{GHY} := \) Gibbons-Hawking-York boundary term.
- Solve the hierarchy problem \( \Rightarrow \) Brane dynamics should fix \( (\phi_0, \phi_1) \) to get \( A(\phi_1) - A(\phi_0) \approx O(35) \Rightarrow M_{\text{Planck}} \sim 10^{15} M_{\text{TeV}} \).
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The soft-wall model

[Cabrera et al. NJP ’10], [C. Csaki et al. ’19], [E.M., M. Quirós, JHEP ’19 & 2106.09598]

Superpotential: \( W(\phi) = \frac{6k}{\kappa^2} \left( 1 + e^{\nu \phi} \right) \).

- Critical case \( \nu = \nu_c \equiv \kappa / \sqrt{3} \) \( \rightarrow \) Gapped continuum KK spectra.
  \[ A(z) \simeq \begin{cases} 
  \log(kz) & 1/k \leq z \leq z_1 \\
  \log(kz_1) + \rho(z - z_1) & z_1 < z < \infty
  \end{cases} \]

- For \( \nu > \nu_c \) \( \rightarrow \) discrete KK spectra with TeV spacing.
- For \( \nu < \nu_c \) \( \rightarrow \) ungapped continuum KK spectra.

Effective potential for gauge bosons:

\( \rho \equiv k e^{-ky_1} \sim \text{TeV} \) \( \rightarrow \) \( A_1 \simeq 35 \).

Conformally flat coordinates: \( ds^2 = e^{-2A(z)} \left( \eta_{\mu\nu} dx^\mu dx^\nu - dz^2 \right) \).
Green’s functions: gapped continuum spectra

- Up to now, searches of new physics at the LHC → detection of bumps in the invariant mass of final states.
- An exploring possibility to cope with the elusiveness of signals is a continuum of Kaluza-Klein (KK) states beyond a mass gap \( m_g \).
- New physics associated with an excess in the measured cross section with respect to the SM prediction.
- The Green’s functions generalize the particle propagators

\[
\frac{1}{p^2 - m^2 + i\epsilon} = \mathcal{P} \frac{1}{p^2 - m^2} - i\pi\delta(p^2 - m^2)
\]

... to Green’s functions with an isolated pole (the zero mode) and a continuum of states with a mass gap \( m_g \)

\[
G(p^2, m_g^2) = \text{Re} \ G(p^2, m_g^2) + i \ [c_0 \delta(p^2) + \eta(p^2, m_g^2)\Theta(p^2 - m_g^2)] .
\]

- Same behavior as gapped unparticles [M. Pérez-Victoria et al, PRD '09 & JHEP '09]. Here \( m_g \sim \text{TeV} \) is linked to the solution of the hierarchy problem.
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Massless gauge bosons

- Lagrangian for massless gauge bosons (in the gauge $A_5 = 0$):

\[
\mathcal{L} = \int_0^{y_s} dy \left[ -\frac{1}{4} \text{tr} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} e^{-2A} \text{tr} A'_\mu A'_\mu \right],
\]

\[
A_\mu(p, y) = f_A(y) A_\mu(p)/\sqrt{y_s}.
\]

- Schrödinger like form of the EoM of the fluctuations

\[
-\tilde{f}_A''(z) + V_A(z) \tilde{f}_A(z) = p^2 \tilde{f}_A(z), \quad [f_A(z) = e^{A(z)/2} \tilde{f}_A(z)],
\]

with potential

\[
V_A(z) = \frac{1}{4} A'(z)^2 - \frac{1}{2} A''(z), \quad V_A(z) \rightarrow m_g^2 = \left(\frac{\rho}{2}\right)^2.
\]

→ Existence of a mass gap.

→ Continuum of states above the mass gap.
Massless gauge bosons

- The **Green’s function** is given by
  \[ G_A^{\mu\nu}(y, y'; p) = [\eta^{\mu\nu} - (1 - \xi)p^\mu p^\nu / p^2] G_A(y, y'; p). \]

- **EoM for the Green’s function:**
  \[ p^2 G_A(y, y'; p) + \partial_y \left( e^{-2A(y)} \partial_y G_A(y, y'; p) \right) = \delta(y - y'). \]

- All Green’s functions include the **zero-mode** contribution:
  \[ G_A^0 = \frac{1}{y_s p^2} = \lim_{p \to 0} G_A(y, y'; p) \Rightarrow \text{Define: } G_A(y, y'; p) = G_A(y, y'; p) - G_A^0. \]
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Green’s functions in the complex plane: resonances

- Complex plane ($s \equiv p^2$):
  \[ s = M^2 - iM\Gamma = M^2(1 - ir), \quad (r \equiv \Gamma / M). \]

- Riemann sheets:
  \[
  \left\{ \begin{array}{l}
  \text{1st Riemann sheet} \quad \delta_A^I = +\sqrt{1 - s/m_g^2} \\
  \text{2nd Riemann sheet} \quad \delta_A^II = -\sqrt{1 - s/m_g^2}
  \end{array} \right. 
  \]

- $G_A(y, y'; p) \propto \frac{1}{\Phi(p)}$ and $\Phi(p)|_{\text{2nd Riemann}} \propto \left[ e^{i2p/\rho} - 8i(p/\rho)^2 \right]$

- Zeros of $\Phi(p) \equiv$ Poles of $G_A(y, y'; p)$:
  \[ \frac{s}{\rho^2} \simeq -\mathcal{W}_n \left[ \pm \frac{1}{4} (1 + i) \right]^2, \quad n = -1, -2, \ldots \]
  \[ \mathcal{W}_n \equiv \text{Lambert function} \]
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Spectral function (soft-wall model)

- Spectral function:
  \[
  \rho_A(y, y'; s) = -\frac{1}{\pi} \text{Im } G_A(y, y'; s + i\epsilon), \quad s \equiv p^2.
  \]

- Zero mode + continuum:
  \[
  \rho_A(y, y'; s) = \frac{1}{y_s} \delta(s) + \eta_A(s) \Theta(s - m_g^2).
  \]

- Kramers-Kronig relations: \(\text{Re} G_A \iff \text{Im} G_A\) - All the information of \(G_A\) is in \(\rho_A\).
The function $\rho_A(y, y')$ can be understood as a matrix element of a spectral operator [L.L. Salcedo, Private communication]

$$\hat{\rho}_A = -\frac{1}{\pi} \text{Im} \hat{G}_A,$$

where

$$\text{Im} \hat{G}_A = \frac{1}{2i} \left( \hat{G}_A - \hat{G}_A^\dagger \right),$$

$$\rho_A(y, y') = \langle y | \hat{\rho}_A | y' \rangle \equiv \rho_{yy'}.$$

$\hat{\rho}_A$ is positive semidefinite.

In the RS model

$$\lambda_{RS}(p) \equiv \text{tr} \hat{\rho}_{A,RS} = \int_0^{y_1} dy \rho_{A,RS}(y, y; p) = \sum_n \delta(p^2 - m_n^2) \geq 0,$$

is interpreted as the density of states.

The integral of $\lambda_{RS}(s)$:

$$\int_0^\infty ds \lambda_{RS}(s) = \int_0^\infty ds \sum_n \delta(s - m_n^2) = N_{\text{states}} \rightarrow \infty,$$

is the number of states.
\[ \lambda(p) = \lambda_0(p) + \lambda_{\text{resonant}}(p) + \lambda_{\text{unparticles}}(p), \]

where

\[ \lambda_0(p) = \delta(p^2), \]

\[ \lambda_{\text{resonant}}(p) = \xi_A(p) \Theta(p^2 - m_g^2), \]

\[ \lambda_{\text{unparticles}}(p) = - \frac{\log(\rho \epsilon)}{2\pi \rho \sqrt{p^2 - m_g^2}} \Theta(p^2 - m_g^2). \]

See e.g. [A.Delgado, J.R. Espinosa, J.No, M.Quirós, PRD79 ’09] for unparticles.
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Regularized continuum model

[E.M., M. Pérez-Victoria, M. Quirós, to appear ’21]

- Spectrum: $z \in [z_0, z_c] \rightarrow$ Discrete. $z \in [z_0, \infty] \rightarrow$ Continuum.
- ’Discrete’ $\rightarrow$ ’Continuum’ by considering $z \in [z_0, z_c]$ with

$$z_0 < z_1 < z_c \quad \text{and} \quad z_c \rightarrow \infty.$$  

\[\frac{p_n^2}{\rho^2} \approx \frac{1}{4} + \left[ \frac{z_1}{z_c} \pi \left( n - \frac{1}{4} \right) \right]^2, \quad n = 1, 2, 3, \ldots\]

(Eigenvalues tighten as $z_c$ increases).

- **Green’s function** ’$G_A$’ and **spectral density** ’$\sigma$’:

$$G_A(z, z'; s) = \sum_n \frac{1}{||f_n||^2} \frac{f_n(z)f_n^*(z')}{s - p_n^2 + i\epsilon} \quad \xrightarrow{z_c \rightarrow \infty} \quad \int_0^\infty dp^2 \sigma(p^2) \frac{f_{p^2}(z)f_{p^2}^*(z')}{s - p^2 + i\epsilon},$$

$$\lambda(p^2) = ||f_{p^2}||^2 \sigma(p^2) \approx \delta(p^2) + \frac{z_c}{2\pi\sqrt{p^2 - m_g^2}} \Theta(p^2 - m_g^2) \quad \xrightarrow{z_c \rightarrow \infty} \quad \infty.$$
Radion effective potential

[E.M., M. Pérez-Victoria, M. Quirós, to appear ’21] [Preliminary]

- Goldberger-Wise mechanism [W. Goldberger, M. Wise PRD ’99]
  Spontaneous breaking of conformal invariance.

- It then appears a “light state”: the radion/dilaton with interesting Higgs-like phenomenology [C.Csaki et al., PRD63, 065002 ’01].

- In the Linear Dilaton Model: \( A(z) = \rho (z - z_0); \ z \in [z_0, z_c] \). Action:
  \[
  S_{\text{on-shell}} = S_{\text{bulk}} + S_{\text{brane}} + S_{\text{GHY}} = - \int d^4 x \ U_{\text{eff}}.
  \]

\[
U_{\text{eff}}(\bar{\chi}) = \left[ e^{-4A} (W + \Lambda_1) \right]_{z_c} + \left[ e^{-4A} (-W + \Lambda_0) \right]_{z_0} \approx \frac{1}{2} m^2_{\text{rad}} (\bar{\chi} - \bar{\chi}_{\text{min}})^2
\]

Light radion field: \( \bar{\chi}(z_1) \approx e^{-A(z_1)} \).

\( U_{\text{eff}}(\bar{\chi}) \) becomes unstable as \( z_c \to \infty \) (\( m^2_{\text{rad}} < 0 \)).
Brane-to-brane Green’s functions

\[ G_A(y_0, y_0; p^2) \to \text{production of fermions by Drell-Yan via gluon}
\]

KK continuum \((\rho = \sqrt{s} \equiv \text{partonic c.o.m. energy})\):

\[
\sigma(q\bar{q} \to g^* \to f_{UV}\bar{f}_{UV})/\sigma_{SM}(q\bar{q} \to g^{(0)} \to f_{UV}\bar{f}_{UV}) = \left| G_A(y_0, y_0; \hat{s})/G^0_A \right|^2.
\]

\[
\sigma(q\bar{q} \to g^* \to f_{IR}\bar{f}_{IR})/\sigma_{SM}(q\bar{q} \to g^{(0)} \to f_{IR}\bar{f}_{IR}) = \left| G_A(y_0, y_1; \hat{s})/G^0_A \right|^2.\]

\[
\sigma(f_{IR}\bar{f}_{IR} \to g^* \to f_{IR}\bar{f}_{IR}) \propto |G_A(y_1, y_1; \hat{s})|^2 \implies R_{D(*)}\text{anomalies}.
\]

\(g^* \equiv \text{contribution from the gluon continuum}. \quad g^{(0)} \equiv \text{the SM gluon}.
\]

\(q \equiv \text{light fermions localized in the UV brane}.
\]

\(f_{UV}/f_{IR} \equiv \text{light/heavy fermions localized in the UV/IR brane}.
\]
Conclusions

- We have studied 5D warped models solving the hierarchy problem:
  - $\text{AdS}_5$ near the UV brane.
  - Linear dilaton theory near the IR singularity.
- The KK spectra of all particles (gauge bosons, fermions, graviton, radion, Higgs boson) are continua of states with a mass gap.
- We have computed the Green’s functions $G(y, y'; p)$ and spectral functions $\rho(y, y'; p)$:
  - Gauge bosons $\implies$ resonance effects at tree level.
  - Positivity of the spectral function.
  - Connection with unparticles.
- The existence of a continuum spectrum should modify the present searches of new physics.
  - Brane-to-brane Green’s functions.
  - Increase in the cross section $\sigma(pp \to Q\bar{Q})$.
- Other phenomenological applications should be inspired on unparticle phenomenology.
Thank You!