Holography of Strongly Coupled Gauge Theories

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Work with Johanna Erdmenger, Kostas Rigatos and Werner Porod: 1907.09489 [hep-th]
2009.10737 [hep-ph]
2010.10279 [hep-ph]

Southampton Theory Astrophysics Gravity Research Centre

- Holographic descriptions of chiral symmetry breaking by running γ and NJL operators
- "perfected QCD" & the space of SCGT

The SCGT Periodic Table

Asymptotically free gauge theories are key to the SM – SU(3)xSU(2)

They likely play a role in BSM physics: techicolour (WTC), composite higgs, strongly coupled dark matter; susy breaking….

Yet we barely know the behaviours of the set of strongly coupled gauge theories (UV defined (+asymptotic safety))

We should do better…

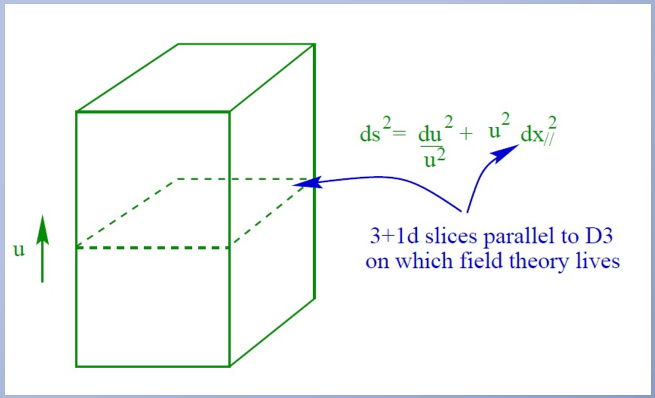
SU(Nc) + F

	Nc = 2	Nc = 3	Nc = 4
Nf = 21			$\gamma_* = 0.04$
Nf = 20			$\gamma_* = 0.09$
Nf = 19			$\gamma_* = 0.16$
Nf = 18			$\gamma_* = 0.24$
Nf = 17			$\gamma_* = 0.34$
Nf = 16		$\gamma_* = 0.03$	$\gamma_* = 0.49$
Nf = 15		$\gamma_* = 0.09$	$\mathcal{R} = 2.15$ $\mathcal{S}=148.4$
Nf = 14		$\gamma_* = 0.18$	$\mathcal{R} = 2.15$ $\mathcal{S}=24.5$
Nf = 13		$\gamma_* = 0.1966(14)$ (0.30)	$\mathcal{R} = 2.15$ $\mathcal{S}=10.7$
Nf = 12		$\gamma_* = 0.1\text{-}0.52$ (0.48)	$\mathcal{R} = 2.15$ $\mathcal{S}=6.6$
Nf = 11		$\mathcal{R} = 2.1$ $\mathcal{S}=55$	$\mathcal{R} = 2.15$ $\mathcal{S}=4.8$
Nf = 10	$\gamma_* = 0.08$	$\gamma_* = 1.10(17)$ ($\mathcal{R} = 2.1$ $\mathcal{S}=12$)	$\mathcal{R} = 2.15$ $\mathcal{S}=3.9$
Nf = 9	$\gamma_* < 0.15$ (0.21)	$\mathcal{R} = 2.1$ $\mathcal{S}=6.0$	$\mathcal{R} = 2.15$ $\mathcal{S}=3.3$
Nf = 8	$\gamma_* = 0.15$ (0.45)	$\mathcal{R} = 2.1$ $\mathcal{S}=4.1$	$\mathcal{R} = 2.15$ $\mathcal{S}=2.9$
Nf = 7	$0.15 < \gamma_* < 0.283$ ($\mathcal{R} = 1.8$ $\mathcal{S}=13$)	$\mathcal{R} = 2.1$ $\mathcal{S}=3.2$	$\mathcal{R} = 2.15$ $\mathcal{S}=2.6$
Nf = 6	$\gamma_* = 0.283$ ($\mathcal{R} = 1.8$ $\mathcal{S}=4.4$)	$\mathcal{R} = 2.1$ $\mathcal{S}=2.7$	$\mathcal{R} = 2.15$ $\mathcal{S}=2.4$
Nf = 5	$\mathcal{R} = 1.8$ $\mathcal{S}=2.8$	$\mathcal{R} = 2.1$ $\mathcal{S}=2.4$	$\mathcal{R} = 2.15$ $\mathcal{S}=2.2$
Nf = 4	$\mathcal{R} = 1.8$ $\mathcal{S}=2.2$	$\mathcal{R} = 2.1$ $\mathcal{S}=2.1$	$\mathcal{R} = 2.15$ $\mathcal{S}=2.1$
Nf = 3	$\mathcal{R} = 1.8$ $\mathcal{S}=1.9$	$\mathcal{R} = 2.1$ $\mathcal{S}=2.0$	$\mathcal{R} = 2.15$ $\mathcal{S}=2.0$
Nf = 2	$\mathcal{R} = 1.8$ $\mathcal{S}=1.7$	$\mathcal{R} = 2.1$ $\mathcal{S}=1.9$	$\mathcal{R} = 2.15$ $\mathcal{S}=1.9$

Figure from SCGT Forum web site

Holographic Magic

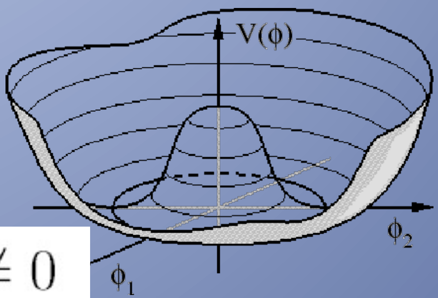
Provides a new description of RG flow at strong coupling…



More Holographic Magic

Describes chiral symmetry breaking

$$SU(2)_L \times SU(2)_R \to SU(2)_V$$



$$\langle \bar{u}_L u_R + \bar{d}_L d_R + h.c. \rangle \neq 0$$

$\Delta m^2 = -1$ corresponds to $\gamma = 1$ and is special – the Breitenlohner Freedman bound instability…

$$\partial_\rho[\rho^3 \partial_\rho L] - \rho \Delta m^2 L = 0 .$$

as a radially dependent higgs mechanism in AdS… with the potential related to the anomalous dimension of the qq operator

$$L = \frac{m_{FP}}{\rho^\gamma} + \frac{c_{FP}}{\rho^{2-\gamma}}, \qquad \gamma(\gamma - 2) = \Delta m^2$$

Dynamic AdS/YM

Timo Alho, NE, KimmoTuominen
1307.4896

$$S = \int d^4x\, d\rho\, \mathrm{Tr}\, \rho^3 \left[\frac{1}{\rho^2 + |X|^2} |DX|^2 \quad + \frac{\Delta m^2}{\rho^2} |X|^2 \right.$$

$$X = L(\rho)\, e^{2i\pi^a T^a}.$$

$$ds^2 = \frac{d\rho^2}{(\rho^2 + |X|^2)} + (\rho^2 + |X|^2) dx^2,$$

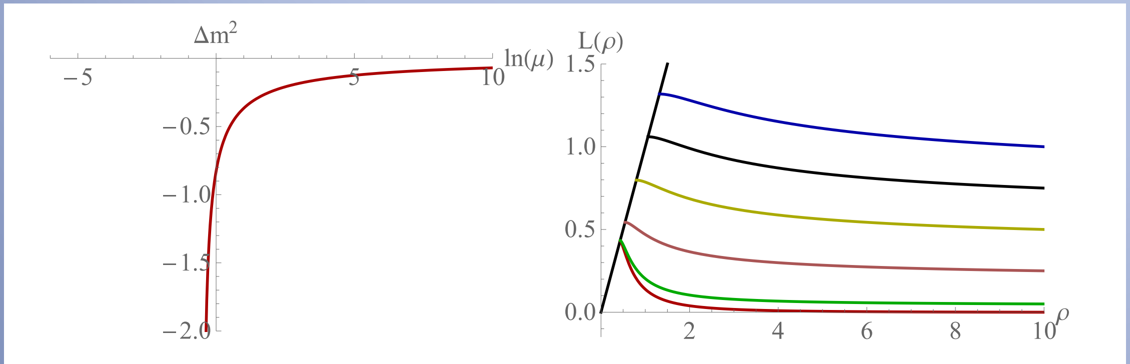
The gauge DYNAMICS is input through a guess for Δm

$$\Delta m^2 = -2\gamma = -\frac{3(N_c^2 - 1)}{2N_c \pi} \alpha$$

The only free parameters are Nc, Nf, Λ

More complex models exist – especially note VQCD of Kiritsis-Jarvinen' group - that attempt to decouple the quarks, include confinement below the chiral symmetry breaking scale, and explain stringy Regge trajectories…. *Heroic* but many built in features….

Formation of the Chiral Condensate



We solve for the vacuum configuration of L

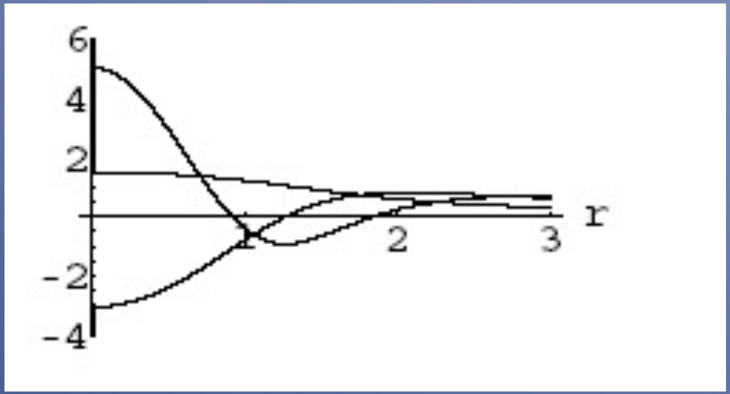
$$\partial_\rho[\rho^3\partial_\rho L] - \rho\Delta m^2 L = 0\,.$$

Yet More Holographic Magic

Fluctuations in the holographic model translate to the effective meson Lagrangian

$$L = L_0 + \delta(\rho)e^{ikx} \qquad k^2 = -M^2$$

$$\partial_\rho(\rho^3\delta') - \Delta m^2\rho\delta - \rho L_0\delta \left.\frac{\partial\Delta m^2}{\partial L}\right|_{L_0} + M^2R^4\frac{\rho^3}{(L_0^2+\rho^2)^2}\delta = 0\,.$$



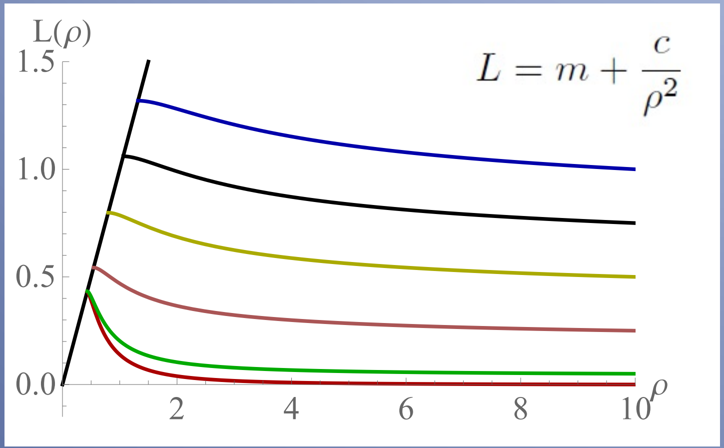
Predicts ρ, a, π ,σ, baryon masses and couplings

Cf AdS/QCD

Higher Dimension/Nambu Jona-Lasinio Operators

$$\mathcal{L} = \bar{\psi}_L \partial\!\!\!/ \psi_L + \bar{\psi}_R \partial\!\!\!/ \psi_R + \frac{g^2}{\Lambda_{UV}^2} \bar{\psi}_L \psi_R \bar{\psi}_R \psi_L$$

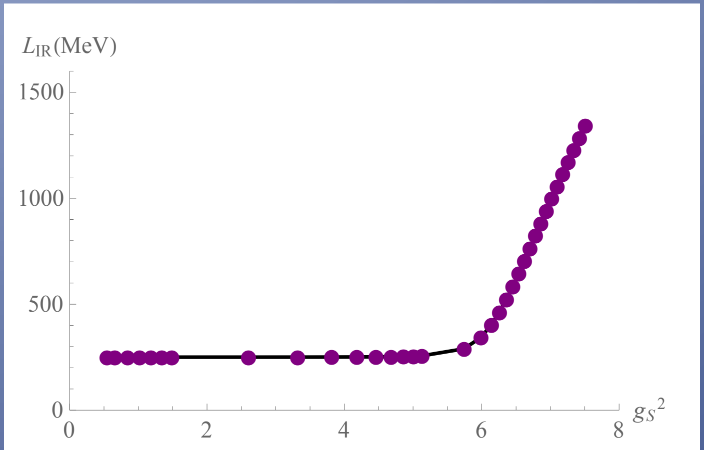
$$\frac{g^2}{\Lambda_{UV}^2} \bar{\psi}_L \psi_R \bar{\psi}_R \psi_L \to \frac{g^2}{\Lambda_{UV}^2} \langle \bar{\psi}_L \psi_R \rangle \bar{\psi}_R \psi_L \to m = \frac{g^2}{\Lambda_{UV}^2} c$$



Witten hep-th/0112258; NE + K Kim 1601.02824 [hep-th]
Jarvinen 1501.07272 [hep-ph]

Witten's multi-trace prescription is to reinterpret the massive solutions as massless + four fermion term

Read off m,c and compute g



QCD Dynamics – Nc=3, Nf=2, m_q=0

$$\mu \frac{d\alpha}{d\mu} = -b_0 \alpha^2, \qquad b_0 = \frac{1}{6\pi}(11 N_c - 2N_F),$$

$$\gamma = \frac{3C_2}{2\pi}\alpha = \frac{3(N_c^2-1)}{4N_c\pi}\alpha\,.$$

Observables (MeV)	QCD	AdS/SU(3) 2 F 2 $\bar{F}$	Deviation
M_ρ	775	775*	fitted
M_A	1230	1183	- 4%
M_S	500/990	973	+64%/-2%
M_B	938	1451	+43%
f_π	93	55.6	-50%
f_ρ	345	321	- 7%
f_A	433	368	-16%
$M_{\rho,n=1}$	1465	1678	+14%
$M_{A,n=1}$	1655	1922	+19%
$M_{S,n=1}$	990 /1200-1500	2009	+64%/+35%
$M_{B,n=1}$	1440	2406	+50%

Table 1: The predictions for masses and decay constants (in MeV) for $N_f = 2$ massless QCD. The ρ-meson mass has been used to set the scale (indicated by the *).

Scale fixed by V-meson

Pattern sensible

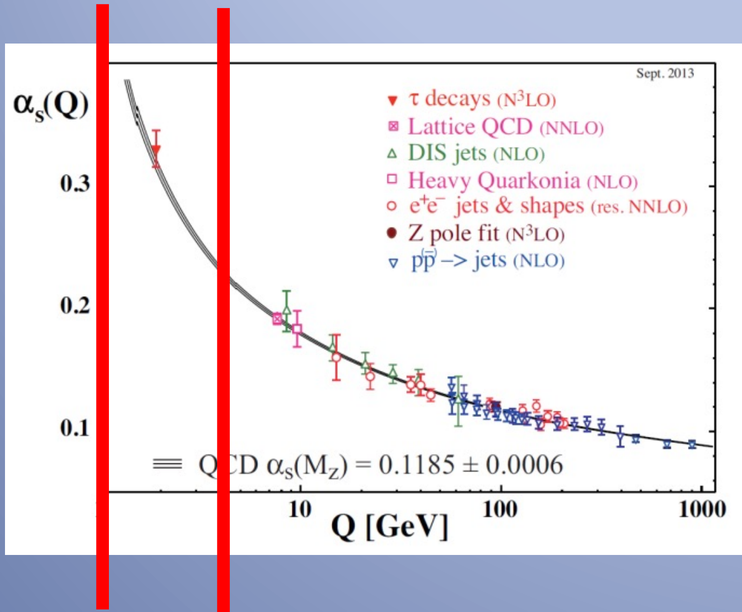
Pion decay constant needs a mass term

Baryon mass high

Radial excitations scale wrongly – no string physics included

Perfecting with HDOs

NE, Johanna Erdmenger, Kostas Rigatos
and Werner Porod: 2010.10279 [hep-ph]



The weakly coupled gravity dual should only live between the red lines… probably we need HDOs at the UV scale to include matching effects… and stringy effects in the gravity model….

$$\frac{g_S^2}{\Lambda_{UV}^2}|\bar{q}q|^2\,, \qquad \frac{g_V^2}{\Lambda_{UV}^2}|\bar{q}\gamma^\mu q|^2\,, \qquad \frac{g_A^2}{\Lambda_{UV}^2}|\bar{q}\gamma^\mu\gamma_5 q|^2\,, \qquad \frac{g_B^2}{\Lambda_{UV}^5}|qqq|^2\,,$$

Observables (MeV)	QCD	Dynamic AdS/QCD	HDO coupling
M_V	775	775	sets scale
M_A	1230	1230	fitted by $g_A^2 = 5.76149$
M_S	500/990	597	prediction $+20\%/-40\%$
M_B	938	938	fitted by $g_B^2 = 25.1558$
f_π	93	93	fitted by $g_S^2 = 4.58981$
f_V	345	345	fitted by $g_V^2 = 4.64807$
f_A	433	444	prediction $+2.5\%$
$M_{V,n=1}$	1465	1532	prediction $+4.5\%$
$M_{A,n=1}$	1655	1789	prediction $+8\%$
$M_{S,n=1}$	990/1200-1500	1449	prediction $+46\%/0\%$
$M_{B,n=1}$	1440	1529	prediction $+6\%$

Table 2: The spectum and the decay constants for two-flavour QCD with HDOs from fig. 7 used to improve the spectrum.

Pretty good… but we've lost some predictivity….

Beyond QCD – Multi-Flavour Theories

Sp(4) 4F $6A_2$

NE, Johanna Erdmenger, Kostas Rigatos
and Werner Porod: 2010.10279 [hep-ph]

We run the model with two scalars – one for the F condensate and one for the A2 condensate.. We input perturbative runnings of γ in each case to fix Δm^2…

		AdS/Sp(4) A2 decouple	AdS/Sp(4) quench	lattice [79] quench
$f_{\pi A_2}$		0.120	0.103	0.1453(12)
$f_{\pi F}$		0.0701	0.0756	0.1079(52)
M_{VA_2}		1*	1*	1.000(32)
f_{VA_2}		0.517	0.518	0.508(18)
M_{VF}		0.814	0.962	0.83(19)
f_{VF}		0.364	0.428	0.411(58)
M_{AA_2}		1.35	1.28	1.75 (13)
f_{AA_2}		0.520	0.524	0.794(70)
M_{AF}		1.19	1.36	1.32(18)
f_{AF}		0.399	0.462	0.54(11)
M_{SA_2}		0.375	1.14	1.65(15) †
M_{SF}		0.902	1.25	1.52 (11)†
M_{BA_2}		1.85	1.86	
M_{BF}		1.53	1.79	

We set the scale in the A2 sector…

the pattern of mass scales is right…

F sector is lighter than the A2s

Again F sector - right pattern

KEY IMPACT: easy for us to unquench – the slower the running the lighter the sigma

[78] E. Bennett, D. K. Hong, J.-W. Lee, C.-J. D. Lin, B. Lucini, M. Mesiti, M. Piai, J. Rantaharju, and D. Vadacchino, "Sp(4) gauge theories on the lattice: quenched fundamental and antisymmetric fermions," arXiv:1912.06505 [hep-lat].

SU(4) 3 F 3 $\bar{\text{F}}$ 5 A_2

NE, Johanna Erdmenger, Kostas Rigatos and Werner Porod: 2010.10279 [hep-ph]

G. Ferretti, "UV Completions of Partial Compositeness: The Case for a SU(4) Gauge Group," JHEP **06** (2014) 142, arXiv:1404.7137 [hep-ph].

The lattice has simulated (unquenched) SU(4) 2 F 2 F 4 A_2

V. Ayyar, T. DeGrand, M. Golterman, D. C. Hackett, W. I. Jay, E. T. Neil, Y. Shamir, and B. Svetitsky, "Spectroscopy of SU(4) composite Higgs theory with two distinct fermion representations," Phys. Rev. D **97** no. 7, (2018) 074505, arXiv:1710.00806 [hep-lat].

	Lattice [80] $4A_2, 2F, 2\bar{F}$ unquench		AdS/$SU(4)$ $4A_2, 2F, 2\bar{F}$ decouple		AdS/$SU(4)$ $5A_2, 3F, 3\bar{F}$ decouple	AdS/$SU(4)$ $5A_2, 3F, 3\bar{F}$ quench
$f_{\pi A_2}$	0.15(4)		0.0997		0.111	0.102
$f_{\pi F}$	0.11(2)		0.0953		0.109	0.892
$M_{V A_2}$	1.00(4)		1*		1*	1*
$f_{V A_2}$	0.68(5)		0.489		0.516	0.517
$M_{V F}$	0.93(7)		0.939		0.904	0.976
$f_{V F}$	0.49(7)		0.461		0.491	0.479
$M_{A A_2}$			1.37		1.32	1.28
$f_{A A_2}$			0.505		0.521	0.522
$M_{A F}$			1.37		1.23	1.28
$f_{A F}$			0.504		0.509	0.492
$M_{S A_2}$			0.873		0.684	1.18
$M_{S F}$			1.02		0.798	1.25
$M_{J A_2}$	3.9(3)		2.21		2.21	2.22
$M_{J F}$	2.0(2)		2.08		2.00	2.17
$M_{B A_2}$	1.4(1)		1.85		1.85	1.86
$M_{B F}$	1.4(1)		1.75		1.68	1.81

The pattern is right…

The A2-F gap is well described…

KEY POINTS: Adding extra flavours is not a huge change…

Scalar masses get lighter as add extra flavours

We've also computed for

These are the sub-set of theories that do not lie in the conformal window at the level of our parametrizations of the runnings.

AdS/G_2 $11F$	AdS/G_2 $15F$	AdS/F_4 $11F$	AdS/F_4 $12F$

$\mathrm{AdS}/Sp(4)$ $5A_2, 6F$	$\mathrm{AdS}/Sp(14)$ $5A_2, 6F$	$\mathrm{AdS}/Sp(4)$ $4F, 6A_2$	$\mathrm{AdS}/Sp(8)$ $4F, 6A_2$

$\mathrm{AdS}/SO(7)$ $5F, 6s$	$\mathrm{AdS}/SO(7)$ $5s, 6F$	$\mathrm{AdS}/SO(9)$ $5F, 6s$	$\mathrm{AdS}/SO(9)$ $5s, 6F$

$\mathrm{AdS}/SO(10)$ $5F, 6s$	$\mathrm{AdS}/SO(11)$ $5F, 6s$	$\mathrm{AdS}/SO(11)$ $4s, 6F$	$\mathrm{AdS}/SO(13)$ $4s, 6F$

$\mathrm{AdS}/SO(8)$ $5F, 3s, 3\tilde{s}$	$\mathrm{AdS}/SO(10)$ $5F, 3s, 3\tilde{s}$	$\mathrm{AdS}/SO(12)$ $5F, 3s, 3\tilde{s}$

$\mathrm{AdS}/SO(10)$ $4s, 4\tilde{s}, 6F$	$\mathrm{AdS}/SU(4)$ $4F, 4\bar{F}, 6A_2$

$\mathrm{AdS}/SU(5)$ $4F, 4\bar{F}, 3A_2, 3\bar{A}_2$	$\mathrm{AdS}/SU(5)$ $4A_2, 4\bar{A}_2, 3F, 3\bar{F}$	$\mathrm{AdS}/SU(7)$ $4F, 4\bar{F}, 3A_3, 3\bar{A}_3$	$\mathrm{AdS}/SU(10)$ $4F, 4\bar{F}, 3S_2, 3\bar{S}_2$	$\mathrm{AdS}/SU(71)$ $3F, 3\bar{F}, 4S_2, 4\bar{S}_2$

which shows the simplicity of the method to get decent stabs at the spectrum….

Holographic Model Building Highlights Key Dynamical Questions

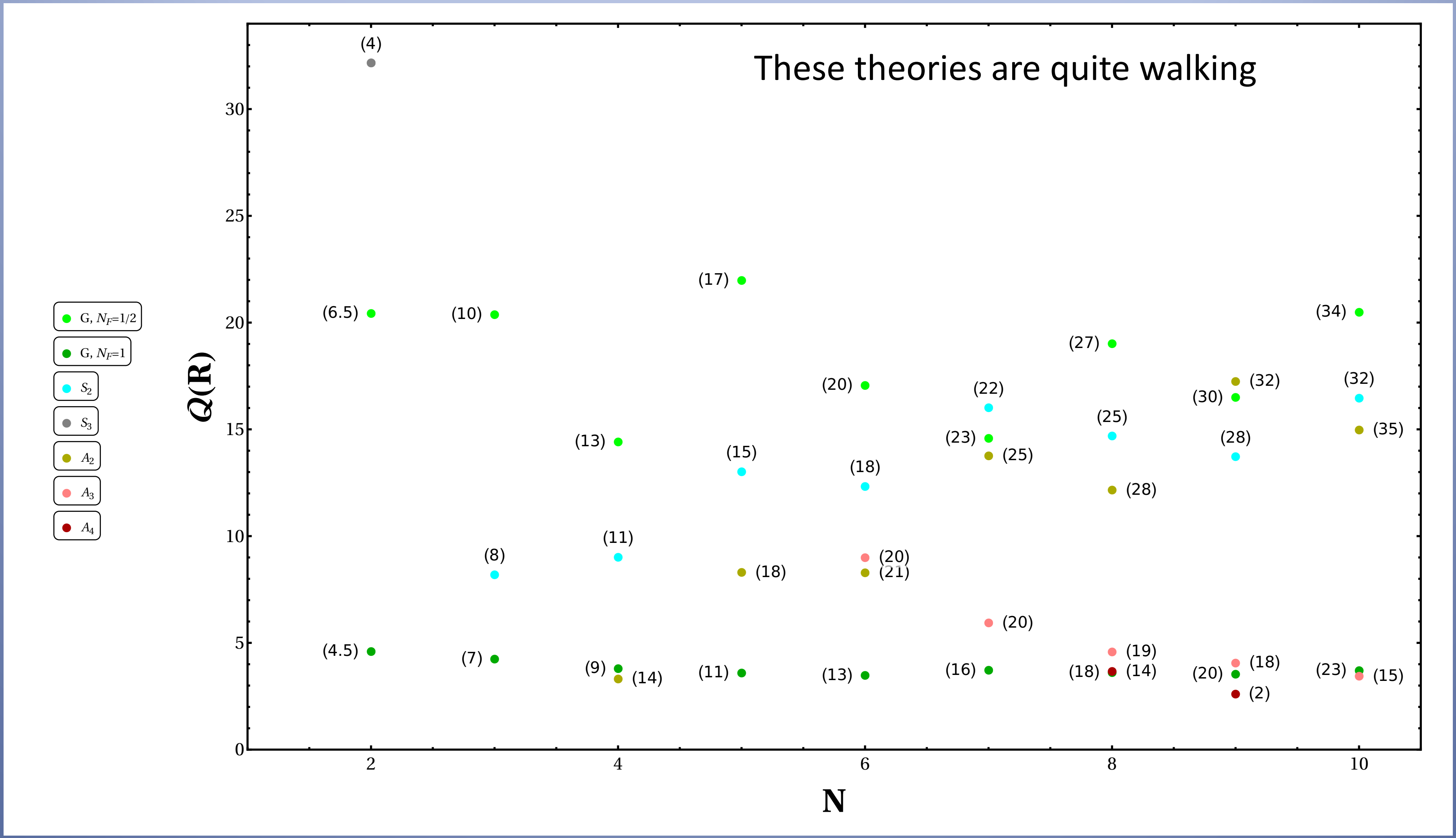
Holographic models are good at telling you how masses & couplings will change as the running changes due to unquenching, added flavours etc…

They high light questions such as the way in which quarks decouple at strong coupling…

We’ve seen multi-rep theories with gaps between chiral symmetry breaking scales…. How big can they be?

This would be a measure of the gap to confinement also…

Two loop results for SU(N) with fundamentals + another rep. γ=1/2 criteria



$$\mathcal{Q}(R) = \frac{\Lambda_{\chi SB\ R}}{\Lambda_{\chi SB\ F}}$$

Nick Evans, Kostas Rigatos. 2012.00032 [hep-ph]

Summary

* we need to understand SCGT better!

* The lattice is key but slow

* holographic modelling provides insight into how spectra change as change the running of a theory

* holographic modelling highlights key dynamics questions – eg separating confinement and chiral symmetry breaking

* studies complement lattice!

* Online SCGT Forum (NE, Biagio Lucini) is bringing results together – do please contribute with your work

https://www.southampton.ac.uk/~evans/SCGT/