Improving the cold-QM pressure via soft interactions at N3LO

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Work in Collaboration with:

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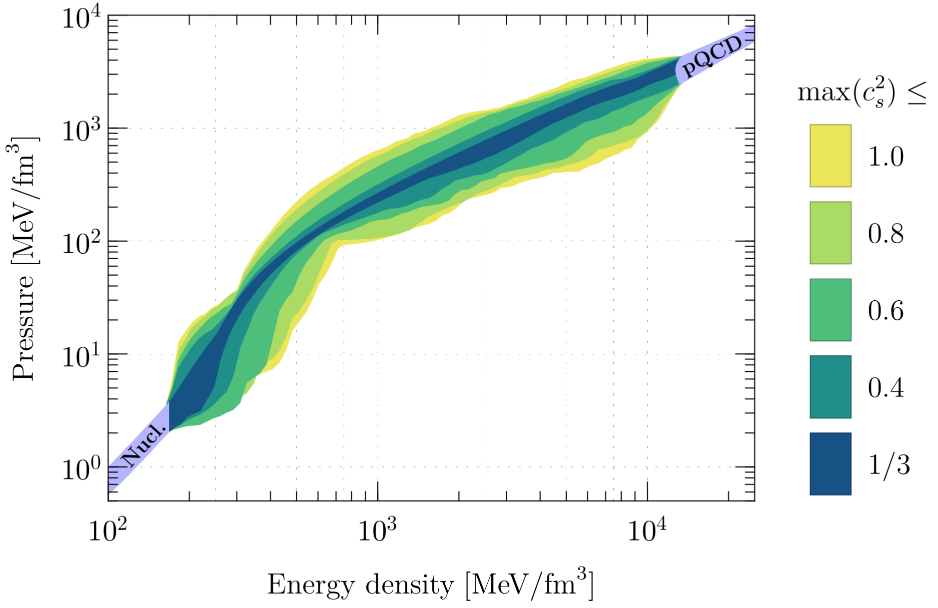
arXiv 2103.05658, 2103.07427

Motivation

- Cold QM pQCD EOS relevant for constraining NS EOS:



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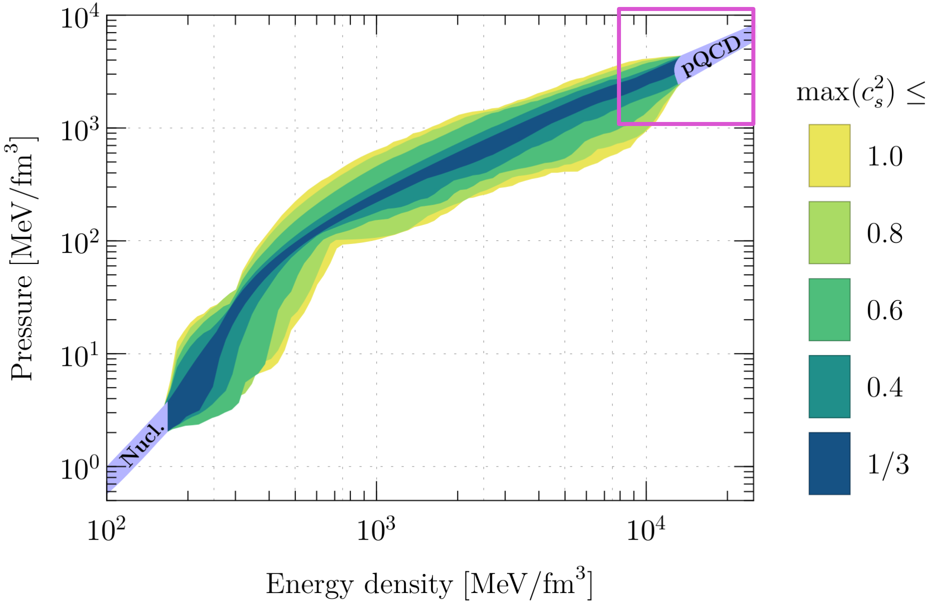

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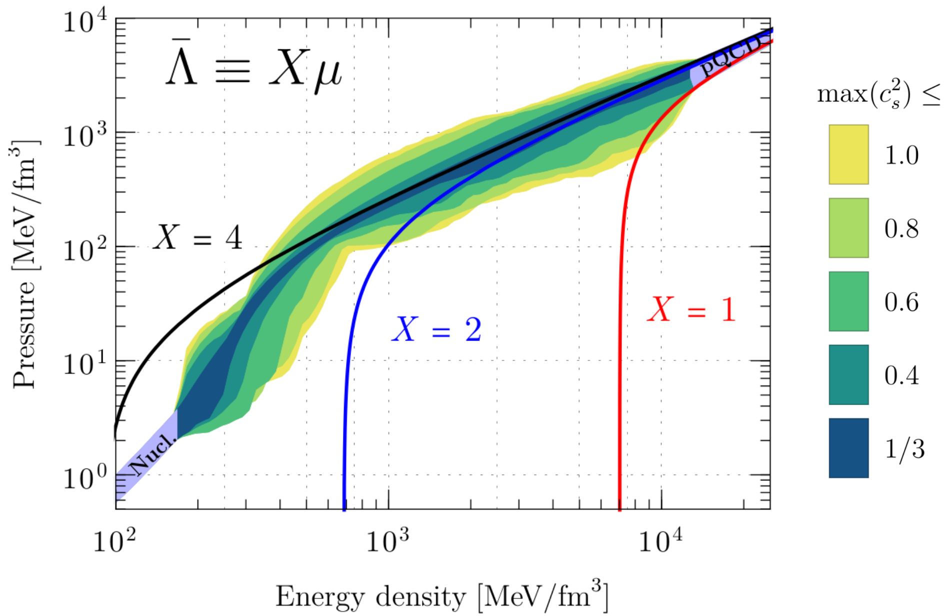
Even though pQCD EOS only valid for $n_\mathrm{B} \gtrsim 40 n_0$, **CONSTRAINS** NS EOS at lower densities

Motivation

- Cold QM pQCD EOS has poor (renorm.) scale sensitivity, which prevents us from using it to lower densities:



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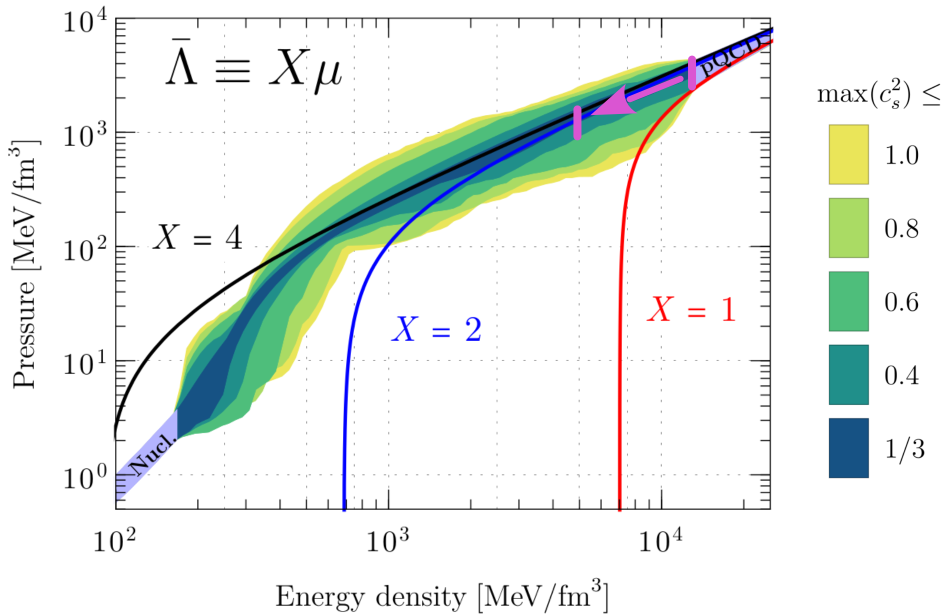

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If we could improve the dependence on the renorm. scale, could have dramatic consequences

Calculating the pressure; a tale of EFTs

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Hot QGP

Three scales:

1) $P \sim T$: Full-theory (hard) diagrams
2) $P \sim \alpha_s^{1/2} T$: EFT for (massive) chromo-electric fields
3) $P \sim \alpha_s T$: Lattice EFT for (massless) chromo-magnetic fields

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Cold QM

Two scales:

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No softer scale b/c gluons not thermally occupied at $T = 0$: Great!

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Not great

Calculating the pressure; a tale of EFTs

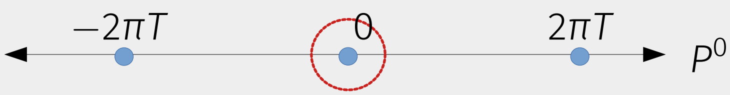
Effective in-medium mass scale: $m_E \sim \alpha_s^{1/2}\mu$

m_E^2 " = " $\lim_{|P|\to 0} \Pi^{\mu\nu}(P)$

(coeff in front; also angular function)

Hot QGP

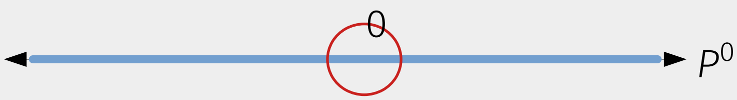
sum-integrals: $T\sum_{n=-\infty}^{\infty}\int \frac{d^3P}{(2\pi)^3}$



- Only zero-mode requires special treatment for $m_E << T$
- 3d EFT of massive zero mode "dimensional reduction"

Cold QM

4d-Euclid. Integrals: $\int \frac{d^4P}{(2\pi)^4}$



- No simple separation
- No simple EFT to deal with IR problems

Pressure contributions in cold QM

$$p = p_{\mathrm{FD}} + p_1^h \alpha_s + p_2^h \alpha_s^2 + p_3^h \alpha_s^3$$

$P \gtrsim \mu$

$$+ p_2^s \alpha_s^2 + p_3^s \alpha_s^3$$

$P \lesssim m_E$

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Both scales

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$$+ p_3^m \alpha_s^3, \qquad \text{Both scales}$$

**Ambiguity in soft/hard split ($m_{\mathrm{E}} \ll K \ll \mu$) gives logarithmic sensitivity to a factorization mass scale Λ_{h}, which cancels out of sum over all kinematic regions (columns!)

Pressure contributions in cold QM

Freedman, McLerran (1977)

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This WORK

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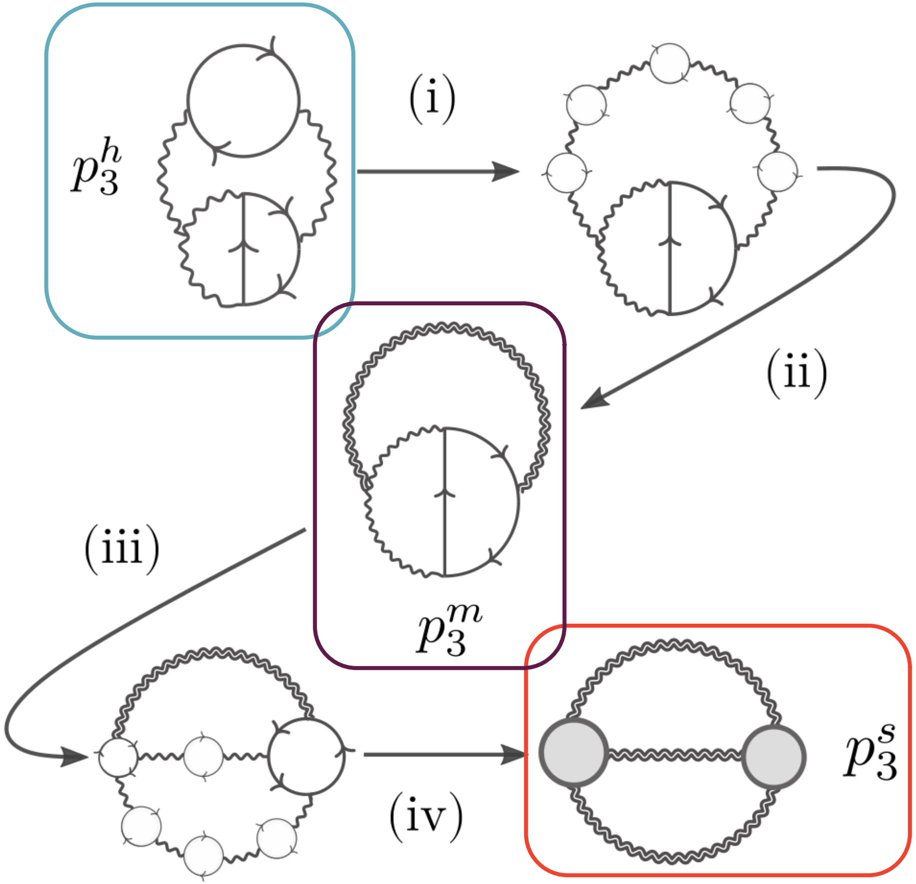
Cold QM approach



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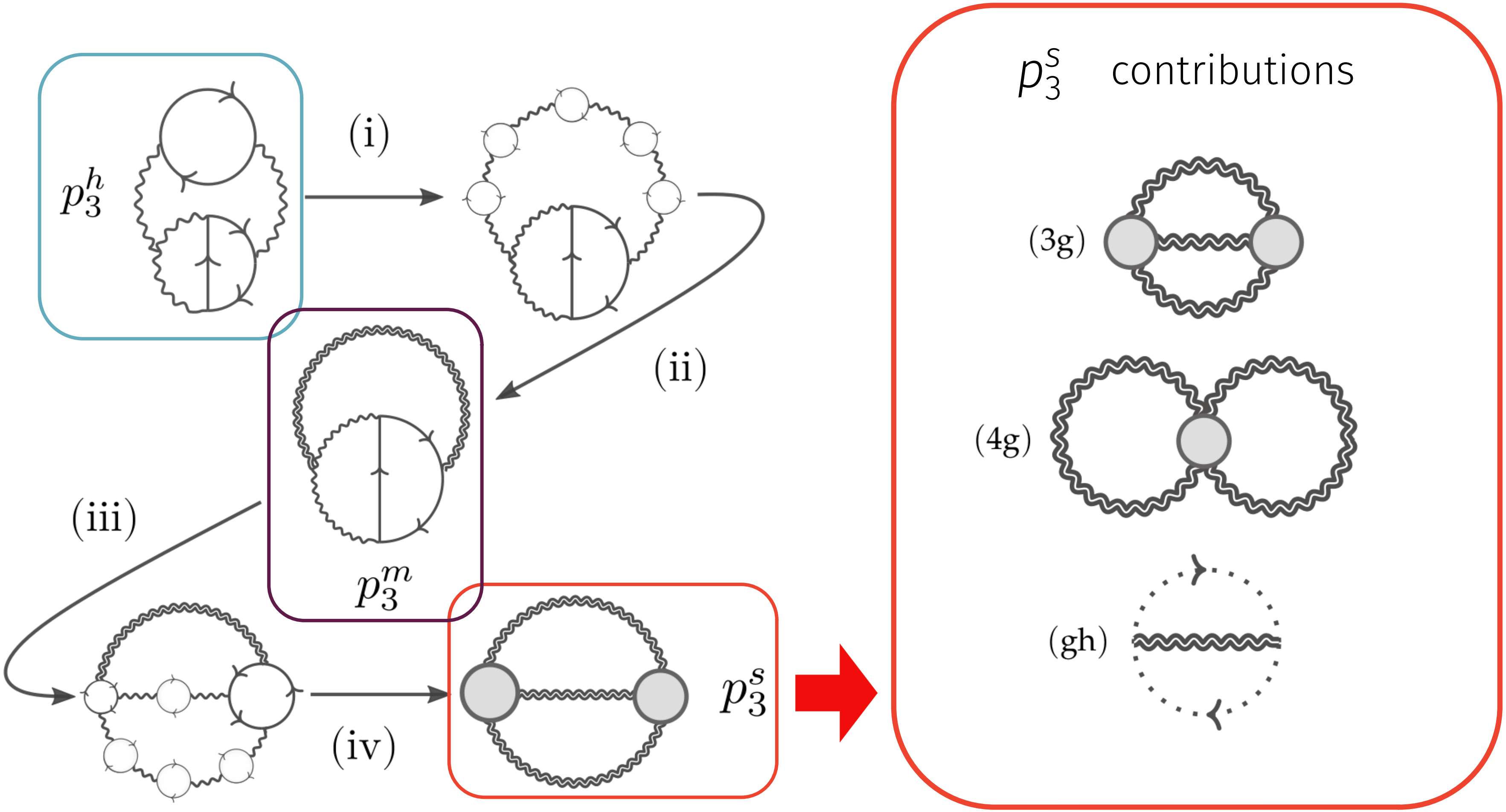


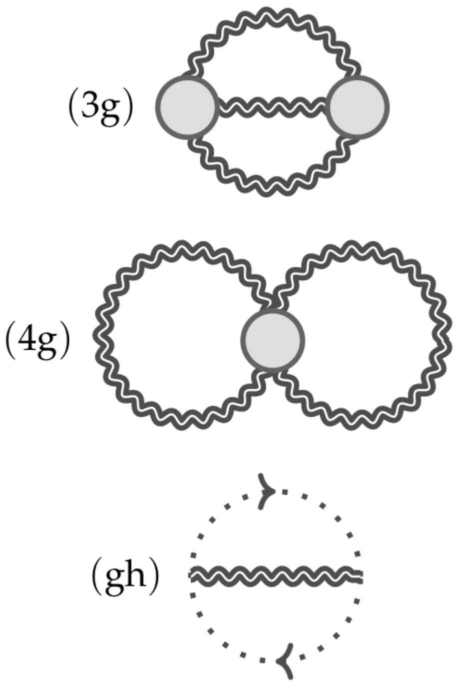
Cold QM approach

Remarks:

- Examined in [Andersen et al. Phys.Rev.D 66 (2002)], but we need the full unexpanded computation
- Here, don't need full quark kinematics, just $\Pi^{\mu\nu}_{\mathrm{HTL}}(\hat{P})$
- Hence, radial integrals over $|K|$, $|P|$ in 2-loop HTL diagrams are simpler

$$\alpha_s^3 p_3^s = \frac{\alpha_s N_c d_A m_{\mathrm{E}}^4}{(8\pi)^2}\left(\frac{m_{\mathrm{E}}}{\Lambda_{\mathrm{h}}}\right)^{-4\epsilon}\left(\frac{p_{-2}}{(2\epsilon)^2}+\frac{p_{-1}}{2\epsilon}+p_0\right)$$

p_3^s contributions



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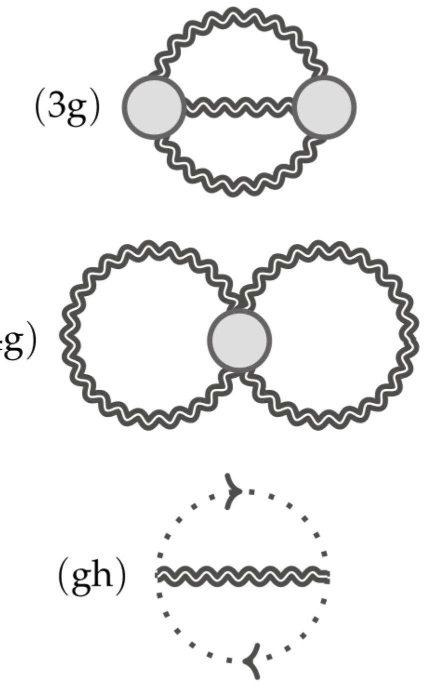

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TG, Kurkela, Romatschke, Säppi, Vuorinen PRL (2018)

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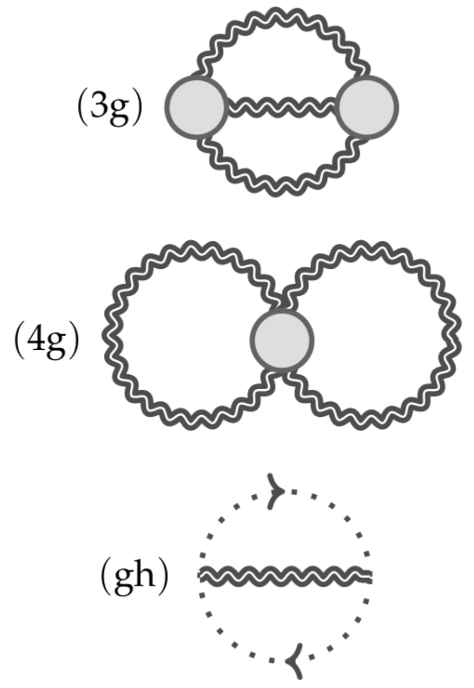

Results

Structure of the soft results

Freedman, McLerran (1977)

$$\alpha_s^2 p_2^s = \frac{d_A m_{\rm E}^4}{(8\pi)^2} \left(\frac{m_{\rm E}}{\Lambda_{\rm h}}\right)^{-2\epsilon} \left(\frac{p_{-1}^{\rm NNLO}}{2\epsilon} + p_0^{\rm NNLO}\right),$$

New:

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Expanded, gives:

UV div.s cancel with IR div.s from other regions

$$\alpha_s^3 p_3^s = \frac{\alpha_s N_c d_A m_{\rm E}^4}{(8\pi)^2} \left(\frac{p_{-2}}{4\epsilon^2} + \frac{p_{-1} - 2p_{-2}\log(m_{\rm E}/\Lambda_{\rm h})}{2\epsilon}\right.$$

$$\left. + p_0 - 2p_1 \log(m_{\rm E}/\Lambda_{\rm h}) + 2p_2 \log^2(m_{\rm E}/\Lambda_{\rm h})\right)$$

Finite results can be plotted and compared to lower-order results

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Can set Λ_h using PMS prescription!

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Values:

Freedman, McLerran (1977)

p_{-1}^{NNLO}	1
p_0^{NNLO}	1.17201
p_{-2}	$11/(6\pi)$
p_{-1}	$1.50731(19)$
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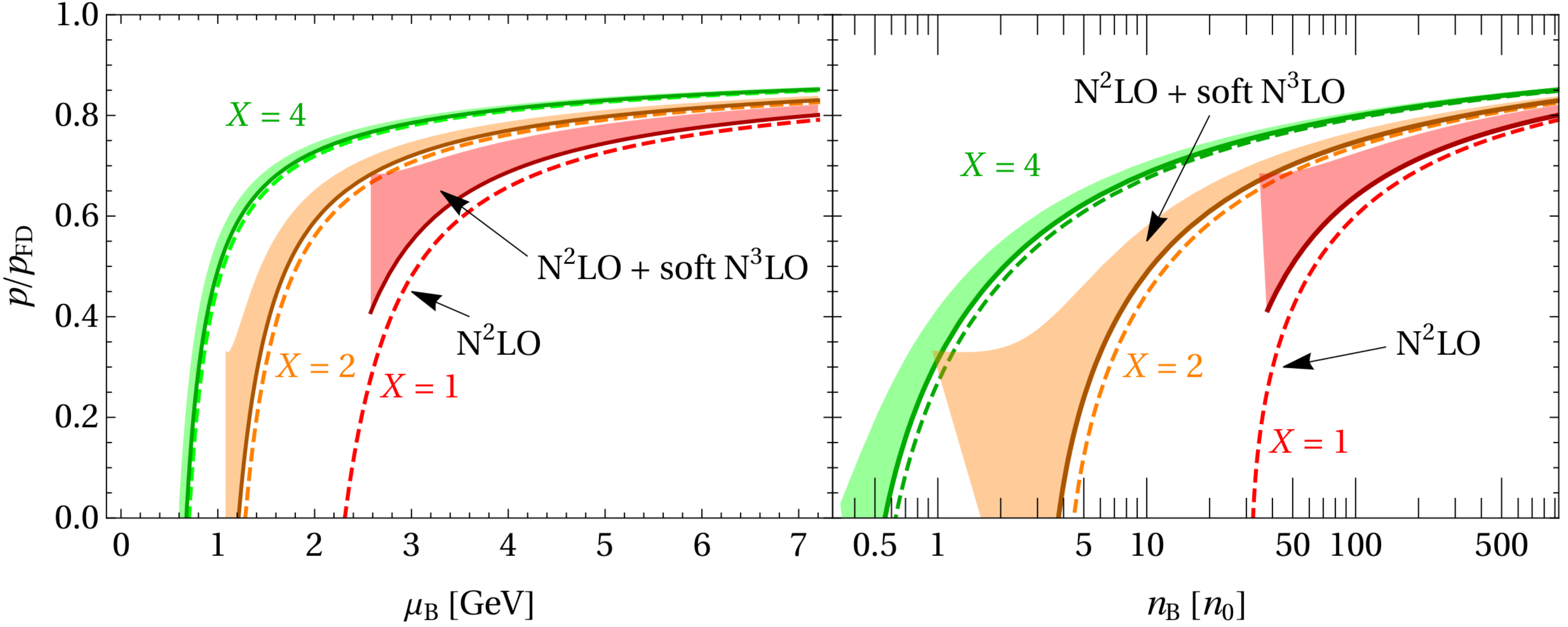
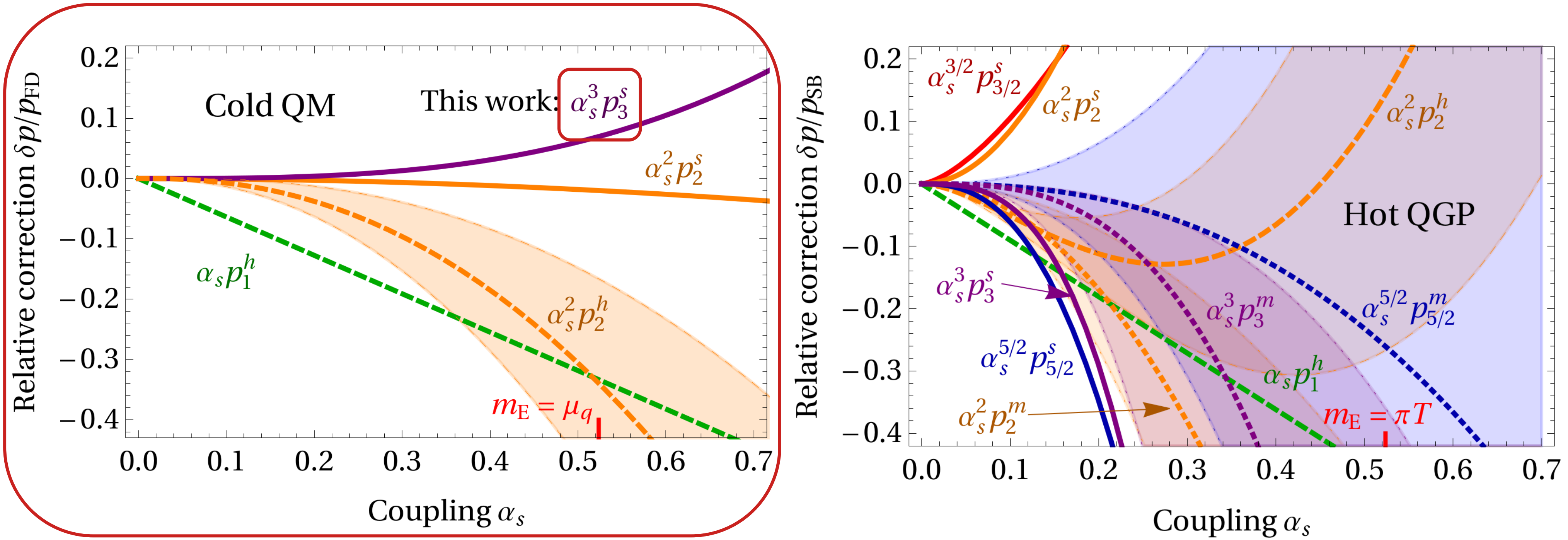
* Makes sense b/c p_{-2} comes from UV limit of soft, pure-glue, theory

Plots of soft results: 1/2



* Shifts EOS to slightly higher pressures;
* Slightly decreases scale dependence; may improve NS applications: $40n_0 \to 35n_0$?

Plots of soft results: 2/2



Cold QM perturbatively well behaved – better than hot QGP

Conclusions + outlook

- Have improved the EOS of perturbative cold QM – new state-of-the-art result
- Result shows fine convergence of the soft sector; contrary to high T behavior. Perturbation theory may be more powerful in cold QM.
- Appearance of β_0: Leading term in soft sector simple to all orders?
- Mixed "*m*" in progress, will improve EOS further. (PMS on renormalization scale too?)
- Constitutes first full calculation of the interactions between soft, screened gluons in QCD thermodynamics

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Thank you for your attention!

Backup Slides

Extracting UV divergences: steps

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1. Isolate UV divergences by looking for: $m_\mathrm{E}^4 \times$ (dimensionless int.) ; power-count to extract them

$$m_\mathrm{E}^4 \int_{\hat{K},\hat{P}} \int_{|K|,|P|} f(|K|, |P|, \Pi(\hat{K}), \Pi(\hat{P}))$$

Extracting UV divergences: steps

1. Isolate UV divergences by looking for: $m_{\mathrm{E}}^4 \times$ (dimensionless int.) ; power-count to extract them
2. $(|P|,|K|)$ to Euclid. polar coordinates: (X, χ). Radial integral gives one divergence, sometimes angle gives another. Often both tractable.

$$m_{\mathrm{E}}^4 \int_{\hat{K},\hat{P}} \int_{|K|,|P|} f(|K|, |P|, \Pi(\hat{K}), \Pi(\hat{P})) \mapsto m_{\mathrm{E}}^4 \int_{\hat{K},\hat{P}} \int_{X,\chi} f(X, \chi, \hat{K}, \hat{P}, \Pi(\hat{K}), \Pi(\hat{P}))$$

$$\mapsto \frac{m_{\mathrm{E}}^4}{\varepsilon^2} \int_{\hat{K},\hat{P}} g(\hat{K}, \hat{P}, \Pi(\hat{K}), \Pi(\hat{P})) + O\left(\frac{1}{\varepsilon}\right)$$

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2. $(|P|,|K|)$ to Euclid. polar coordinates: (X, χ). Radial integral gives one divergence, sometimes angle gives another. Often both tractable.
3. Do tensor reduction to simplify remaining contractions between $\hat{P} \cdot \hat{K}$, since we are averaging over remaining angles

$$
\begin{aligned}
m_E^4 \int_{\hat{K},\hat{P}} \int_{|K|,|P|} f(|K|, |P|, \Pi(\hat{K}), \Pi(\hat{P})) &\mapsto m_E^4 \int_{\hat{K},\hat{P}} \int_{X,\chi} f(X, \chi, \hat{K}, \hat{P}, \Pi(\hat{K}), \Pi(\hat{P})) \\
&\mapsto \frac{m_E^4}{\varepsilon^2} \int_{\hat{K},\hat{P}} g(\hat{K}, \hat{P}, \Pi(\hat{K}), \Pi(\hat{P})) + O\left(\frac{1}{\varepsilon}\right) \\
&\mapsto \frac{m_E^4}{\varepsilon^2} \int_{\hat{K},\hat{P}} h(\Pi(\hat{K}), \Pi(\hat{P})) + O\left(\frac{1}{\varepsilon}\right)
\end{aligned}
$$

Non-UV parts

Also need to deal with nasty HTL vertex corrections:

1. Introduce Feynman parametrization
2. Use extensive Ward identities
3. Analytically perform some angular integrals
4. Numerically compute final 6d numerical integral using Monte-Carlo integration

Quick aside about logarithms

Ambiguity in soft/hard split ($m_E \ll K \ll \mu$) gives logarithmic sensitivity to a factorization mass scale Λ_h;

When summing over all kinematic regions, gives logs of coupling:

$$p_2^h + p_2^s = p_2^{\rm LL} \log \alpha_s + p_2^{\rm const},$$
$$p_3^h + p_3^s + p_3^m = p_3^{\rm LL} (\log \alpha_s)^2 + p_3^{\rm NLL} \log \alpha_s + p_3^{\rm const},$$

Each individually depend on new mass scale Λ_h; cancels out of sum

TG, Kurkela, Romatschke, Säppi, Vuorinen PRL (2018)