Non-equilibrium dynamics in weakly coupled gauge theories. (attractors and hydrodynamisation)

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A Virtual Tribute to Quark Confinement and the Hadron Spectrum 2021
Aug 02, 2021
Dynamical evolution of ultra-relativistic heavy ion collisions

- Phenomenological demand: quantify the uncertainties in transition between various stages

\[ \sim 0^{-} \text{ fm/c} \quad \sim 0.1 \text{ fm/c} \quad \sim 1 \text{ fm/c} \quad \sim 10 \text{ fm/c} \quad \sim 20 \text{ fm/c} \]

- Hydrodynamization
- Improve out-of-equilibrium hydrodynamical descriptions
- Insights into URHIC phenomenology
Non-equilibrium effects in ultra-relativistic heavy ion collisions

\[ \sim 0^\circ \text{ fm/c} \quad \sim 0.1 \text{ fm/c} \quad \sim 1 \text{ fm/c} \quad \sim 10 \text{ fm/c} \quad \tau \sim 20 \text{ fm/c} \]

Initial state \quad Preequilibrium \quad Hydrodynamics \quad Hadronization \quad Hadronic cascade

QCD kinetic transport

- weakly coupled approach valid at high T
- Physically motivated to study the pre-equilibrium dynamics
Formalism
QCD medium at high temperatures: Effective kinetic theory

\[ P^\mu \partial_\mu f_{q,g}(p) = -C[f(p)], \quad f_{q,g} \propto \frac{dN_{g,q}}{d^3x d^3p} \]

\[ \frac{df_{q,g}(p)}{d\tau} - \frac{p_z}{\tau} \partial_{p_z} f_{q,g}(p) = -C_2 f_{q,g}(p) - C_1 f_{q,g}(p) \]

At Leading order, transport at different momentum scales “hard” \( p \sim T \) and “soft” \( p \sim gT \)

regulated by HTL

LPM suppression

QCD medium at high temperatures: Effective kinetic theory

\[ P^\mu \partial_\mu f_{q,g}(p) = -C[f(p)], \quad f_{q,g} \propto \frac{dN_{g,q}}{d^3x d^3p} \quad g,q(u,d,s,\bar{u},\bar{d},\bar{s}) \]

\[ \frac{df_{q,g}(p)}{d\tau} - \frac{p_z}{\tau} \partial_{p_z} f_{q,g}(p) = -C_{2\leftrightarrow 2}[f_{q,g}(p)] - C_{1\leftrightarrow 2}[f_{q,g}(p)] \]

0 + 1d Bjorken

“Bottom up thermalisation”
Baier, Mueller, Schiff, and Son (2001)

Relaxation time approximation: RTA


- Approach to equilibrium set by an equilibration rate.

\[
- \frac{df_g(p)}{d\tau} + \frac{p_z}{\tau} \partial_{p_z} f_g(p) = \frac{p^\mu u_\mu}{\tau R} (f_g(p) - f_g^{eq}(p))
\]


Popular approach in URHICs phenomenology \(\Rightarrow\) affects the extraction of transport coefficients
Non-equilibrium dynamics
Non-equilibrium effects and Hydrodynamization

Uncertainty in our understanding of the regime applicability of fluid dynamics

- **Collectivity:** system sizes/ collision energies
- **Hydrodynamics constitutive in AdS/CFT**
  

- Large order hydrodynamic gradient expansion can be divergent!
  
  Heller, Janik, Witaszczyk PRL 110, (2013)

- System lives most of its life-time in a state of out of equilibrium
  
  (Niemi, Denicol arxiv.1404.7327) (Noronha-Hostler, Noronha, Gyulassy PRC 93(2016))

  especially true for small systems!
Non-equilibrium effects and Hydrodynamization

Criteria of the regime applicability of fluid dynamics

\[ Kn = \frac{\text{microscopic scale}}{\text{macroscopic scale}} \leq 0.5 \]

\[ \sqrt{s} = 2.76 \text{ TeV Pb+Pb} \]

Noronha-Hostler, Noronha, Gyulassy PRC 93(2016)

\[ \sqrt{s} = 5 \text{ TeV Pb+Pb} \]


How can we improve the validity of the hydrodynamical approach?
The attractor: a proper way to think of hydrodynamics

Hydrodynamics as a universal attractor
Heller and Spalinski. PRL.115 (2015)

- Memory loss of initial conditions
- Competition: expansion and interaction rate
- Microscopic model dependent
- Non-hydro modes as regulators to ensure causality

Onset of hydrodynamics set by decay of non-hydrodynamic modes

\[ W = \frac{\tau}{\tau_R} = \frac{\tau T}{4\pi \eta} = K_n^{-1} \]
Attractors in different microscopic theories

Kurkela, van der Schee, Wiedemann, Wu PRL. 124, (2020)

Decay of non-hydro modes depends on the underlying microscopic theory

Strong coupling

How can one access the underlying information?
Structure of non-hydrodynamics modes


Non-hydrodynamic excitations make large contribution to $v_2$ in small systems.

\[ G_{R_{\mu\nu,\alpha\beta}}^k (x; t) = \langle [T^\mu_\nu(x, t), T^{\alpha\beta}(0, 0)] \rangle \]

- **Kinetic theory**
  - Hydrodynamic pole
  - Non-hydrodynamic cut
  - $Dk^2 + ...$

- **Israel-Stewart hydro**
  - Hydrodynamic pole
  - Non-hydrodynamic pole $\omega = i/\tau$
  - $Dk^2 + ...$

- **AdS/CFT**
  - Hydrodynamic pole
  - Non-hydrodynamic poles $\omega \sim 2\pi T(1/2 \pm i)n$

▶ Small systems?
▶ Collectivity arises in small system from microscopic interactions?.
▶ Can we disentangle their effects from the hydrodynamical flow?
Universal scaling
Giuliano Giacalone, Aleksas Mazeliauskas, and Sören Schlichting
Phys. Rev. Lett. 123, 262301

Qualitatively similar description from different microscopic theories

\[ \tilde{\omega} = \frac{\tau T_{\text{eff}}}{4\pi \eta/s} \]

Attractors provide connections between the early time dynamics and final state observables
Non-equilibrium attractor beyond hydrodynamics?

Almaalol, Kurkela, Strickland PRL. 125, (2020)
General moments of the distribution function

M. Strickland, JHEP2018, 128; 1809.01200.

Solving higher moments of the Boltzmann equation

\( N_C = 3 \) in \( 0 + 1d \) Bjorken flow, 2D grid \( \{x_i, p_j\} \) with \( 250 \times 2000 \) grid points

(Kurkela and Zhu PRL 115, 182301 (2015))

▶ A general moment of the distribution function is defined by

\[
\mathcal{M}^{nm}[f] \equiv \int dP (p.u)^n (p.z)^{2m} f(x, p)
\]

▶ The hydrodynamics degrees of freedom are

- \( \mathcal{M}^{10} = \) number density
- \( \mathcal{M}^{20} = \) energy density
- \( \mathcal{M}^{01} = \) longitudinal pressure

▶ Study the deviations from equilibrium

The corresponding equilibrium values using a Bose distribution,

\[
\mathcal{M}_{eq}^{nm} = \frac{T^{n+2m+2} \Gamma(n + 2m + 2) \zeta(n + 2m + 2)}{2\pi^2(2m + 1)}
\]

\[
\overline{\mathcal{M}}^{nm}(\tau) \equiv \frac{\mathcal{M}^{nm}(\tau)}{\mathcal{M}_{eq}^{nm}(\tau)}.
\]
Initial distribution $-\frac{df_p}{d\tau} = C_{1 \leftrightarrow 2}[f_p] + C_{2 \leftrightarrow 2}[f_p] + C_{\exp}[f_p]$.

▶ Thermal Romatschke-Strickland

\[ f_{0,RS}(p) = f_{Bose} \left( \sqrt{p^2 + \xi_0 p_z^2}/\Lambda_0 \right) \]

anisotropy parameter \((-1 < \xi_0 < \infty)\)

\(\Lambda_0\) is set by Landau matching


▶ Non-thermal distribution

\[ f_{0,CGC}(p) = \frac{2A}{\lambda} \frac{\tilde{\Lambda}_0}{\sqrt{p^2 + \xi_0 p_z^2}} \exp^{-\frac{2}{3}(p^2 + \xi_0 \tilde{p}_z^2)/\tilde{\Lambda}_0^2} \]

The initial scale \(\tilde{\Lambda}_0\) is related to the saturation scale \(\tilde{\Lambda}_0 = \langle p_T \rangle_0 \approx 1.8 Q_s\)

\(A\) is set by fixing the initial energy density to match an expectation value estimated from a CYM simulation


Non-equilibrium QCD attractor at high temperature

DA, Kurkela, Strickland PRL. 125, (2020)

Non-equilibrium evolution becomes insensitive to initial conditions at very early times

\[ \tau_R(\tau) = \frac{4\pi \bar{\eta}}{T(\tau)} \]

▶ Forward attractor

Pressure anisotropy

\[ M^{01}[f] = \langle p^{-1} p_z^2 \rangle \]

\[ M^{21}[f] = \langle p^1 p_z^2 \rangle \]

\[ M^{33}[f] = \langle p^2 p_z^6 \rangle \]
Non-equilibrium QCD attractor at high temperature

DA, Kurkela, Strickland PRL. 125, (2020)

An attractor for the momentum phase space distribution function

- Pullback attractor
- EKT extends beyond hydro degrees of freedom
- RTA fails to capture the dynamics at high moments

### Pressure anisotropy

\[ \mathcal{M}^{01}[f] = \langle p^{-1} p_z^2 \rangle \]
\[ \mathcal{M}^{21}[f] = \langle p^1 p_z^2 \rangle \]
\[ \mathcal{M}^{33}[f] = \langle p^2 p_z^6 \rangle \]

<table>
<thead>
<tr>
<th>( t/t_R )</th>
<th>( \tau )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>0.32 fm/c</td>
</tr>
<tr>
<td>0.5</td>
<td>0.86 fm/c</td>
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<tr>
<td>1</td>
<td>1.88 fm/c</td>
</tr>
<tr>
<td>2</td>
<td>4.23 fm/c</td>
</tr>
<tr>
<td>5</td>
<td>14.1 fm/c</td>
</tr>
<tr>
<td>10</td>
<td>38.5 fm/c</td>
</tr>
</tbody>
</table>
Beyond Bjorken flow?
Bjorken flow attractors with transverse dynamics (conformal)

Which aspects of the attractor behavior are accessible in collisions with a finite transverse extent?

\[-C[F] = - \left( -v_\mu u^\mu \right) \frac{F - F_{iso}}{\tau_{iso}},\]

\[\tau_{iso} = \frac{1}{\gamma \epsilon^{1/4}}, \quad \hat{\gamma} = R^{3/4} \gamma (\epsilon_0 \tau_0)^{1/4}\]

\[\delta \tau_H = (\tau_H - \tau_0) / \tau_0\]

What remains universal is the early-time dynamics
Non-conformal attractor
Non-conformal attractor in weakly coupled RTA (0+1d)

Breaking conformality

\[ C[f] = -\frac{1}{\tau_R} (f - f_{eq}) \quad , \quad \tau_R = 5 \frac{C}{T(\tau)} \quad , \quad C = 10/4\pi, \]

No early-time attractor for \( \pi, \Pi \)

\[
\begin{align*}
\Pi/(\epsilon + P) & \quad (a) \\
\pi/(\epsilon + P) & \quad (b)
\end{align*}
\]

\[
\begin{align*}
P_L &= P + \Pi - \pi
\end{align*}
\]

Modifications to both shear and bulk channels

See also: Romatchke JHEP 12 (2017), Florkowski, Maksymiuk, Ryblewski, PRC 97 (2018),

Chemical equilibration?
QGP chemical equilibration

Kurkela.Mazeliauskas.PRL, 122,(2019)

\[ \tau_{\text{hydro}} < \tau_{\text{chem}} < \tau_{\text{therm}} \]

- QCD transport of \( N_f = 3 \) massless fermions.
- Quarks are dynamically produced through fusion \( gg \rightarrow q\bar{q} \) and splitting \( g \rightarrow q\bar{q} \)
- energy transfer from gluonic to quark sectors
Quarks?

[Image]

Inclusion of quarks increases anisotropy

- QCD transport of $N_f = 3$ massless fermions.
- Quarks are dynamically produced through fusion $gg \to q\bar{q}$ and splitting $g \to q\bar{q}$

An attractor exists for all moments in QCD with $N_f = 3$ quarks
QGP chemical equilibration

Giacalone, Mazeliauskas, Schlichting PRL. 123(202)
Du, Schlichting (2012.09068), (2012.09079)

Non-equilibrium corrections dominate the system's lifetime at lower beam energies

- Slower isotropization with quarks
- Moderate dependence on $\mu_B/T$
Non-equilibrium effects at Freeze out
Almaalol, Kurkela, Strickland PRL. 125, (2020)
Non-equilibrium effects at freezeout

(Almaalol, Kurkela, Strickland PRL 125, (2020))

- $\delta f$ corrections at freezeout directly affect the anisotropic flow $v_2(p_T)$

$$E \frac{d^3 N_s}{d^3 p} = \nu_s \frac{1}{(2\pi)^3} \int \sigma (f_s(\vec{p}) + \delta f))$$

- $\delta f$ can be computed for a particular form of $C[f]$

(Dusling, Moore, Teaney PRC 81, (2008))

The quadratic ansatz ($\alpha = 0$)

$$\frac{\delta f(i)}{f_{eq}(1 + f_{eq})} = \frac{3\Pi}{16T^2} (p^2 - 3p_z^2)$$

The LPM ansatz ($\alpha = 0.5$)

$$\frac{\delta f(ii)}{f_{eq}(1 + f_{eq})} = \frac{16\Pi}{21\sqrt{\pi} T^{3/2}} \left( p^{3/2} - \frac{3p_z^2}{\sqrt{p}} \right)$$

The aHydro freeze-out ansatz

$$f(p) = f_{Bose}(\sqrt{\mathbf{p}^2 + \xi p_z^2}/\Lambda)$$

$$\mathcal{M}_{aHydro}^{nm}(\tau) = 2^{(n+2m-2)/4}(2m+1) \frac{\mathcal{H}^{nm}(\alpha)}{\mathcal{H}^{20}(\alpha)(n+2m+2)/4}$$
Insights into the freezeout prescription
(Almaalol,Kurkela,Strickland PRL 125, (2020))

- Disagreement increases for higher moments and for earlier times.
- Good agreement between aHydro ansatz and EKT at all times

\( M^{11}[f] = \langle p_z^2 \rangle \)
\( M^{21}[f] = \langle p^1 p_z^2 \rangle \)
\( M^{22}[f] = \langle p^1 p_z^4 \rangle \)

For earlier implementation: (Pratt,Torrieri PRC 82(2010) (Weller, Romatchke PLB 774 (2017))
Future directions

To what limit we can extend the weakly coupled analysis?

- Extrapolation towards higher couplings?
- Convergence of the perturbative series?

(Kurkela, Zhu PRL 774 (2015)

(Ghiglieri, Moore, and Teaney PRL 121 (2018)
Conclusions

- Weakly coupled kinetic transport theory provides a clean limit to study the non-equilibrium dynamics of the QCD medium at high temperatures.

- EKT also provides insights into phenomenology and different stages of the ultra-relativistic heavy ion collisions evolution.

- How can we extrapolate results from the weakly coupled dynamics to higher couplings? Is it possible? Path to it?

- Breaking conformality in QCD kinetic theory: critical for applying EKT in phenomenology.
Thank you for your attention!
hydrodynamic model results are dependent on initialization time, and different hydrodynamic codes regulate these extreme initial conditions in different ad hoc ways.

The subsequent hydrodynamic evolution becomes independent of the hydrodynamic initialization time!!

\begin{equation}
T^\mu_\nu(\tau_{\text{hydro}}, x) = T^\mu_\nu(\tau_{\text{EKT}}, x) + \delta T^\mu_\nu(\tau_{\text{EKT}}, x').
\end{equation}

\begin{equation}
f_{x,p} = \bar{f}_{p} + \int \frac{d^2 k}{(2\pi)^2} \delta f_{k,p} e^{i k \cdot x}.
\end{equation}

\begin{equation}
T^\mu_\nu(\tau_{\text{hydro}}, x) = T^\mu_\nu(\tau_{\text{hydro}}) + \frac{T^\mu_\nu(\tau_{\text{hydro}})}{T^\mu_\nu(\tau_{\text{EKT}})} \times
\end{equation}

\begin{equation}
\times \int d^2 x' \, C^{\mu_\nu}_{\alpha\beta}(x, x', \tau_{\text{hydro}}, \tau_{\text{EKT}}) \delta T_{x}^{\alpha\beta}(\tau_{\text{EKT}}, x').
\end{equation}
Scaling and Entropy production


Macroscopic description from hydrodynamics attractors. Integrate equations of motion to find energy attractor

\[
\frac{\partial \tau}{e} = -\frac{e}{\tau} + \frac{P_L}{e}; \quad \frac{P_L}{e} = f \left[ \tilde{w} = \frac{\tau T_{eff}}{4\pi \eta/s} \right]
\]

\[
(s\tau)_{\text{hydro}} = \frac{4}{3} C^3 \left( \frac{4\pi \eta}{s} \right)^{1/3} \left( \frac{\pi^2}{30} \nu_{\text{eff}} \right)^{1/3} (e\tau)^{2/3},
\]

\[
\frac{dN_{\text{ch}}}{d\eta} \approx \frac{1}{J} A_{\perp} (s\tau)_{\text{hydro}} \frac{N_{\text{ch}}}{S}.
\]

where \( A_{\perp} \approx \pi R^2 S/N_{\text{ch}} \approx 7 \) is a constant of hadron gas

\[
\frac{dE_{\perp}}{d\eta} \approx A_{\perp} (e\tau)_0.
\]

Macroscopic description from hydrodynamics attractors!
KoMPoST: future improvement

- Inclusion of kompost influence the extraction of transport coefficients


- Insensitivity of PCA to the pre-equilibrium stage


⇒ Call for non-conformal treatment
Kinetic based hydrodynamics equations

Moment integral operator

\[ \hat{O}_n g = \mathcal{O}^{\mu_1 \mu_2 \cdots \mu_n}[g] \equiv \int dP \: p^{\mu_1} p^{\mu_2} \cdots p^{\mu_n} \: g(p) n^{th} \]

\[ p^{\mu} \partial_\mu f_p = \mathcal{C}[f_p] \]

\[ \partial_\mu I^{\mu \nu_1 \nu_2 \cdots \nu_n} = C^{\nu_1 \nu_2 \cdots \nu_n} \]

\[ I^{\mu \nu_1 \nu_2 \cdots \nu_n} \equiv \int dP \: p^{\mu} p^{\nu_1} p^{\nu_2} \cdots p^{\nu_n} \: f \]

\[ C^{\nu_1 \nu_2 \cdots \nu_n} \equiv \int dP \: p^{\nu_1} p^{\nu_2} \cdots p^{\nu_n} \: \mathcal{C}[f] \]

Landau Matching

\[ \partial_\mu n^\mu = \mathcal{C} \]

\[ \partial_\mu T^{\mu \nu} = C^\nu \]

\[ \mathcal{C}_{RTA} = \frac{u^\mu \cdot p^\mu}{\tau_R} \left[ f_{eq}(p/T) - f_p \right] \]

\[ \partial_\mu n^\mu = \frac{1}{\tau_{eq}} [n_{eq} - n] \]

\[ \partial_\mu T^{\mu \nu} = C^\nu = \frac{1}{\tau_{eq}} [\epsilon_{eq} - \epsilon] \]
Kinetic based hydrodynamics equations

Moment method


- Popular approach in phenomenology
- Direct impact on transport coefficients!

Landau Matching

\[
\partial_\mu n^\mu = C \\
\partial_\mu T^{\mu\nu} = C^\nu
\]

\[
C_{RTA} = \frac{u^\mu p^\mu}{\tau_R} \left[ f_{eq}(p/T) - f_p \right]
\]

\[
\partial_\mu n^\mu = \frac{1}{\tau_R} [ n_{eq} - n ] \\
\partial_\mu T^{\mu\nu} = \frac{1}{\tau_R} [ \epsilon_{eq} - \epsilon ]
\]
How microscopic details affect macroscopic quantities far from equilibrium?

"renormalized transport coefficients"?