Non-equilibrium dynamics in weakly coupled gauge theories.
(attractors and hydrodynamisation)

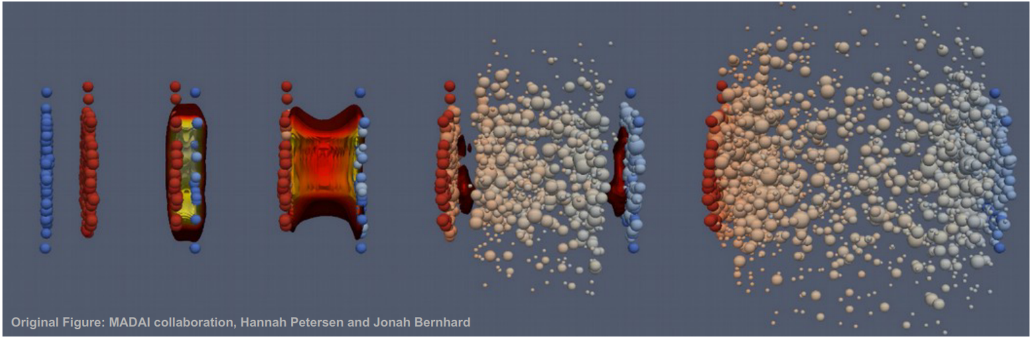
Dekrayat Almaalol

A Virtual Tribute to Quark Confinement and the Hadron Spectrum 2021
Aug 02, 2021

Dynamical evolution of ultra-relativistic heavy ion collisions

- Phenomenological demand: quantify the uncertainties in transition between various stages

$\sim 0^-$ fm/c ~ 0.1 fm/c ~ 1 fm/c ~ 10 fm/c ~ 20 fm/c

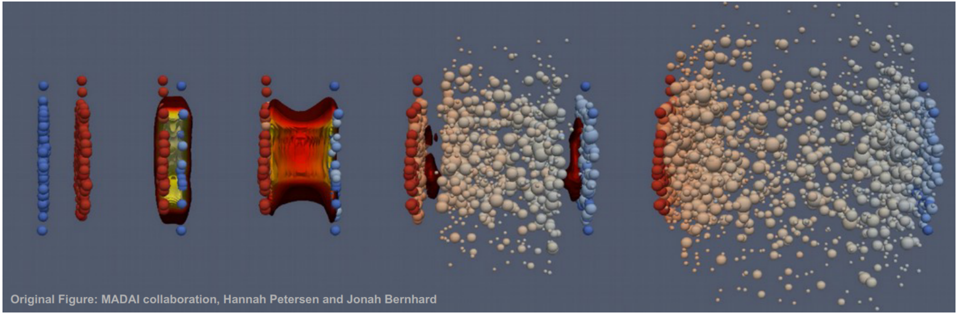


initial state   preequilibrium   Hydrodynamics   Hadronization   Hadronic cascade

- Hydrodynamization
- Improve out-of-equilibrium hydrodynamical descriptions
- Insights into URHIC phenomenology

Non-equilibrium effects in ultra-relativistic heavy ion collisions

$\sim 0^-$ fm/c ~ 0.1 fm/c ~ 1 fm/c ~ 10 fm/c $\tau \sim 20$ fm/c



initial state preequilibrium Hydrodynamics Hadronization Hadronic cascade

QCD kinetic transport

- weakly coupled approach valid at high T
- Physically motivated to study the pre-equilibrium dynamics

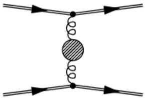
Formalism

QCD medium at high temperatures: Effective kinetic theory AMY JHEP0301 (2003)

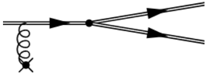
$$P^{\mu}\partial_{\mu} f_{q,g}(\mathbf{p}) = -C[f(\mathbf{p})], \qquad f_{q,g} \propto \frac{dN_{g,q}}{d^3x d^3p} \qquad g, q(u, d, s, \bar{u}, \bar{d}, \bar{s}))$$

$$\frac{df_{q,g}(\mathbf{p})}{d\tau} - \frac{p_z}{\tau}\partial_{p_z} f_{q,g}(\mathbf{p}) = -\mathcal{C}_{\mathbf{2}\leftrightarrow\mathbf{2}}[f_{q,g}(\mathbf{p})] - \mathcal{C}_{\mathbf{1}\leftrightarrow\mathbf{2}}[f_{q,g}(\mathbf{p})] \qquad \underline{0 + 1\mathrm{d\,Bjorken}}$$

At Leading order, transport at different momentum scales “hard” p$\sim$ T and “soft” p $\sim$ gT)



regulated by HTL



LPM suppression

Baier, Mueller, Schiff, and Son (2001); J.Berges, M.Heller, A.Mazeliauskas and R.Venugopalan arxiv.2005.12299 (2020); Schlichting, Teaney, Ann. Rev. of Nuc Part. Sci.(2019); Arnold, P. Gorda, T. Iqbal, S. JHEP. 2020, 53

QCD medium at high temperatures: Effective kinetic theory

AMY JHEP0301 (2003) 030

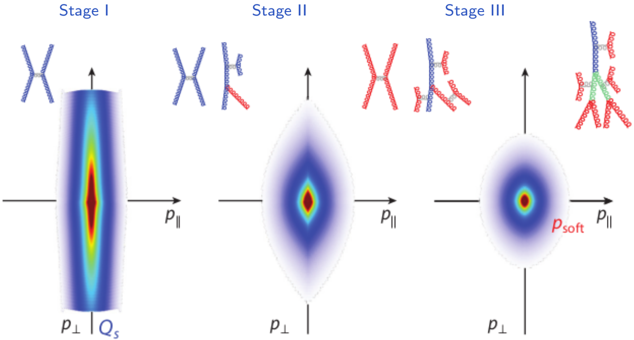
$$P^{\mu}\partial_{\mu}\, f_{q,g}(\mathbf{p}) = -C[f(\mathbf{p})], \qquad f_{q,g} \propto \frac{dN_{g,q}}{d^3x d^3p} \qquad g, q(u, d, s, \bar{u}, \bar{d}, \bar{s}))$$

$$\frac{df_{q,g}(\mathbf{p})}{d\tau} - \frac{p_z}{\tau}\partial_{p_z} f_{q,g}(\mathbf{p}) = -\mathcal{C}_{\mathbf{2}\leftrightarrow\mathbf{2}}[f_{q,g}(\mathbf{p})] - \mathcal{C}_{\mathbf{1}\leftrightarrow\mathbf{2}}[f_{q,g}(\mathbf{p})]$$

0 + 1d Bjorken

" Bottom up thermalisation"

Baier, Mueller, Schiff, and Son (2001)



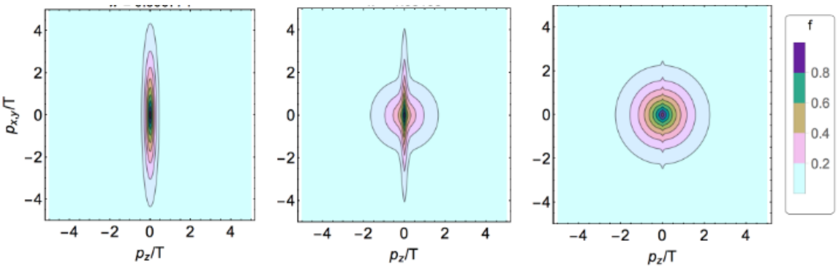
J.Berges, M.Heller, A.Mazeliauskas and R.Venugopalan arxiv.2005.12299 (2020); Schlichting, Teaney, Ann. Rev. of Nuc Part. Sci.(2019)

Relaxation time approximation: RTA

J.L. Anderson, H.R. Witting Physica, 74(1974)

- Approach to equilibrium set by an equilibration rate.

$$-\frac{df_g(\mathbf{p})}{d\tau} + \frac{p_z}{\tau}\partial_{p_z} f_g(\mathbf{p}) = \frac{p^\mu u_\mu}{\tau_R}\left(f_g(\mathbf{p}) - f_g^{eq}(\mathbf{p})\right)$$



M. Strickland. JHEP.2018,128 (2018)

Popular approach in URHICs phenomenology ⇒ affects the extraction of transport coefficients

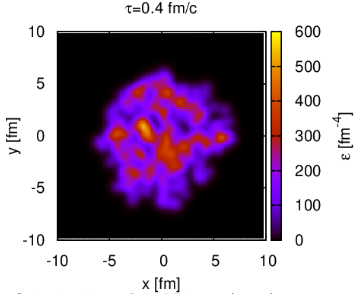
Non-equilibrium dynamics

Non-equilibrium effects and Hydrodynamization

Uncertainty in our understanding of the regime applicability of fluid dynamics

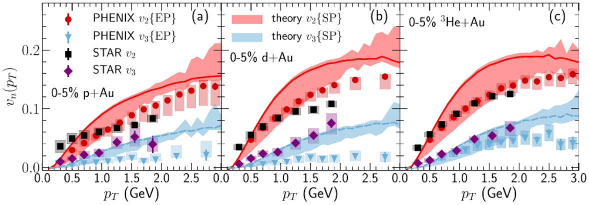
- Collectivity: system sizes/ collision energies
- Hydrodynamics constitutive in AdS/CFT

  Chesler,Yaffe PRD 82,(2010) , Heller,Janik,Witaszczyk PRL.108,(2012)



Schenke, Jeon, Gale PRL.106 (2010)

Schenke.Chun.Prithwish PRB 803 (2020)



- Large order hydrodynamic gradient expansion can be divergent!

  Heller, Janik, Witaszczyk PRL. 110,(2013)

- System lives most of its life-time in a state of out of equilibrium

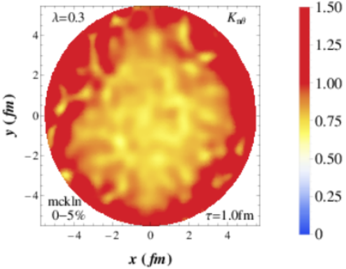
  (Niemi,Denicol arxiv.1404.7327) (Noronha-Hostler,Noronha,Gyulassy PRC 93(2016))

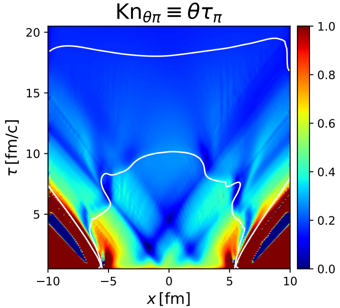
especially true for small systems!.

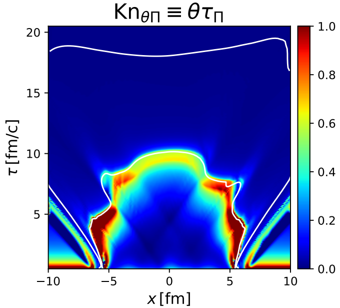
Non-equilibrium effects and Hydrodynamization

Criteria of the regime applicability of fluid dynamics

$$\mathrm{Kn} = \frac{\text{microscopic scale}}{\text{macroscopic scale}} \leq 0.5$$







$\sqrt{s} = 2.76$ TeV Pb+Pb

Noronha-Hostler,Noronha,Gyulassy PRC 93(2016)

$\sqrt{s} = 5$ TeV Pb+Pb

Bazow, Heinz, Strickland Comp. Phys. Comm. 225 (2018)

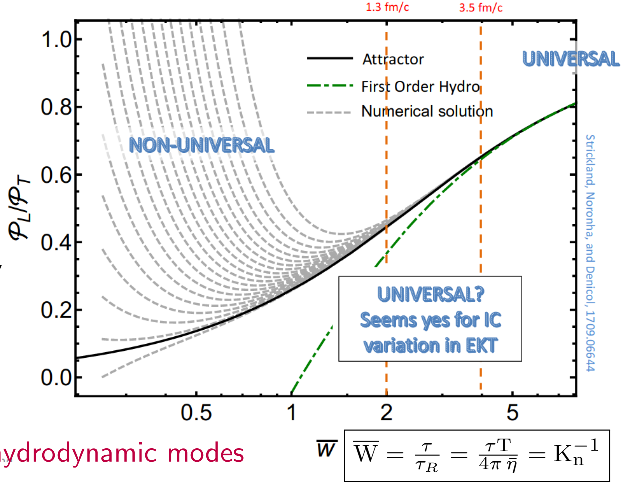
How can we improve the validity of the hydrodynamical approach?

The attractor: a proper way to think of hydrodynamics

Hydrodynamics as a universal attractor

Heller and Spalinski. PRL.115 (2015)

- Memory loss of initial conditions
- Competition: expansion and interaction rate
- Microscopic model dependent
- Non-hydro modes as regulators to ensure causality

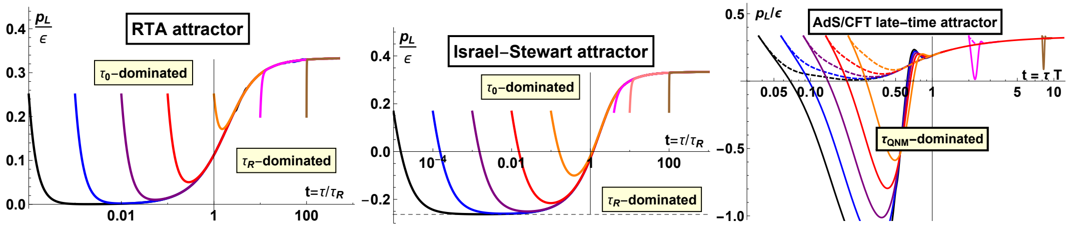


Onset of hydrodynamics set by decay of non-hydrodynamic modes

Attractors in different microscopic theories

Kurkela,van der Schee,Wiedemann,Wu PRL.124,(2020)

Decay of non-hydro modes depends on the underlying microscopic theory



Strong coupling

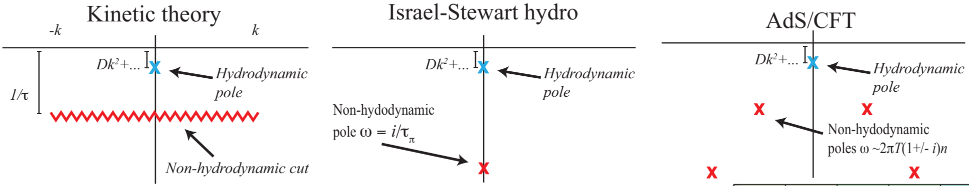
How can one access the underlying information?

Structure of non-hydrodynamics modes

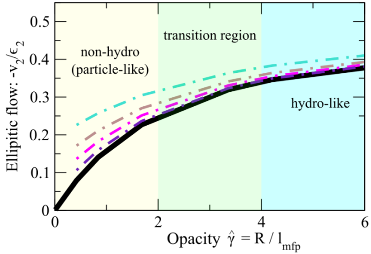
Kurkela,Wiedemann,Wu, EPJ.C79,(2019)

Non-hydrodynamic excitations make large contribution to v_2 in small systems.

$$G_R^{\mu\nu,\alpha\beta}(x;t) = \langle [T^{\mu\nu}(x,t),\, T^{\alpha\beta}(0,0)] \rangle$$



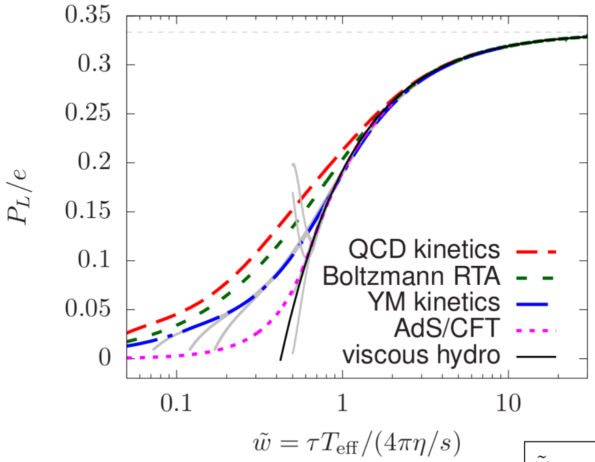
- Small systems?
- Collectivety arises in small system from microscopic interactions?.
- Can we disentangle their effects from the hydrodynamical flow?



Universal scaling

Giuliano Giacalone, Aleksas Mazeliauskas, and Sören Schlichting Phys. Rev. Lett. 123, 262301

Qualitatively similar description from different microscopic theories



$$\tilde{\omega} = \frac{\tau}{\tau_R} = \frac{\tau T}{4\pi\,\bar{\eta}} = \mathrm{K_n}^{-1}$$

Attractors provide connections between the early time dynamics and final state observables

Non-equilibrium attractor beyond hydrodynamics?

Almaalol,Kurkela, Strickland PRL. 125,(2020)

General moments of the distribution function

M. Strickland, JHEP2018, 128; 1809.01200.

Solving higher moments of the Boltzmann equation

$\mathrm{N_C} = 3$ in $0+1\mathrm{d}$ Bjorken flow, 2D grid $\{x_i, p_j\}$ with 250×2000 grid points

(Kurkela and Zhu PRL 115, 182301 (2015))

- A general moment of the distribution function is defined by

$$\mathcal{M}^{nm}[f] \equiv \int dP\,(p.u)^n\,(p.z)^{2m}\,f(x,p)$$

- The corresponding equilibrium values using a Bose distribution,

$$\mathcal{M}^{nm}_{\text{eq}} = \frac{T^{n+2m+2}\Gamma(n+2m+2)\zeta(n+2m+2)}{2\pi^2(2m+1)}$$

- the hydrodynamics degrees of freedom are

$$\mathcal{M}^{10} = \quad \text{number density}$$
$$\mathcal{M}^{20} = \quad \text{energy density}$$
$$\mathcal{M}^{01} = \quad \text{longitudinal pressure}$$

- Study the deviations from equilibrium

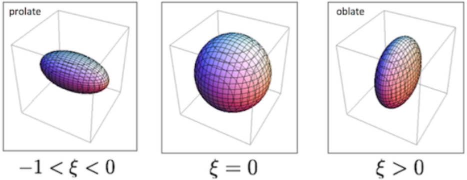
.

$$\overline{\mathcal{M}}^{nm}(\tau) \equiv \frac{\mathcal{M}^{nm}(\tau)}{\mathcal{M}^{nm}_{\text{eq}}(\tau)}$$

Initial distribution $-\frac{d\mathbf{f_p}}{d\tau} = \mathcal{C}_{1\leftrightarrow 2}[\mathbf{f_p}] + \mathcal{C}_{2\leftrightarrow 2}[\mathbf{f_p}] + \mathcal{C}_{\exp}[\mathbf{f_p}].$

▶ Thermal Romatschke-Strickland

$$\mathrm{f}_{0,\mathrm{RS}}(\mathbf{p}) = f_{\mathrm{Bose}}\left(\sqrt{\mathbf{p}^2 + \xi_0 p_z^2}/\Lambda_0\right)$$



anisotropy parameter $(-1 < \xi_0 < \infty)$

Λ_0 is set by Landau matching

Romatschke,Strickland,PRD68, (2003)

▶ Non-thermal distribution

$$\mathrm{f}_{0,\mathrm{CGC}}(\mathbf{p}) = \frac{2A}{\lambda}\frac{\tilde{\Lambda}_0}{\sqrt{\mathbf{p^2} + \xi_0 p_z^2}} \exp^{-\frac{2}{3}\left(\mathbf{p}^2 + \xi_0 \hat{p}_z^2\right)/\tilde{\Lambda}_0^2}$$

The initial scale $\tilde{\Lambda}_0$ is related to the saturation scale $\tilde{\Lambda}_0 = \langle \mathrm{p_T} \rangle_0 \approx 1.8\,\mathrm{Q_s}$

A is set by fixing the initial energy density to match an expectation value estimated from a CYM simulation

A. Kurkela and Y. Zhu, Phys. Rev. Lett.115, 182301(2015)

T. Lappi, Phys. Lett.B703, 325-330 (2011)

Non-equilibrium QCD attractor at high temperature

DA, Kurkela,Strickland PRL. 125, (2020)

Non-equilibrium evolution becomes insensitive to initial conditions at very early times

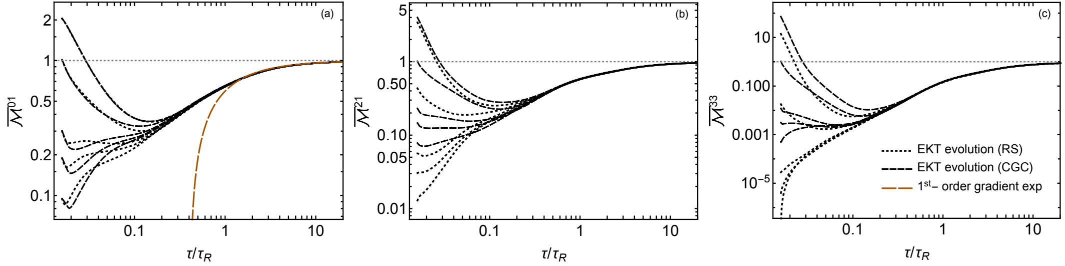
$\tau_R(\tau) = 4\pi\bar{\eta}/T(\tau)$

τ/τ_R	τ
0.2	0.32 fm/c
0.5	0.86 fm/c
1	1.88 fm/c
2	4.23 fm/c
5	14.1 fm/c
10	38.5 fm/c

- Forward attractor

Pressure anisotropy

$\mathcal{M}^{01}[f] =< p^{-1}p_z^2 >$ $\mathcal{M}^{21}[f] =< p^{1}p_z^2 >$ $\mathcal{M}^{33}[f] =< p^{2}p_z^6 >$



Non-equilibrium QCD attractor at high temperature

DA, Kurkela,Strickland PRL. 125, (2020)

An attractor for the momentum phase space distribution function

$$\tau_R(\tau) = 4\pi\bar{\eta}/T(\tau)$$

τ/τ_R	τ
0.2	0.32 fm/c
0.5	0.86 fm/c
1	1.88 fm/c
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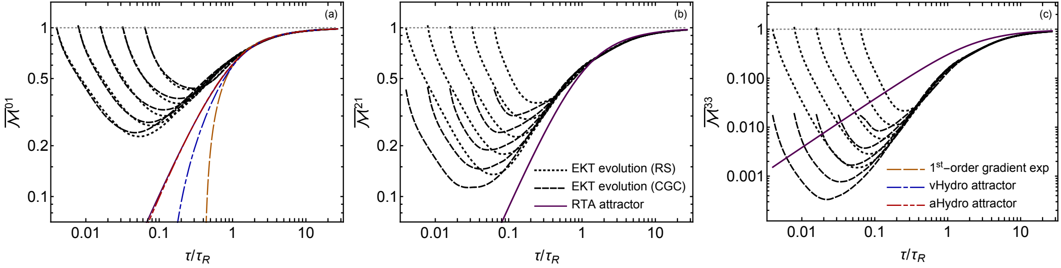
- Pullback attractor
- EKT extends beyond hydro degrees of freedom
- RTA fails to capture the dynamics at high moments

Pressure anisotropy

$\mathcal{M}^{01}[f] = < p^{-1} p_z^2 >$ $\quad$ $\mathcal{M}^{21}[f] = < p^{1} p_z^2 >$ $\quad$ $\mathcal{M}^{33}[f] = < p^{2} p_z^6 >$



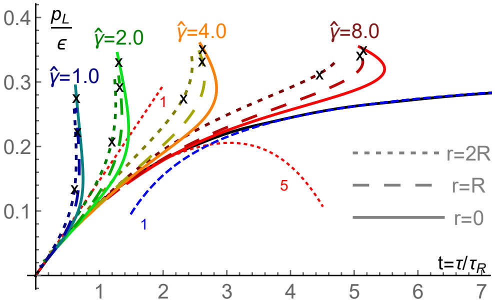
Beyond Bjorken flow?

Bjorken flow attractors with transverse dynamics (conformal)

Which aspects of the attractor behavior are accessible in collisions with a finite transverse extent?

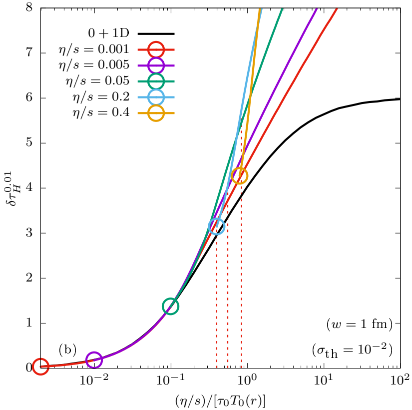
$$-C[F] = -\frac{(-v_\mu u^\mu)}{\tau_{iso}}(F - F_{iso}),$$

$$\tau_{iso} = \frac{1}{\gamma \varepsilon^{1/4}}, \qquad \hat{\gamma} = R^{3/4}\gamma(\varepsilon_0 \tau_0)^{1/4}$$



Kurkela, van der Schee, Wiedemann Wu, PRL.124(2020)

$$\delta\tau_H = (\tau_H - \tau_0)/\tau_0$$



Ambrus, Busuioc, Fotakis, Gallmeister, Greiner arXiv:2102.117805(2021)

What remains universal is the early-time dynamics

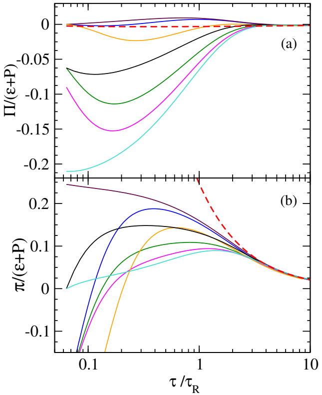
Non-conformal attractor

Non-conformal attractor in weakly coupled RTA (0+1d)

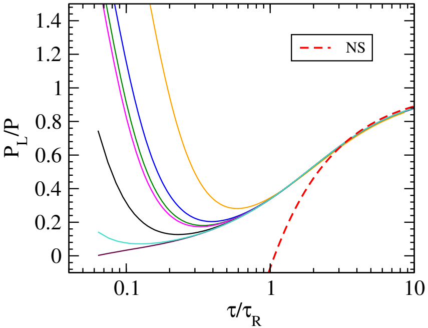
Breaking conformality

$$C[f] = -\frac{1}{\tau_R}(f - f_{eq}) \qquad , \tau_R = 5\,C/T(\tau) \qquad , C = 10/4\pi,$$

No early-time attractor for π, Π



$P_L = P + \Pi - \pi$



Chattopadhyay, Jaiswal, Du, Heinz, and Pal, arXiv:2107.05500 and 2107.10248 (2021)

Modifications to both shear and bulk channels

See also: Romatchke JHEP 12 (2017), Florkowski, Maksymiuk, Ryblewski, PRC 97 (2018),

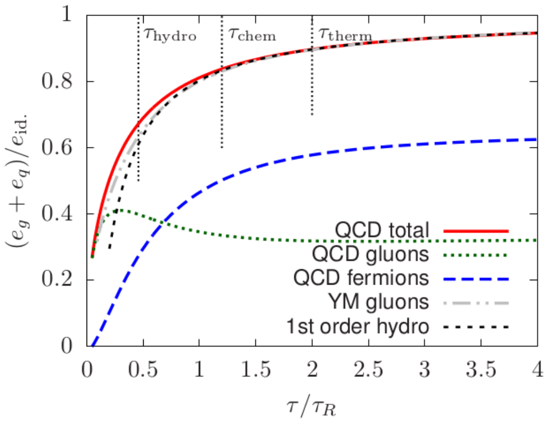
Chemical equilibration?

QGP chemical equilibration

Kurkela.Mazeliauskas.PRL, 122,(2019)

$\tau_{\text{hydro}} < \tau_{\text{chem}} < \tau_{\text{therm}}$

- QCD transport of $N_f = 3$ massless fermions.
- Quarks are dynamically produced through fusion $\mathrm{gg} \to \mathrm{q\bar{q}}$ and splitting $\mathrm{g} \to \mathrm{q\bar{q}}$
- energy transfer from glounic to quark sectors

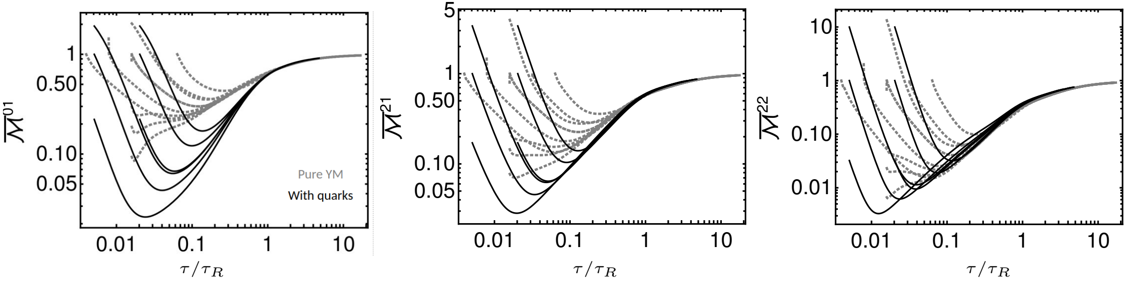


Quarks?

DA, Mazeliauskas, Strickland. forthcoming

Inclusion of quarks increases anisotropy

- QCD transport of $N_f = 3$ massless fermions.
- Quarks are dynamically produced through fusion $\mathrm{gg} \to \mathrm{q\bar{q}}$ and splitting $\mathrm{g} \to \mathrm{q\bar{q}}$



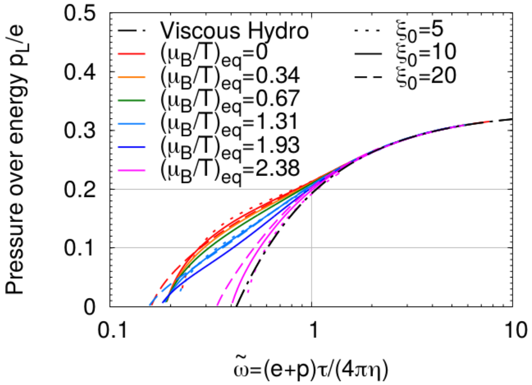
- An attractor exists for all moments in QCD with $N_f = 3$ quarks

QGP chemical equilibration

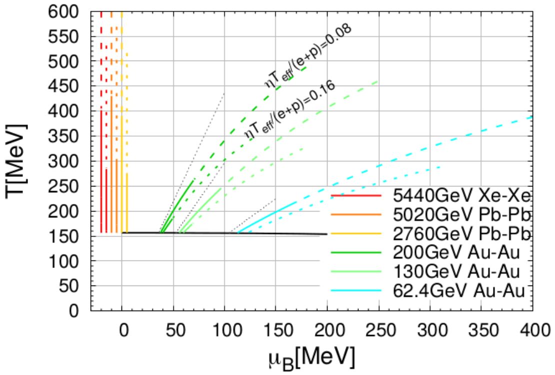
Giacalone,Mazeliauskas,Schlichting PRL.123(202)

Du, Schlichting (2012.09068), (2012.09079)

Non-equilibrium corrections dominate the system's lifetime at lower beam energies



Slower isotropization with quarks



moderate dependence on μ_B/T

Non-equilibrium effects at Freeze out

Almaalol,Kurkela, Strickland PRL. 125,(2020)

Non-equilibrium effects at freezeout

(Almaalol,Kurkela, Strickland PRL 125,(2020))

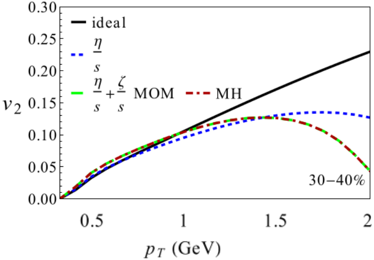
- δf corrections at freezeout directly affect the anisotropic flow $v_2(p_T)$

$$E\frac{d^3N_s}{d^3p} = \frac{\nu_s}{(2\pi)^3}\int_\sigma (f_s(\tilde{p}) + \delta f))$$

- δf can be computed for a particular form of $C[f]$

(Dusling,Moore,Teaney PRC 81, (2008))

(Noronha-Hostler, Noronha,Grassi, PRC 90 (2014))



The *quadratic ansatz* $(\alpha = 0)$

$$\frac{\delta f_{(i)}}{f_{\rm eq}(1+f_{\rm eq})} = \frac{3\overline{\Pi}}{16T^2}(p^2 - 3p_z^2)$$

The *LPM ansatz* $(\alpha = 0.5)$

$$\frac{\delta f_{(ii)}}{f_{\rm eq}(1+f_{\rm eq})} = \frac{16\overline{\Pi}}{21\sqrt{\pi}\,T^{3/2}}\left(p^{3/2} - \frac{3p_z^2}{\sqrt{p}}\right)$$

The *aHydro freeze-out ansatz*

$$f(p) = f_{\rm Bose}(\sqrt{\mathbf{p}^2 + \xi p_z^2}/\Lambda)$$

$$\overline{\mathcal{M}}^{nm}_{\rm aHydro}(\tau) = 2^{(n+2m-2)/4}(2m+1)\frac{\mathcal{H}^{nm}(\alpha)}{[\mathcal{H}^{20}(\alpha)]^{(n+2m+2)/4}}$$

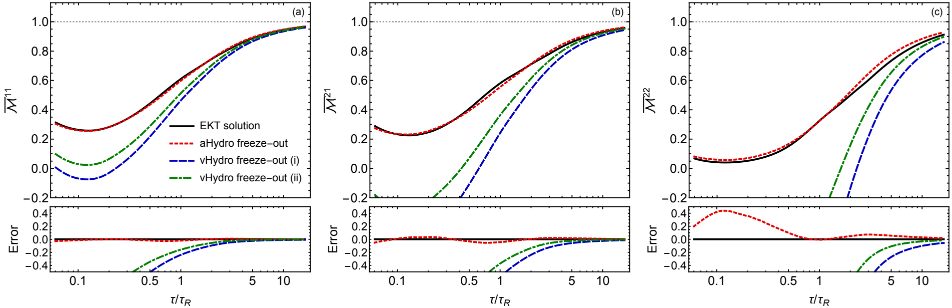
Insights into the freezeout perscription

(Almaalol,Kurkela,Strickland PRL 125, (2020))

τ/τ_R	τ
0.2	0.32 fm/c
0.5	0.86 fm/c
1	1.88 fm/c
2	4.23 fm/c
5	14.1 fm/c
10	38.5 fm/c

- Disagreement increases for higher moments and for earlier times.
- Good agreement between aHydro ansatz and EKT at all times

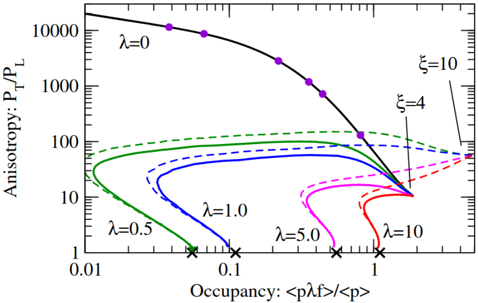
$\mathcal{M}^{11}[f] =< p_z^2 >$ $\mathcal{M}^{21}[f] =< p^1 p_z^2 >$ $\mathcal{M}^{22}[f] =< p^1 p_z^4 >$



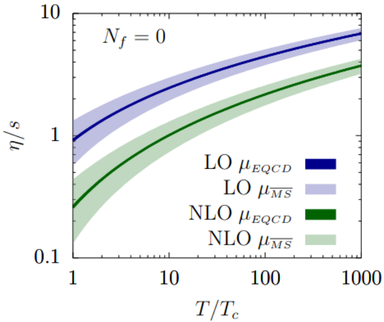
For earlier implementation: (Pratt,Torrieri PRC 82(2010)) (Weller, Romatchke PLB 774 (2017))

Future directions

To what limit we can extend the weakly coupled analysis?



(Kurkela, Zhu PRL 774 (2015))



(Ghiglieri, Moore, and Teaney PRL 121 (2018))

- Extrapolation towards higher couplings?
- Convergence of the perturbative series?

Conclusions

- Weakly coupled kinetic transport theory provides a clean limit to study the non-equilibrium dynamics of the QCD medium at high temperatures
- EKT also provides insights into phenomenology and different stages of the ultra-relativistic heavy ion collisions evolution

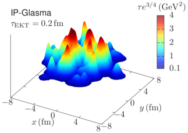
- How can we extrapolate results from the weakly coupled dynamics to higher couplings?. Is it possible? path to it?
- Breaking conformality in QCD kinetic theory: critical for applying EKT in phenomenology

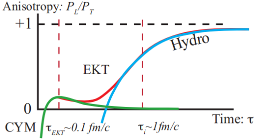
Thank you for your attention!

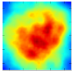
KoMPoST

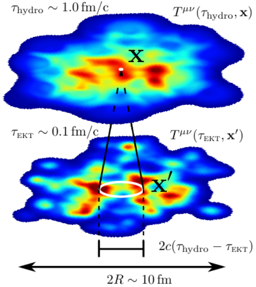
A. Kurkela, A. Mazeliauskas, J.F. Paquet, S. Schlichting, D. Teaney, Phys.Rev.Lett. 122 (2019) 12, 122302

- hydrodynamic model results are dependent on initialization time, and different hydrodynamic codes regulate these extreme initial conditions in different ad hoc ways









$$T^{\mu\nu}(\tau_{\rm EKT}, \mathbf{x}') = \overline{T}^{\mu\nu}_{\mathbf{x}}(\tau_{\rm EKT}) + \delta T^{\mu\nu}_{\mathbf{x}}(\tau_{\rm EKT}, \mathbf{x}').$$

$$f_{\mathbf{x},\mathbf{p}} = \bar{f}_{\mathbf{p}} + \int \frac{d^2\mathbf{k}}{(2\pi)^2}\, \delta f_{\mathbf{k},\mathbf{p}}\, e^{i\mathbf{k}\cdot\mathbf{x}}.$$

$$T^{\mu\nu}(\tau_{\rm hydro}, \mathbf{x}) = \overline{T}^{\mu\nu}_{\mathbf{x}}(\tau_{\rm hydro}) + \frac{\overline{T}^{\tau\tau}_{\mathbf{x}}(\tau_{\rm hydro})}{\overline{T}^{\tau\tau}_{\mathbf{x}}(\tau_{\rm EKT})} \times$$
$$\times \int d^2\mathbf{x}'\, G^{\mu\nu}_{\alpha\beta}\left(\mathbf{x}, \mathbf{x}', \tau_{\rm hydro}, \tau_{\rm EKT}\right) \delta T^{\alpha\beta}_{\mathbf{x}}(\tau_{\rm EKT}, \mathbf{x}').$$

the subsequent hydrodynamic evolution becomes independent of the hydrodynamic initialization time !!

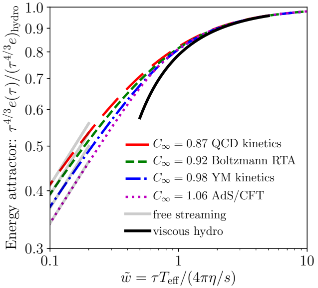
Scaling and Entropy production

Giuliano Giacalone, Aleksas Mazeliauskas, and Sören Schlichting Phys. Rev. Lett. 123, 262301;P. Hanus, A. Mazeliauskas, and K. Reygers, Phys. Rev.C100, 064903 (2019)

- Macroscopic description from hydrodynamics attractors. Integrate equations of motion to find energy attractor

$$\partial_\tau e = -\frac{e}{\tau} + \frac{P_L}{e}; \qquad \frac{P_L}{e} = f\left[\tilde{w} = \frac{\tau T_{\text{eff}}}{4\pi\eta/s}\right] \quad \Rightarrow \quad \frac{e(\tau)\tau^{4/3}}{e_{\text{hydro}}\tau_{\text{hydro}}^{4/3}} = \mathcal{E}\left(\tilde{w} = \frac{T_{\text{eff}}(\tau)\tau}{4\pi\eta/s}\right)$$

$$(s\tau)_{\text{hydro}} = \frac{4}{3} C_\infty^{3/4} \left(4\pi\frac{\eta}{s}\right)^{1/3} \left(\frac{\pi^2}{30}\nu_{\text{eff}}\right)^{1/3} (e\tau)_0^{2/3},$$



$$\frac{dN_{\text{ch}}}{d\eta} \approx \frac{1}{J} A_\perp \, (s\tau)_{\text{hydro}} \frac{N_{\text{ch}}}{S} .$$

where $A_\perp \approx \pi R^2 \; S/N_{\text{ch}} \approx 7$ is a constant of hadron gas

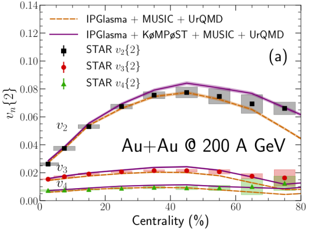
$$\frac{dE_\perp}{d\eta} \approx A_\perp \, (e\tau)_0 .$$

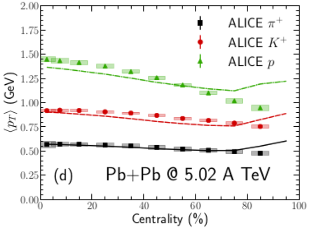
Macroscopic description from hydrodynamics attractors!

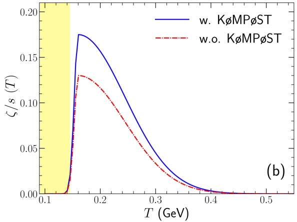
KoMPoST : future improvement

- Inclusion of kompost influence the extraction of transport coefficients

Paquet,Shen, Schenke, Gale. arxiv.2002.05191(2020)

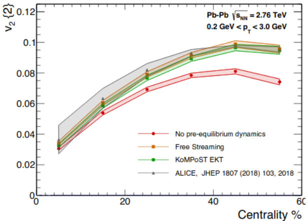






- Insensitivity of PCA to the pre-equilibrium stage

da Silva. Chinellato, Hippert , Serenone,Takahashi,Denicol,Luzum,Noronha arxiv.2006.02324 (2020)



$\Rightarrow$ Call for non-conformal treatment

Kinetic based hydrodynamics equations

Moment integral operator

$$\hat{\mathcal{O}}_n\, g = \mathcal{O}^{\mu_1\mu_2\cdots\mu_n}[g] \equiv \int dP\, p^{\mu_1} p^{\mu_2} \cdots p^{\mu_n}\, g(p) n^{th}$$

$$p^\mu \partial_\mu f_p = C[f_p]$$

$$\partial_\mu I^{\mu\nu_1\nu_2\cdots\nu_n} = \mathcal{C}^{\nu_1\nu_2\cdots\nu_n}$$

$$I^{\mu\nu_1\nu_2\cdots\nu_n} \equiv \int dP\, p^\mu p^{\nu_1} p^{\nu_2} \cdots p^{\nu_n}\, f$$

$$\mathcal{C}^{\nu_1\nu_2\cdots\nu_n} \equiv \int dP\, p^{\nu_1} p^{\nu_2} \cdots p^{\nu_n}\, C[f]$$

Landau Matching

$$\partial_\mu n^\mu = \mathcal{C}$$

$$\partial_\mu T^{\mu\nu} = \mathcal{C}^\nu$$

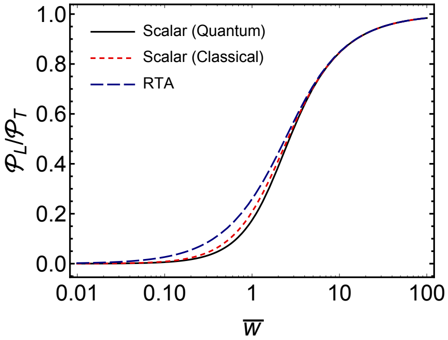
$$\mathcal{C}_{RTA} = \frac{u^\mu . p^\mu}{\tau_\mathrm{R}} \left[f_\mathrm{eq}(p/T) - f_p\right]$$

$$\partial_\mu n^\mu = \frac{1}{\tau_\mathrm{R}} \left[n_\mathrm{eq} - n\right]$$

$$\partial_\mu T^{\mu\nu} = \frac{1}{\tau_\mathrm{R}} \left[\epsilon_\mathrm{eq} - \epsilon\right]$$

Kinetic based hydrodynamics equations

Moment method



Almaalol, Alqahtani, Strickland PRC 99, (2019)

- Popular approach in phenomenolgy
- Direct impact on transpot coefficients!

Landau Matching

$$\partial_\mu n^\mu = \mathcal{C}$$
$$\partial_\mu T^{\mu\nu} = \mathcal{C}^\nu$$

$$\mathcal{C}_{RTA} = \frac{u^\mu . p^\mu}{\tau_\mathrm{R}} \left[f_\mathrm{eq}(p/T) - f_p \right]$$

$$\partial_\mu n^\mu = \frac{1}{\tau_\mathrm{R}} \left[n_\mathrm{eq} - n \right]$$
$$\partial_\mu T^{\mu\nu} = \frac{1}{\tau_\mathrm{R}} \left[\epsilon_\mathrm{eq} - \epsilon \right]$$

Insights into far from equilibrium transport coefficients

Kamata, Martinez, Plaschke, Ochsenfeld, Schlicting PRD 102 (2020)

Both RTA and QCD capture the macroscopic response of the system similarly!

- How microscopic details affect macroscopic quantities far from equilibrium?
- "renormalized transport coefficients"?