Transport near the chiral critical point

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Based on: 2005.02885, 2101.10847

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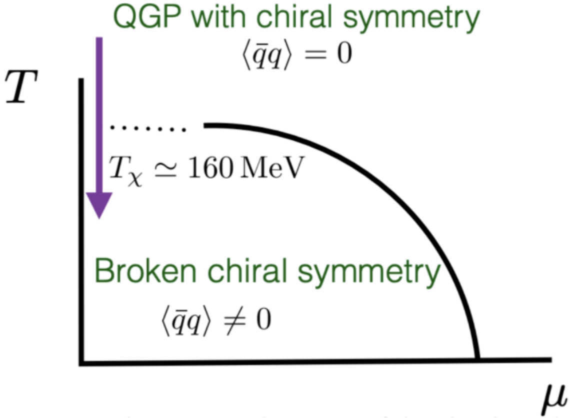
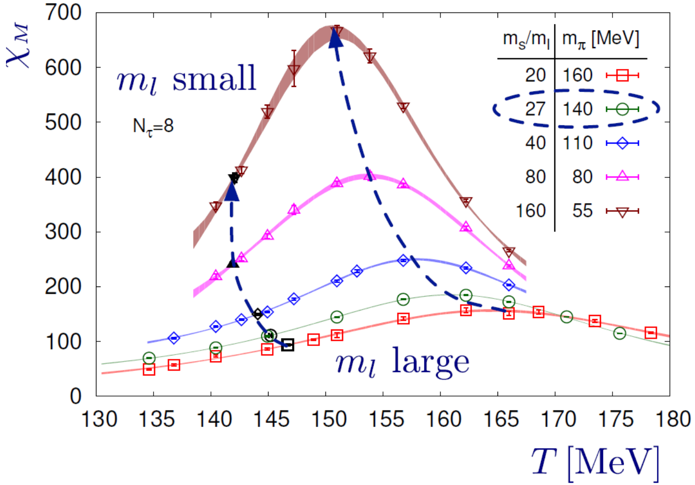
FWF Der Wissenschaftsfonds.

Physical motivation



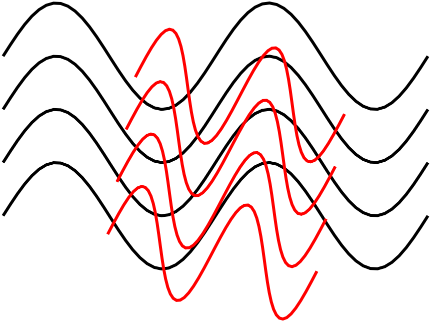
Approximate chiral symmetry $SU(2)_L \times SU(2)_R \sim O(4)$
What role does chiral symmetry play in heavy ion collisions?

$O(4)$ scaling as seen on the lattice



Chiral susceptibility: $\chi_M \propto \frac{\partial\langle\bar{\psi}\psi\rangle}{\partial m_l} \sim m_l^{1/\delta-1} f_\chi(z)$, $z \equiv t m_l^{-1/\beta\delta}$
Plot from HotQCD collaboration: arXiv:1903.04801

Hydrodynamics in the chiral limit[1]



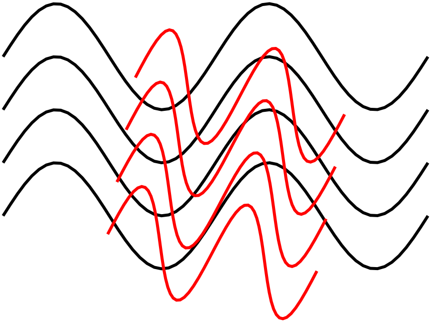
equilibrated hydro modes, $k \ll m_\pi$

superfluid modes, $k \sim m_\pi$

At long distances, effective theory of QCD is hydrodynamics
At finite quark mass, theory should be superfluid-like for $L \sim m_\pi^{-1}$

[1] Son hep-ph/9912267,Son and Stephanov hep-ph/020422

Hydrodynamics in the chiral limit[1]



equilibrated hydro modes, $k \ll m_\pi$

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Question: How do these modes contribute to hydrodynamic variables and transport?

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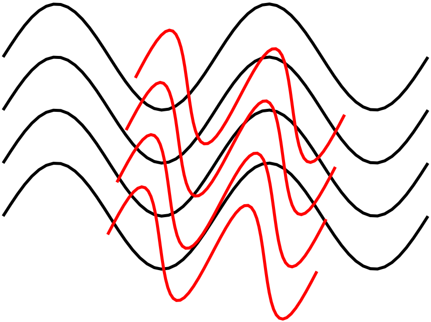

equilibrated hydro modes, $k \ll m_\pi$

superfluid modes, $k \sim m_\pi$

Hydrodynamic variables get correction due to pions, e.g.

$$\eta = \eta_{\text{hydro}} + \Delta\eta$$

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Equilibrium statics near O(4) critical point[2]

- Start from generating functional

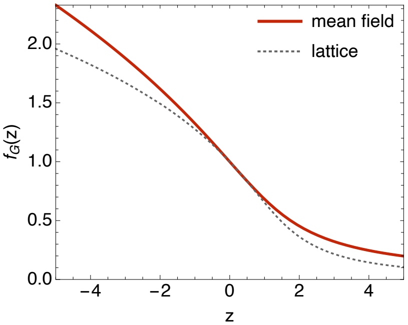
$$W = \int_x p(T) + \frac{\chi_0}{4}\mu_{ab}^2 - \frac{1}{2}\Delta^{\mu\nu} D_\mu \phi_a D_\nu \phi_a - V(\phi)$$

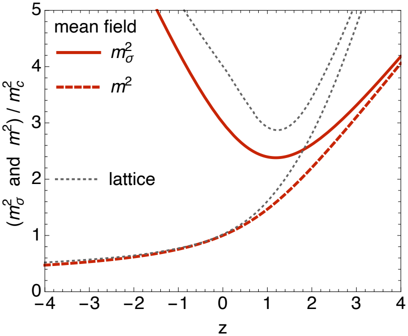
  - μ_{ab} is the $O(4)$ chemical potential (e.g. $\mu_{0i} = \mu_A$)
  - $O(4)$ vector: $\phi_a = (\sigma, \pi_i)$
  - $V(\phi_\alpha \phi_\alpha) = m_0^2(t)\phi^2 + \lambda\phi^4 - H\phi$
  - $m_0^2(t) \sim (T - T_c)$

[2] Rajagopal/Wilczek arXiv:9210253, Son/Stephanov arXiv:0204226, Jensen et al 1203.3556

Mean field compared to lattice[3]

$$0 = \frac{dV}{d\phi} = m_0^2(t)\,\bar{\sigma} + \frac{\lambda}{3!}\bar{\sigma}^3 - H.$$





O(4) scaling function $f_G \propto \bar{\sigma}$

Pion and sigma masses

Note: $z \propto \frac{T - T_c}{T_c}$

[3] Engels Vogt arXiv:0911.1939, Engels Karsch arXiv:1105.0584

Dynamics near O(4) critical point[2]

- Start from generating functional

$$W = \int_x p(T) + \frac{\chi_0}{4}\mu_{ab}^2 - \frac{1}{2}\Delta^{\mu\nu} D_\mu \phi_a D_\nu \phi_a - V(\phi)$$

- Compute ideal equations of motion
  Josephson constraint follows from entropy conservation:

$$u \cdot \partial \phi_a + \mu_{ab}\phi_b = 0$$

  Coupling between pions and chemical potential!

[2] Rajagopal/Wilczek arXiv:9210253, Son/Stephanov arXiv:0204226, Jensen et al 1203.3556

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- Compute ideal equations of motion

$$u \cdot \partial\phi_a + \mu_{ab}\phi_b = 0$$

- Add dissipation in standard way:

$$\begin{aligned} T^{\mu\nu} &= T^{\mu\nu}_{\rm ideal} + \Pi^{\mu\nu} \\ J^\mu &= J^\mu_{\rm ideal} + q^\mu \\ u \cdot \partial\phi_a + \mu_{ab}\phi_b &= \Xi_a \end{aligned}$$

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- Linearize equations around mean field: $\phi_a = (\bar\sigma + \delta\sigma, \bar\sigma\varphi_i)$

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  $\Rightarrow$ Compute change in transport coefficients

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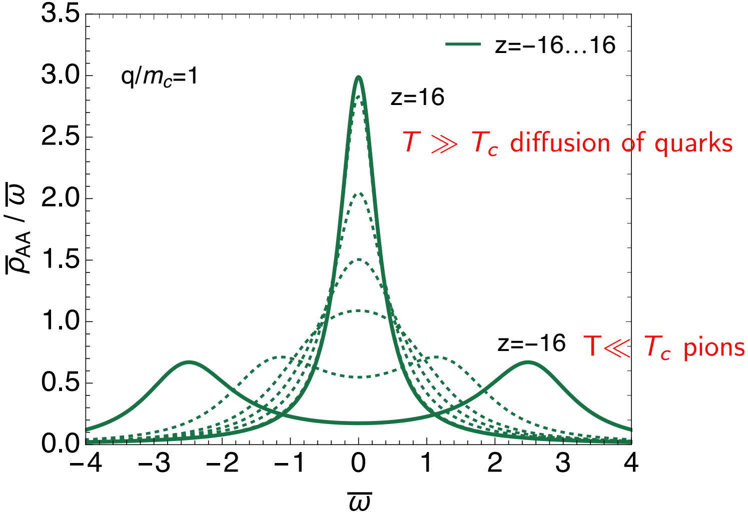
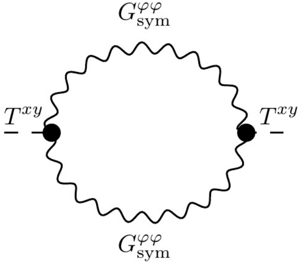
Spectral density for axial charge density-density correlator

$(\chi_0\omega_k)^{-2} G_{sym}^{\varphi\varphi} = \frac{T}{\omega}\rho_{AA}$



Can see hadronization from QGP to soft pions in the propagator!

Modification of shear viscosity



$$2T\eta = \int d^4x \left\langle \tfrac{1}{2}\{T^{xy}(t,\boldsymbol{x}), T^{xy}(0,\mathbf{0})\}\right\rangle$$

In linearized regime, relevant components for shear viscosity:

$$\Delta T^{xy} = \underbrace{\partial^x \delta\sigma \partial^y \delta\sigma}_{\text{condensate contribution}} + \underbrace{\bar{\sigma}^2 \partial^x \varphi_a \partial^y \varphi_a}_{\text{pion contribution}}$$

Break up computation into two pieces:

$$\Delta\eta \equiv I^{xy}_{\sigma\sigma} + I^{xy}_{\varphi\varphi}$$

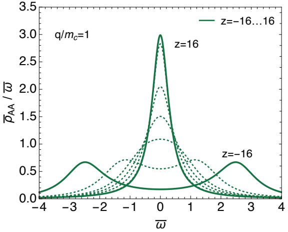
Modification of shear viscosity

Break up computation into two pieces:

$$\Delta\eta \equiv I^{xy}_{\sigma\sigma} + I^{xy}_{\varphi\varphi}$$

Pion correlator:

$$G^{\varphi\varphi}_{sym}(\omega, k) = 2\frac{T}{\omega}\mathrm{Im}\, G^{\varphi\varphi}_{R}(\omega, k) =$$

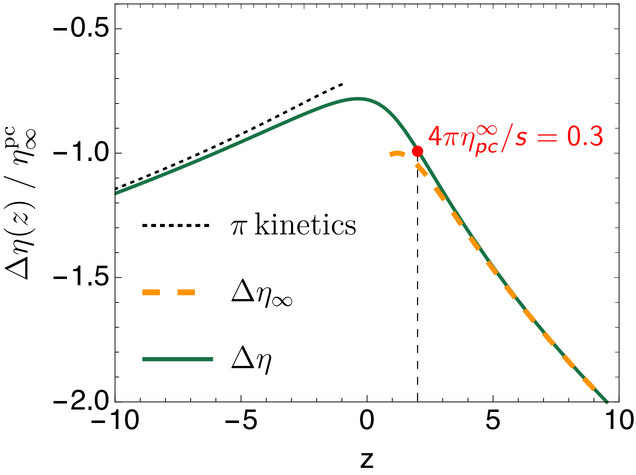


$$I^{xy}_{\varphi\varphi} = 2\int \frac{d^3k}{(2\pi)^3}\frac{d\omega}{(2\pi)}(k^x k^y G^{\varphi\varphi}_{sym})^2$$

Two pieces: divergence which can be absorbed and a universal finite piece, $\Delta\eta$

Results: change in shear viscosity

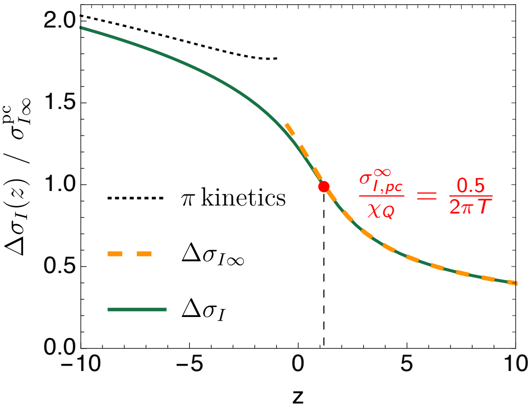
Note: $z \propto \frac{T-T_c}{T_c}$



Large z simple form $\Delta\eta_\infty = -\frac{Tm_\sigma}{8\pi\Gamma}$

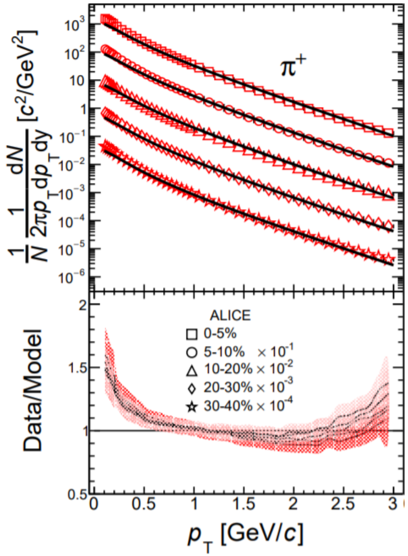
Results: change in isovector conductivity

$$2T\sigma_I = \int d^4x \frac{1}{d_A} \left\langle \tfrac{1}{2}\{J^x_{V,a}(t,\boldsymbol{x}), J^x_{V,a}(0,\boldsymbol{0})\} \right\rangle ,$$
$$J^x_{V,a} = \sigma_0^2 f_{abc}\varphi_b \partial^x \varphi_c$$



Large z simple form $\Delta\sigma_\infty = -\frac{T}{16\pi m\Gamma}$

Outlook: soft pions in experiment?



Plot from arXiv:1909.10485

Soft pions are harder to fit to hydrodynamic models

Estimating soft pion yield enhancement near critical point

Want to compare soft pion dispersion: $E_p^2 = v^2(p)p^2 + m^2(p)$
to vacuum dispersion: $E_{\mathrm{vac}}^2 = p^2 + m_\pi^2$

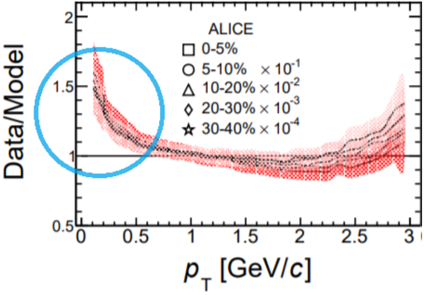
Quick model:

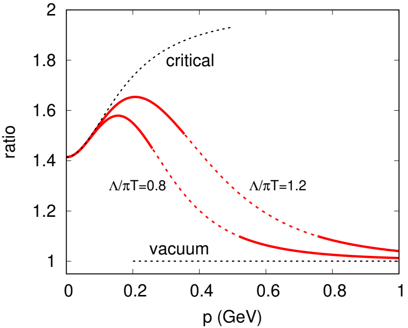
- Critical theory valid at low momentum
- Vacuum for high momentum
- Interpolating form in between.

Compare to vacuum by computing ratio:

$$\text{ratio} = \frac{n(E_p)}{n(E_{\mathrm{vac}})} \approx \frac{E_{\mathrm{vac}}}{E_p}$$

Estimating soft pion yield enhancement near critical point





- Promising, enhancement is where it should be!
- Needs more robust treatment, e.g. constraining dispersion via lattice, second order hydro, including resonance decays, etc.
- Future: upgrade to Inner Tracking System at ALICE allows to see more low p_T particles, especially pions...