Transport near the chiral critical point

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Physical motivation

Approximate chiral symmetry $SU(2)_L \times SU(2)_R \sim O(4)$

What role does chiral symmetry play in heavy ion collisions?
$O(4)$ scaling as seen on the lattice

Chiral susceptibility: $\chi_M \propto \frac{\partial \langle \bar{\psi} \psi \rangle}{\partial m_l} \sim m_l^{1/\delta-1} f_\chi(z), \quad z \equiv t m_l^{-1/\beta \delta}$

Plot from HotQCD collaboration: arXiv:1903.04801
Hydrodynamics in the chiral limit

At long distances, effective theory of QCD is hydrodynamics
At finite quark mass, theory should be superfluid-like for $L \sim m_{\pi}^{-1}$

equilibrated hydro modes, $k \ll m_{\pi}$
superfluid modes, $k \sim m_{\pi}$

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Hydrodynamics in the chiral limit\(^1\)

equilibrated hydro modes, \(k \ll m_\pi\)
superfluid modes, \(k \sim m_\pi\)

Question: How do these modes contribute to hydrodynamic variables and transport?

\(^1\)Son hep-ph/9912267, Son and Stephanov hep-ph/020422
Hydrodynamics in the chiral limit\(^1\)

equilibrated hydro modes, \( k \ll m_\pi \)

superfluid modes, \( k \sim m_\pi \)

Hydrodynamic variables get correction due to pions, e.g.

\[
\eta = \eta_{\text{hydro}} + \Delta \eta
\]

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Equilibrium statics near $O(4)$ critical point

- Start from generating functional

\[
W = \int_x p(T) + \frac{\chi_0}{4} \mu_{ab}^2 - \frac{1}{2} \Delta^{\mu\nu} D_\mu \phi_a D_\nu \phi_a - V(\phi)
\]

- $\mu_{ab}$ is the $O(4)$ chemical potential (e.g. $\mu_{0i} = \mu_A$)
- $O(4)$ vector: $\phi_a = (\sigma, \pi_i)$
- $V(\phi_\alpha \phi_\alpha) = m_0^2(t) \phi^2 + \lambda \phi^4 - H\phi$
- $m_0^2(t) \sim (T - T_c)$

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\(^2\text{Rajagopal/Wilczek arXiv:9210253, Son/Stephanov arXiv:0204226, Jensen et al 1203.3556}\)
Mean field compared to lattice\(^3\)

\[ 0 = \frac{dV}{d\phi} = m_0^2(t) \bar{\sigma} + \frac{\lambda}{3!} \bar{\sigma}^3 - H. \]

O(4) scaling function \( f_G \propto \bar{\sigma} \)

Pion and sigma masses

Note: \( z \propto \frac{T - T_c}{T_c} \)

Dynamics near O(4) critical point

- Start from generating functional

\[ W = \int_x p(T) + \frac{\chi_0}{4} \mu_{ab}^2 - \frac{1}{2} \Delta^{\mu\nu} D_\mu \phi_a D_\nu \phi_a - V(\phi) \]

- Compute ideal equations of motion

Josephson constraint follows from entropy conservation:

\[ u \cdot \partial \phi_a + \mu_{ab} \phi_b = 0 \]

Coupling between pions and chemical potential!

\(^2\text{Rajagopal/Wilczek arXiv:9210253, Son/Stephanov arXiv:0204226, Jensen et al 1203.3556}\)
Dynamics near $O(4)$ critical point\textsuperscript{2}

- Start from generating functional

\[ W = \int_x p(T) + \frac{\chi_0}{4} \mu_{ab}^2 - \frac{1}{2} \Delta^{\mu\nu} D_\mu \phi_a D_\nu \phi_a - V(\phi) \]

- Compute ideal equations of motion

\[ u \cdot \partial \phi_a + \mu_{ab} \phi_b = 0 \]

- Add dissipation in standard way:

\[ T^{\mu\nu} = T^{\mu\nu}_{\text{ideal}} + \Pi^{\mu\nu} \]
\[ J^\mu = J^\mu_{\text{ideal}} + q^\mu \]
\[ u \cdot \partial \phi_a + \mu_{ab} \phi_b = \Xi_a \]

\textsuperscript{2}Rajagopal/Wilczek arXiv:9210253, Son/Stephanov arXiv:0204226, Jensen et al 1203.3556
Dynamics near O(4) critical point

- Start from generating functional

\[ W = \int_x p(T) + \frac{\chi_0}{4} \mu_{ab}^2 - \frac{1}{2} \Delta^{\mu\nu} D_\mu \phi_a D_\nu \phi_a - V(\phi) \]

- Compute ideal equations of motion

\[ u \cdot \partial \phi_a + \mu_{ab} \phi_b = 0 \]

- Add dissipation in standard way

- Linearize equations around mean field: \( \phi_a = (\bar{\sigma} + \delta \sigma, \bar{\sigma} \varphi_i) \)

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Dynamics near $O(4)$ critical point

- Start from generating functional

$$W = \int x p(T) + \frac{\chi_0}{4} \mu_{ab}^2 - \frac{1}{2} \Delta^{\mu\nu} D_\mu \phi_a D_{\nu} \phi_a - V(\phi)$$

- Compute ideal equations of motion

$$u \cdot \partial \phi_a + \mu_{ab} \phi_b = 0$$

- Add dissipation in standard way

- Linearize equations around mean field: $\phi_a = (\bar{\sigma} + \delta \sigma, \bar{\sigma} \varphi_i)$

$\Rightarrow$ Compute change in transport coefficients

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Spectral density for axial charge density-density correlator

\[(\chi_0 \omega_k)^{-2} G_{\text{sym}}^{\phi \phi} = \frac{T}{\omega} \rho_{AA}\]

Can see hadronization from QGP to soft pions in the propagator!
Modification of shear viscosity

\[2 T \eta = \int d^4x \langle \frac{1}{2} \{ T^{xy}(t, x), T^{xy}(0, 0) \} \rangle\]

In linearized regime, relevant components for shear viscosity:

\[\Delta T^{xy} = \underbrace{\partial^x \delta \sigma \partial^y \delta \sigma}_{\text{condensate contribution}} + \underbrace{\bar{\sigma}^2 \partial^x \varphi_a \partial^y \varphi_a}_{\text{pion contribution}}\]

Break up computation into two pieces:

\[\Delta \eta \equiv I^{xy}_{\sigma \sigma} + I^{xy}_{\varphi \varphi}\]
Modification of shear viscosity

Break up computation into two pieces:

\[ \Delta \eta \equiv l^{xy}_{\sigma \sigma} + l^{xy}_{\phi \phi} \]

Pion correlator:

\[ G_{\phi \phi}^{\text{sym}}(\omega, k) = 2 \frac{T}{\omega} \text{Im} G_{R}^{\phi \phi}(\omega, k) = \]

\[ I_{\phi \phi}^{xy} = 2 \int \frac{d^3 k}{(2\pi)^3} \frac{d\omega}{(2\pi)} (k^x k^y G_{\phi \phi}^{\text{sym}})^2 \]

Two pieces: divergence which can be absorbed and a universal finite piece, \( \Delta \eta \)
Results: change in shear viscosity

Note: \( z \propto \frac{T - T_c}{T_c} \)

Large \( z \) simple form \( \Delta \eta_\infty = -\frac{Tm_\sigma}{8\pi \Gamma} \)
Results: change in isovector conductivity

\[ 2T \sigma_I = \int d^4x \frac{1}{dA} \left\langle \frac{1}{2} \{ J^x_{V,a}(t, x), J^x_{V,a}(0, 0) \} \right\rangle, \]

\[ J^x_{V,a} = \sigma_0^2 f_{a b c} \varphi_b \partial^x \varphi_c \]

Large \( z \) simple form \( \Delta \sigma_\infty = -\frac{T}{16\pi mT} \)
Outlook: soft pions in experiment?

Soft pions are harder to fit to hydrodynamic models.

Plot from arXiv:1909.10485
Estimating soft pion yield enhancement near critical point

Want to compare soft pion dispersion: \( E_p^2 = v^2(p)p^2 + m^2(p) \)
to vacuum dispersion: \( E_{\text{vac}}^2 = p^2 + m^2_\pi \)

Quick model:

- Critical theory valid at low momentum
- Vacuum for high momentum
- Interpolating form in between.

Compare to vacuum by computing ratio:

\[
\text{ratio} = \frac{n(E_p)}{n(E_{\text{vac}})} \approx \frac{E_{\text{vac}}}{E_p}
\]
Estimating soft pion yield enhancement near critical point

- Promising, enhancement is where it should be!
- Needs more robust treatment, e.g. constraining dispersion via lattice, second order hydro, including resonance decays, etc.
- Future: upgrade to Inner Tracking System at ALICE allows to see more low $p_T$ particles, especially pions...