Hadron Structure in Lattice QCD
Status and Challenges

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A Virtual Tribute to Quark Confinement and the Hadron Spectrum
2–6 August 2021

HIM Helmholtz-Institut Mainz

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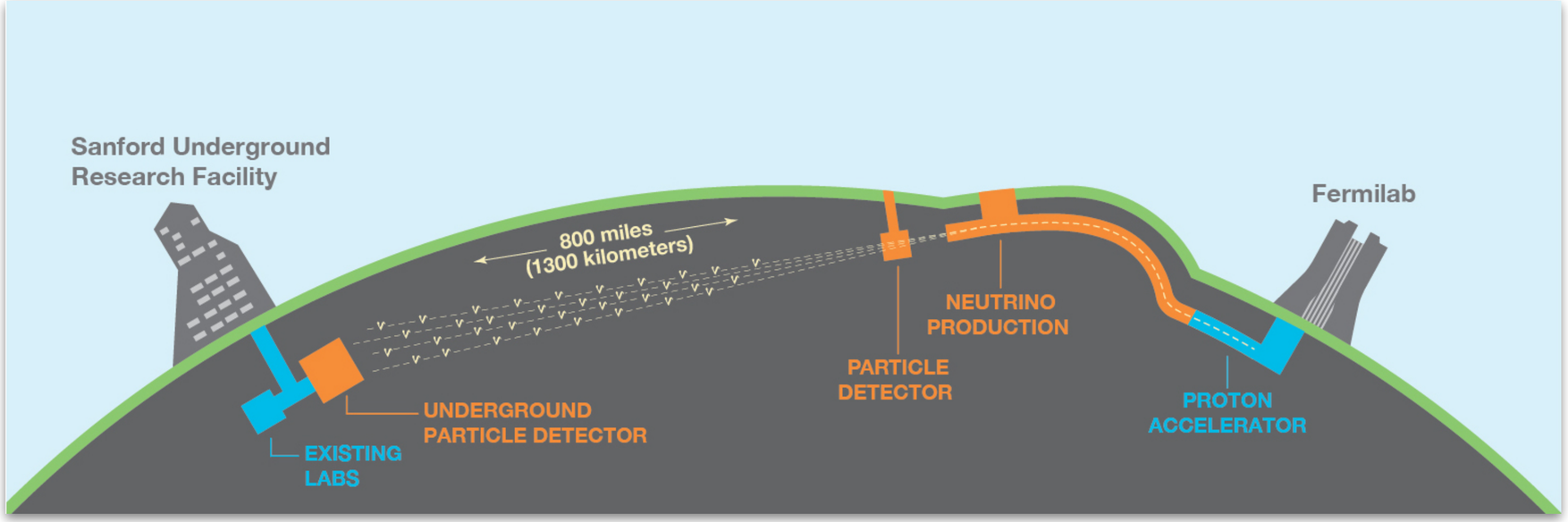
JG|U

Nucleon structure observables and BSM physics searches

Scattering experiments probe interactions of $e^-, p, \nu's$, DM particles with nuclear targets

$\rightarrow$ precise knowledge of nucleon / nuclear matrix elements of currents, quark bilinears

DUNE — neutrino oscillation experiment: (anti-)neutrino beam onto $\mathrm{C, O, Ar}$ targets

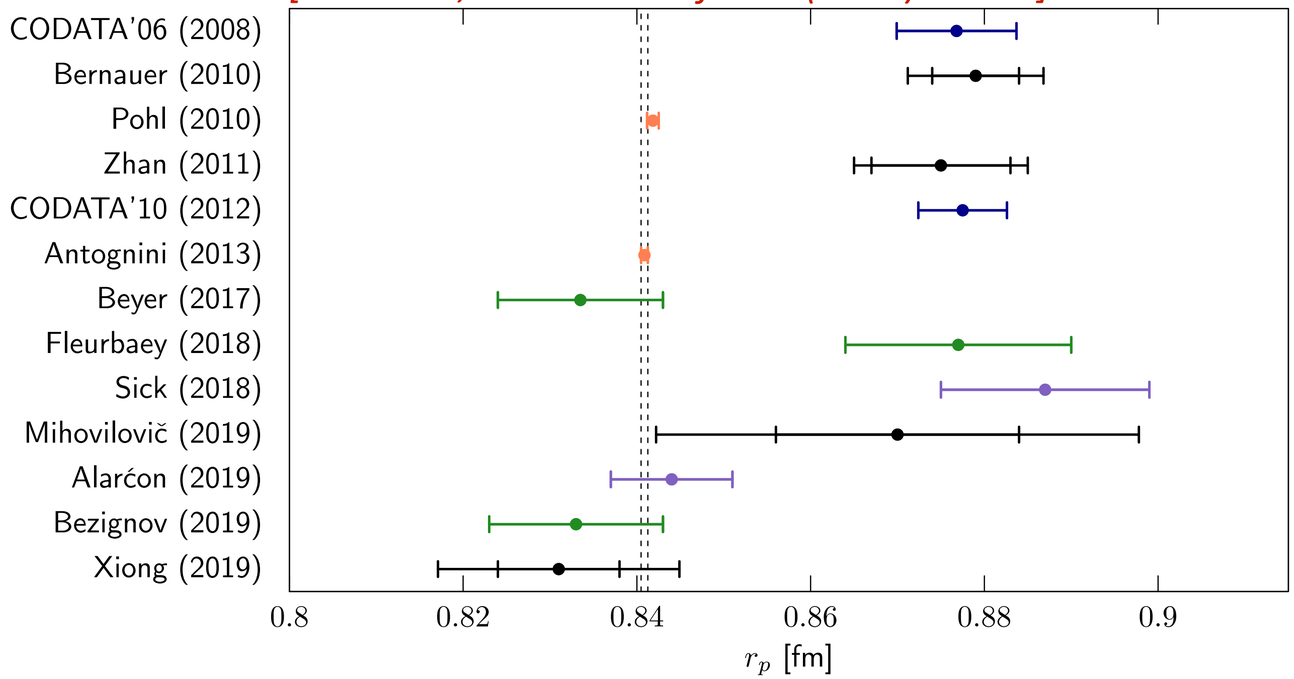


Neutrino-nucleus cross section dominates uncertainty

Is there a proton radius puzzle?

Discrepant measurements of r_p in muonic / electronic hydrogen and ep scattering

[Bernauer, EPJ Web Conf. 234 (2020) 01001]



Muonic hydrogen: *[Antognini et al., 2013]*

$$r_\mathrm{E} = 0.84087 \pm 0.00039\,\mathrm{fm}$$

CODATA:

$$r_\mathrm{E} = 0.8414 \pm 0.0019\,\mathrm{fm}$$ *[CODATA 2018]*

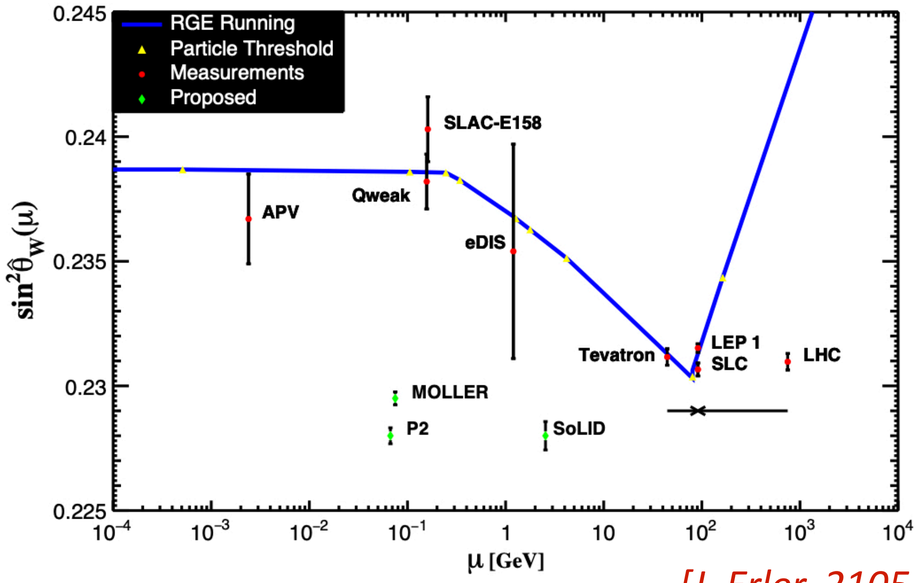
$$r_\mathrm{E} = 0.8775 \pm 0.0051\,\mathrm{fm}$$ *[CODATA 2012]*

Signal for new physics or poorly understood systematic effects?

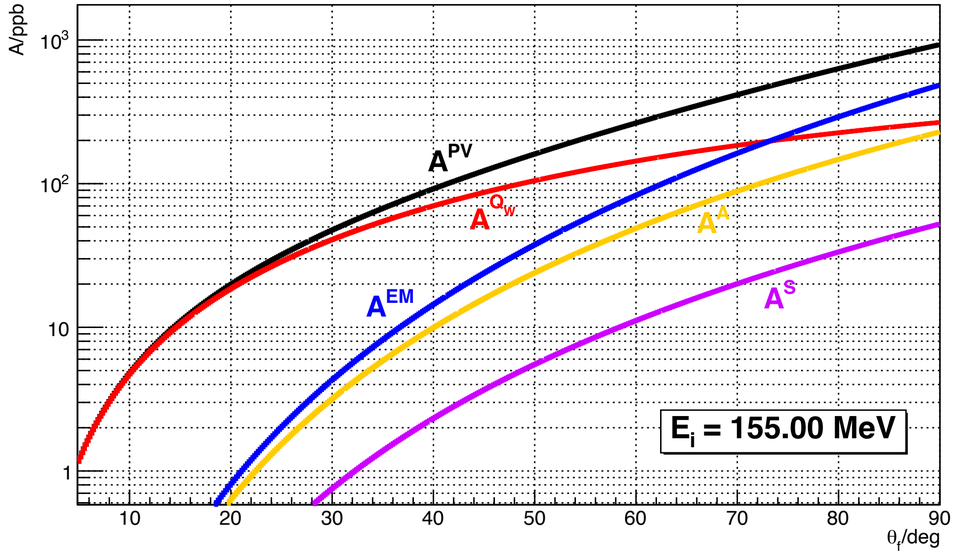
→ calls for *ab initio* calculation of the proton radius from QCD

Weak charge of the proton and the running of $\sin^2\theta_W$

Running of electroweak mixing angle at low energies constrains BSM physics models



[J. Erler, 2105.00217]



[D. Becker et al., 1802.04759]

P2@MESA: parity-violating ep scattering

$$A_{LR} \equiv \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = -\frac{G_F\, Q^2}{4\pi\sqrt{2}\alpha}\left(Q_W^P - F(Q^2)\right),$$

$$Q_W^P = 1 - 4\sin^2\theta_W \quad \text{(tree level)}$$

Hadronic contributions

$$F(Q^2) = F_{\rm EM}(Q^2) + F_{\rm A}(Q^2) + F_{\rm str}(Q^2)$$

$$Q^2 \approx 4E_iE_f\sin^2(\theta_f/2)$$

This talk

Methodology: The noise problem and excited states

Nucleon charges and pion-nucleon σ-term

Electromagnetic form factors

Axial and strange form factors

Summary and Outlook

Related talks at this conference:

Krzysztof Cichy, Mon 15:50

Boram Yoon, Wed 17:30

Nucleon form factors

Dirac and Pauli form factors:

$$\left\langle N(p',s')\left|J_\mu^{\rm em}(0)\right|N(p,s)\right\rangle = \bar{u}(p',s')\left[\gamma_\mu F_1(Q^2) + \sigma_{\mu\nu}\frac{Q_\nu}{2m_{\rm N}}F_2(Q^2)\right]u(p,s)$$

$$G_{\rm E}(Q^2) = F_1(Q^2) - \frac{Q^2}{(am_{\rm N})^2}F_2(Q^2)\,, \qquad G_{\rm M}(Q^2) = F_1(Q^2) + F_2(Q^2)$$

Axial and induced pseudoscalar form factors:

$$\left\langle N(p',s')\left|A_\mu(0)\right|N(p,s)\right\rangle = \bar{u}(p',s')\left[\gamma_\mu\gamma_5 G_{\rm A}(Q^2) - i\gamma_5\frac{Q_\mu}{2m_{\rm N}}\widetilde{G}_{\rm P}(Q^2)\right]u(p,s)$$

Charge radii, magnetic moment, axial charge:

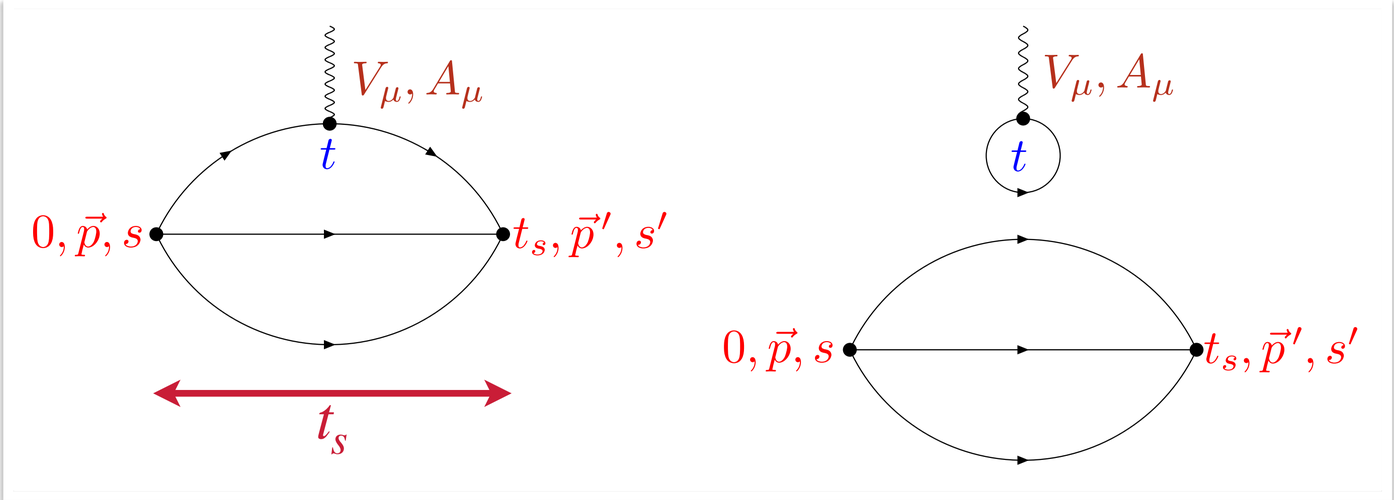
$$G_{\rm E}(Q^2) = \left(1 - \tfrac{1}{6}\left\langle r_{\rm E}^2\right\rangle Q^2 + {\rm O}(Q^2)\right)\,, \quad G_{\rm M}(Q^2) = \mu\left(1 - \tfrac{1}{6}\left\langle r_{\rm M}^2\right\rangle Q^2 + {\rm O}(Q^2)\right)$$

$$G_{\rm A}(Q^2) = g_{\rm A}\left(1 - \tfrac{1}{6}\left\langle r_{\rm A}^2\right\rangle Q^2 + {\rm O}(Q^2)\right)$$

Challenges for lattice QCD

Quark-disconnected diagrams

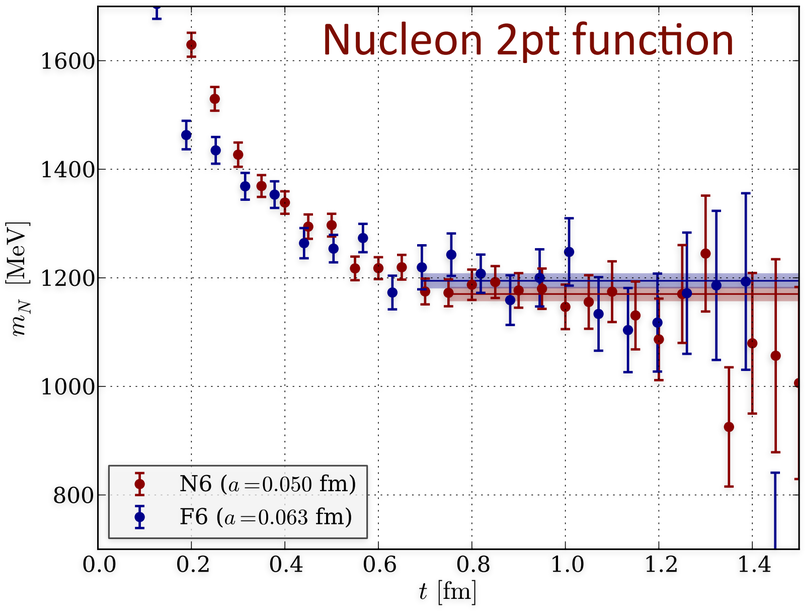
- large inherent statistical noise
- contribute to isoscalar quantities and sigma-terms
- contribute exclusively to strange form factors



"Noise problem"

- exponentially decreasing signal-to-noise ratio in baryonic correlators
- calculations of baryonic three-point functions limited to source-sink separations $t_s \lesssim 1.7\,\mathrm{fm}$

⇒ potential bias from unsuppressed excited-state contributions

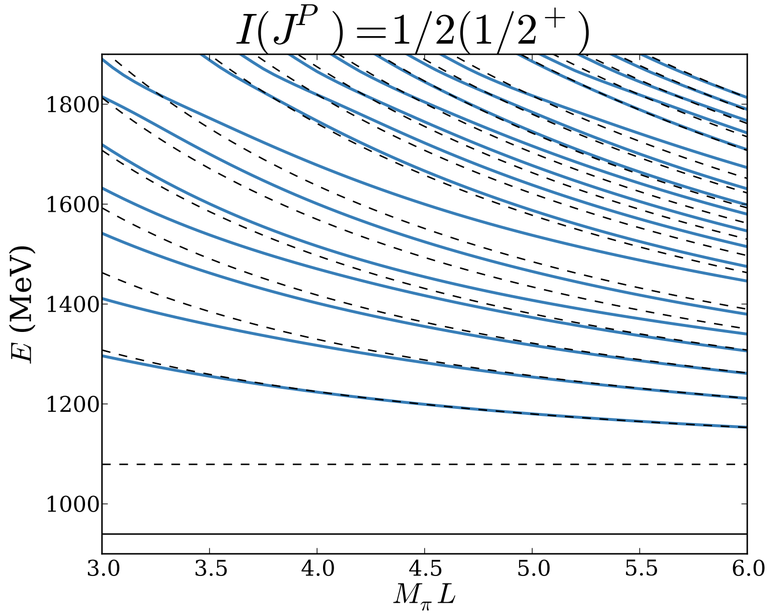


Excitation spectrum

Nucleon charges from ratios of three- and two-point functions: $\Delta = (E_1 - E_0)\,,\ \Gamma = A,\ S,\ T,\ \ldots$

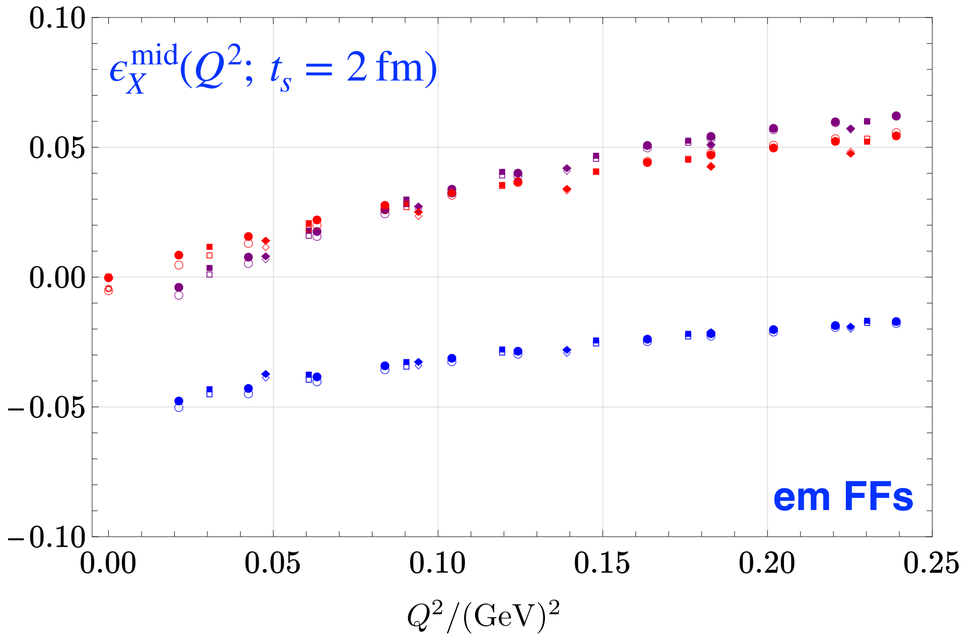
$$R_\Gamma(t,\ t_s) \equiv \frac{C_3^\Gamma(\boldsymbol{q}=0; t,\ t_s)}{C_2(\boldsymbol{p}=0;\ t_s)} = g_\Gamma + c_{01}\,\mathrm{e}^{-\Delta t} + c_{10}\,\mathrm{e}^{-\Delta(t_s-t)} + c_{11}\,\mathrm{e}^{-\Delta t_s} + \ldots$$

Dense spectrum of $N\pi, N\pi\pi,\ \ldots$ states:



[Hansen & Meyer, 1610.03843]

ChPT analyses of excited-state contamination:

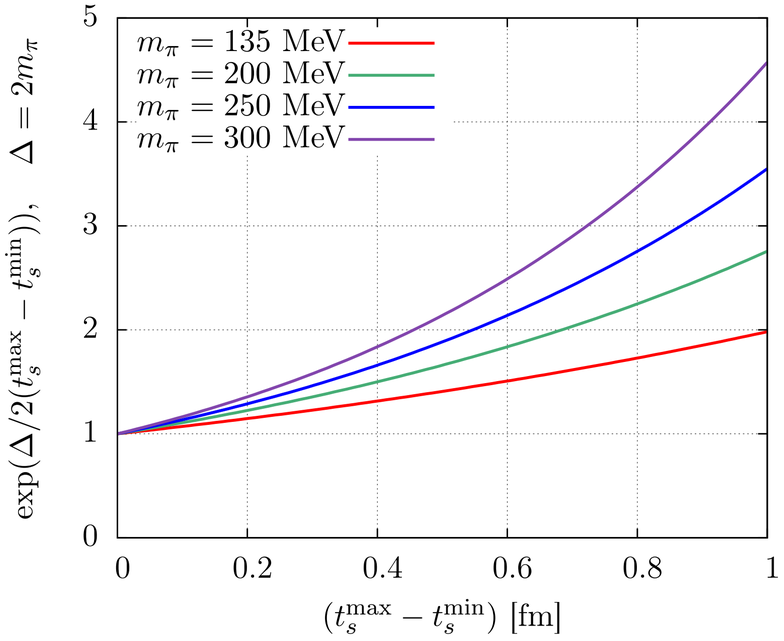


[O. Bär, 1705.02806, 1802.10442, 1812.09191, 1906.03652, 1912.05873]

Fighting the noise problem

$$R_\Gamma(t,\, t_s) = g_\Gamma + c_{01}\, \mathrm{e}^{-\Delta t} + c_{10}\, \mathrm{e}^{-\Delta(t_s - t)} + c_{11}\, \mathrm{e}^{-\Delta t_s} + \ldots\,,$$

"Sensitivity" to excited-state effects:



Multi-state fits

Include sub-leading terms in $R_\Gamma(t, t_s)$,
(or individual two- and three-point functions)
with or without priors for the excitation spectrum

"Summation Method"

Excited-state contributions more strongly suppressed

$$S_\Gamma(t_s) \equiv \sum_{t=0}^{t_s - a} R_\Gamma(t,\, t_s) = K_\Gamma + (t_s - a)\, g_\Gamma + (t_s - a)\, \mathrm{e}^{-\Delta t_s}\, d_\Gamma + \mathrm{e}^{-\Delta t_s}\, f_\Gamma + \ldots$$

Variational approach

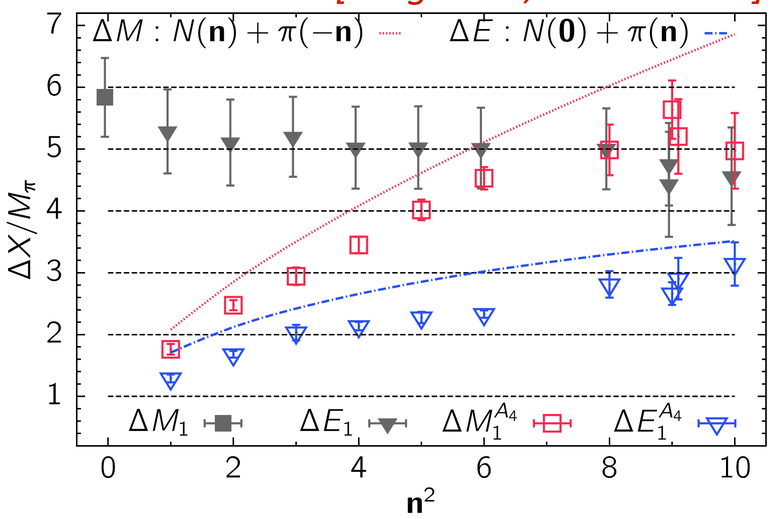
Compute correlator matrices; solve GEVP; optimise projection on ground state

Multi-state fits

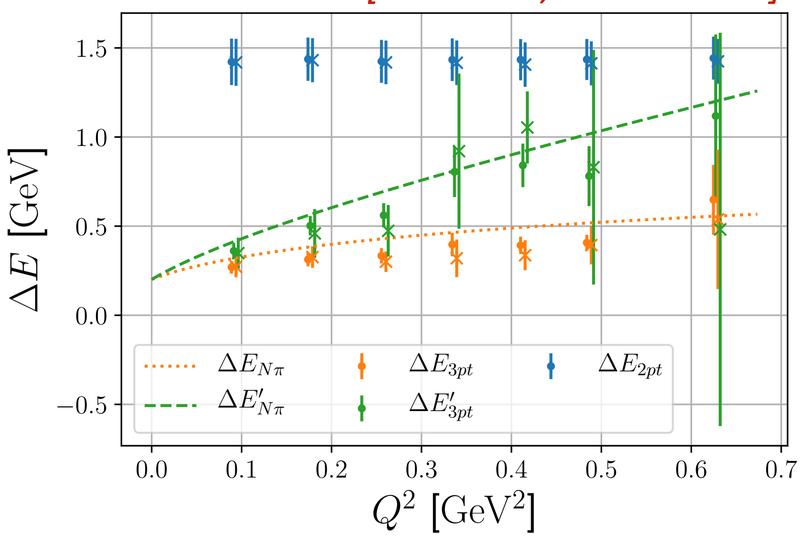
Nucleon interpolating operators may have small overlap onto multi-particle states: $N\pi, N\pi\pi, \ldots$

Energy gaps from nucleon two-functions do not capture $N\pi$ states: $\Delta^{2\mathrm{pt}} > (E_{N\pi} - E_N)$

[Jang et al., 1905.06470]



[Bali et al., 1911.13150]



Violations of PCAC relation (Goldberger-Treiman) as indicator of excited-state contamination

$$\partial_\mu A^a_\mu(x) = 2m\, P^a(x) \quad \Leftrightarrow \quad 2M_\mathrm{N}\, G_\mathrm{A}(Q^2) - \frac{Q^2}{2M_\mathrm{N}}\, \widetilde{G}_\mathrm{P}(Q^2) = 2\hat{m}\, G_\mathrm{P}(Q^2)$$

Needs further investigation — dedicated calculation using multi-hadron interpolators

Summed operator insertions

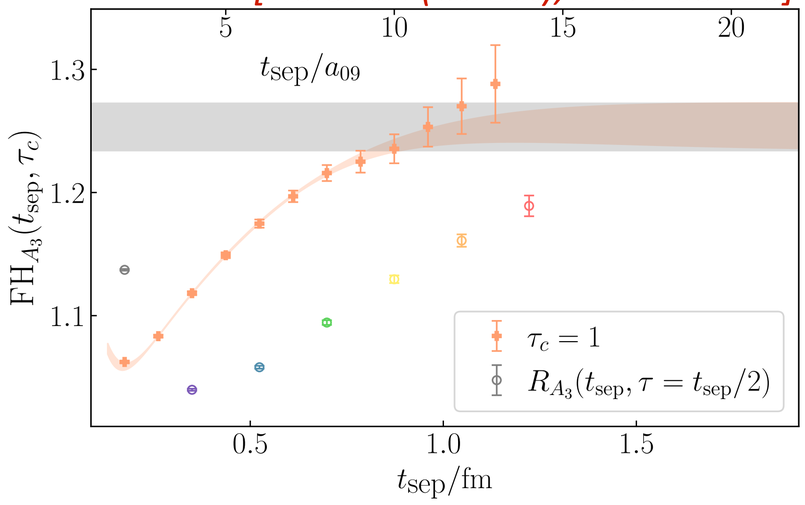
Variants: Fixed sink ("summation method") versus fixed operator ("Feynman-Hellman")

$$S_\Gamma(t_s) = K_\Gamma + (t_s - a)\, g_\Gamma + (t_s - a)\, \mathrm{e}^{-\Delta t_s}\, d_\Gamma + \mathrm{e}^{-\Delta t_s}\, f_\Gamma + \ldots$$

"Summed-subtracted" ratio:

$$g_\Gamma^{\mathrm{eff}}(t_s) \equiv \frac{1}{t_s}\left(S_\Gamma(t_s + a) - S_\Gamma(t_s)\right)$$

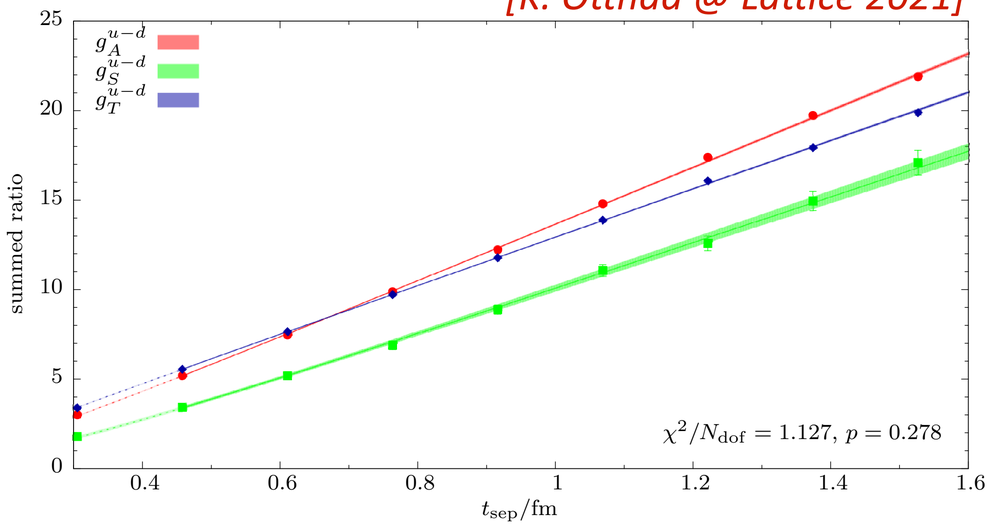
[He et al. (CalLat), 2104.05226]



Faster convergence to ground state

Extend source-sink separations into region with sensitivity to sub-leading terms:

[K. Ottnad @ Lattice 2021]



Improved statistical precision

FLAG Report

2019 edition ("FLAG 4") contains section on nucleon matrix elements *[Aoki et al., 1902.08191]*

Quantities include

- Isovector axial, scalar and tensor charges: g_A^{u-d}, g_S^{u-d}, g_T^{u-d}
- Flavour-diagonal charges: $g_A^{u,d,s}$, $g_S^{u,d,s}$, $g_T^{u,d,s}$
- Sigma terms: $\sigma_q = m_q \langle N|\bar{q}q|N\rangle \equiv m_q\, g_S^q$ $\qquad \sigma_{\pi N} = m_{ud} \langle N|\, \bar{u}u + \bar{d}d\, |N\rangle \approx m_\pi^2 \dfrac{\partial m_N}{\partial m_\pi^2}$

Control over systematics assessed according to quality criteria:

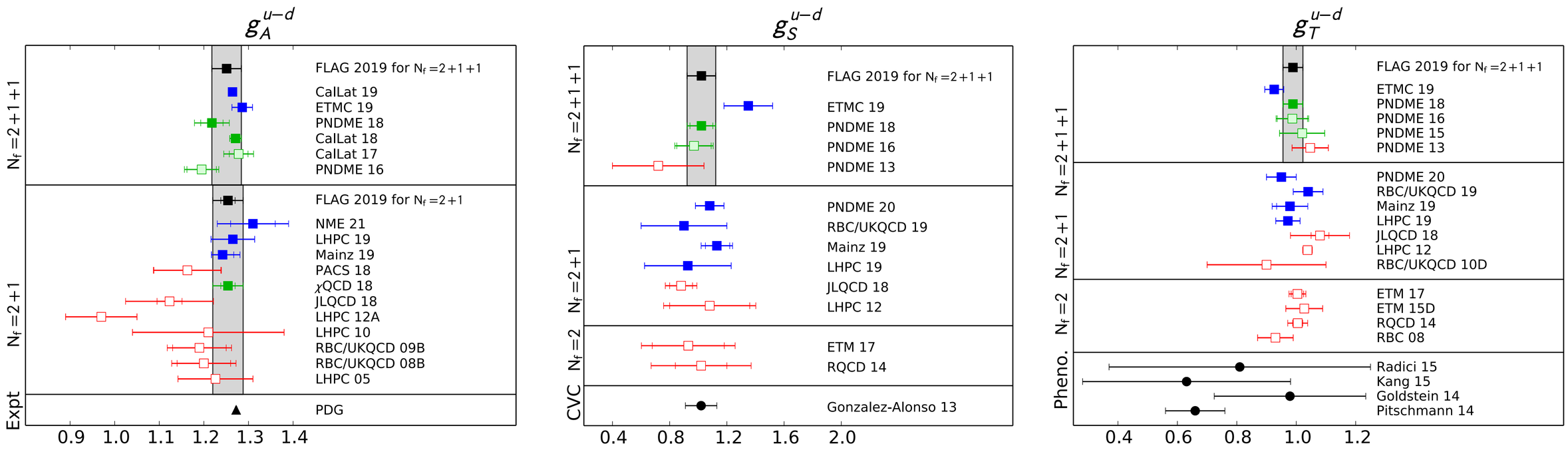
- Chiral extrapolation
- Finite-volume effects
- Continuum extrapolation
- Renormalisation
- Excited states

FLAG 5:

- ★ Three or more source-sink separations τ, at least two of which must be above 1.0 fm.
- ~~○ Two or more source-sink separations, τ, with at least one value above 1.0 fm.~~
- ■ Otherwise

Form factors and non-forward matrix elements, not (yet) included

Isovector charges: preliminary FLAG 5 update



[**blue**: new results since FLAG 4; **grey**: FLAG 4 averages; **solid green**: basis for FLAG 4 average]

Many new results since FLAG 4 — confirmation of previous global estimates

- Axial charge: percent-level precision reached — agreement with experimental values
- Scalar and tensor charges: consistent picture; larger errors for g_S^{u-d}

Pion-nucleon sigma-term

Two different methods:

$$\sigma_{\pi N} = m_{ud} \langle N | \bar{u}u + \bar{d}d | N \rangle \approx m_\pi^2 \frac{\partial m_N}{\partial m_\pi^2}$$

"direct" ◀ Feynman-Hellman ■

FLAG 4 average for $N_f = 2 + 1$:

$$\sigma_{\pi N} = (39.7 \pm 3.6)\,\mathrm{MeV}$$

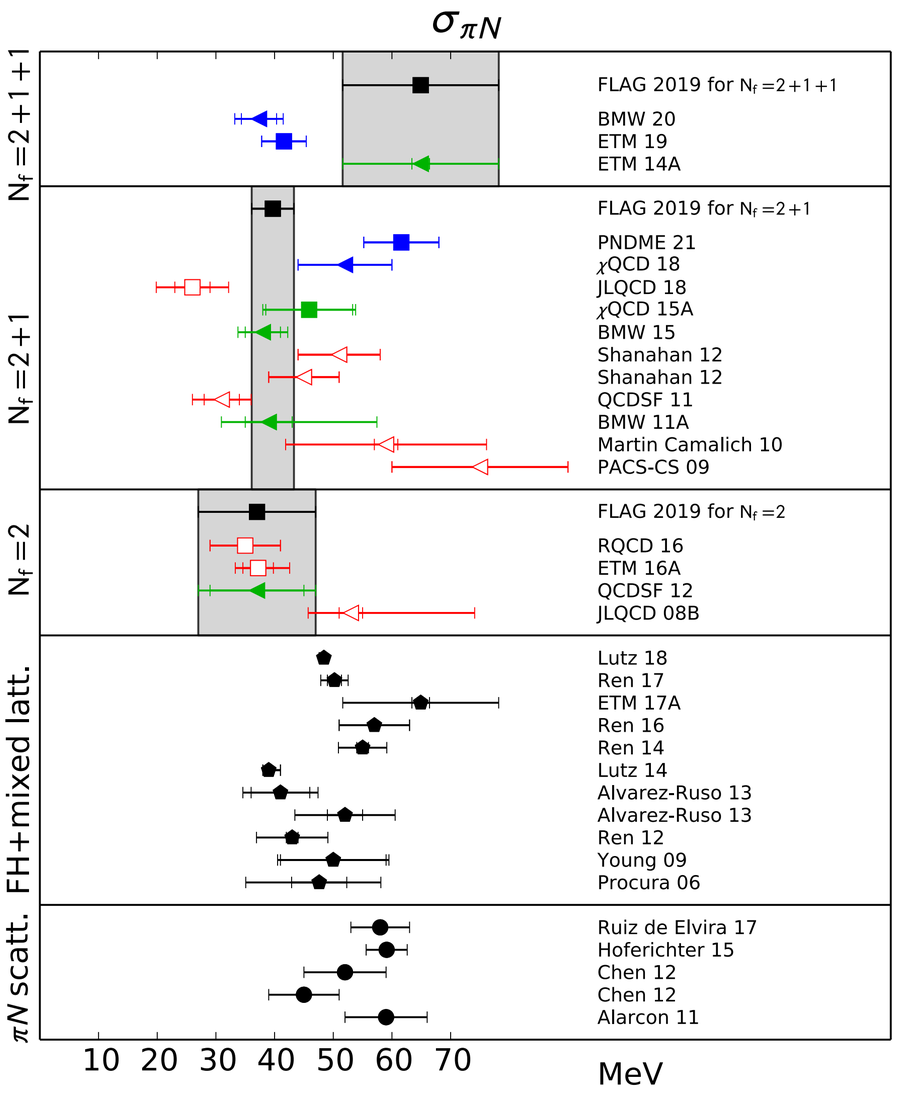
2.5σ tension with result from $N\pi$-scattering:

$$\sigma_{\pi N} = (58 \pm 5)\,\mathrm{MeV}$$

[Ruiz de Elvira et al., 1706.01465]

Bias from excited-state contributions?

→ talk by Boram Yoon



Form factor calculations

Form factors obtained for a discrete set of Q^2-values, at a given value of m_π and non-zero lattice spacing

- Describe the Q^2-dependence: dipole fits or z-expansion *[Hill & Paz, PRD 82 (2010) 113005]*

$$G_{\mathrm{E/M}}(Q^2) = \sum_k a_k^{\mathrm{E/M}} z(Q^2)^k, \quad z(Q^2) = \frac{\sqrt{t_{\mathrm{cut}} + Q^2} - \sqrt{t_{\mathrm{cut}}}}{\sqrt{t_{\mathrm{cut}} + Q^2} + \sqrt{t_{\mathrm{cut}}}}$$

$\rightarrow$ Fits yield electric, magnetic and axial charge radii, magnetic moment, axial charge

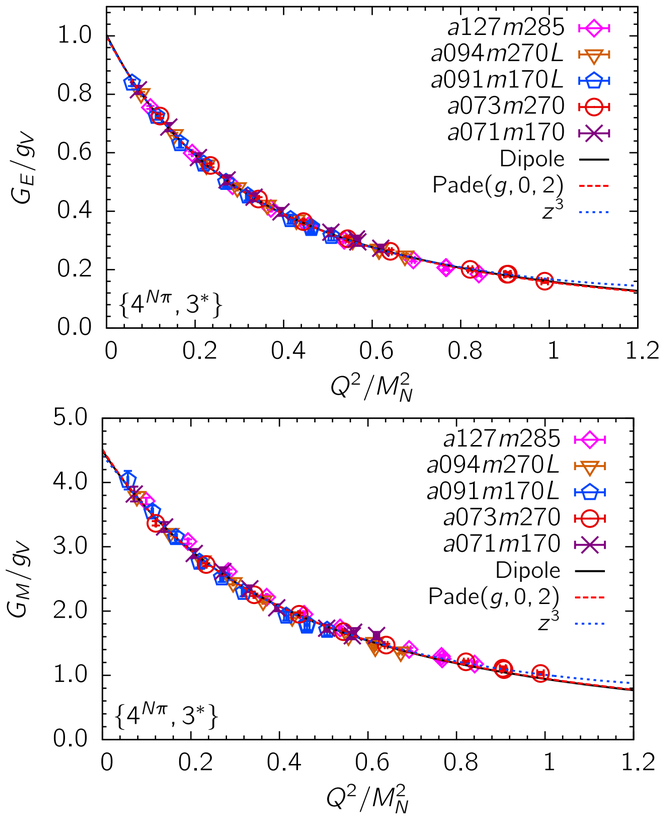
- Extrapolate to the physical point: continuum and infinite-volume limits, physical m_π
- Systematic error estimate by performing variations in the procedures and applying cuts to the data
- Alternative: Direct fits to the dependence of form factors on Q^2 and m_π, supplemented by terms describing the a-dependence and finite-volume corrections

  [Bauer et al., PRC 86 (2012) 065206; Capitani et al., 1504.04628, Djukanovic et al., 2102.07460]

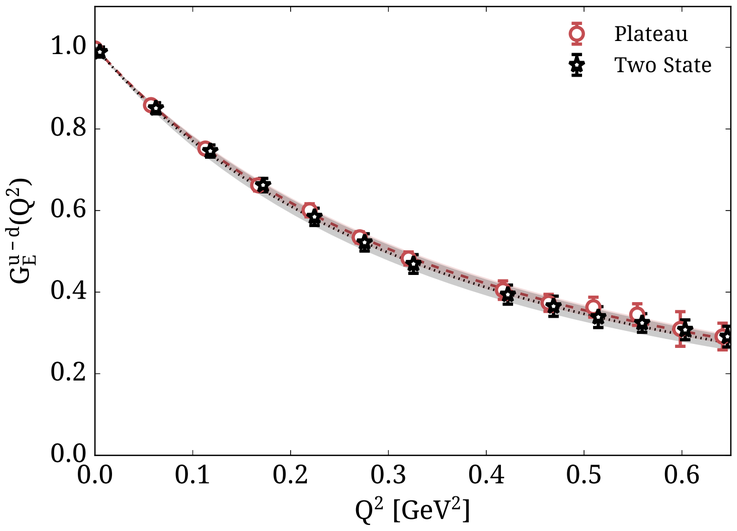
Electromagnetic form factors

Recent calculations for a variety of different discretisations

Isovector form factors: Q^2-dependence



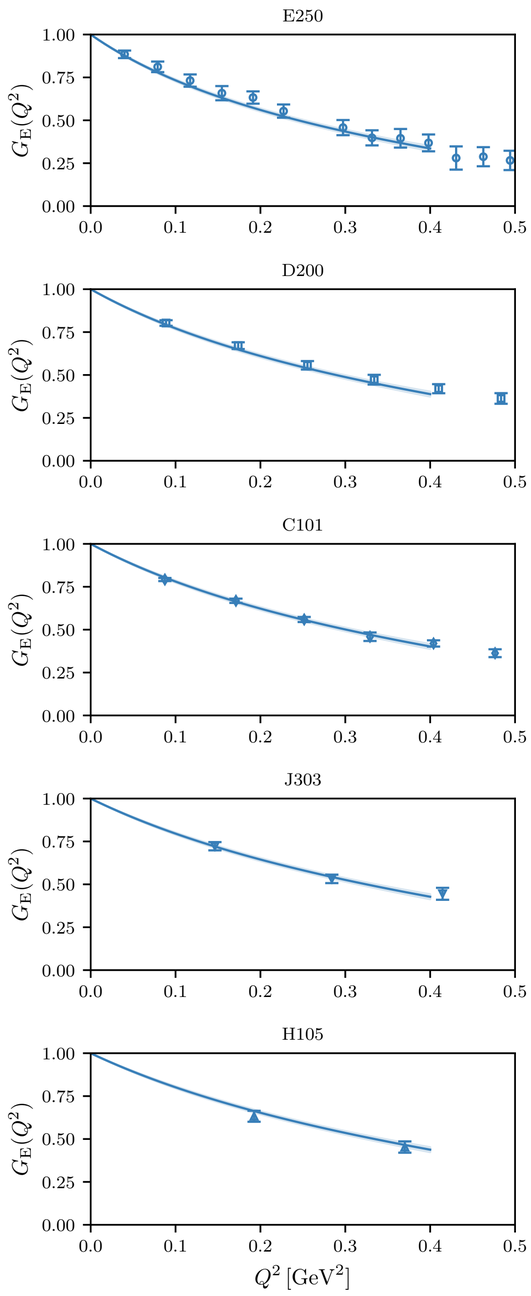
[Park et al. (NME), 2103.05599]



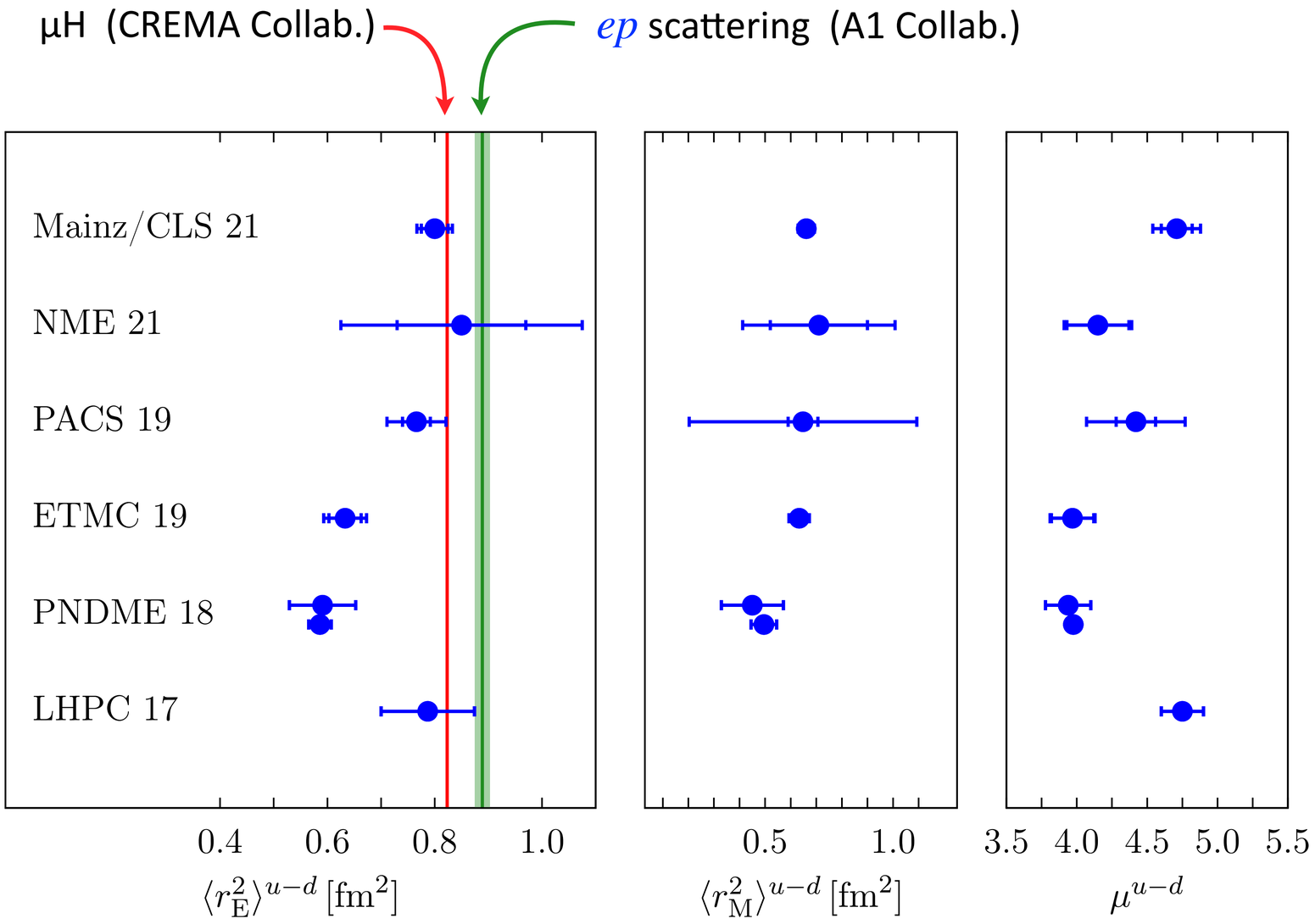
[Alexandrou et al. (ETMC), 1812.10311]

Direct chiral EFT fits to Q^2 and m_π dependence

[Djukanovic et al. (Mainz/CLS), 2102.07460]



Isovector electric and magnetic charge radii & magnetic moment



Comparison with experiment:

- combine proton and neutron charge radii

$$\langle r_E^2 \rangle^{u-d} = \langle r_E^2 \rangle_p - \langle r_E^2 \rangle_n$$

$$\langle r_E^2 \rangle_n^{exp} = -0.1161(22)\,\text{fm}^2$$

- Tension of 2.7σ between A1 and Mainz/CLS 21

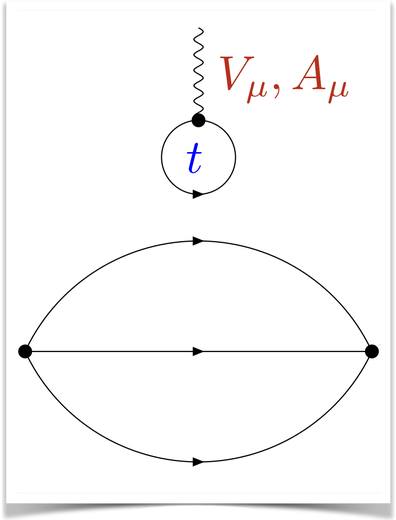
Isoscalar form factors

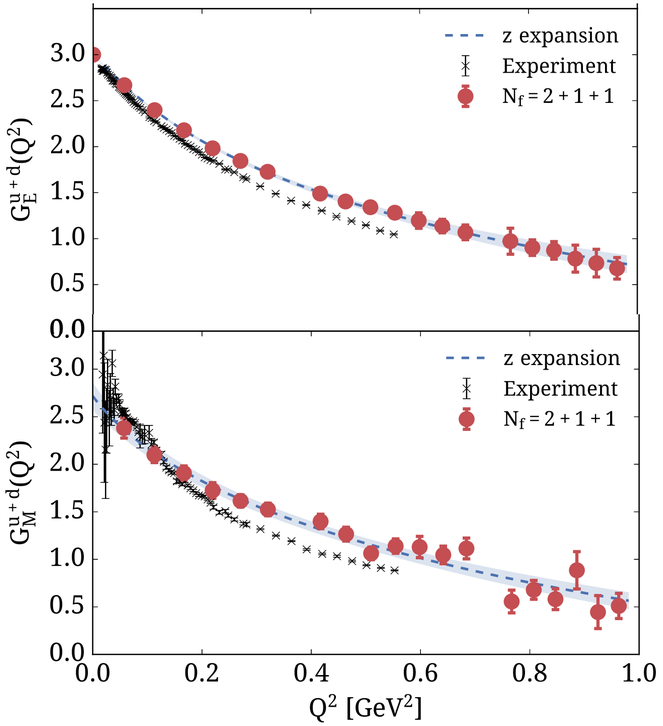
Requires the calculation of quark-disconnected diagrams:

→ stochastic sources, hierarchical probing, “one-end trick”, frequency splitting

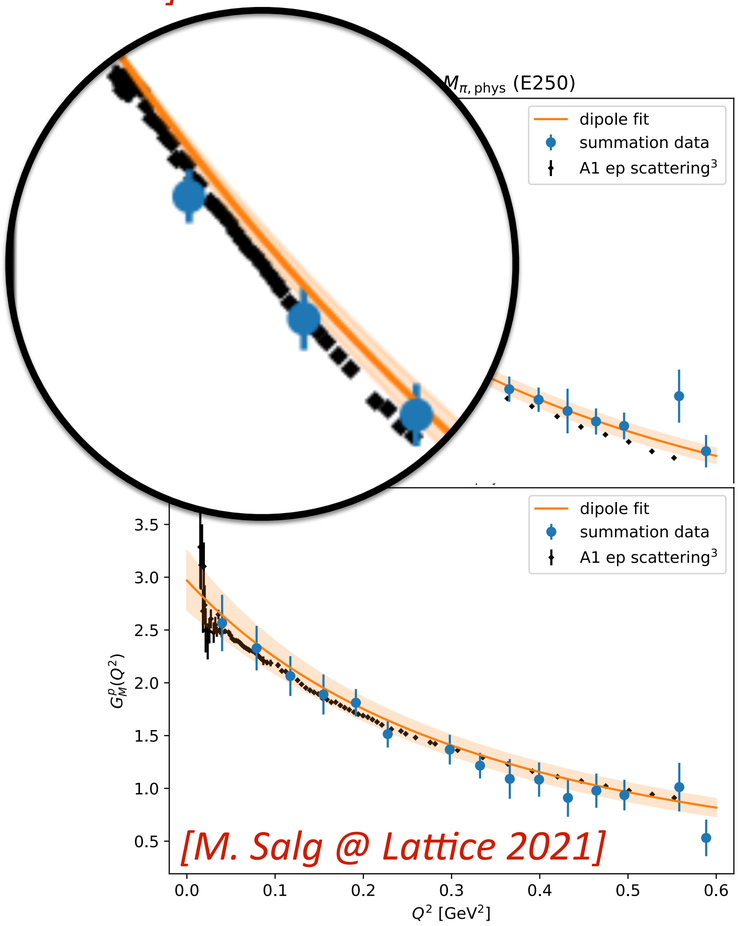
[Dinter et al., 1202.1480; Stathopoulos et al., 1302.4018; Giusti et al., 1903.10447]

Lattice results at physical m_π versus experimental data:



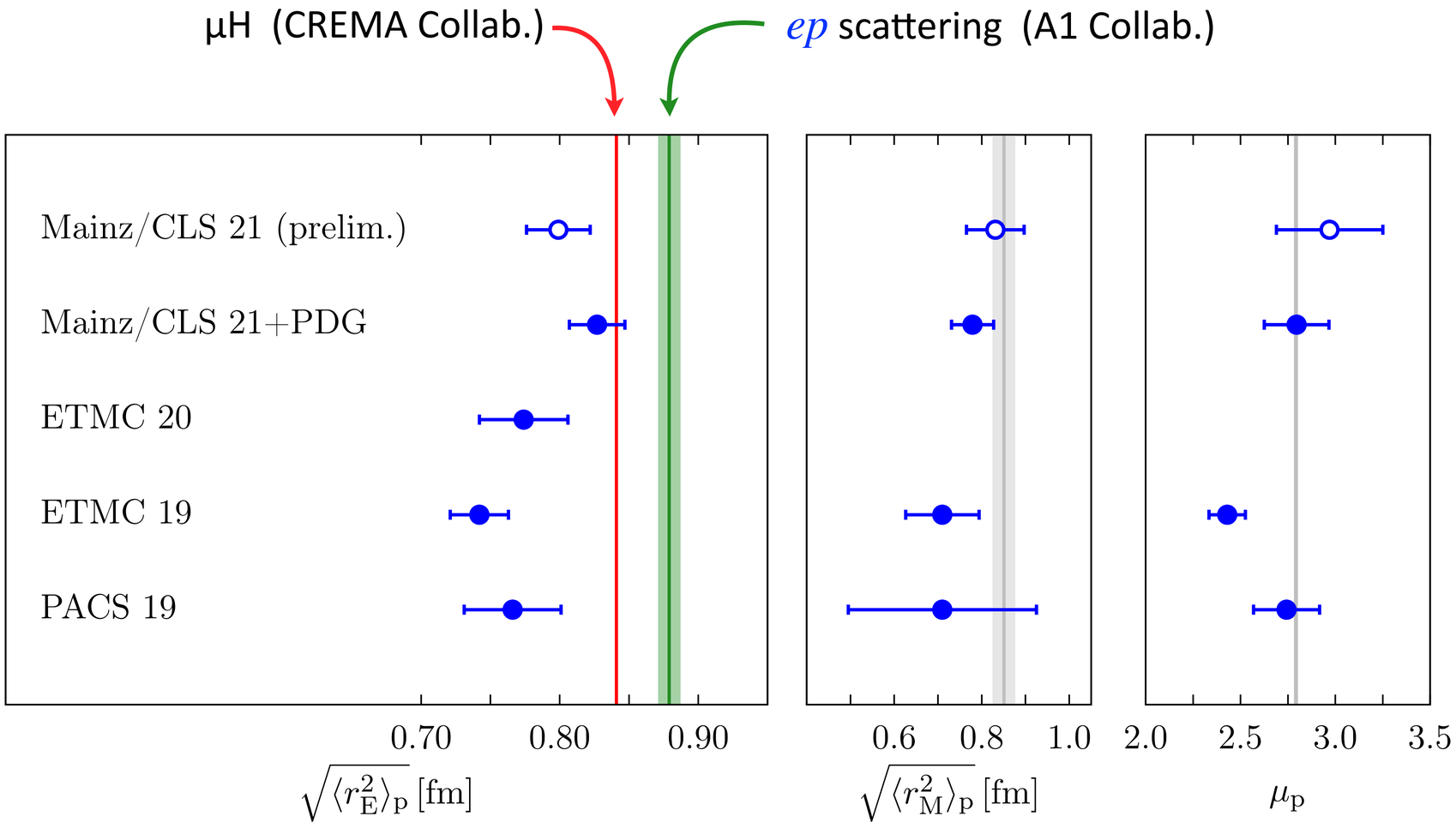


[Alexandrou et al., 1812.10311]



[M. Salg @ Lattice 2021]

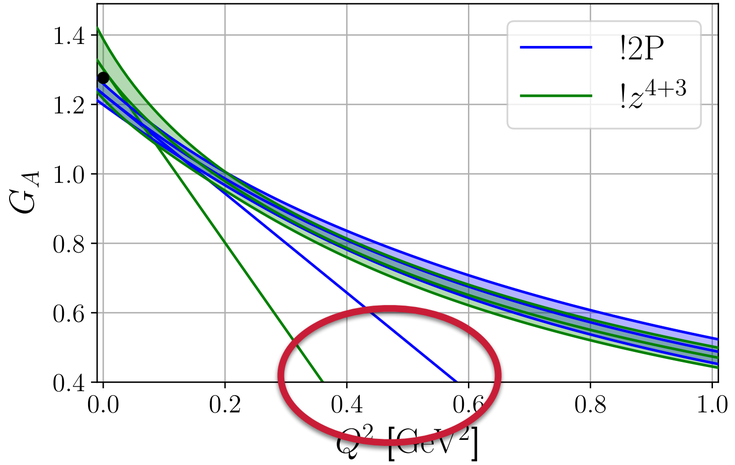
Isoscalar form factors

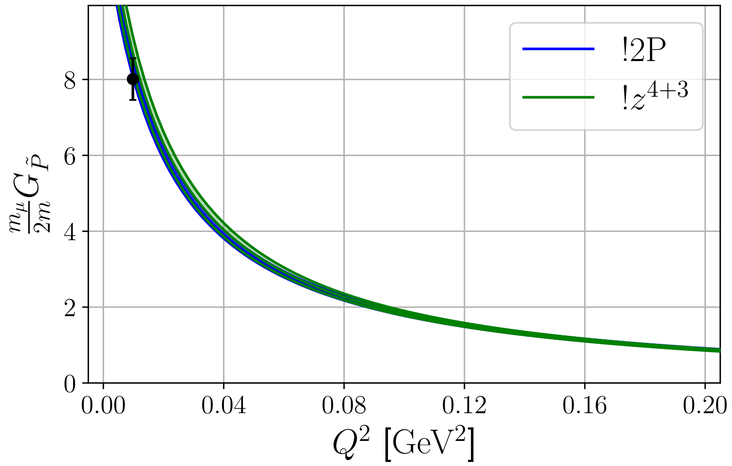


Higher precision required to discriminate between scenarios

Axial form factor and charge radius

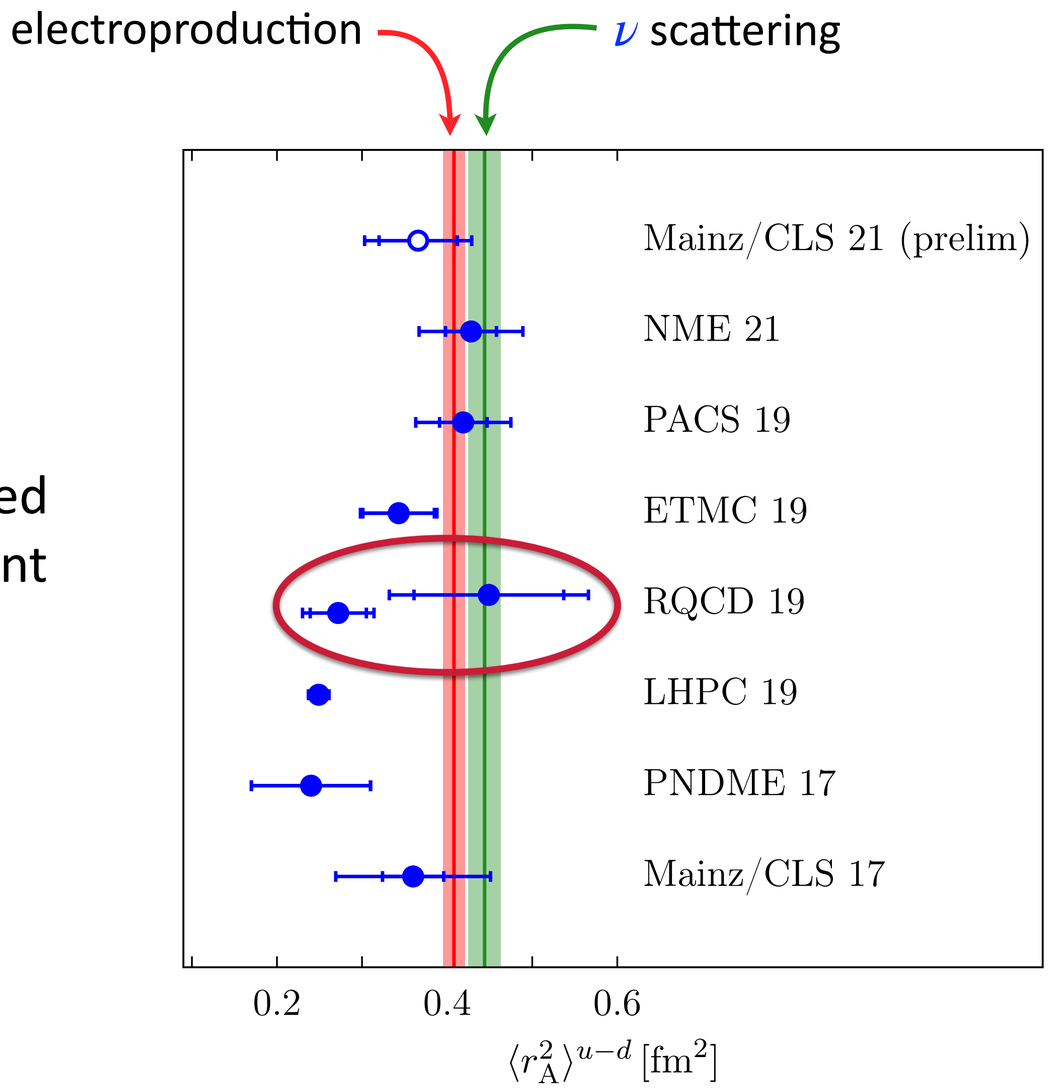
Results in the continuum limit:



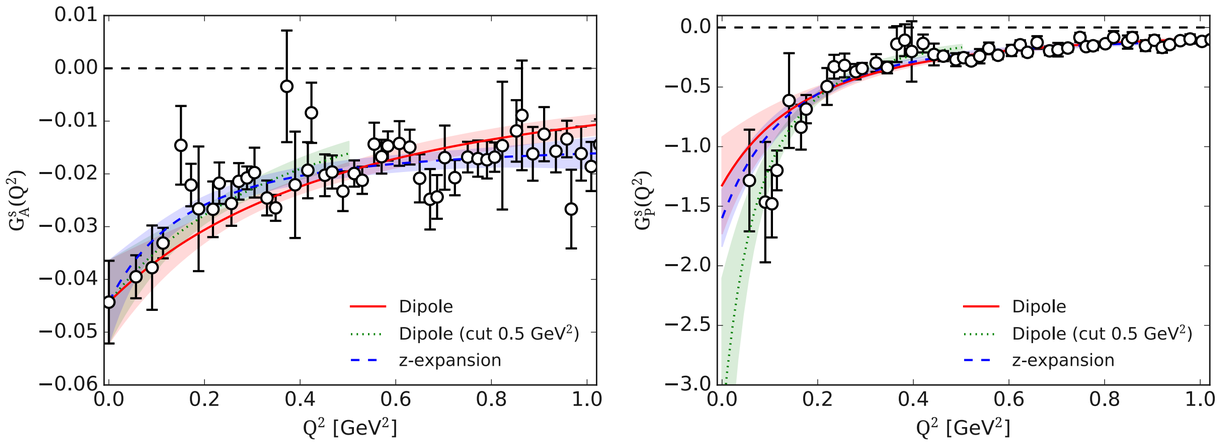


[Bali et al. (RQCD), 1911.13150]

Treatment of excited states has significant impact on $\langle r_A^2 \rangle$



Strange form factors



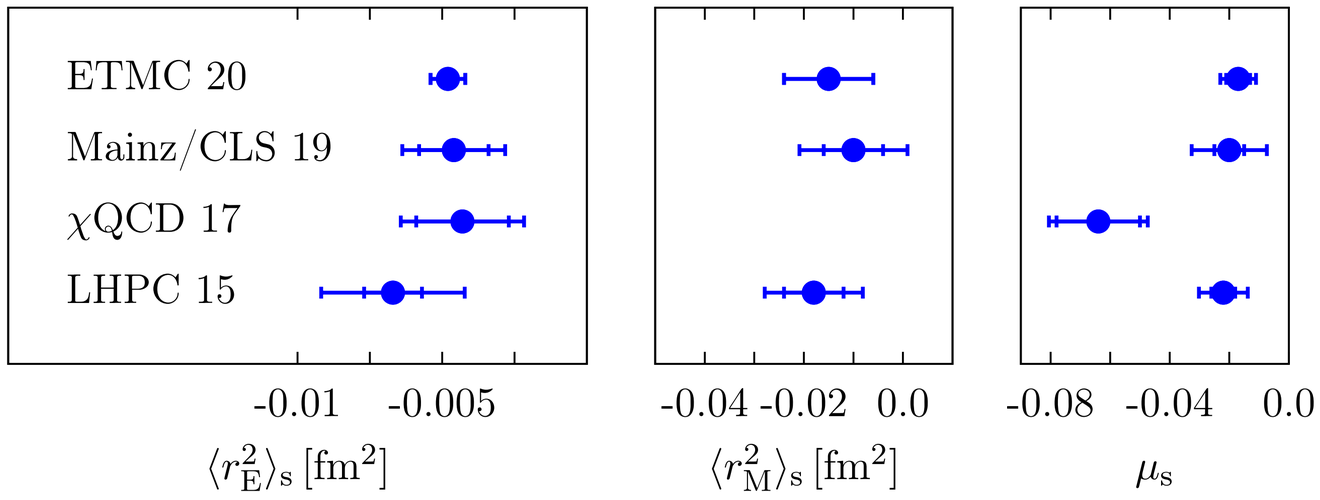
[Alexandrou et al. (ETMC), 2106.13468]

Measured by SAMPLE, HAPPEX, G0, A4 experiments, e.g.

$$G_E^s(Q^2 = 0.22\,\mathrm{GeV}^2) = 0.050 \pm 0.038 \pm 0.019$$

$$G_M^s(Q^2 = 0.22\,\mathrm{GeV}^2) = -0.14 \pm 0.11 \pm 0.11$$

[Baunack et al. (A4 Collab.), PRL 102 (2009) 151803]



Lattice QCD exceeds experimental precision

Summary

* Lattice QCD calculations of nucleon matrix elements have made substantial progress
* Systematic effects such as excited state contamination must be explored further
* Percent-level precision achieved for nucleon isovector axial charge
* Lattice calculations “favour” small values for the proton radius, but precision must be further increased
* Strange form factors and magnetic moment obtained with high precision

Thank you!