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Quark and pion condensates at finite isospin density in χPT

A Virtual Tribute to Quark Confinement and the Hadron Spectrum 2021
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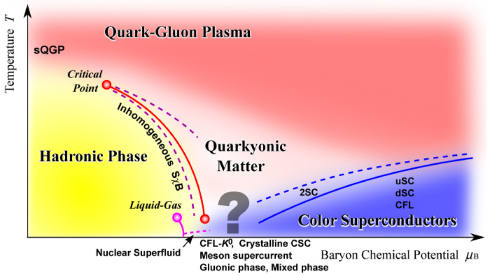
References: EPJC **79** 879 (2019) EPJC **80** 11 1028 (2020).

Introduction

— **QCD phase diagram**

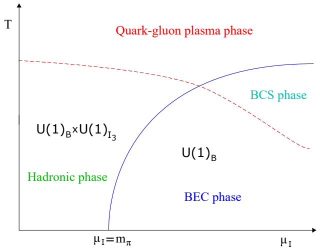


Fukushima and Hatsuda ’11

— **More dimensions**

- Strong magnetic field B - no sign problem
- Finite isospin chemical potential μ_I - no sign problem (Kogut and Sinclair 2002, Brandt et al 2018)

Phase diagram



— **This talk** [2]

- Pion condensation at $T = 0$ using two and three-flavor χPT
- Kaon condensation at $T = 0$ using three-flavor χPT

[2] No inhomogeneous phases, see talks by Mannarelli and Carignano.

χPT at finite isospin μ_I

- **Low-energy effective theory for QCD based on symmetries and relevant degrees of freedom** [3]
- **Two-flavor QCD, pions and** $SU(2)_L \times SU(2)_R$
- **Three-flavor QCD, pions, kaons, and** η **and** $SU(3)_L \times SU(3)_R$
- **Leading order Lagrangian and addition of quark chemical potentials**

$$
\begin{aligned}
\mathcal{L}_2 &= \frac{f^2}{4} \mathrm{Tr}\left[\nabla^\mu \Sigma^\dagger \nabla_\mu \Sigma\right] + \frac{f^2 m^2}{4} \mathrm{Tr}\left[\Sigma + \Sigma^\dagger\right] , \\
\Sigma &= e^{i\frac{\phi_a \tau_a}{f}} , \qquad \nabla_\mu \Sigma \equiv \partial_\mu \Sigma - i\,[v_\mu, \Sigma] , \\
v_\mu &= \delta_{\mu,0} \mathrm{diag}(\mu_u, \mu_d) = \delta_{\mu,0}(\tfrac{1}{3}\mu_B + \tfrac{1}{2}\mu_I, \tfrac{1}{3}\mu_B - \tfrac{1}{2}\mu_I) , \\
f &\sim f_\pi \qquad m \sim m_\pi
\end{aligned}
$$

- **Results independent of** μ_B

[3] Weinberg '79, Gasser and Leutwyler '85

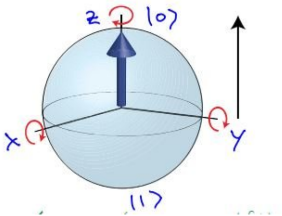
— **Rotated ground state and pion condensation** [4]

$$\Sigma_\alpha = A_\alpha \mathbb{1} A_\alpha \ ,$$
$$A_\alpha = e^{i\frac{\alpha}{2}\tau_1} = \cos\frac{\alpha}{2} + i\tau_1 \sin\frac{\alpha}{2} \ .$$

— **Naive parametrization of fluctuations**

- **Noncanonical kinetic term (not problematic?)**
- **Divergences that cannot be cancelled by standard counterterms (disaster)**

— **Visualizing problem via** $SO(3) \to SO(2)$



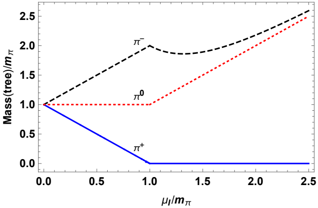
[4] Son and Stephanov '01

— **Static Hamiltonian**

$$\mathcal{H}^{\text{static}} = \frac{1}{8} f^2 \mu_I^2 \,\text{Tr}\left[\tau_3 \Sigma \tau_3 \Sigma^\dagger - 1\right] - \frac{1}{2} f^2 m^2 \,\text{Tr}\left[\Sigma + \Sigma^\dagger\right]$$
$$= -f^2 m^2 \cos\alpha - \frac{1}{2}\mu_I^2 f^2 \sin^2\alpha \; .$$

— **Leading-order Lagrangian for two-flavor χPT**

$$\mathcal{L}_2^{\text{static}} = f^2 m^2 \cos\alpha + \frac{1}{2} f^2 \mu_I^2 \sin^2\alpha \; ,$$
$$\mathcal{L}_2^{\text{quadratic}} = \frac{1}{2}(\partial_\mu \phi_a)(\partial^\mu \phi_a) + \mu_I \cos\alpha(\phi_1 \partial_0 \phi_2 - \phi_2 \partial_0 \phi_1)$$
$$-\frac{1}{2}\Big[(m^2\cos\alpha - \mu_I^2 \cos 2\alpha)\phi_1^2 + (m^2 \cos\alpha - \mu_I^2 \cos^2\alpha)\phi_2^2$$
$$+(m^2\cos\alpha + \mu_I^2 \sin^2\alpha)\phi_3^2\Big] \; .$$



— **Next-to-leading order Lagrangian**

$$\mathcal{L}_4 = \tfrac{1}{4} l_1 \left(\mathrm{Tr}\left[D_\mu \Sigma^\dagger D^\mu \Sigma \right] \right)^2 + \tfrac{1}{4} l_2 \,\mathrm{Tr}\left[D_\mu \Sigma^\dagger D_\nu \Sigma \right] \mathrm{Tr}\left[D^\mu \Sigma^\dagger D^\nu \Sigma \right]$$
$$+ \tfrac{1}{16} (l_3 + l_4) m^4 \left(\mathrm{Tr}\left[\Sigma + \Sigma^\dagger \right] \right)^2 + \tfrac{1}{8} l_4 m^2 \mathrm{Tr}\left[D_\mu \Sigma^\dagger D^\mu \Sigma \right] \mathrm{Tr}[\Sigma + \Sigma^\dagger]$$

— **Static part**

$$\mathcal{L}_4^{\mathrm{static}} = (l_1 + l_2) \mu_I^4 \sin^4 \alpha + l_4 m^2 \mu_I^2 \cos\alpha \sin^2 \alpha + (l_3 + l_4) m^4 \cos^2 \alpha \ ,$$

— **Renormalization of parameters**

$$l_i = l_i^r(\Lambda) - \frac{\gamma_i \Lambda^{-2\epsilon}}{2(4\pi)^2} \left[\frac{1}{\epsilon} + 1 - \bar{l}_i \right] \ ,$$

— **Effective potential at NLO**

$$V_1 = V_{1,\pi^+} + V_{1,\pi^-} + V_{1,\pi^0}$$
$$= \tfrac{1}{2} \int_p \left(E_{\pi^+} + E_{\pi^-} + E_{\pi^0} \right)$$

— **Isolate divergences by adding and subtracting divergent terms that can be calculated in dimensional regularization**

$$V_1^{\mathrm{div}} = -\frac{1}{2(4\pi)^2}\left[\frac{1}{\epsilon}+\frac{3}{2}+\log\left(\frac{\Lambda^2}{m_3^2}\right)\right]\left(m^2\cos\alpha+\mu_I^2\sin^2\alpha\right)^2$$
$$-\frac{1}{4(4\pi)^2}\left[\frac{1}{\epsilon}+\frac{3}{2}+\log\left(\frac{\Lambda^2}{\tilde{m}_2^2}\right)\right](m^2\cos\alpha)^2 \ .$$

— **Renormalized effective potential**

$$V_{\mathrm{eff}} = -f^2m^2\cos\alpha-\frac{1}{2}f^2\mu_I^2\sin^2\alpha$$
$$-\frac{1}{4(4\pi)^2}\left[\frac{3}{2}-\bar{l}_3+4\bar{l}_4+\log\left(\frac{m^2}{\tilde{m}_2^2}\right)+2\log\left(\frac{m^2}{m_3^2}\right)\right]m^4\cos^2\alpha$$
$$-\frac{1}{(4\pi)^2}\left[\frac{1}{2}+\bar{l}_4+\log\left(\frac{m^2}{m_3^2}\right)\right]m^2\mu_I^2\cos\alpha\sin^2\alpha$$
$$-\frac{1}{4(4\pi)^2}\left[1+\frac{2}{3}\bar{l}_1+\frac{4}{3}\bar{l}_2+2\log\left(\frac{m^2}{m_3^2}\right)\right]\mu_I^4\sin^4\alpha+V_{1,\pi^+}^{\mathrm{fin}}+V_{1,\pi^-}^{\mathrm{fin}} \ .$$

Parameter fixing in two-flavor χPT

— **Low-energy constants**

$$\bar{l}_1 = -0.4 \pm 0.6 \,, \qquad \bar{l}_2 = 4.3 \pm 0.1 \,,$$
$$\bar{l}_3 = 2.9 \pm 2.4 \,, \qquad \bar{l}_4 = 4.4 \pm 0.2 \,.$$

— **Physical masses** [5]

$$m_\pi = m\left[1 - \frac{m^2}{4(4\pi)^2 f^2}\bar{l}_3\right] = 131 \pm 3\mathrm{MeV} \,,$$
$$f_\pi = f\left[1 + \frac{m^2}{(4\pi)^2 f^2}\bar{l}_4\right] = \frac{128 \pm 3}{\sqrt{2}}\mathrm{MeV} \,.$$

— **Parameters**

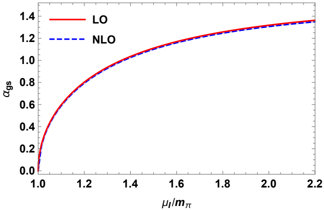
$$m_{\rm cen} = 132.4884\,\mathrm{MeV} \,, \qquad f_{\rm cen} = 84.9342\,\mathrm{MeV} \,,$$

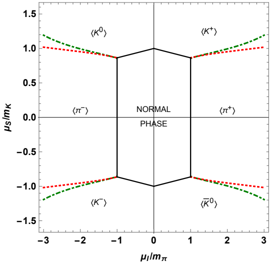
[5] B. B. Brandt, G. Endrodi, E. S. Fraga, M. Hippert, J.Schaffner-Bielich, and S. Schmalzbauer, '18.

Results

— **Rotation angle α and phase diagram**

$$\begin{aligned} \cos\alpha &= \frac{m^2}{\mu_I^2} & \mu_I \geq m \\ \alpha &= 0\,, & \mu_I < m\,. \end{aligned}$$





— **Silver Blaze property. Onset of pion condensation exactly at m_π**

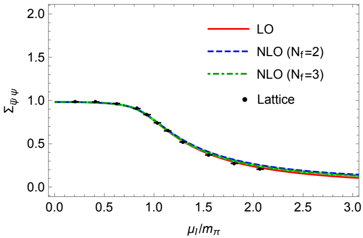
— **Second-order transition from the vacuum. First-order transition between pion and kaon-condensed phases**

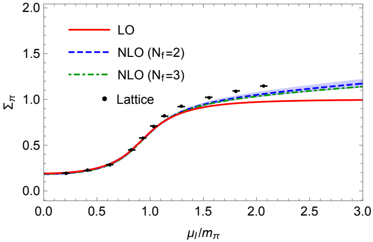
Results

— **Quark condensate and pion condensates** [6]

$$\langle\bar{\psi}\psi\rangle = \frac{1}{2}\frac{\partial V}{\partial m}\,, \qquad \langle\pi^+\rangle = \frac{1}{2}\frac{\partial V}{\partial j}\,.$$

$$\langle\bar{\psi}\psi\rangle^2_{\mathrm{tree},\mu_\mathrm{I}} + \langle\pi^+\rangle^2_{\mathrm{tree},\mu_\mathrm{I}} = \langle\bar{\psi}\psi\rangle^2_{\mathrm{tree},0}\,.$$



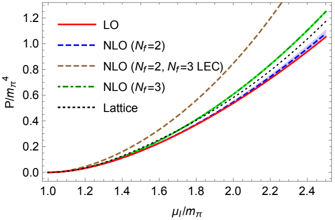


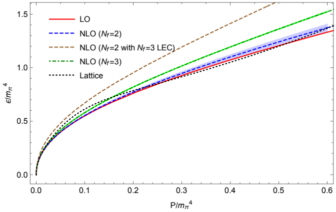
[6] Lattice data by Brandt, Endrodi, Schmalzbauer '18

Results

— **Pressure and equation of state** [7]

$$
\begin{aligned}
P &= -V_{\rm eff}\ , \\
\varepsilon &= -P - \frac{\partial V_{\rm eff}}{\partial \mu_I}\mu_I\ .
\end{aligned}
$$





[7] Lattice data by Brandt, Endrodi, Schmalzbauer '18

Conclusions and Outlook

— Conclusions

- **First calculation of thermodynamic functions at next-to-leading order in the pion-condensed phase in two -and three-flavor χPT**
- **Good agreement with lattice data at $T = 0$ First precision test of χPT at NLO with nonzero μ_I**
- **Good agreement with lattice data at $T \neq 0$ only for very low temperatures (phase diagram not shown)**

— Outlook

- **Phase diagram in an external magnetic field**
  - — **Quantum and thermal corrections to vortices**
  - — **Competition with other phases such as the chiral soliton lattice**