Warm dense QCD matter in strong magnetic fields

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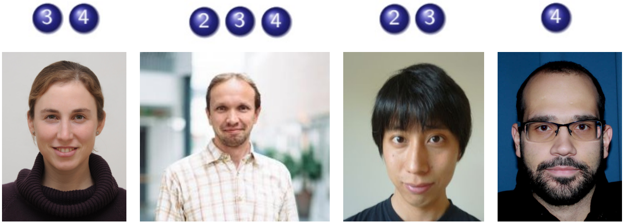
Joint work with Tomáš Brauner, Naoki Yamamoto and Georgios Filios

Outline

1. Motivation: ⟨ QCD phase diagram / Inhomogeneous phases of QCD matter
2. Chiral soliton lattice phase at LO
3. Chiral soliton lattice phase at finite temperature
4. Chiral soliton lattice phase seen in lattice simulations?



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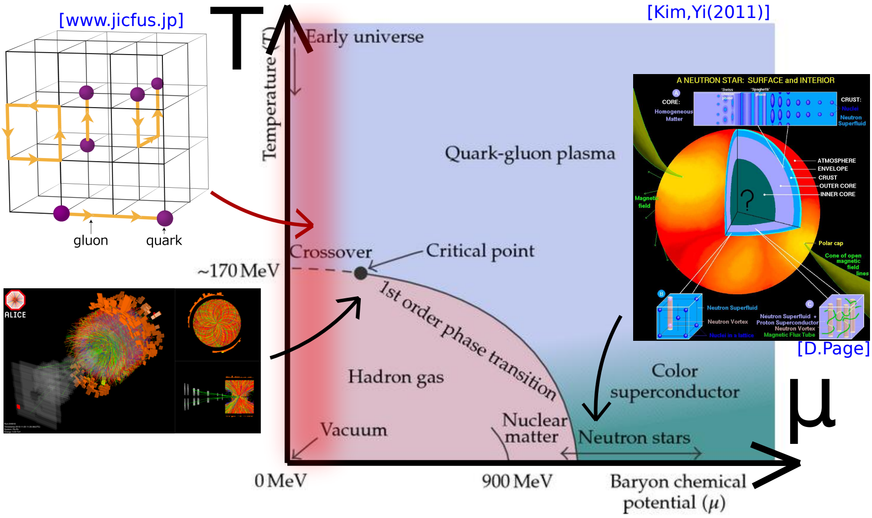

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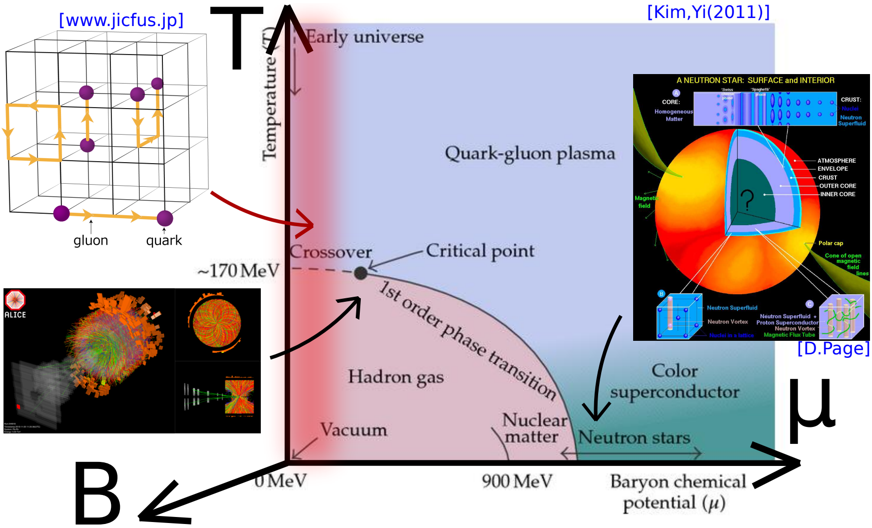
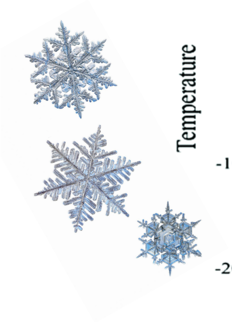
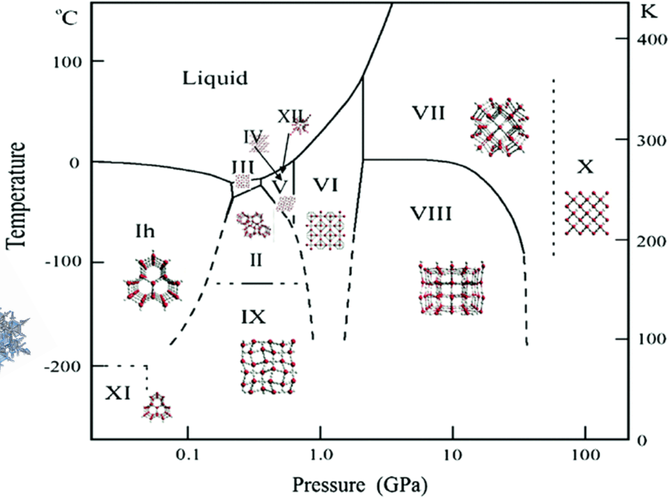


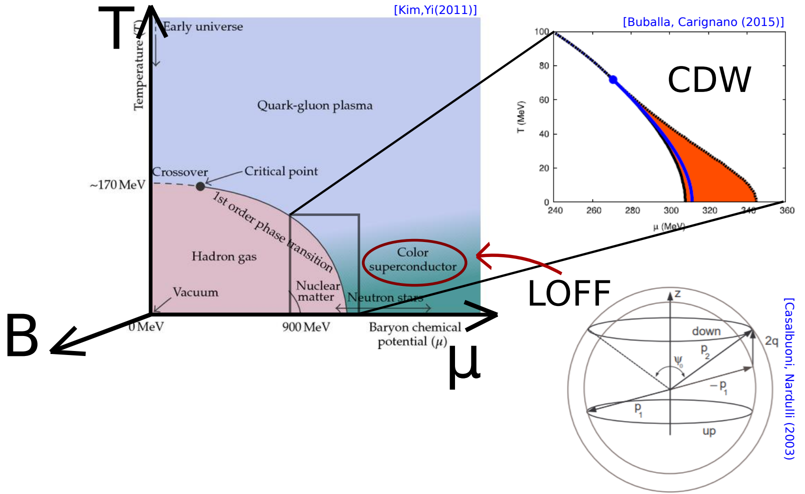
Motivation: Inhomogeneous phases of matter





Inhomogeneous phases in the QCD phase diagram?

[Alford,Bowers,Rajagopal(2001)], review: [Buballa,Carignano(2015)]



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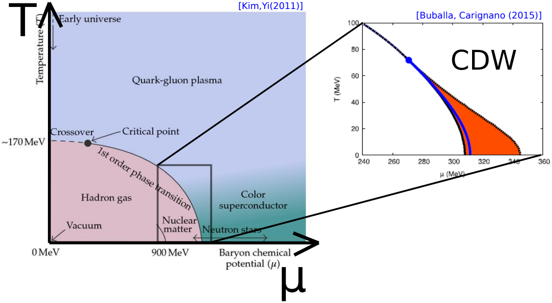

- Chiral Density Wave: $\langle\overline{\psi}\psi\rangle = \Delta\cos(\vec{q}\cdot\vec{x})$, $\langle\overline{\psi}i\gamma_5\tau_3\psi\rangle = \Delta\sin(\vec{q}\cdot\vec{x})$

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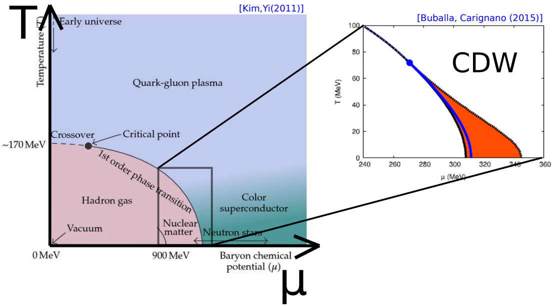

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- 1D modulation in spherically symmetric 3D system unstable at any non-zero temperature! [Peierls (1934), Landau]
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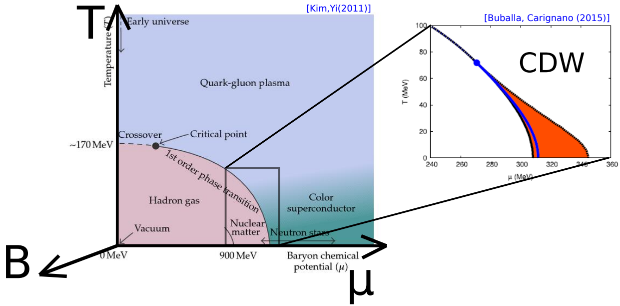

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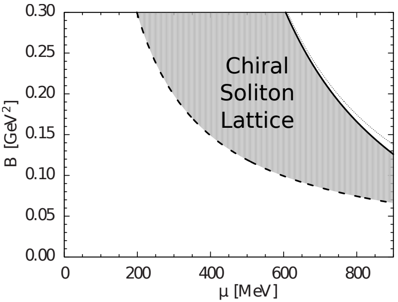
  (but quasi-long-range order may be sustained [Hidaka et al. (2015)])
- If the spherical symmetry broken by magnetic field, 1D modulations stable! [Tatsumi,Nishiyama,Karasawa(2015)] [Ferrer,Incera(2020)]

QCD matter in strong B: “new” inhomogeneous phase!

[Son,Stephanov(2008)][Brauner,Yamamoto(2017)]

Ground state of QCD matter for sufficiently strong magnetic field and large enough baryon chemical potential: inhomogeneous condensate of neutral pions

Shown using chiral perturbation theory!



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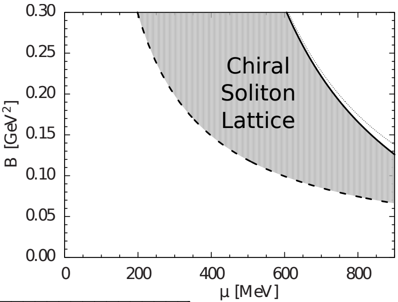
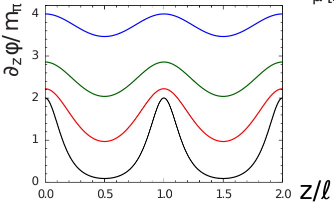
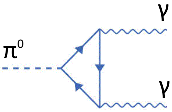


Spatially varied neutral pion background field:



Neutral pions coupled to the magnetic field due to chiral anomaly!



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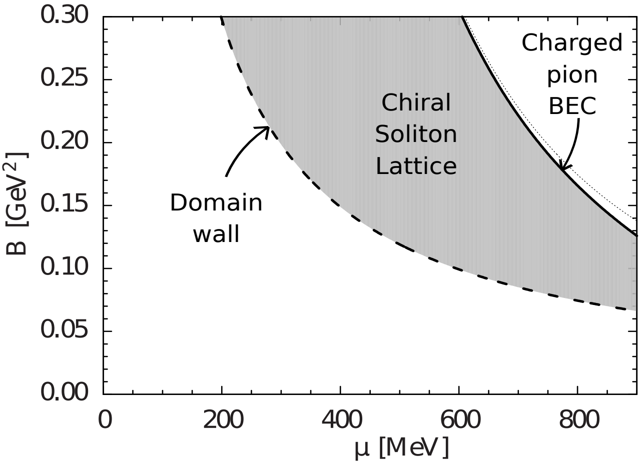

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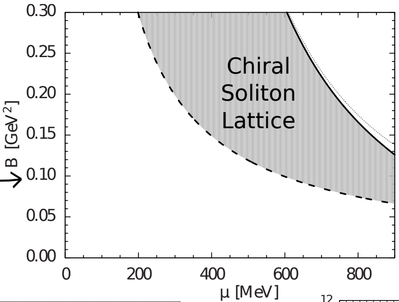
$\approx 10^{19}$G

[Ferrer, Incera, Keith, Portillo, Springsteen(2010)]
[Cardall, Prakash, Lattimer (2001)]
[Eto, Hashimoto, Hatsuda (2013)]



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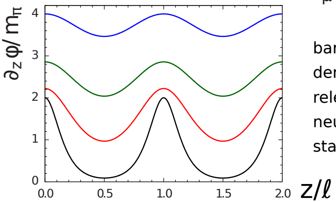
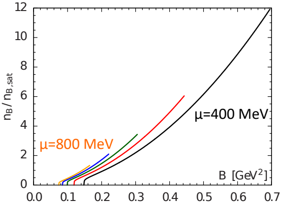


baryon densities relevant for neutron stars:



Chiral soliton lattice phase @LO

[Brauner,Yamamoto(2017)]

- In strong magnetic fields charged pions get large effective masses $\Rightarrow$ only neutral pions remain relevant degrees of freedom!
- Ground state configuration of the neutral pion field? Minimize the energy functional (based on chiral perturbation theory):

$$\mathcal{H}_{\text{eff}} = \frac{f_\pi^2}{2}(\boldsymbol{\nabla}\phi)^2 + m_\pi^2 f_\pi^2 (1 - \cos\phi) - \frac{\mu}{4\pi^2}\boldsymbol{B}\cdot\boldsymbol{\nabla}\phi \qquad (\phi = \frac{\pi^0}{f_\pi})$$

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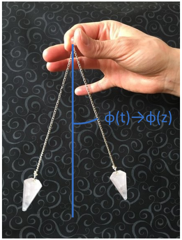


Ground state for $\mu B \geq 16\pi m_\pi f_\pi^2$:

$$\cos\frac{\phi(z)}{2} = \text{sn}\left(\frac{m_\pi z}{k}, k\right),$$

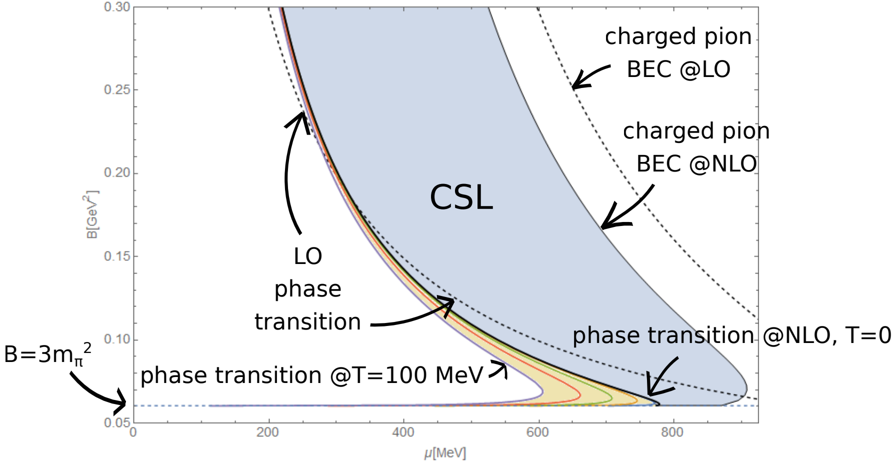
with elliptic modulus k fixed as

$$\frac{E(k)}{k} = \frac{\mu B}{16\pi m_\pi f_\pi^2}$$

Chiral soliton lattice phase at finite temperature

[Brauner,H.K.,Yamamoto (soon)]

Chiral soliton lattice phase stabilized by finite temperature!



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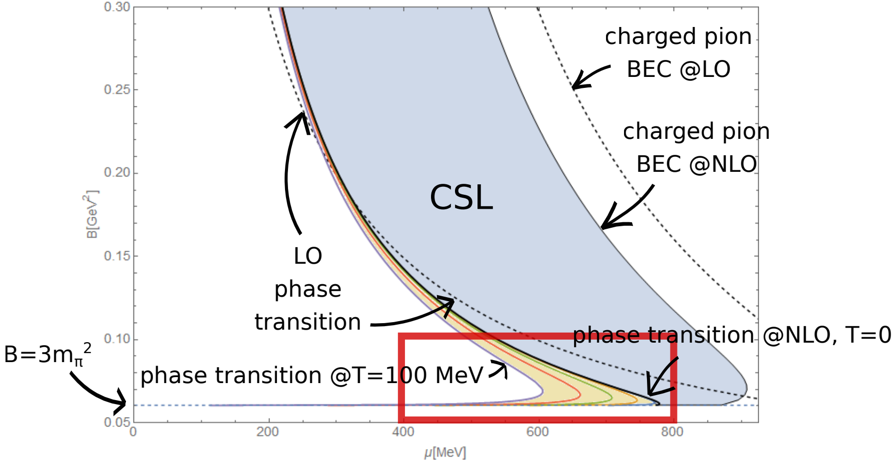

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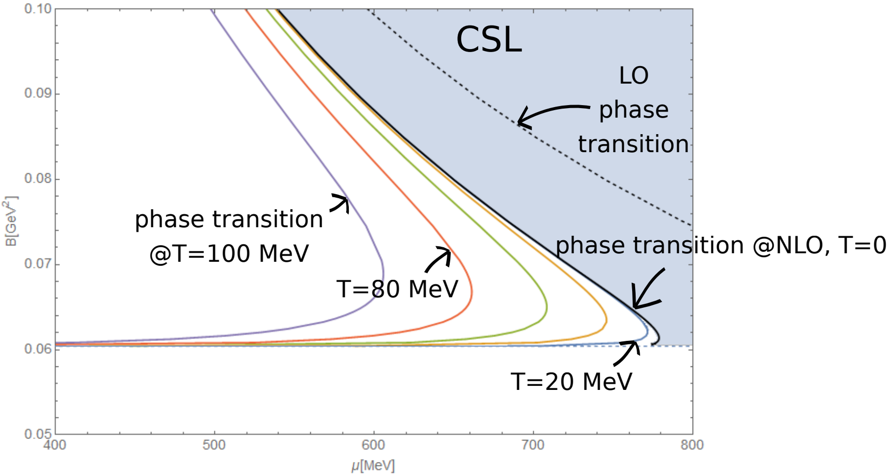

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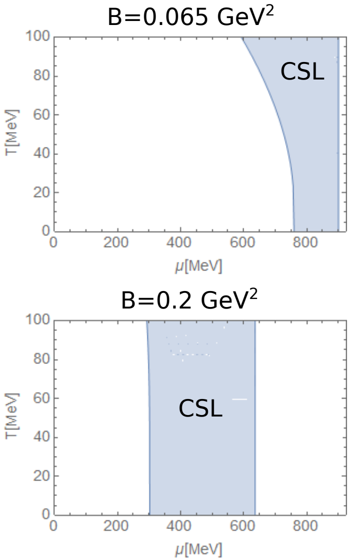

Chiral soliton lattice phase at finite temperature

- LO: domain wall formed at the phase transition ⇒
  1. 1-loop free energy for domain wall background $\mathcal{F}^{NLO}(B,T)$ calculated
  2. phase transition occurs at such points in parameter space where $\mathcal{F}^{LO+NLO}(\mu,B,T)=0$

LO:
$$\frac{\mathscr{E}_{\rm wall}}{S} = 8m_\pi f_\pi^2 - \frac{\mu B}{2\pi}.$$

NLO, T=0

$$\frac{\mathscr{F}^{T=0}_{\rm wall,fin}}{S} = \frac{B^{3/2}}{\sqrt{2\pi}}\Bigg\{\zeta(-\tfrac{1}{2},\tfrac{1}{2}-\tfrac{3m_\pi^2}{2B})+\zeta(-\tfrac{1}{2},\tfrac{1}{2}) - \int\frac{\mathrm{d}P}{2\pi}\frac{2}{1+P^2}\left[\zeta(-\tfrac{1}{2},\tfrac{1}{2}+\tfrac{m_\pi^2(1+P^2)}{2B})+\frac{2}{3}\left(\frac{m_\pi^2}{2B}\right)^{3/2}(1+P^2)^{3/2}\right] - \int\frac{\mathrm{d}P}{2\pi}\frac{4}{4+P^2}\left[\zeta(-\tfrac{1}{2},\tfrac{1}{2}+\tfrac{m_\pi^2(1+P^2)}{2B})+\left(\frac{m_\pi^2}{2B}\right)^{3/2}\left(\frac{2}{3}(4+P^2)^{3/2}-3(4+P^2)^{1/2}\right)\right]\Bigg\}.$$

$$\frac{\mathscr{F}^{T=0}_{\rm wall,ren}}{S} = \frac{m_\pi^3}{24\pi^2}\left(-17\log\frac{4\pi\Lambda^2_{\rm RG}}{m_\pi^2}+40\log 2-\frac{121}{3}+17\gamma_{\rm E}\right)+m_\pi^3\left(-\frac{64}{3}\bar{\ell}_1-\frac{8}{3}\bar{\ell}_2+\frac{8}{3}\bar{\ell}_3\right).$$

NLO, finite T:

$$\frac{\mathscr{F}^{T,(\pi^0)}_{\rm wall}}{S} = -\frac{\zeta(3)T^3}{2\pi}-\frac{m_\pi^2 T}{\pi^2}\int_0^\infty Q\arctan Q \times\log\left(1-e^{-x\sqrt{1+Q^2}}\right)\mathrm{d}Q,$$

$$\frac{\mathscr{F}^{T,(\pi^\pm)}_{\rm wall}}{S} = \frac{BT}{\pi}\sum_{m=0}^\infty\Bigg\{\log\left[1-e^{-\beta\sqrt{(2m+1)B-3m_\pi^2}}\right]+\log\left[1-e^{-\beta\sqrt{(2m+1)B}}\right] - \frac{1}{\pi}\int_0^\infty\mathrm{d}P\left(\frac{2}{1+P^2}+\frac{4}{4+P^2}\right)\log\left[1-e^{-\beta\epsilon(m,P^2)}\right]\Bigg\}.$$

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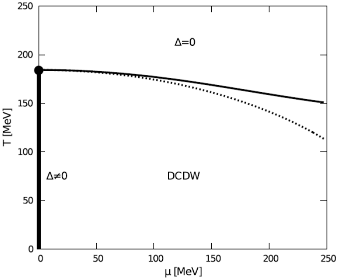
NB: Connection to Chiral Density Wave?

[Tatsumi,Nishiyama,Karasawa(2015)] [Ferrer,Incera(2020)]

- QCD at finite B, μ, T studied within NJL-like models, CDW found to be preferred in certain parameter range
- Important effect of chiral anomaly observed
- Only chiral limit considered
- CSL at chiral limit is equivalent to CDW! [Brauner,Yamamoto(2017)]

B=(300 MeV)^2:

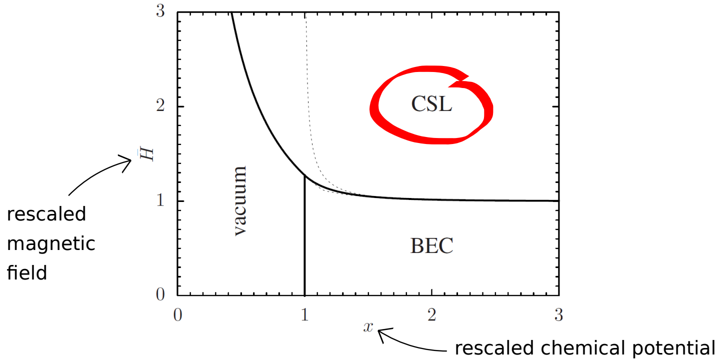


[Tatsumi,Nishiyama,Karasawa(2015)]

CSL phase in lattice simulations?

[Tomáš Brauner,Georgios Filios,H.K.;Phys.Rev.Lett.123(2019), JHEP 1912 (2019) 029]

- In certain QCD-like theories (e.g., two-color QCD) the sign problem is absent ⇒ lattice simulations possible
- CSL-like phase present for sufficiently large magnetic fields! (Shown using chiral perturbation theory.)
- Conjecture of [Splittorff, Son, Stephanov (2001)] that the inhomogeneous phases exist only in theories with the sign problem disproved!



Conclusions

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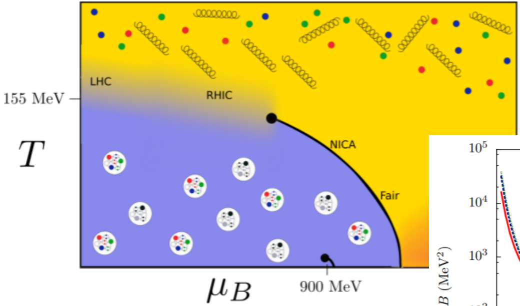
Thank you for your attention!

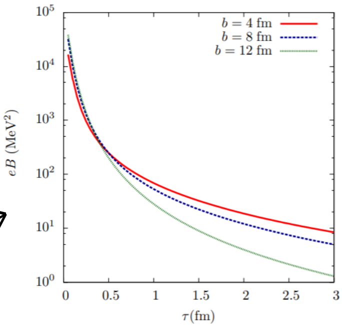
Backup Slides

Relevance of CSL for heavy ion collisions?

[Guenther(2020)]



[Kharzeev et al.(2007)]



Time evolution of the magnetic field created in heavy ion collisions with different impact parameters