The unified equation of state with induced surface tension consistent with astrophysical, gravitational, high- and low-energy nuclear physics data

Violetta Sagun
CFisUC, University of Coimbra, Portugal

In collaboration with Oleksii Ivanytskyi, Constança Providência
Strongly Interacting Matter Phase Diagram

Constraints on realistic EoS
IST EoS
Modelling of neutron stars
Conclusions
Constraints on the EoS

HEP

- **proton flow**
anisotropic expansion is caused by gradient of pressure, which gives an access to EoS

- **hadron multiplicities**
hard core radii of hadrons control the rate of their production in thermal medium: R ∼ 0.3 – 0.5 fm

- **nucleon-nucleon scattering**
hard core radius of nucleons extracted as a parameter of microscopic interaction potential: R = 0.5 fm
  - M. Nagels, Phys. Part. Nucl. 5, 524 (2014)

Astro

- ~ 2 M\(_\odot\)
  - PSR J0348-0436: M = 2.01(4)\(\ M_\odot\)
    - J. Demorest et al., Science, 341, 1214 (2013)
  - PSR J0740+6620: M = 2.14(3)\(\ M_\odot\)
- NICER results
- NSs cooling
- observations of pulsars
  - M-R relation

Nucl. Phys.

- **nuclear matter ground state**
  - binding energy per nucleon at saturation density \(n_0\):
    \[ E(n_0)/A = -16.0 \pm 1.0 \text{ MeV} \]
  - incompressibility at \(n_0\):
    \[ K_\text{Q} = 200 - 260 \text{ MeV} \]
  - symmetry energy at \(n_0\):
    \[ S(n_0) = 30 \pm 4 \text{ MeV} \]
  - symmetry energy slope at \(n_0\):
    \[ L = 3n_0 \left( \frac{\partial S(n)}{\partial n_B} \right)_{n_B=n_0} = 20 - 115 \text{ MeV} \]

Grav. Phys.

- 2 NS+NS mergers
  - GW170817
  - GW190425

Love numbers and tidal polarizability are highly sensitive to EoS

LIGO and Virgo collaborations, PRL 119, 161101 (2017)

General Requirements

- causality
- thermodynamic consistency
- multicomponent character (n, p, e, ...)
- electric neutrality
- \(\beta\)-equilibrium
- realistic interaction between the constituents
Constraints on the EoS

HEP

- proton flow
  anisotropic expansion is caused by gradient of pressure, which gives an access to EoS
  "F. Baudrillol et al., Science 290, 1583 (2001)"

- hadron multiplicities
  hard core radii of hadrons control the rate of their production in therer

- nucleon
  hard core radius of of microscopic inte

Realistic EoS has to fulfil

all these constraints

Astro

- ~ 2 Msun
  PSR J0348-0432 : \( M = 2.01(4) M_\odot \)
  PSR J0740+6620 : \( M = 2.14 \pm 0.01 M_\odot \)

- NICER results
- NSs cooling

2 NS+NS mergers

GW170817
GW190425

- GW170817

LIGO and Virgo collaborations, PRL 119, 161101 (2017)

GW190425


- Love numbers and tidal polarizability are highly sensitive to EoS

General Requirements

- causality
- thermodynamic consistency
- multicomponent character (n, p, e, ...)
- electric neutrality
- \( \beta \)-equilibrium
- realistic interaction between the constituents
EoS with hard core repulsion

- Hard core reduces volume available for motion of particles by $V_{\text{excl}} = Nb$

  \[ V \rightarrow V - V_{\text{excl}} \Rightarrow p = nT = \frac{NT}{V} \rightarrow \frac{NT}{V - V_{\text{excl}}} = \frac{NT}{V - Nb} = \frac{nT}{1 - nb} \]

- Van der Waals EoS ($b = \text{const}$) in the Grand Canonical Ensemble

  \[
  \begin{cases}
    p = p(T, \mu) \\
    n = \frac{\partial p}{\partial \mu}
  \end{cases} \Rightarrow \quad p = T \int \frac{1}{k} \exp\left(\frac{\mu - pb - \sqrt{m^2 + k^2}}{T}\right) = p_{\text{id}}(T, \mu - pb)
  \]

- Excluded volume (per particle) depends on density ($b \neq \text{const}$)

**Low densities**

- $b \approx \frac{1}{2} \cdot \frac{4\pi}{3} (2R)^3 = 4v$

**High densities**

- $b \approx v$
The Induced Surface Tension (IST) EoS

- Quantities of the Boltzmann ideal gas
  \[ p = nT, \quad n = \sum_i n_i^{id}, \quad n_i^{id} = \frac{p_i^{id}}{T} \]

- Virial expansion for one particle species
  \[ \frac{p}{T} = n + a_2 n^2 + a_3 n^3 + a_4 n^3 + ... \]

  Ideal gas: \( a_2 = 0, \quad a_3 = 0, \quad a_4 = 0, \quad ... \)

  Hard spheres: \( a_2 = 4v, \quad a_3 = 10v^2, \quad a_4 = 18.365v^3, \quad ... \)

  \[ R.K.\text{Pathria, Statistical Mechanics, Pergamon Press, Oxford, 1972} \]

- Excluded volume at low density (\( v_i = \frac{4\pi}{3} R_i^3 \) and \( s_i = 4\pi R_i^2 \))
  \[ a_{ij}^{2} = \frac{1}{2} \cdot \frac{4\pi}{3} (R_i + R_j)^3 = \frac{1}{2} \cdot (v_i + s_i R_j + R_i s_j + v_j) \]

- Virial expansion for many particle species
  \[ \frac{p}{T} = \sum_i \frac{n_i^{id}}{T} - \sum_{i,j} a_{ij}^{2} n_i^{id} n_j^{id} + ... = \sum_i \frac{p_i^{id}}{T} \left( 1 - v_i \sum_j \frac{p_j^{id}}{T} - s_i \sum_j \frac{p_j^{id}}{T} R_j \right) + ... \]
EoS with induced surface tension (IST)

\[ p = \sum_i p_i^{id} \left( 1 - \frac{v_i}{T} \sum_j p_j^{id} - \frac{S_i}{T} \sum_j p_j^{id} R_j \right) + \ldots \]

- \( \Sigma \) is conjugated to \( s_i \) – induced surface tension (IST)

\[
\begin{align*}
p &= \sum_i p_i^{id} \left( 1 - \frac{pv_i}{T} - \frac{\Sigma s_i}{T} \right) \\
\Sigma &= \sum_i p_i^{id} \left( 1 - \frac{pv_i}{T} - \alpha \frac{\Sigma s_i}{T} \right) R_i
\end{align*}
\]

\( \alpha \) accounts for not uniqueness of extrapolation to high densities

- High density extrapolation (gives exponentials)

\[
\begin{align*}
p &= \sum_i p_i^{id} \exp \left( - \frac{p v_i + \Sigma s_i}{T} \right) \\
\Sigma &= \sum_i p_i^{id} \exp \left( - \frac{p v_i + \alpha \Sigma s_i}{T} \right) R_i \\
\rightarrow & \\
p &= \sum_i p_i^{id} (\mu_i - pv_i - \Sigma s_i) \\
\Sigma &= \sum_i p_i^{id} (\mu_i - pv_i - \alpha \Sigma s_i) R_i
\end{align*}
\]

VS, A. Ivanytskyi, K. Bugaev, I. Mishustin, NPA 924, 24 (2014)
Physical Origin of the Induced Surface Tension

- Hard core repulsion only in part is accounted by eigen volume
- The rest corresponds to surface tension and curvature tension
  Curvature tension can be accounted explicitly or implicitly
- Physical clusters tend to have spherical (in average) shape
Determination of $\alpha$

- One component EoS with IST and $\alpha > 1$

$$\begin{aligned}
    p &= p^{id} \exp\left(-\frac{pv + \Sigma s}{T}\right) \\
    \Sigma &= p^{id} \exp\left(-\frac{pv + \alpha \Sigma s}{T}\right) R
\end{aligned} \Rightarrow \begin{aligned}
    p &= p^{id} \exp\left(-\frac{pb}{T}\right) R \\
    \Sigma &= pR \exp\left(\frac{(1-\alpha)\Sigma s}{T}\right)
\end{aligned}$$


Excluded volume: $\frac{b}{V} = 1 + 3e^{\frac{(1-\alpha)\Sigma s}{T}} \rightarrow \begin{cases}
    4, & \Sigma \rightarrow 0 \\
    1, & \Sigma \rightarrow \infty
\end{cases}$

$\alpha > 1$ switches different regimes of excluded volume

- Virial expansion of one component EoS with IST

**Second virial coefficient:** $a_2 = 4V$ is reproduced always

**Third virial coefficient:** $a_3 = 10V^2 \Rightarrow \alpha = \frac{4}{3}$

$a_4$ is not reproduce

**Fourth virial coefficient:** $a_4 \simeq 18.365V^3 \Rightarrow \alpha \simeq 1.245$

$a_3$ – reproduced with 16% accuracy

$\alpha > 1.245$ reproduces two (3rd and 4th) virial coefficients
Effect of the IST

Thermodynamic parameters with and without IST
FIG. 3. Values of incompressibility constant $K_0$ and critical temperature $T_c$, which obey the proton flow constraint are located between the lines ABC and FED. The lines ABC and FED are, respectively, generated by the lower and upper branches of the proton flow constraint. The vertical lines AF, BE, and CD correspond to $K_0$ values 200 MeV, 250 MeV, and 315 MeV, respectively.


A. Ivanytskyi et al., PRC 97, 064905 (2018)
Hadron Resonance Gas Model

- Hadrons with masses \( \leq 2.5 \text{ GeV} \) (widths, strong decays, zero strangeness)
- 111 independent particle ratios measured at 14 energies (from 2.7 GeV to 200 GeV)
- \( 14 \times 4 \) local parameters (\( T, \mu_B, \mu_{I3}, \gamma_s \)) + 5 global parameters (hard core radii)

\[
\begin{align*}
\sqrt{s_{NN}} & = 6.3 \text{ GeV} \\
\sqrt{s_{NN}} & = 130 \text{ GeV}
\end{align*}
\]

\[
\begin{align*}
R_b & = 0.365 \text{ fm} ,
R_m & = 0.42 \text{ fm} ,
R_\pi & = 0.15 \text{ fm} ,
R_K & = 0.395 \text{ fm} ,
R_\Lambda & = 0.085 \text{ fm} \\
\text{Overall } \chi^2 / \text{dof} & \approx 1.038
\end{align*}
\]

Constraints on realistic EoS

IST EoS

Modelling of neutron stars

Conclusions

Hadron Resonance Gas at ALICE Energies

- 11 independent particle yields, 6 parameters (temperature + 5 hard core radii)
- Overall $\chi^2/dof \simeq 0.89$
- Freeze out temperature $T_{FO} = 148 \pm 7$ MeV

![Graph showing particle yields and deviations](chart.png)

$\sqrt{s_{NN}} = 2.76$ TeV

$\alpha = 1.25$

$R_K = 0.365$, $R_m = 0.42$ fm, $R_\pi = 0.15$ fm, $R_\bar{K} = 0.395$ fm, $R_\Xi = 0.085$ fm

The Induced Surface Tension (IST) EoS

\[
\begin{align*}
\rho = \sum_{\text{all particles}} p_{id}(T, \mu_i - pV_i - \Sigma S_i + U_{\text{at}} \pm U_{\text{sym}}) + p_{id}(\mu_e) - p_{\text{at}} + p_{\text{sym}} \\
\Sigma = \sum_{\text{all particles}} p_{id}(T, \mu_i - pV_i - \alpha \Sigma S_i + U_0) R_i
\end{align*}
\]

- \( p_{id} \) – pressure of the ideal gas for quantum statistics
- \( \Sigma \) – induced surface tension
- \( U_0, \alpha \) – model parameters

**Thermodynamic consistency of the model:**

\[
\frac{\partial p_{\text{int}}}{\partial n_{id}} = n_{id} \frac{\partial U(n_{id})}{\partial n_{id}}
\]

**Parametrization of the mean field potential:**

\[
U_{\text{at}} = -C_d^2 n_{id}^\kappa
\]


A. Ivanytskyi et al., PRC 97, 064905 (2018)
Mean field interaction for nuclear matter

- Thermodynamic consistency provides identity \( \frac{\partial p}{\partial \mu} = n \)

\[
p(\mu) = p^{id}(\mu - U(x)) + p_{int}(x), \quad p_{int}(x) = \int_0^x dx' x' \frac{\partial U(x')}{\partial x'}
\]

\( x \) – any quantity (density, asymmetry parameter, …)


Long range attraction (negative contribution to pressure)

\[
p_{at}(x) = -\frac{\kappa}{\kappa + 1} C_d^2 x^{1+\kappa}, \quad x = n_{id}^n + n_{id}^p
\]

\( \kappa < 1, \quad C_d^2 \) – fitted to flow constraint and properties of ground state

A. Ivanytskyi et al., PRC 97, 064905 (2018)

- Repulsion due to symmetry energy (positive contribution to pressure)

binding energy of stable nuclei: \( E_{sym} = a_{sym} \frac{(N-Z)^2}{A}, \quad a_{sym} = 30 \pm 4 \text{ MeV} \)

\[
p_{sym}(x) = \frac{A_{sym}^2 x^2}{[1+(B_{sym} x)^2]^2}, \quad x = n_{id}^n - n_{id}^p
\]

\( A_{sym}, B_{sym} \) – fitted to \( a_{sym} \) and slope of symmetry energy at ground state
EoS for NS interiors

\[
\begin{align*}
\{ & \quad \text{all particles} \\
\quad p &= \sum_i \left[ p_{id}(T, \mu_i - pV_i - \Sigma S_i + U_{at} \pm U_{sym}) + p_{id}(\mu_e) - p_{at} + p_{sym} \right] \\
\quad \Sigma &= \sum_i p_{id}(T, \mu_i - pV_i - \alpha \Sigma S_i + U_0)R_i
\end{align*}
\]

- **Degrees of freedom:** neutrons, protons, electrons
- **Quantum statistics** is accounted by construction of \( p_{id} \) and \( n_{id} \)
- **Realistic interaction:** HC repulsion, MF attraction, symmetry energy repulsion
- **Virial coefficients** of classic hard spheres are reproduced
- **Causal behaviour** up to densities where QGP is expected and **Flow constraint**

VS, G. Panotopoulos, I. Lopes, PRD 101, 063025 (2020)
M-R relation and compactness of NS

<table>
<thead>
<tr>
<th>Set</th>
<th>$R_{\text{nuc}}$ (fm)</th>
<th>$\alpha$</th>
<th>$\kappa$</th>
<th>$B^{\text{sym}}$ (fm$^3$)</th>
<th>$A^{\text{sym}}$ (MeV · fm$^3$)</th>
<th>$C_d^2$ (MeV · fm$^{3\kappa}$)</th>
<th>$U_0$ (MeV)</th>
<th>$K_0$ (MeV)</th>
<th>$J$ (MeV)</th>
<th>$L$ (MeV)</th>
<th>$M_{\text{max}}$ ($M_\odot$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A (magenta curve)</td>
<td>0.477</td>
<td>1.245</td>
<td>0.254</td>
<td>14.0</td>
<td>111.87</td>
<td>145.90</td>
<td>157.35</td>
<td>202.36</td>
<td>30.0</td>
<td>96.05</td>
<td>2.15</td>
</tr>
<tr>
<td>B (magenta curve)</td>
<td>0.463</td>
<td>1.245</td>
<td>0.25</td>
<td>16.0</td>
<td>138.30</td>
<td>146.30</td>
<td>162.87</td>
<td>201.02</td>
<td>30.0</td>
<td>93.19</td>
<td>2.08</td>
</tr>
</tbody>
</table>

VS, G. Panotopoulos, I. Lopes, PRD 101, 063025 (2020)
New formulation of the IST EoS with density dependent mass

**Total pressure and surface tension**

\[
\begin{align*}
p &= \sum_{\text{all particles}} [p_{id}(T, \mu_i - pV_i - \Sigma S_i \pm U_{sym}) + p_{id}(\mu_e) - p_{at} + p_{sym}] \\
\Sigma &= \sum_{\text{all particles}} p_{id}(T, \mu_i - pV_i - \alpha \Sigma S_i \pm U_{sym} + U_0) R_i
\end{align*}
\]

**Effective masses**

\[
\begin{align*}
m_p^p &= m_{nucl} - U_{at} \\
m_f^f &= m_{nucl} - U_{at}
\end{align*}
\]
Summary

IST approach was successfully applied to the description of
- compressible nuclear matter properties near the (3)CEP
  

- heavy-ion collision data between $\sqrt{S_{NN}} = 2.7$ GeV – 2.76 TeV
  

- NS properties at T=0
  

Advantages of the IST EoS

- can be easily generalized to any number of particle species
- can be formulated to finite temperatures $\Rightarrow$ proto-neutron stars
- provide a unified description of hadron and nuclear matter
- extract the surface tension of nuclear matter $\Rightarrow$ pasta phase
- IST approach can be applied to the description of the deconfinement phase transition

Future prospects

- will be available soon on CompOSE
- add hyperons and heavy leptons
- formulate a hybrid star model with QGP core
thank you for your attention!