Constraints on realistic EoS

IST EoS

Modelling of neutron stars

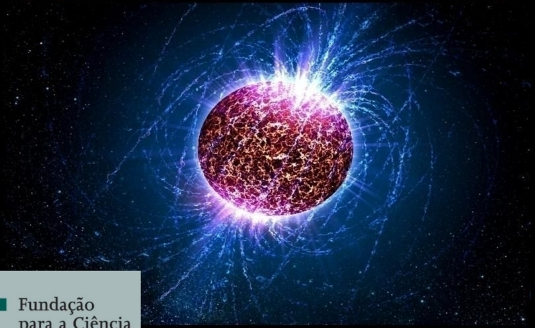
Conclusions

# The unified equation of state with induced surface tension consistent with astrophysical, gravitational, high- and low- energy nuclear physics data


Violetta Sagun

CFisUC, University of Coimbra, Portugal

In collaboration with Oleksii Ivanytskyi, Constança Providência

Fundação para a Ciência e a Tecnologia

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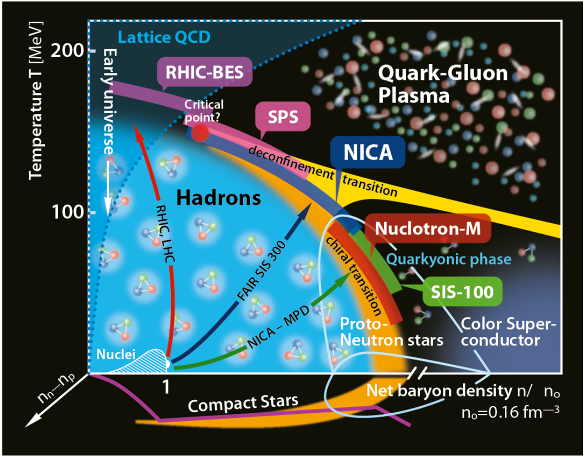
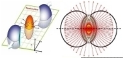
# Strongly Interacting Matter Phase Diagram



# Constraints on the EoS



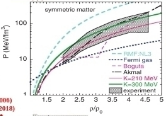
## HEP



- **proton flow**

anisotropic expansion is caused by gradient of pressure, which gives an access to EoS

P. Danielewicz et al., Science 298, 1593 (2002)

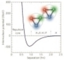



- **hadron multiplicities**

hard core radii of hadrons control the rate of their production in thermal medium: R= 0.3 - 0.5 fm

A. Andronic et al., Nucl. Phys. A 772, 167 (2006)
K. A. Bugaev et al., Nucl. Phys. A 970, 133 (2018)

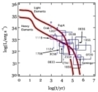
- **nucleon-nucleon scattering**



hard core radius of nucleons extracted as a parameter of microscopic interaction potential: R = 0.5 fm

M. Naghdi, Phys. Part. Nucl. 5, 924 (2014)

## Astro



- **~ 2 Msun**

PSR J0348-0432 : $M = 2.01(4) M_\odot$

J. Antoniadis et al., Science, 340, 448 (2013)

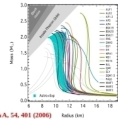
PSR J0740+6620: $M = 2.14^{+0.20}_{-0.18} M_\odot$

H. T. Cromartie et al. Nature, 996, 822 (2019)

- **NICER results**
- **NSs cooling**
- **observations of pulsars**

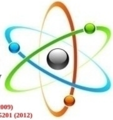
⇩

**M-R relation**




F. Ozel, P. Freire, A&A, 54, 401 (2006)

## Nucl. Phys.



- **nuclear matter ground state**

- binding energy per nucleon at saturation density $n_0$:

$n_0 = 0.16 \pm 0.01 \text{ fm}^{-3}$, $E(n_0)/A = -16.0 \pm 1.0$ MeV

- incompressibility at $n_0$:

$K_0$= 200 - 260 MeV

E. Khan, Phys.Rev. C, 80, 011307 (2009)
M. Dutra et al., Phys.Rev. C, 85, 035201 (2012)

- symmetry energy at $n_0$:

$S(n_0) \equiv J = 30 \pm 4$ MeV

Zhang, Z., Chen, L.-W., Phys. Lett. B, 726, 234 (2013)

- symmetry energy slope at $n_0$: $L \equiv 3n_0 \left(\frac{\partial S(n_B)}{\partial n_B}\right)_{n_B=n_0}$ = 20 -115 MeV

## Grav. Phys.

**2 NS+NS mergers**

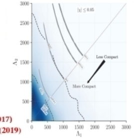
**GW170817**

**GW190425**

⇩

Love numbers and tidal polarizability are highly sensitive to EoS

LIGO and Virgo collaborations, PRL 119, 161101 (2017)
LIGO and Virgo collaborations, arXiv:2001.01761 (2019)

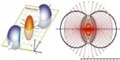



## General Requirements

- causality
- thermodynamic consistency
- multicomponent character (n, p, e, ...)
- electric neutrality
- β-equilibrium
- realistic interaction between the constituents

# Constraints on the EoS

## HEP



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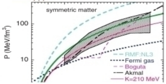


- **hadron multiplicities**

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- **nucleon**

hard core radius of of microscopic inte

## Realistic EoS has to fulfil all these constraints

- **nuclear**

- binding energy per nucleon at saturation density $n_0$: $n_0 = 0.16 \pm 0.01\ \mathrm{fm}^{-3}$, $E(n_0)/A = -16.0 \pm 1.0$ MeV
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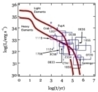


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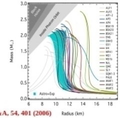


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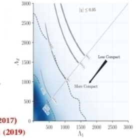


## General Requirements

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# EoS with hard core repulsion

- Hard core reduces volume available for motion of particles by $V_{excl} = Nb$

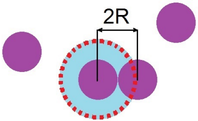
$$V \to V - V_{excl} \;\Rightarrow\; \overbrace{p = nT}^{ideal\ \ gas\ \ EoS} = \frac{NT}{V} \to \frac{NT}{V - V_{excl}} = \frac{NT}{V - Nb} = \overbrace{\frac{nT}{1 - nb}}^{Van\ \ der\ \ Waals\ \ EoS}$$

- Van der Waals EoS ($b = const$) in the Grand Canonical Ensemble

$$\begin{cases} p = p(T,\mu) \\ n = \frac{\partial p}{\partial \mu} \end{cases} \;\Rightarrow\; p = T \int_{\vec{k}} \exp\left(\frac{\mu - pb - \sqrt{m^2 + k^2}}{T}\right) = p_{id}(T, \mu - pb)$$

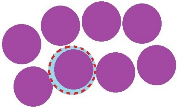
- Excluded volume (per particle) depends on density ($b \neq const$)

**Low densities**

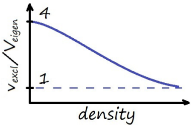



$b \simeq \frac{1}{2} \cdot \frac{4\pi}{3}(2R)^3 = 4v$

**High densities**



$b \simeq v$



# The Induced Surface Tension (IST) EoS



- Quantitites of the Boltzmann ideal gas

$$p = nT, \quad n = \sum_i n_i^{id}, \quad n_i^{id} = \frac{p_i^{id}}{T}$$

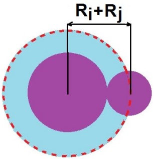
- Virial expansion for one particle species

$$\frac{p}{T} = n + a_2 n^2 + a_3 n^3 + a_4 n^3 + \ldots$$

Ideal gas: $a_2 = 0, \; a_3 = 0, \; a_4 = 0, \; \ldots$

Hard spheres: $a_2 = 4v, \; a_3 = 10v^2, \; a_4 = 18.365v^3, \; \ldots$

R.K.Pathria, Statistical Mechanics, Pergamon Press, Oxford, 1972




- Excluded volume at low density ($v_i = \frac{4\pi}{3} R_i^3$ and $s_i = 4\pi R_i^2$)

$$a_2^{ij} = \frac{1}{2} \cdot \frac{4\pi}{3} (R_i + R_j)^3 = \frac{1}{2} \cdot (v_i + s_i R_j + R_i s_j + v_j)$$

- Virial expansion for many particle species

$$\frac{p}{T} = \overbrace{\sum_i n_i^{id}}^{\simeq n} - \overbrace{\sum_{i,j} a_2^{ij} n_i^{id} n_j^{id}}^{\simeq a_2 n^2} + \ldots = \sum_i \frac{p_i^{id}}{T} \Bigg( 1 - \overbrace{v_i \sum_j \frac{p_j^{id}}{T}}^{bulk \;\; term} - \overbrace{s_i \sum_j \frac{p_j^{id}}{T} R_j}^{surface \;\; term} \Bigg) + \ldots$$

# EoS with induced surface tension (IST)

$$p = \sum_i p_i^{id}\Big(1 - \overbrace{\underbrace{\frac{v_i}{T}\sum_j p_j^{id}}_{p+\mathcal{O}(n^2)}}^{bulk\ \ term} - \overbrace{\underbrace{\frac{s_i}{T}\sum_j p_j^{id} R_j}_{\Sigma+\mathcal{O}(n^2)}}^{surface\ \ term}\Big) + \ldots$$

- $\Sigma$ is conjugated to $s_i$ – **induced surface tension (IST)**

$$\begin{cases} p = \sum_i p_i^{id}\left(1 - \frac{pv_i}{T} - \frac{\Sigma s_i}{T}\right) \\ \Sigma = \sum_i p_i^{id}\left(1 - \frac{pv_i}{T} - \frac{\alpha \Sigma s_i}{T}\right) R_i \end{cases}$$

$\alpha$ accounts for not uniqueness of extrapolation to high densities

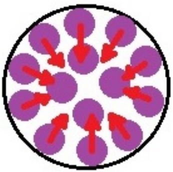
- High density extrapolation (gives exponentials)

$$\begin{cases} p = \sum_i p_i^{id} \exp\left(-\frac{pv_i + \Sigma s_i}{T}\right) \\ \Sigma = \sum_i p_i^{id} \exp\left(-\frac{pv_i + \alpha\Sigma s_i}{T}\right) R_i \end{cases} \rightarrow \begin{cases} p = \sum_i p_i^{id}(\mu_i - pv_i - \Sigma s_i) \\ \Sigma = \sum_i p_i^{id}(\mu_i - pv_i - \alpha\Sigma s_i) R_i \end{cases}$$

VS, A. Ivanytskyi, K. Bugaev, I. Mishustin, NPA 924, 24 (2014)

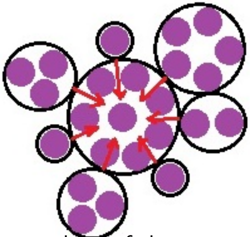
# Physical Origin of the Induced Surface Tension

Vacuum



attraction of constituents
⇒ **eigen surface tension**

Medium



repulsion of clusters
⇒ **induced surface tension**

- Hard core repulsion only in part is accounted by eigen volume
- The rest corresponds to surface tension and curvature tension
  Curvature tension can be accounted explicitly or implicitly
- Physical clusters tend to have spherical (in average) shape

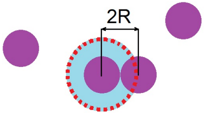
# Determination of $\alpha$

- One component EoS with IST and $\alpha > 1$

$$\begin{cases} p = p^{id} \exp\left(-\frac{pv+\Sigma s}{T}\right) \\ \Sigma = p^{id} \exp\left(-\frac{pv+\alpha\Sigma s}{T}\right) R \end{cases} \Rightarrow \begin{cases} p = p^{id} \exp\left(-\frac{pb}{T}\right) R \\ \Sigma = pR \exp\left(\frac{(1-\alpha)\Sigma s}{T}\right) \end{cases}$$

VVS, A. I. Ivanytskyi, K. A. Bugaev, I. N. Mishustin, Nucl. Phys. A, 924, 24 (2014)

**Excluded volume:** $\frac{b}{v} = 1 + 3e^{\frac{(1-\alpha)\Sigma s}{T}} \rightarrow \begin{cases} 4, & \Sigma \rightarrow 0 \\ 1, & \Sigma \rightarrow \infty \end{cases}$




$\alpha > 1$ **switches different regimes of excluded volume**

- Virial expansion of one component EoS with IST

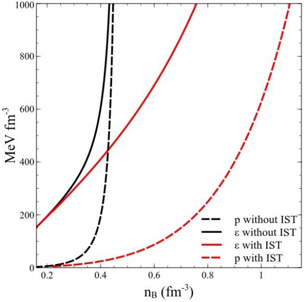
**Second virial coefficient:** $a_2 = 4V$ is reproduced always

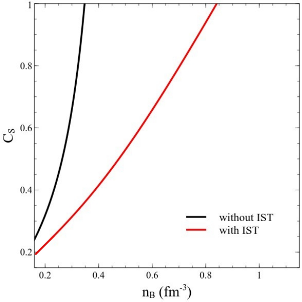
**Third virial coefficient:** $a_3 = 10V^2 \Rightarrow \alpha = \frac{4}{3}$
$a_4$ is not reproduce

**Fourth virial coefficient:** $a_4 \simeq 18.365V^3 \Rightarrow \alpha \simeq 1.245$
$a_3$ – reproduced with 16% accuracy

$\alpha > 1.245$ **reproduces two (3rd and 4th) virial coefficients**

Thermodynamic parameters with and without IST



# Nuclear Matter Properties Near the (3)CEP



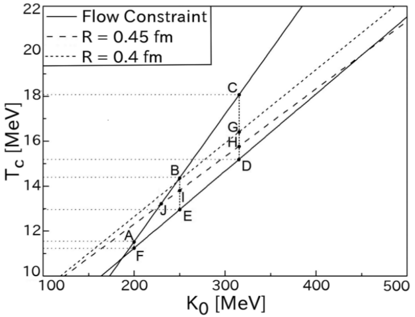


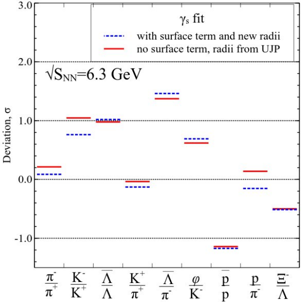

FIG. 3. Values of incompressibility constant $K_0$ and critical temperature $T_C$, which obey the proton flow constraint are located between the lines ABC and FED. The lines ABC and FED are, respectively, generated by the lower and upper branches of the proton flow constraint. The vertical lines AF, BE, and CD correspond to $K_0$ values 200 MeV, 250 MeV, and 315 MeV, respectively.

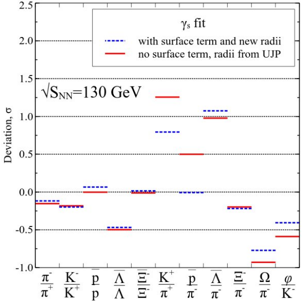
VVS, et al., Nucl. Phys. A, 924, 24 (2014)

A. Ivanytskyi et al., PRC 97, 064905 (2018)

# Hadron Resonance Gas Model

- Hadrons with masses $\leq 2.5$ GeV (widths, strong decays, zero strangeness)
- 111 independent particle ratios measured at 14 energies (from 2.7 GeV to 200 GeV)
- $14 \times 4$ local parameters $(T, \mu_B, \mu_{I3}, \gamma_s)$ + 5 global parameters (hard core radii)



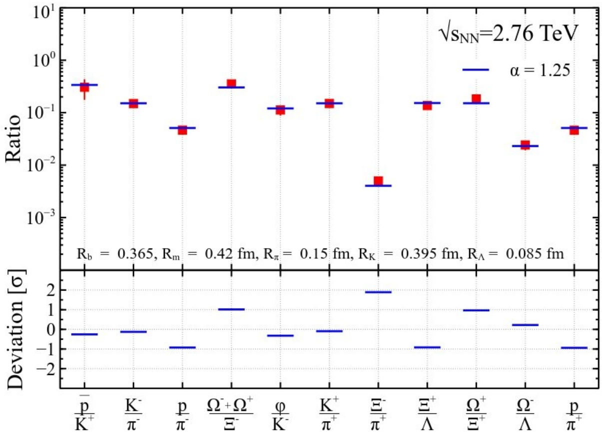

$R_b = 0.365\ fm,\ R_m = 0.42\ fm,\ R_\pi = 0.15\ fm,\ R_K = 0.395\ fm,\ R_\Lambda = 0.085\ fm$

$Overall\ \chi^2/dof \simeq 1.038$

K.A. Bugaev, et al., NPA 970, p. 133-155, (2018)
VVS, Ukr. J. Phys. 59, 755 (2014)

# Hadron Resonance Gas at ALICE Energies

- 11 independent particle yields, 6 parameters (temperature + 5 hard core radii)
- Overal $\chi^2/dof \simeq 0.89$
- Freeze out temperature $T_{FO} = 148 \pm 7$ $MeV$




K.A. Bugaev, et al., NPA 970, p. 133-155, (2018)

# The Induced Surface Tension (IST) EoS



$$\begin{cases} p = \sum_{i}^{all\ \ particles} \left[ p_{id}(T, \mu_i - pV_i - \Sigma S_i + U_{at} \pm U_{sym}) + p_{id}(\mu_e) - p_{at} + p_{sym} \right] \\ \Sigma = \sum_{i}^{all\ \ particles} p_{id}(T, \mu_i - pV_i - \alpha \Sigma S_i + U_0) R_i \end{cases}$$

$p_{id}$ – pressure of the ideal gas for quantum statistics
$\Sigma$ – induced surface tension
$U_0, \alpha$ – model parameters

$$Thermodynamic\ \ consistency\ \ of\ \ the\ \ model: \quad \frac{\partial p_{int}}{\partial n_{id}} = n_{id} \frac{\partial U(n_{id})}{\partial n_{id}}$$

$$Parametrization\ \ of\ \ the\ \ mean\ \ field\ \ potential: U_{at} = -C_d^2 n_{id}^{\kappa}$$

VVS, et al., Nucl. Phys. A, 924, 24 (2014)
A. Ivanytskyi et al., PRC 97, 064905 (2018)

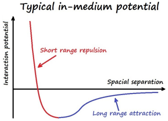
# Mean field interaction for nuclear matter

- Thermodynamic consistency provides identity $\frac{\partial p}{\partial \mu} = n$

$$p(\mu) = p^{id}(\mu - U(x)) + p_{int}(x), \quad p_{int}(x) = \int_0^x dx' x' \frac{\partial U(x')}{\partial x'}$$

$x$ – any quantity (density, asymmetry parameter, ...)

K. A. Bugaev and M. I. Gorenstein, Z. Phys. C 43, 261 (1989)
D.H. Rischke, et al. Z. Phys. C 51, 485 (1991)




Long range attraction (negative contribution to pressure)

$$p_{at}(x) = -\frac{\kappa}{\kappa+1} C_d^2 x^{1+\kappa}, \quad x = n_n^{id} + n_p^{id}$$

$\kappa < 1, \; C_d^2$ – fitted to flow constraint and properties of ground state

A. Ivanytskyi et al., PRC 97, 064905 (2018)

- Repulsion due to symmetry energy (positive contribution to pressure)

**binding energy of stable nuclei:** $E_{sym} = a_{sym} \frac{(N-Z)^2}{A}, \; a_{sym} = 30 \pm 4 \; MeV$

$$p_{sym}(x) = \frac{A^{sym} x^2}{[1+(B^{sym} x)^2]^2}, \quad x = n_n^{id} - n_p^{id}$$

$A^{sym}, \; B^{sym}$ – fitted to $a_{sym}$ and slope of symmetry energy at ground state

Constraints on realistic EoS

**IST EoS**

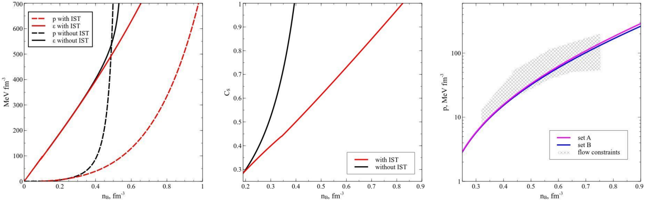
Modelling of neutron stars

Conclusions

# EoS for NS interiors

$$\begin{cases} p = \sum_i^{all\ \ particles} [p_{id}(T,\mu_i - pV_i - \Sigma S_i + U_{at} \pm U_{sym}) + p_{id}(\mu_e) - p_{at} + p_{sym}] \\ \Sigma = \sum_i^{all\ \ particles} p_{id}(T,\mu_i - pV_i - \alpha\Sigma S_i + U_0)R_i \end{cases}$$

- **Degrees of freedom:** neutrons, protons, electrons
- **Quantum statistics** is accounted by construction of $p_{id}$ and $n_{id}$
- **Realistic interaction**: HC repulsion, MF attraction, symmetry energy repulsion
- **Virial coefficients** of classic hard spheres are reproduced
- **Causal behaviour** up to densities where QGP is expected and **Flow constraint**

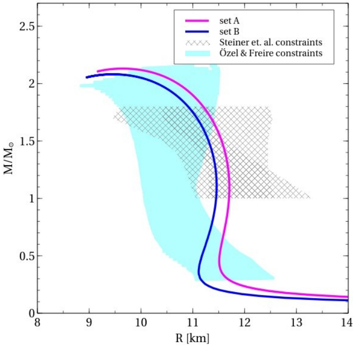


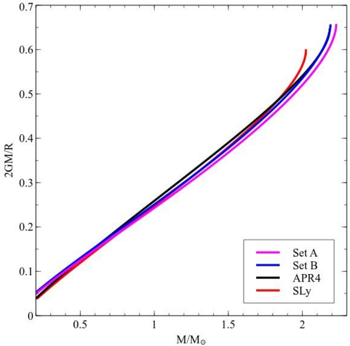


VS, I. Lopes, A. Ivanytskyi, APJ, 871, 157 (2019)
VS, G. Panotopoulos, I. Lopes, PRD 101, 063025 (2020)

# M-R relation and compactness of NS






| Set | $R_{nucl}$ $(fm)$ | $\alpha$ – | $\kappa$ – | $B^{sym}$ $(fm^3)$ | $A^{sym}$ $(MeV \cdot fm^3)$ | $C_d^2$ $(MeV \cdot fm^{3\kappa})$ | $U_0$ (MeV) | $K_0$ (MeV) | $J$ (MeV) | $L$ (MeV) | $M_{max}$ $(M_\odot)$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| A (magenta curve) | 0.477 | 1.245 | 0.254 | 14.0 | 111.87 | 145.90 | 157.35 | 202.36 | 30.0 | 96.05 | 2.15 |
| B (magenta curve) | 0.463 | 1.245 | 0.25 | 16.0 | 138.30 | 146.30 | 162.87 | 201.02 | 30.0 | 93.19 | 2.08 |

VS, I. Lopes, A. Ivanytskyi, APJ 871, 157 (2019)
VS, G. Panotopoulos, I. Lopes, PRD 101, 063025 (2020)

# New formulation of the IST EoS with density dependent mass

**Total pressure and surface tension**

$$\begin{cases} p = \sum_{i}^{all\ \ particles} [p_{id}(T, \mu_i - pV_i - \Sigma S_i \pm U_{sym}) + p_{id}(\mu_e) - p_{at} + p_{sym}] \\ \Sigma = \sum_{i}^{all\ \ particles} p_{id}(T, \mu_i - pV_i - \alpha\Sigma S_i \pm U_{sym} + U_0)R_i \end{cases}$$

**Effective masses**

$$\begin{cases} m_i^p = m_{nucl} - U_{at} \\ m_i^\Sigma = m_{nucl} - U_{at} \end{cases}$$

VS, O. Ivanytskyi, A. Maselli, C. Providência, In preparation 2021

# Summary



IST approach was successfully applied to the description of

- compressible nuclear matter properties near the (3)CEP

  VVS, et al., Nucl. Phys. A, 924, 24 (2014)
- heavy-ion collision data between $\sqrt{S_{NN}} = 2.7$ GeV – 2.76 TeV

  K.A. Bugaev, et al., NPA 970, p. 133-155, (2018)
- NS properties at T=0

  VVS, I. Lopes, A. Ivanytskyi, APJ, 871, 157 (2019)

## Advantages of the IST EoS

- can be easily generalized to any number of particle species
- can be formulated to finite temperatures ⇒ proto-neutron stars
- provide a unified description of hadron and nuclear matter
- extract the surface tension of nuclear matter ⇒ pasta phase
- IST approach can be applied to the description of the deconfinement phase transition

## Future prospects

- will be available soon on CompOSE
- add hyperons and heavy leptons
- formulate a hybrid star model with QGP core


thank you for your attention!