Special point and onset of deconfinement in the M-R diagram of neutron stars.

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   - What is the special point?

2 Properties of the special point
   - Hadronic EoS invariance
   - Mass relation
   - Invariance w.r.t. the phase transition construction

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Introduction

What is the special point?
The TOV equation
The TOV equation

$$\frac{\partial P(r)}{\partial r} = -\frac{\epsilon(r)M(r)\left(1 + \frac{P(r)}{\epsilon(r)}\right)\left(1 + \frac{4\pi r^3 P(r)}{M(r)}\right)}{r^2 \left(1 - \frac{2M(r)}{r}\right)},$$
The TOV equation

\[
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\]
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Fig. 1. Mass-radius diagram for a star made of ordinary matter (thick line) and purely quark stars (thin lines). The numbers at the lines indicate the parameter \(B\).

\cite{1} Yudin et al., Astron. Lett. 40 (2014), 201
The TOV equation

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\]

Fig. 1. Mass–radius diagram for a star made of ordinary matter (thick line) and purely quark stars (thin lines). The numbers at the lines indicate the parameter \(B\).

Fig. 2. Mass–radius diagram of hybrid stars for various values of the parameter \(B\).

\(^{1}\)Yudin et al., Astron. Lett. 40 (2014), 201
Properties of the special point

Hadronic EoS invariance
The constant–speed–of–sound (CSS) model:

2Alford, Han, Prakash, Phys. Rev. D 88 (2013) no.8, 083013
The constant–speed–of–sound (CSS) model:

- dimensionless baryochemical potential

\[ \hat{\mu}_B = \frac{\mu_B}{\mu_{\text{scale}}} = \left( \frac{p+B}{A} \right)^{1/(1+\beta)}, \]

- pressure

\[ p(\mu_B) = A\hat{\mu}_B^{1+\beta} - B, \]

- baryon density

\[ n_B(\mu_B) = (1 + \beta) \frac{A}{\mu_{\text{scale}}} \hat{\mu}_B^\beta, \]

- energy density

\[ \epsilon = B + \beta A\hat{\mu}_B^{1+\beta}, \]

- \( p(\epsilon) \) relation: \( \epsilon = \beta p + (1 + \beta)B. \)

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\(^3\text{Cierniak, Blaschke, Eur.Phys.J.ST 229 (2020) no.22-23, 3663-3673}\)
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Properties of the special point

Mass relation
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Conclusions and outlook

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$M_{\odot}$

\[ R \text{ [km]} \]

\[ M_{\odot} \text{ [M_{\odot}]} \]

\[ c_2^i = 0.7 \]

GW 170817 excluded (Rezzolla et al.)

PSR J0740+6620

GW 170817 excluded (Bauswein et al.)

GW 170817 excluded (Annala et al.)

\[ M_{\max} - M_{\text{SP}} [M_{\odot}] \]

\[ M_{\text{onset}} [M_{\odot}] \]

\[ c_2^i = 1, A = 3 \text{ MeV/fm}^3 \]

\[ c_2^i = 1, A = 2.5 \text{ MeV/fm}^3 \]

\[ c_2^i = 1, A = 2 \text{ MeV/fm}^3 \]

\[ c_2^i = 0.9, A = 2 \text{ MeV/fm}^3 \]

\[ c_2^i = 0.9, A = 1.5 \text{ MeV/fm}^3 \]

\[ c_2^i = 0.9, A = 1 \text{ MeV/fm}^3 \]

\[ c_2^i = 0.9, A = 0.8 \text{ MeV/fm}^3 \]

\[ c_2^i = 0.8, A = 0.5 \text{ MeV/fm}^3 \]

\[ c_2^i = 0.8, A = 0.2 \text{ MeV/fm}^3 \]

\[ c_2^i = 0.7, A = 0.1 \text{ MeV/fm}^3 \]

\[ c_2^i = 0.7, A = 0.05 \text{ MeV/fm}^3 \]

\[ c_2^i = 0.3, A = 0.12 \text{ MeV/fm}^3 \]

\[ c_2^i = 0.3, A = 0.08 \text{ MeV/fm}^3 \]

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Mateusz Cierniak
Special Point
\[ M_{\text{max}} = M_{\text{SP}} + 0.208 M_{\odot} - 0.104 M_{\text{onset}} \]

\(^3\)Cierniak, Blaschke, Eur.Phys.J.ST **229** (2020) no.22-23, 3663-3673
\[ M_{\text{max}} = M_{\text{SP}} + 0.208 M_\odot - 0.104 M_{\text{onset}} \]

\(^3\text{Cierniak, Blaschke, Eur.Phys.J.ST 229 (2020) no.22-23, 3663-3673}\)
Properties of the special point

Invariance w.r.t. the phase transition construction
The mixed phase parabolic ansatz:

\[ \text{4Abgaryan, et al., Universe 4 (2018), 94} \]
The mixed phase parabolic ansatz:

\[ P_M(\mu) = \alpha_2(\mu - \mu_c)^2 + \alpha_1(\mu - \mu_c) + (1 + \Delta_P)P_c, \]

Gibbs condition for phase equilibrium:

\[
P_H(\mu_H) = P_M(\mu_H), \quad P_Q(\mu_Q) = P_M(\mu_Q),
\]

\[
\frac{\partial^k}{\partial \mu^k} P_H(\mu_H) = \frac{\partial^k}{\partial \mu^k} P_M(\mu_H),
\]

\[
\frac{\partial^k}{\partial \mu^k} P_Q(\mu_Q) = \frac{\partial^k}{\partial \mu^k} P_M(\mu_Q).
\]

Derived parameters \((k = 1)\):

\[
\alpha_1 = \frac{-2\kappa_1 + \kappa_2(\mu_c - \mu_H)}{2(\mu_c - \mu_Q)(\mu_H - \mu_Q)},
\]

\[
\alpha_2 = \frac{-2\kappa_1 + \kappa_2(\mu_c - \mu_Q)}{2(\mu_c - \mu_H)(\mu_H - \mu_Q)},
\]

\[
\kappa_1 = n_Q(\mu_c - \mu_Q) - n_H(\mu_c - \mu_H) + P_Q - P_H,
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\kappa_2 = n_Q - n_H.
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\(^4\) Abgaryan, et al., Universe 4 (2018), 94
The mixed phase parabolic ansatz:

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Gibbs condition for phase equilibrium:

\[
\begin{align*}
P_H(\mu_H) &= P_M(\mu_H), \\
P_Q(\mu_Q) &= P_M(\mu_Q), \\
\frac{\partial^k}{\partial \mu^k} P_H(\mu_H) &= \frac{\partial^k}{\partial \mu^k} P_M(\mu_H), \\
\frac{\partial^k}{\partial \mu^k} P_Q(\mu_Q) &= \frac{\partial^k}{\partial \mu^k} P_M(\mu_Q).
\end{align*}
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Derived parameters \( (k = 1) \):

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\begin{align*}
\alpha_1 &= \frac{-2\kappa_1 + \kappa_2(\mu_c - \mu_H)}{2(\mu_c - \mu_Q)(\mu_H - \mu_Q)}, \\
\alpha_2 &= \frac{-2\kappa_1 + \kappa_2(\mu_c - \mu_Q)}{2(\mu_c - \mu_H)(\mu_H - \mu_Q)}, \\
\kappa_1 &= n_Q(\mu_c - \mu_Q) - n_H(\mu_c - \mu_H) + P_Q - P_H, \\
\kappa_2 &= n_Q - n_H.
\end{align*}
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\[ \kappa_2 = n_Q - n_H. \]

\[^4\text{Abgaryan, et al., Universe 4 (2018), 94}\]
Invariance w.r.t. Maxwell $\rightarrow$ mixed phase construction (pasta phases)

Cierniak, Blaschke, Astron. Nachr. (2021), arXiV: 2106.06986
Invariance w.r.t. Maxwell → interpolation construction (soft - stiff transition)

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5Cierniak, Blaschke, Astron. Nachr. (2021), arXiV: 2106.06986
Special point locations for constant sound speed

![Image of graph showing special point locations for constant sound speed]

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Introduction
Properties of the special point
Conclusions and outlook

Hadronic EoS invariance
Mass relation
Invariance w.r.t. the phase transition construction

<table>
<thead>
<tr>
<th>(c_s^2)</th>
<th>(M_{SP} [M_\odot])</th>
<th>(R_{min} [\text{km}])</th>
<th>(R_{max} [\text{km}])</th>
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<td>1.00</td>
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<td>9.95</td>
<td>14.84</td>
</tr>
</tbody>
</table>

The values of \(c_s^2\), largest possible \(M_{SP}\) and the radii range \((R_{min} - R_{max})\) of a 2 \(M_\odot\) hybrid star.

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\(^5\) Cierniak, Blaschke, Astron. Nachr. (2021), arXiV: 2106.06986
The values of $c^2_s$, largest possible $M_{SP}$ and the radii range ($R_{min} - R_{max}$) of a 2 $M_{\odot}$ hybrid star. \textbf{Bold red rows correspond to the nlNJL fit from [6].}
Conclusions:

- A special point is a feature of neutron star mass radius relations unique to hybrid stars.
- The special point does not depend on the choice of the hadronic equation of state, phase transition onset density or the type of transition, thus
- The novel NICER PSR J0740+6620 measurement suggests a stiff high density equation of state, in tension with hyperon hadron models. The existence of hybrid stars solves this problem and hints at a possible hybrid neutron star nature of the 2.6 $M_\odot$ constituent of GW190814.

Outlook:

- Special point of rotating neutron stars?
- Relation between the special point and tidal deformability.
- The special point at a finite temperature.