Special point and onset of deconfinement in the M-R diagram of neutron stars.

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Overview

Introduction

What is the special point?

The TOV equation

The TOV equation

$$\frac{\partial P(r)}{\partial r} = -\frac{\epsilon(r)M(r)\left(1+\frac{P(r)}{\epsilon(r)}\right)\left(1+\frac{4\pi r^3 P(r)}{M(r)}\right)}{r^2\left(1-\frac{2M(r)}{r}\right)},$$

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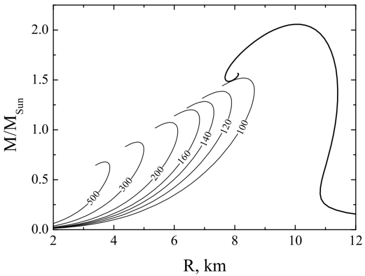

Fig. 1. Mass–radius diagram for a star made of ordinary matter (thick line) and purely quark stars (thin lines). The numbers at the lines indicate the parameter B.

[1]Yudin et al., Astron. Lett. **40** (2014), 201

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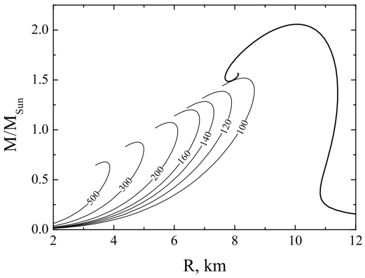

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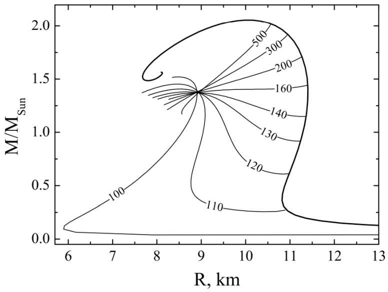

Fig. 2. Mass–radius diagram of hybrid stars for various values of the parameter B

[1] Yudin et al., Astron. Lett. **40** (2014), 201

Properties of the special point

Hadronic EoS invariance

The constant–speed–of–sound (CSS) model:

[2] Alford, Han, Prakash, Phys. Rev. **D 88** (2013) no.8, 083013

The constant–speed–of–sound (CSS) model:

– dimensionless baryochemical potential

$$\hat{\mu}_B = \frac{\mu_B}{\mu_{scale}} = \left(\frac{p+B}{A}\right)^{1/(1+\beta)},$$

– pressure

$$p(\mu_B) = A\hat{\mu}_B^{1+\beta} - B,$$

– baryon density

$$n_B(\mu_B) = (1+\beta)\frac{A}{\mu_{scale}}\hat{\mu}_B^{\beta},$$

– energy density

$$\epsilon = B + \beta A\hat{\mu}_B^{1+\beta},$$

– $p(\epsilon)$ relation: $\epsilon = \beta p + (1+\beta)B$.

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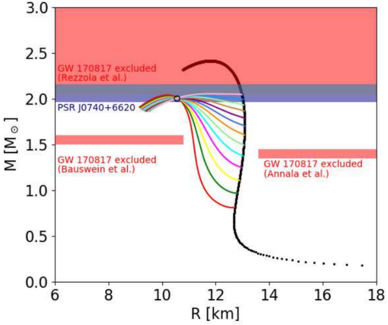

[3] Cierniak, Blaschke, Eur.Phys.J.ST **229** (2020) no.22-23, 3663-3673

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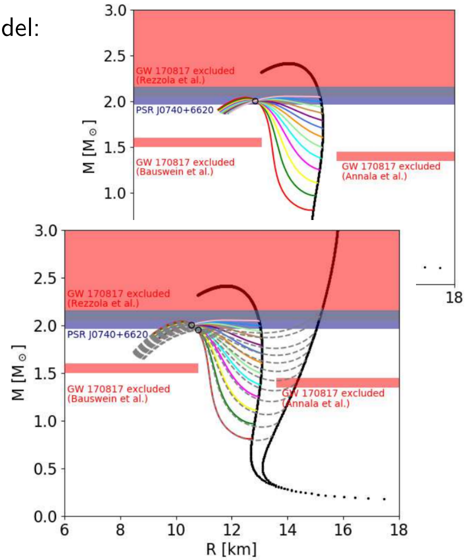

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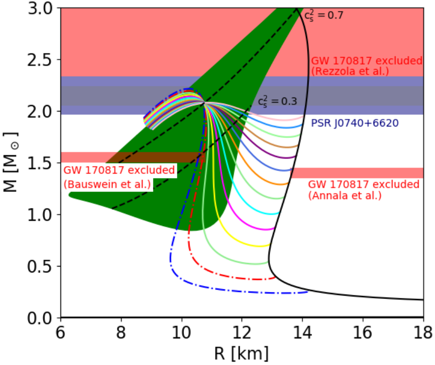
Properties of the special point

Mass relation



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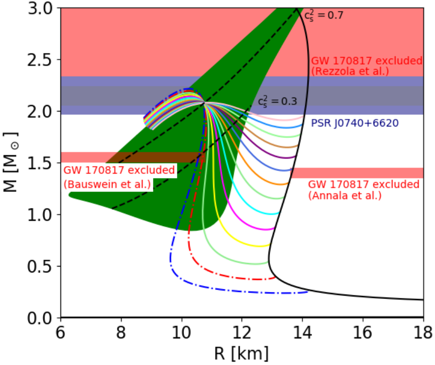

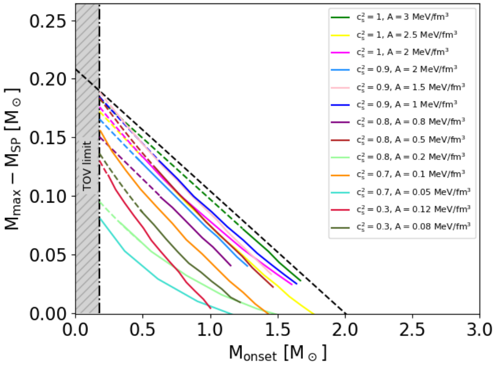

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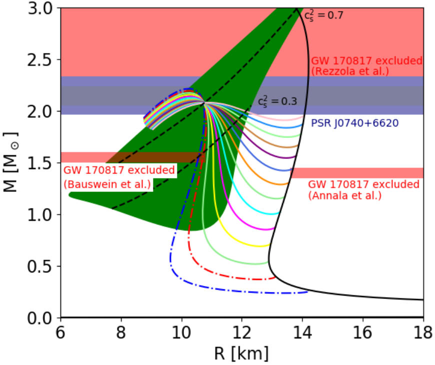

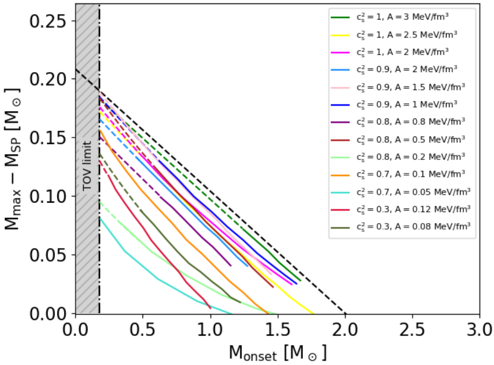

$$M_{max} = M_{SP} + 0.208M_\odot - 0.104M_{onset}$$

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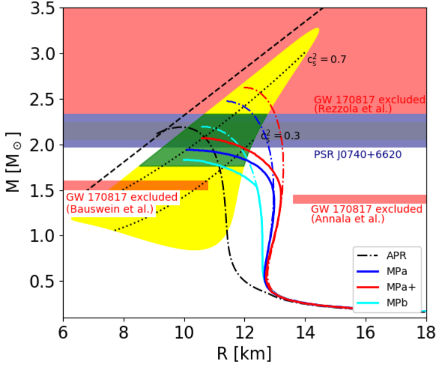

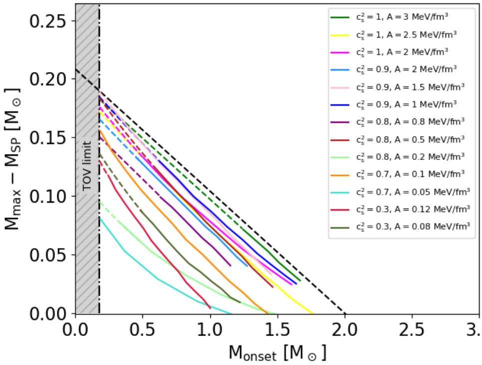

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Properties of the special point

Invariance w.r.t. the phase transition construction

The mixed phase parabolic ansatz:

[4] Abgaryan, et al., Universe **4** (2018), 94

The mixed phase parabolic ansatz:

$$P_M(\mu) \;=\; \alpha_2(\mu-\mu_c)^2 \;+\; \alpha_1(\mu-\mu_c) \;+\; (1+\Delta_P)P_c,$$

Gibbs condition for phase equilibrium:

$$P_H(\mu_H) = P_M(\mu_H)\ ,$$
$$P_Q(\mu_Q) = P_M(\mu_Q)\ ,$$
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Derived parameters (k = 1):

$$\alpha_1 = \frac{-2\kappa_1+\kappa_2(\mu_c-\mu_H)}{2(\mu_c-\mu_Q)(\mu_H-\mu_Q)},$$

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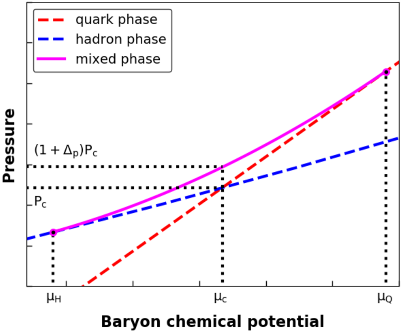

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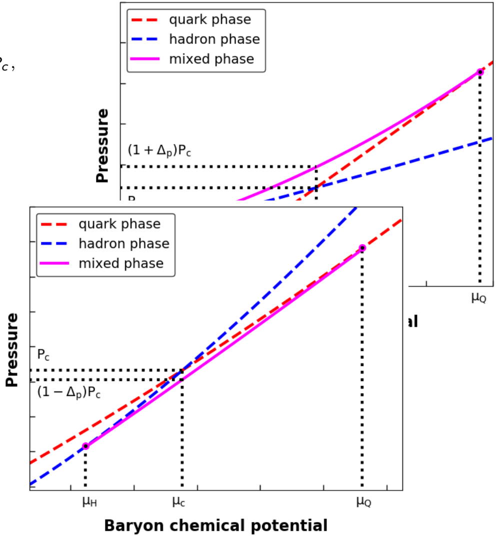

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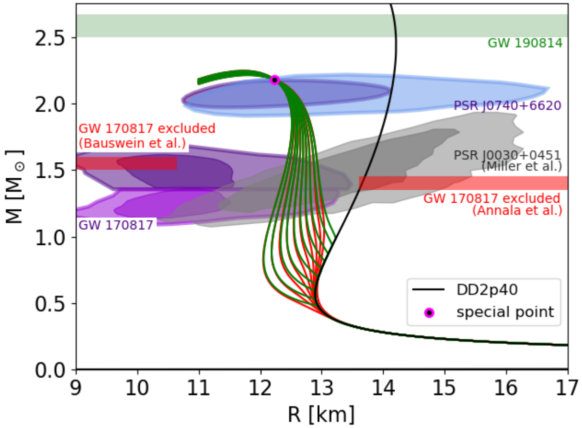
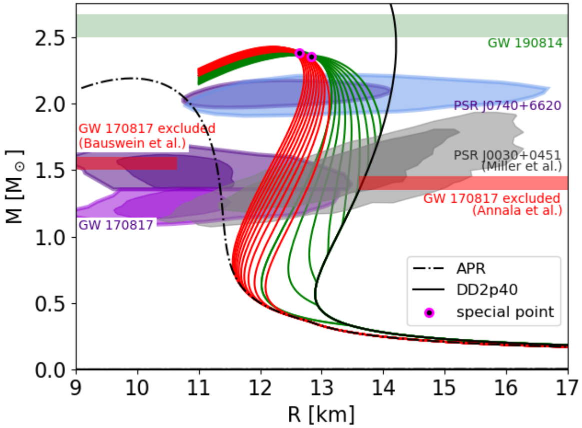
Invariance w.r.t. Maxwell $->$ mixed phase construction (pasta phases)



[5] Cierniak, Blaschke, Astron. Nachr. (2021), arXiV: 2106.06986

Invariance w.r.t. Maxwell $->$ interpolation construction (soft - stiff transition)



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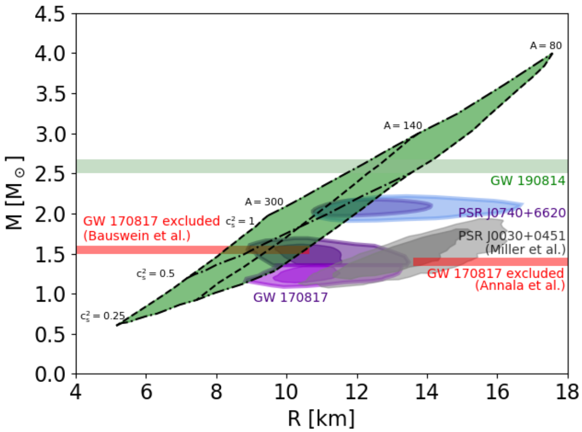
Special point locations for constant sound speed



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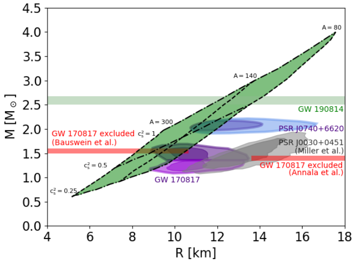

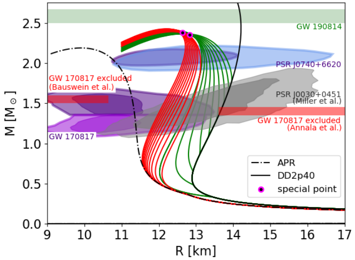

c_s^2	$M_{\rm SP}$ $[M_\odot]$	$R_{\rm min}$ [km]	$R_{\rm max}$ [km]
0.35	1.82	-	-
0.40	2.07	12.18	12.29
0.45	2.30	11.84	13.41
0.50	2.50	11.56	13.91
0.55	2.68	11.30	14.20
0.60	2.86	11.05	14.45
0.70	3.22	10.67	14.67
1.00	4.00	9.95	14.84

The values of c_s^2, largest possible M_{SP} and the radii range ($R_{min} - R_{max}$) of a 2 $M_\odot$ hybrid star.

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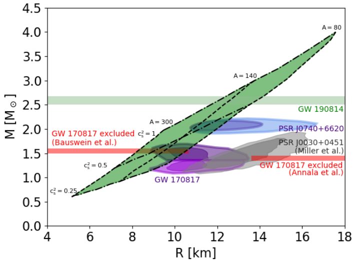

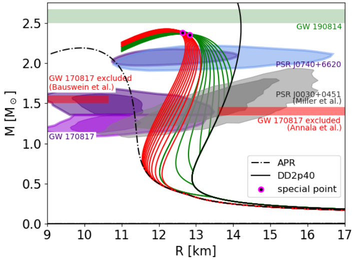

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[6] Antić, Shahrbaf, Blaschke, Grunfeld, arXiV: 2105.00029

Conclusions:

- A special point is a feature of neutron star mass radius relations unique to hybrid stars.
- The special point does not depend on the choice of the hadronic equation of state, phase transition onset density or the type of transition, thus
- The novel NICER PSR J0740+6620 measurement suggests a stiff high density equation of state, in tension with hyperon hadron models. The existence of hybrid stars solves this problem and hints at a possible hybrid neutron star nature of the 2.6 $M_{\odot}$ constituent of GW190814.

Outlook:

- Special point of rotating neutron stars?
- Relation between the special point and tidal deformability.
- The special point at a finite temperature.