# Special point and onset of deconfinement in the M-R diagram of neutron stars.

Mateusz Cierniak, David Blaschke

Division of Elementary Particle Theory,
Institute of Theoretical Physics,
University of Wroclaw.

Uniwersytet Wrocławski

NATIONAL SCIENCE CENTRE
POLAND

# Overview

# Introduction

## What is the special point?

# The TOV equation

## The TOV equation

$$\frac{\partial P(r)}{\partial r} = -\frac{\rho(r)M(r)\left(1+\frac{P(r)}{\rho(r)}\right)\left(1+\frac{4\pi r^3 P(r)}{M(r)}\right)}{r^2\left(1-\frac{2M(r)}{r}\right)},$$

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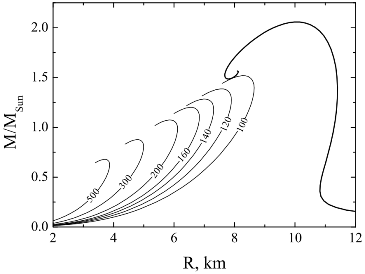


**Fig. 1**. Mass–radius diagram for a star made of ordinary matter (thick line) and purely quark stars (thin lines). The numbers at the lines indicate the parameter $B$.

[1] Yudin et al., Astron. Lett. **40** (2014), 201

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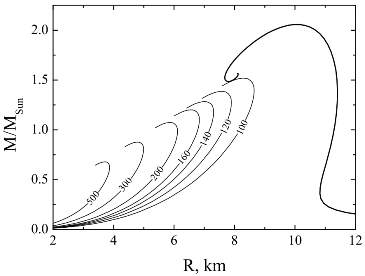


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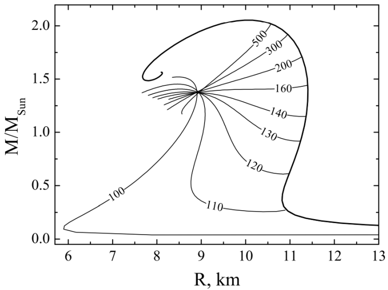


**Fig. 2.** Mass–radius diagram of hybrid stars for various values of the parameter $B$

[1] Yudin et al., Astron. Lett. **40** (2014), 201

# Properties of the special point

## Hadronic EoS invariance

The constant–speed–of–sound (CSS) model:

[2] Alford, Han, Prakash, Phys. Rev. **D 88** (2013) no.8, 083013

The constant–speed–of–sound (CSS) model:

– dimensionless baryochemical potential

$$\hat{\mu}_B = \frac{\mu_B}{\mu_{scale}} = \left(\frac{p+B}{A}\right)^{1/(1+\gamma)};$$

– pressure

$$p(\mu_B) = A\hat{\mu}_B^{1+\gamma} - B;$$

– baryon density

$$n_B(\mu_B) = (1+\gamma)\frac{A}{\mu_{scale}}\hat{\mu}_B^{\gamma};$$

– energy density

$$\varepsilon = B + \gamma A\hat{\mu}_B^{1+\gamma};$$

– $p(\varepsilon)$ relation: $\varepsilon = \gamma p + (1+\gamma)B.$

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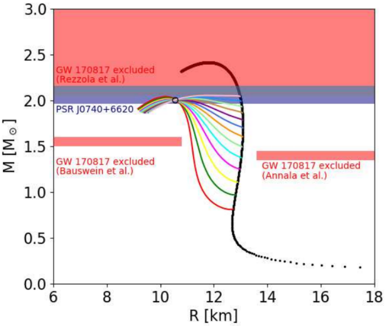


[3]Cierniak, Blaschke, Eur.Phys.J.ST **229** (2020) no.22-23, 3663-3673

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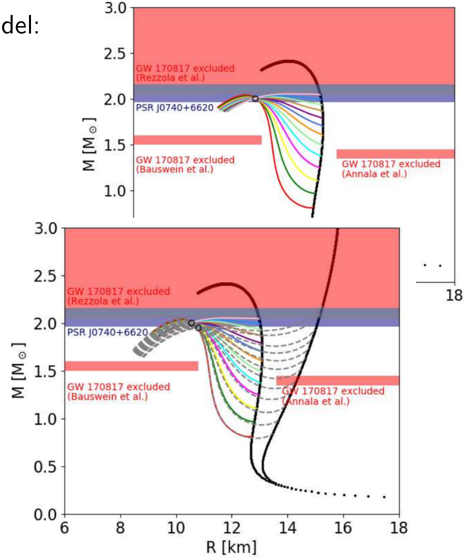
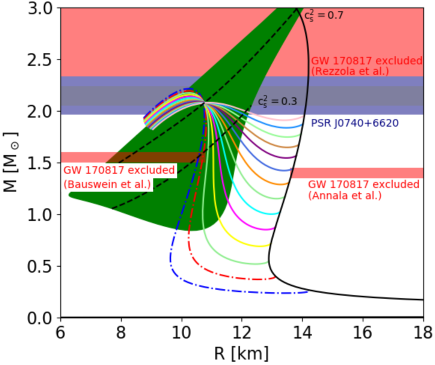



[3]Cierniak, Blaschke, Eur.Phys.J.ST **229** (2020) no.22-23, 3663-3673

# Properties of the special point

## Mass relation

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# Properties of the special point

# Invariance w.r.t. the phase transition constructio

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[4] Abgaryan, et al., Universe 4 (2018), 94

# The mixed phase parabolic ansatz:

$$P_M(\ ) = \ _2(\ \ _c)^2 + \ _1(\ \ _c) + (1 + \ _P)P_c;$$

# Gibbs condition for phase equilibrium:

$$P_H(\ _H) = P_M(\ _H);$$
$$P_Q(\ _Q) = P_M(\ _Q);$$
$$\frac{@^k}{@\ ^k}P_H(\ _H) = \frac{@^k}{@\ ^k}P_M(\ _H);$$
$$\frac{@^k}{@\ ^k}P_Q(\ _Q) = \frac{@^k}{@\ ^k}P_M(\ _Q):$$

# Derived parameters (k = 1):

$$_1 = \frac{2\ _1 + \ _2(\ _c\ \ _H)}{2(\ _c\ \ _Q)(\ _H\ \ _Q)};$$

$$_2 = \frac{2\ _1 + \ _2(\ _c\ \ _Q)}{2(\ _c\ \ _H)(\ _H\ \ _Q)};$$

$$_1 = n_Q(\ _c\ \ _Q)\ \ n_H(\ _c\ \ _H) + P_Q\ \ P_H;$$

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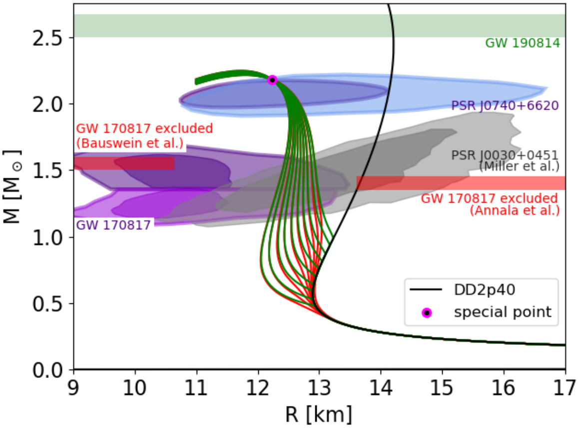
## Invariance w.r.t. Maxwell > mixed phase construction (pasta phases)




[5] Cierniak, Blaschke, Astron. Nachr. (2021), arXiV: 2106.06986

## Invariance w.r.t. Maxwell > interpolation construction (soft - stiff transition)

[5] Cierniak, Blaschke, Astron. Nachr. (2021), arXiV: 2106.06986

## Special point locations for constant sound speed

[5] Cierniak, Blaschke, Astron. Nachr. (2021), arXiV: 2106.06986

| $c_s^2$ | $M_{SP}$ [$M$ ] | $R_{min}$ [km] | $R_{max}$ [km] |
|---|---|---|---|
| 0.35 | 1.82 | - | - |
| 0.40 | 2.07 | 12.18 | 12.29 |
| 0.45 | 2.30 | 11.84 | 13.41 |
| 0.50 | 2.50 | 11.56 | 13.91 |
| 0.55 | 2.68 | 11.30 | 14.20 |
| 0.60 | 2.86 | 11.05 | 14.45 |
| 0.70 | 3.22 | 10.67 | 14.67 |
| 1.00 | 4.00 | 9.95 | 14.84 |

The values of $c_s^2$, largest possible $M_{SP}$ and the radii range ($R_{min} - R_{max}$) of a 2 $M$ hybrid star.

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The values of $c_s^2$, largest possible $M_{SP}$ and the radii range ($R_{min} - R_{max}$) of a 2 $M$ hybrid star. **Bold red rows correspond to the nlNJL  t from [6].**

[6] Antic, Shahrbaf, Blaschke, Grunfeld, arXiV: 2105.00029

Conclusions:

- A special point is a feature of neutron star mass radius relations unique to hybrid stars.
- The special point does not depend on the choice of the hadronic equation of state, phase transition onset density or the type of transition, thus
- The novel NICER PSR J0740+6620 measurement suggests a stiff high density equation of state, in tension with hyperon hadron models. The existence of hybrid stars solves this problem and hints at a possible hybrid neutron star nature of the 2.6 $M$ constituent of GW190814.

Outlook:

- Special point of rotating neutron stars?
- Relation between the special point and tidal deformability.
- The special point at a finite temperature.