Overview of lattice calculations of the x-dependence of PDFs, GPDs and TMDs

Krzysztof Cichy

Adam Mickiewicz University, Poznań, Poland

Supported by the National Science Center of Poland
SONATA BIS grant No 2016/22/E/ST2/00013 (2017-2022)

Outline:

Introduction/motivation

Review of results:

Nucleon twist-2 PDFs/GPDs

twist-3 PDFs/GPDs

Pion PDFs/DAs

TMDs

Prospects/conclusion

Many thanks to my Collaborators:

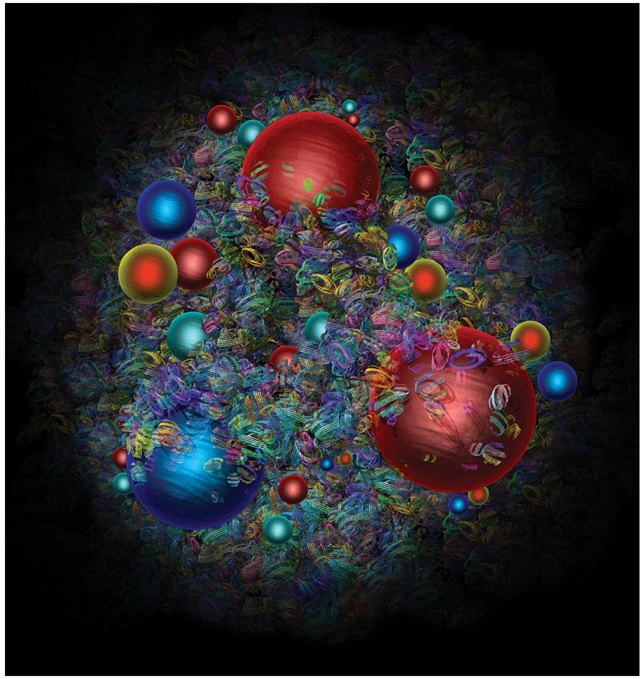
C. Alexandrou, M. Bhat, S. Bhattacharya, Y. Chai, M. Constantinou
L. Del Debbio, J. Dodson, X. Feng, T. Giani, J. Green,
K. Hadjiyiannakou, K. Jansen, G. Koutsou, Y. Li, Ch. Liu,
F. Manigrasso, A. Metz, A. Scapellato, F. Steffens, S.-C. Xia

Acknowledgment for discussions/material for this talk:

M. Constantinou, R. Sufian, S. Zafeiropoulos, Y. Zhao

Nucleon structure

One of the central aims of hadron physics: to understand better nucleon structure.



- This is one of the crucial expectations from the approved Electron-Ion Collider (EIC).
- In particular, we want to probe the 3D structure.
- Thus, we need to access new kinds of functions: GPDs, TMDs.
- Also higher-twist is of growing importance for the full picture.
- Both theoretical and experimental input needed.

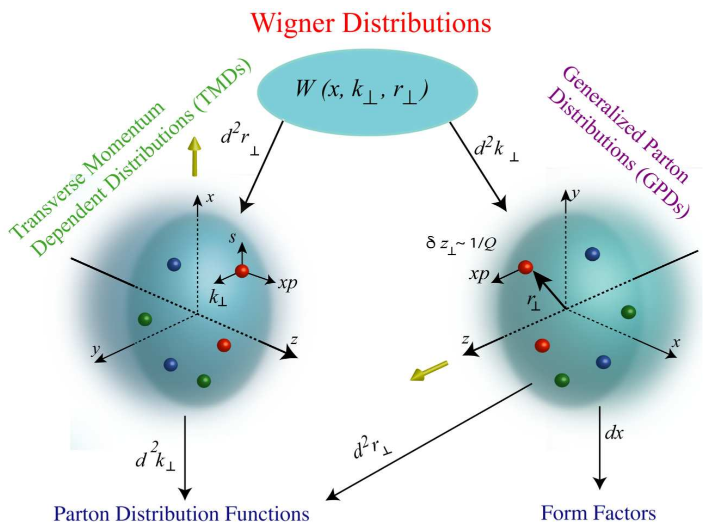
E. C. Aschenauer Mon 15:00

Lattice can provide *qualitative* and eventually *quantitative* knowledge of different functions and their moments:

D. Hackett Mon 17:20

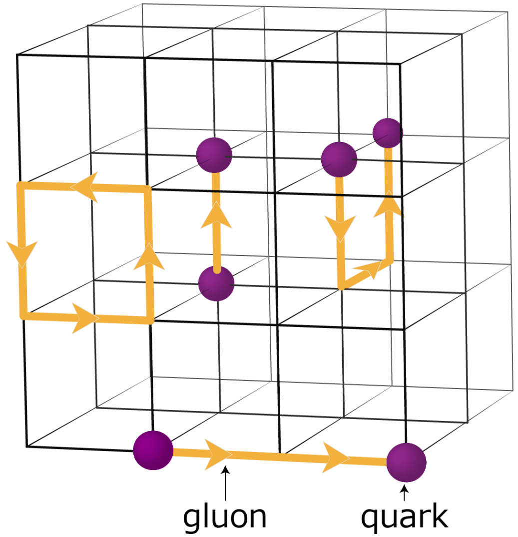
H. Wittig Wed 17:00, B. Yoon Wed 17:30

- 1D: form factors
- 1D: parton distribution functions (PDFs)
- 3D: generalized parton distributions (GPDs)
- 3D: transverse momentum dependent PDFs (TMDs)
- 5D: Wigner function / generalized TMDs



Lattice QCD – brief reminder

- needed because of non-perturbative aspects of QCD
- allows for a quantitative *ab initio* study of QCD
- QCD d.o.f.'s put on a **Euclidean** lattice
  - ⋆ quarks → sites, gluons → links
- **Euclidean** – no direct access to partonic distributions!
  only moments accessible with standard lattice methods
- various discretizations can be used for quarks and gluons
- typical lattice parameters:
  - ⋆ $a \in [0.04, 0.15]$ fm
  - ⋆ $L/a = 32, 48, 64, 80, 96, 128 \quad \Rightarrow \quad L \in [2, 10]$ fm
  - ⋆ $m_\pi \in [1, 4] \times m_\pi^{\text{physical}}, \qquad m_\pi L \geq 3-4$
  - ⋆ $\Rightarrow$ ∞-dim path integral → $10^8 - 10^9$-dim integral
- Monte Carlo simulations to evaluate the discretized path integral
- feasible, but still requires huge computational resources of $\mathcal{O}(1-1000)$ million core-hours, depending on the question asked
- formally, evaluation of a thermodynamic expectation value with respect to the Boltzmann factor $e^{-S_{\text{QCD}}}$
- lattice regulates IR and UV divergences; the regulator needs to be removed $\Rightarrow L \to \infty$, $a \to 0$
- prior to regulator removal – (non-perturbative) renormalization
- **key aspect: control over various systematic effects**
- $\Rightarrow$ exploratory studies vs. precision studies



Approaches to x-dependence

- Recent years (since ≈ 2013): breakthrough in accessing x-dependence.
  X. Ji, *Parton Physics on a Euclidean Lattice*, Phys. Rev. Lett. **110** (2013) 262002

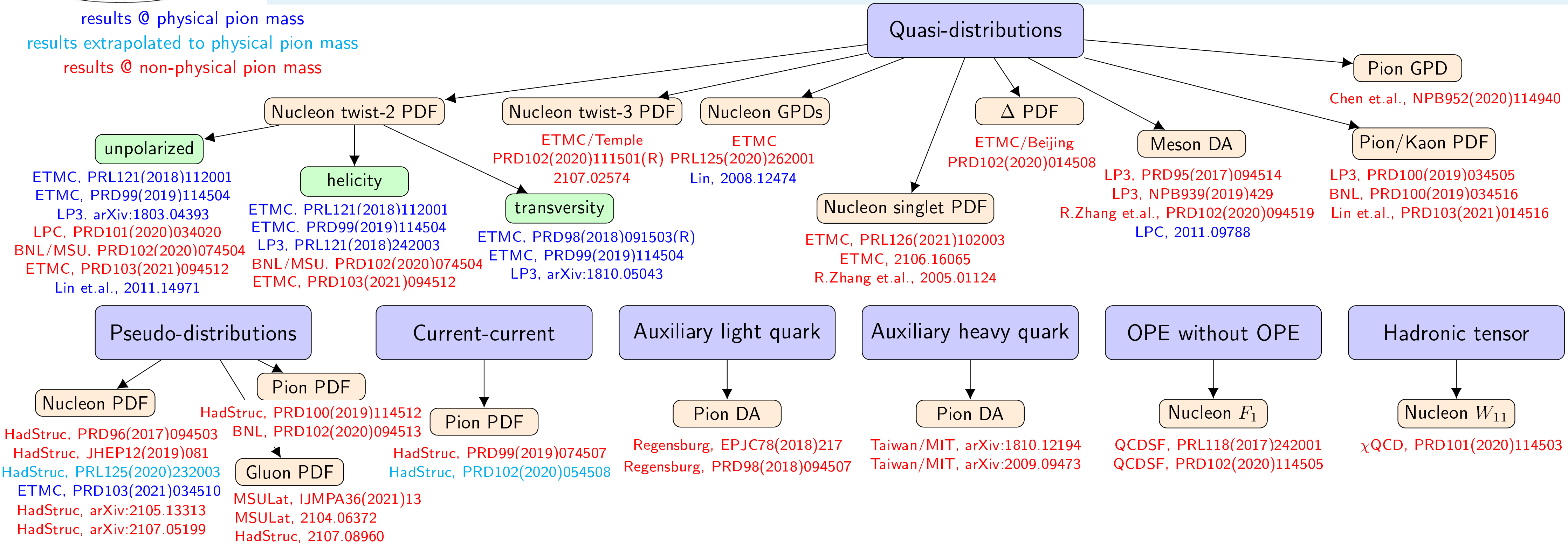
- The common feature of all the approaches is that they rely to some extent on the factorization framework:

$$\underbrace{Q(x,\mu_R)}_{\text{some lattice observable}} = \int_{-1}^{1} \frac{dy}{y}\, C\left(\frac{x}{y},\mu_F,\mu_R\right) q(y,\mu_F),$$

- Matrix elements: $\langle N|\bar{\psi}(z)\Gamma F(z)\Gamma'\psi(0)|N\rangle$
  with different choices of Γ,Γ' Dirac structures and objects $F(z)$.

  ⋆ **hadronic tensor** – K.-F. Liu, S.-J. Dong, 1993
  ⋆ **auxiliary scalar quark** – U. Aglietti et al., 1998
  ⋆ **auxiliary heavy quark (HOPE)** – W. Detmold, C.-J. D. Lin, 2005
  ⋆ **auxiliary light quark** – V. Braun, D. Müller, 2007
  ⋆ **quasi-distributions** – X. Ji, 2013
  ⋆ **"good lattice cross sections"** – Y.-Q. Ma, J.-W. Qiu, 2014,2017
  ⋆ **pseudo-distributions** – A. Radyushkin, 2017
  ⋆ **"OPE without OPE"** – QCDSF, 2017

Lattice PDFs/GPDs: dynamical progress



Reviews: K. Cichy, M. Constantinou, *A guide to light-cone PDFs from Lattice QCD: an overview of approaches, techniques and results*, special issue of Adv. High Energy Phys. 2019 (2019) 3036904, 1811.07248

update: M. Constantinou, *The x-dependence of hadronic parton distributions: A review on the progress of lattice QCD*, (would-be) plenary talk of LATTICE 2020, EPJA 57 (2021) 77, 2010.02445

X. Ji, Y. Liu, Y.-S. Liu, J.-H. Zhang, Y. Zhao, *Large-Momentum Effective Theory*, 2004.03543

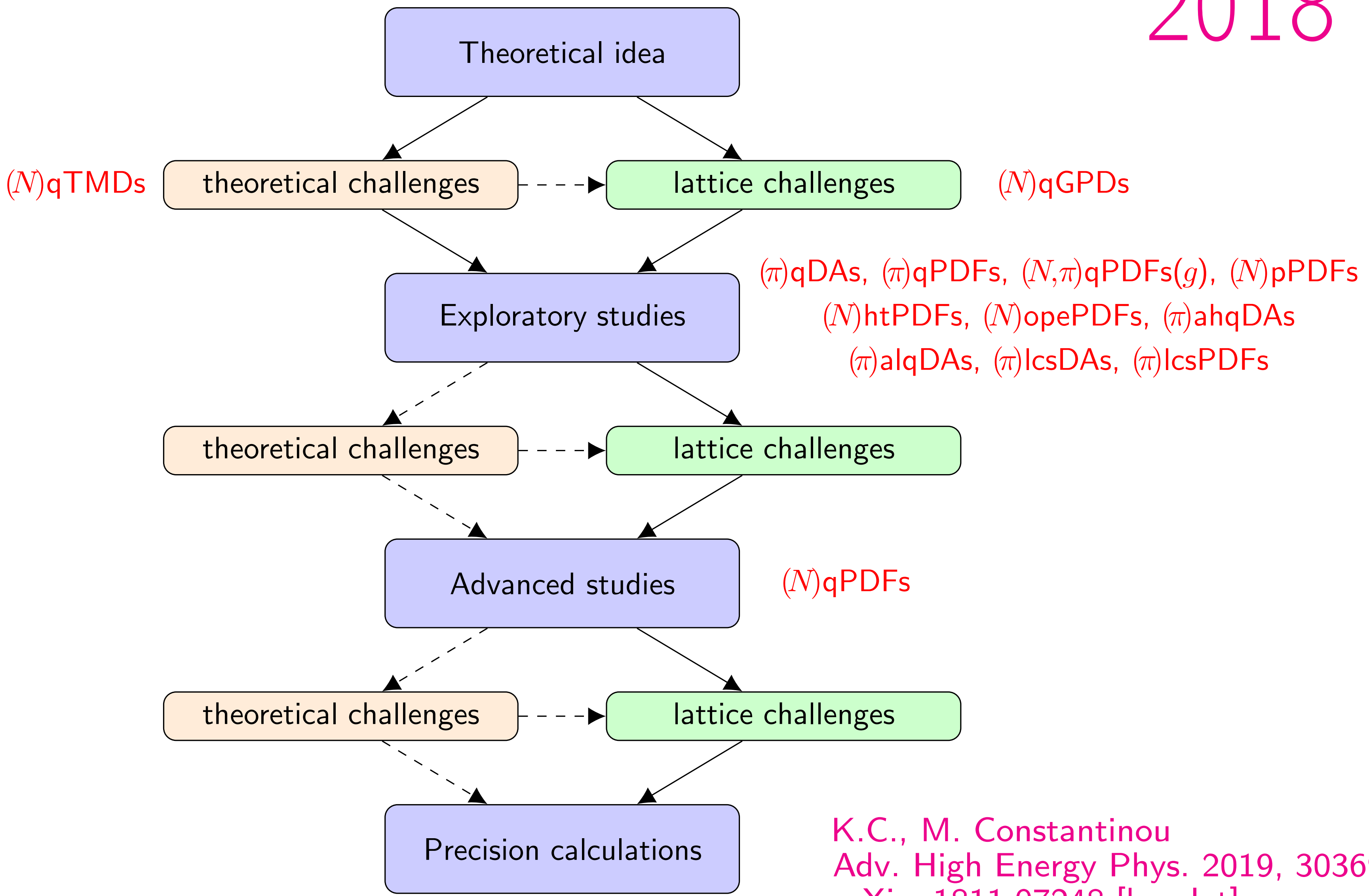
M. Constantinou et al., *Parton distributions and LQCD calculations: toward 3D structure*, 2006.08636

Some studies already advanced, but still full systematics needs to be investigated

Many exploratory directions: GPDs, twist-3 PDFs/GPDs, singlet PDFs, TMDs

Progress of approaches to x-dependence

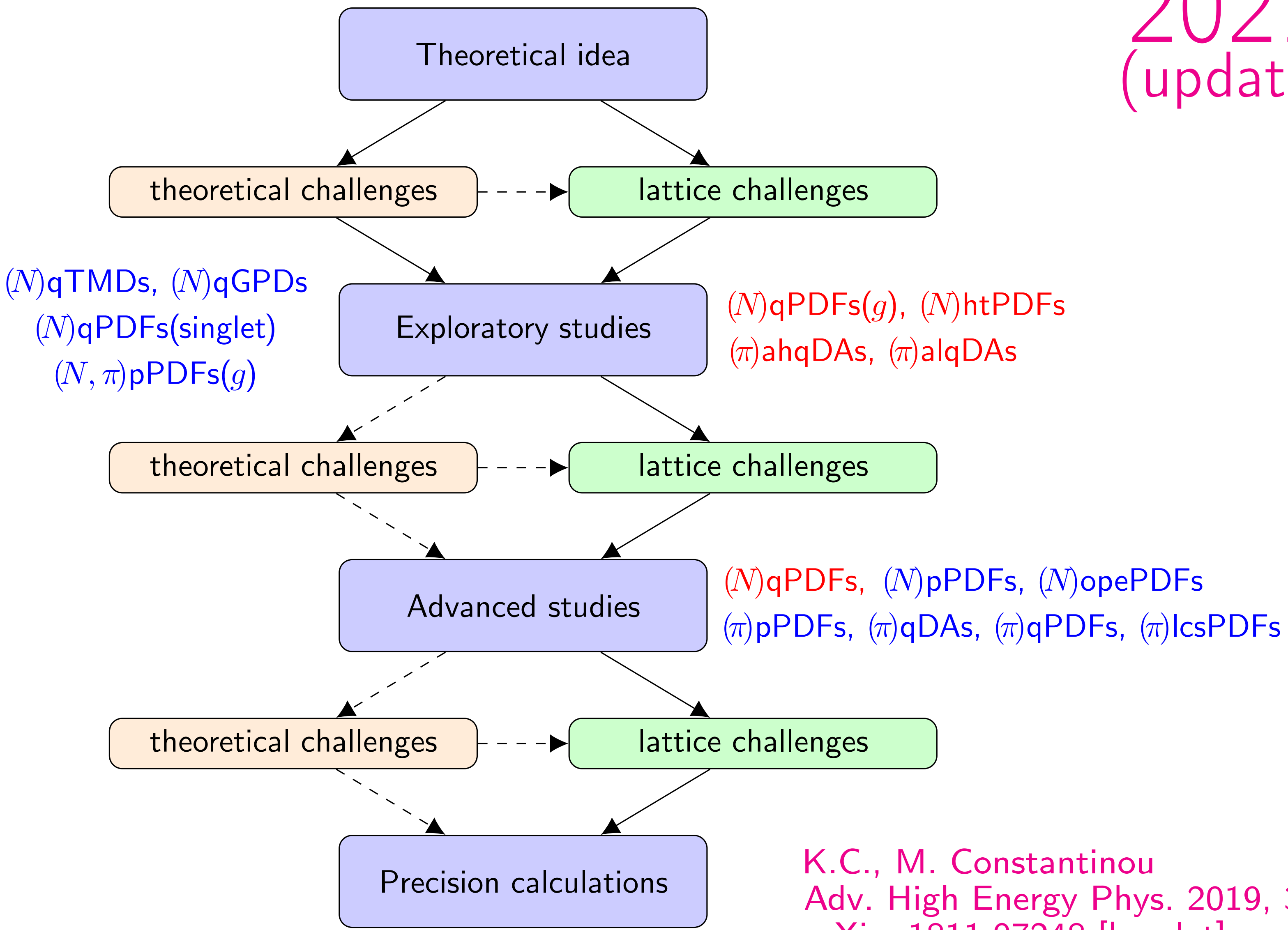
2018



K.C., M. Constantinou
Adv. High Energy Phys. 2019, 3036904
arXiv: 1811.07248 [hep-lat]

Progress of approaches to x-dependence

2021
(update)



K.C., M. Constantinou
Adv. High Energy Phys. 2019, 3036904
arXiv: 1811.07248 [hep-lat]

Quasi-PDFs

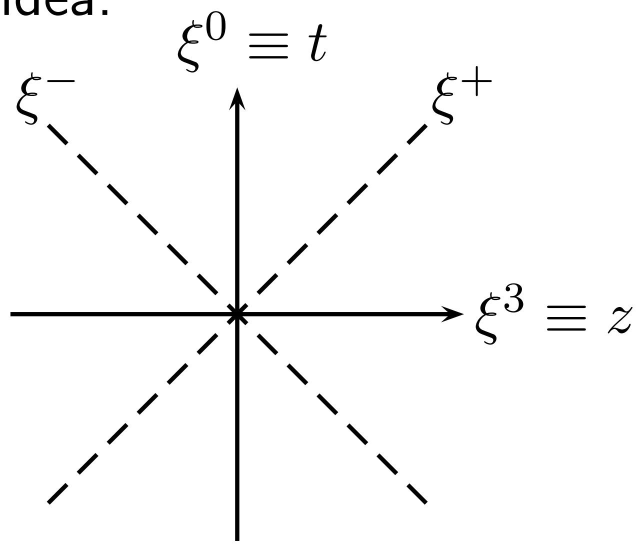
Quasi-distribution approach:

X. Ji, *Parton Physics on a Euclidean Lattice*, Phys. Rev. Lett. **110** (2013) 262002

Quasi-distribution approach:

X. Ji, *Parton Physics on a Euclidean Lattice*, Phys. Rev. Lett. **110** (2013) 262002

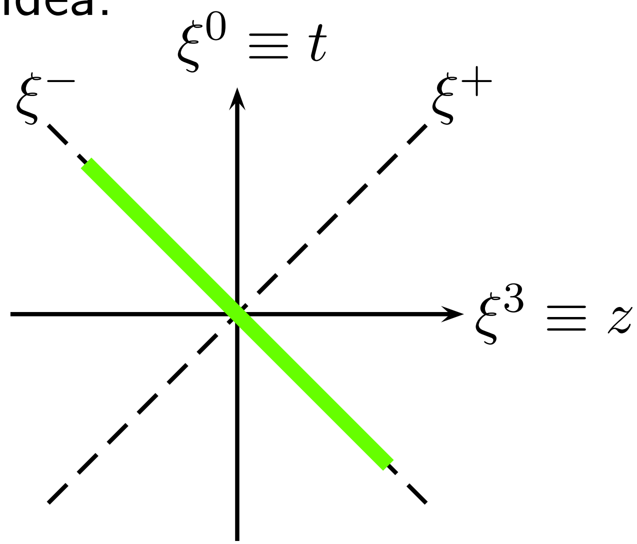
Main idea:



Quasi-distribution approach:

X. Ji, *Parton Physics on a Euclidean Lattice*, Phys. Rev. Lett. **110** (2013) 262002

Main idea:



Correlation along the ξ^--direction:

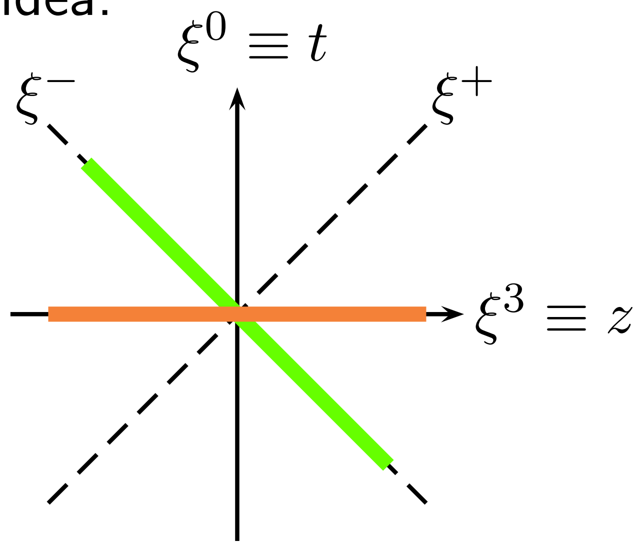
$$q(x) = \frac{1}{2\pi} \int d\xi^- e^{-ixp^+\xi^-} \langle N|\overline{\psi}(\xi^-)\Gamma\mathcal{A}(\xi^-,0)\psi(0)|N\rangle$$

$|N\rangle$ – nucleon at rest in the light-cone frame

Quasi-distribution approach:

X. Ji, *Parton Physics on a Euclidean Lattice*, Phys. Rev. Lett. **110** (2013) 262002

Main idea:



Correlation along the ξ^--direction:

$$q(x) = \frac{1}{2\pi}\int d\xi^- e^{-ixp^+\xi^-}\langle N|\overline{\psi}(\xi^-)\Gamma\mathcal{A}(\xi^-,0)\psi(0)|N\rangle$$

$|N\rangle$ – nucleon at rest in the light-cone frame

Correlation along the $\xi^3 \equiv z$-direction:

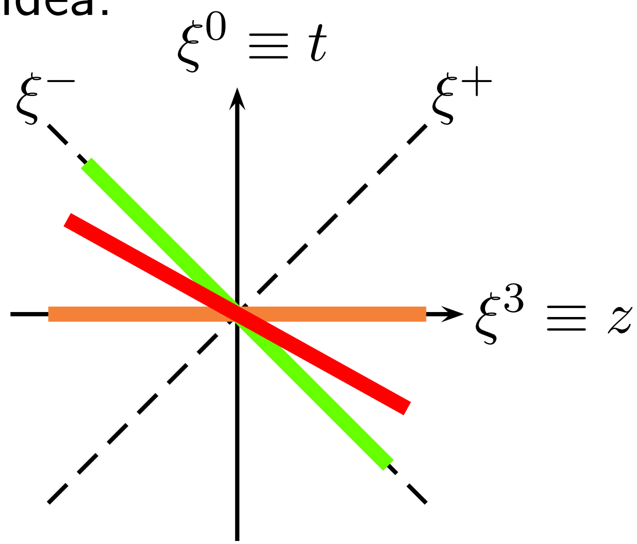
$$\tilde{q}(x) = \frac{1}{2\pi}\int dz\, e^{ixP_3 z}\langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle$$

$|N\rangle$ – nucleon at rest in the standard frame

Quasi-distribution approach:

X. Ji, *Parton Physics on a Euclidean Lattice*, Phys. Rev. Lett. **110** (2013) 262002

Main idea:



Correlation along the ξ^--direction:

$$q(x) = \frac{1}{2\pi} \int d\xi^- e^{-ixp^+\xi^-} \langle N|\overline{\psi}(\xi^-)\Gamma \mathcal{A}(\xi^-,0)\psi(0)|N\rangle$$

$|N\rangle$ – nucleon at rest in the light-cone frame

Correlation along the $\xi^3 \equiv z$-direction:

$$\tilde{q}(x) = \frac{1}{2\pi} \int dz\, e^{ixP_3 z} \langle N|\overline{\psi}(z)\Gamma \mathcal{A}(z,0)\psi(0)|N\rangle$$

$|N\rangle$ – nucleon at rest in the standard frame

Correlation along the ξ^3-direction:

$$\tilde{q}(x) = \frac{1}{2\pi} \int dz\, e^{ixP_3 z} \langle P|\overline{\psi}(z)\Gamma \mathcal{A}(z,0)\psi(0)|P\rangle$$

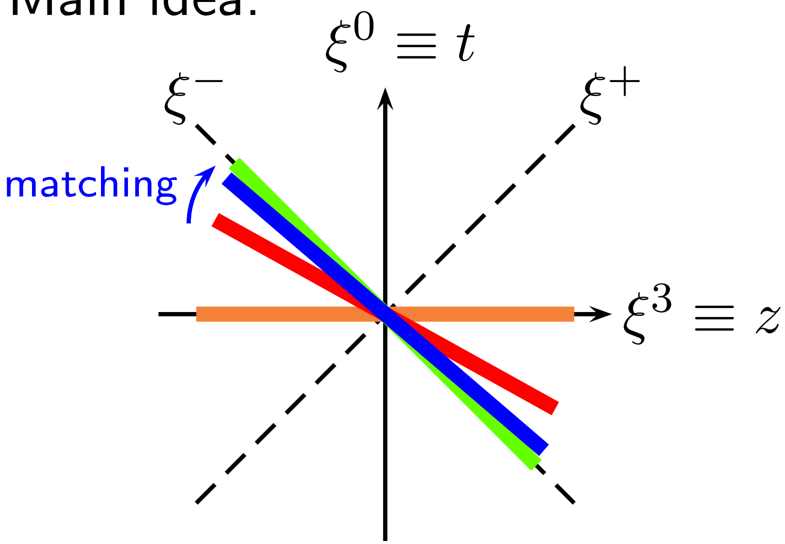
$|P\rangle$ – boosted nucleon

Quasi-PDFs

Quasi-distribution approach:

X. Ji, *Parton Physics on a Euclidean Lattice*, Phys. Rev. Lett. **110** (2013) 262002

Main idea:



Correlation along the ξ^--direction:

$$q(x) = \frac{1}{2\pi} \int d\xi^- e^{-ixp^+\xi^-} \langle N|\overline{\psi}(\xi^-)\Gamma\mathcal{A}(\xi^-,0)\psi(0)|N\rangle$$

$|N\rangle$ – nucleon at rest in the light-cone frame

Correlation along the $\xi^3 \equiv z$-direction:

$$\tilde{q}(x) = \frac{1}{2\pi} \int dz\, e^{ixP_3 z} \langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle$$

$|N\rangle$ – nucleon at rest in the standard frame

Correlation along the ξ^3-direction:

$$\tilde{q}(x) = \frac{1}{2\pi} \int dz\, e^{ixP_3 z} \langle P|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|P\rangle$$

$|P\rangle$ – boosted nucleon

Matching (Large Momentum Effective Theory (LaMET)

X. Ji, *Parton Physics from Large-Momentum Effective Field Theory*, Sci.China Phys.Mech.Astron. 57 (2014) 1407

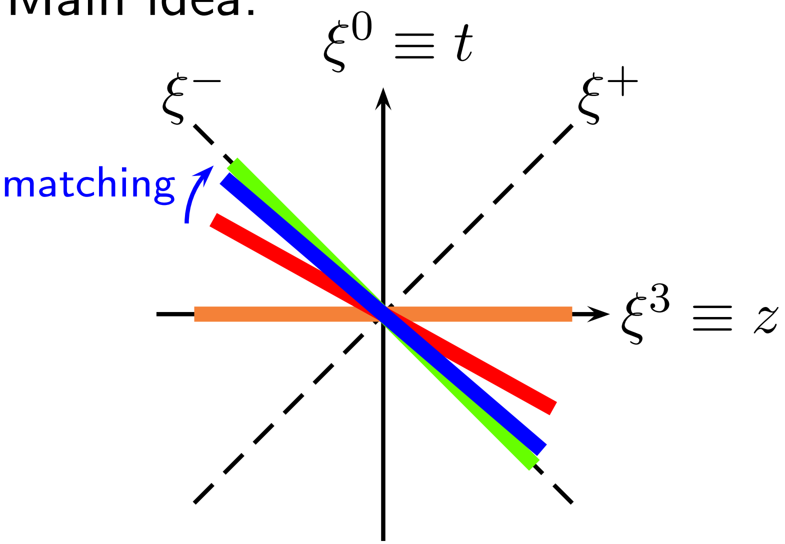
$\rightarrow$ brings quasi-distribution to the light-cone distribution, up to power-suppressed effects:

$$\tilde{q}(x,\mu,P_3) = \int_{-1}^{1} \frac{dy}{|y|}\, C\left(\frac{x}{y}, \frac{\mu}{P_3}\right) q(y,\mu) + \mathcal{O}\left(\Lambda^2_{\rm QCD}/P_3^2, M_N^2/P_3^2\right)$$

Quasi-distribution approach:

X. Ji, *Parton Physics on a Euclidean Lattice*, Phys. Rev. Lett. **110** (2013) 262002

Main idea:



Correlation along the ξ^--direction:

$$q(x) = \frac{1}{2\pi}\int d\xi^- e^{-ixp^+\xi^-}\langle N|\overline{\psi}(\xi^-)\Gamma\mathcal{A}(\xi^-,0)\psi(0)|N\rangle$$

$|N\rangle$ – nucleon at rest in the light-cone frame

Correlation along the $\xi^3 \equiv z$-direction:

$$\tilde{q}(x) = \frac{1}{2\pi}\int dz\, e^{ixP_3z}\langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle$$

$|N\rangle$ – nucleon at rest in the standard frame

Correlation along the ξ^3-direction:

$$\tilde{q}(x) = \frac{1}{2\pi}\int dz\, e^{ixP_3z}\langle P|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|P\rangle$$

$|P\rangle$ – boosted nucleon

Matching (Large Momentum Effective Theory (LaMET)

X. Ji, *Parton Physics from Large-Momentum Effective Field Theory*, Sci.China Phys.Mech.Astron. 57 (2014) 1407

→ brings quasi-distribution to the light-cone distribution, up to power-suppressed effects:

$$\tilde{q}(x,\mu,P_3) = \int_{-1}^{1}\frac{dy}{|y|}\,C\left(\frac{x}{y},\frac{\mu}{P_3}\right)q(y,\mu) + \mathcal{O}\left(\Lambda^2_{\rm QCD}/P_3^2, M_N^2/P_3^2\right)$$

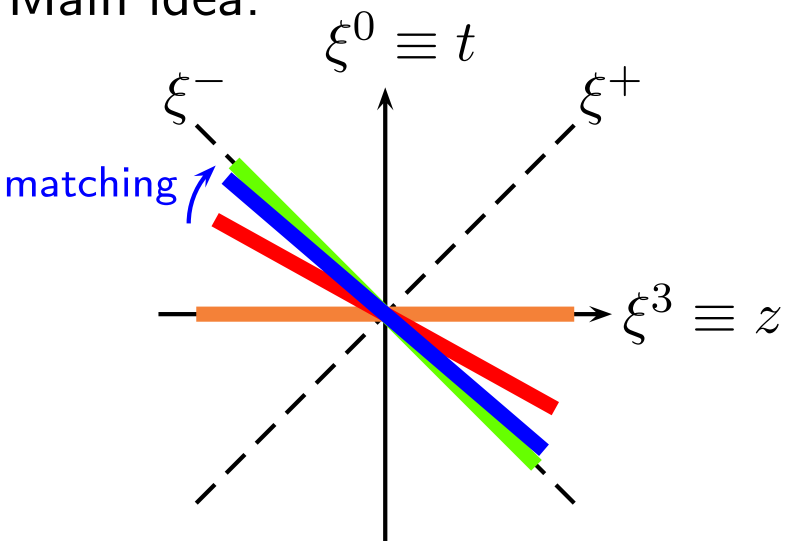
quasi-PDF

Quasi-PDFs

Quasi-distribution approach:

X. Ji, *Parton Physics on a Euclidean Lattice*, Phys. Rev. Lett. **110** (2013) 262002

Main idea:



Correlation along the ξ^--direction:

$$q(x) = \frac{1}{2\pi}\int d\xi^- e^{-ixp^+\xi^-}\langle N|\overline{\psi}(\xi^-)\Gamma\mathcal{A}(\xi^-,0)\psi(0)|N\rangle$$

$|N\rangle$ – nucleon at rest in the light-cone frame

Correlation along the $\xi^3 \equiv z$-direction:

$$\tilde{q}(x) = \frac{1}{2\pi}\int dz\, e^{ixP_3 z}\langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle$$

$|N\rangle$ – nucleon at rest in the standard frame

Correlation along the ξ^3-direction:

$$\tilde{q}(x) = \frac{1}{2\pi}\int dz\, e^{ixP_3 z}\langle P|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|P\rangle$$

$|P\rangle$ – boosted nucleon

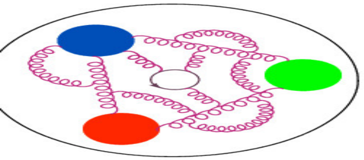
Matching (Large Momentum Effective Theory (LaMET)

X. Ji, *Parton Physics from Large-Momentum Effective Field Theory*, Sci.China Phys.Mech.Astron. 57 (2014) 1407

→ brings quasi-distribution to the light-cone distribution, up to power-suppressed effects:

$$\tilde{q}(x,\mu,P_3) = \int_{-1}^{1}\frac{dy}{|y|}\,C\left(\frac{x}{y},\frac{\mu}{P_3}\right)q(y,\mu) + \mathcal{O}\left(\Lambda^2_{\rm QCD}/P_3^2, M_N^2/P_3^2\right)$$

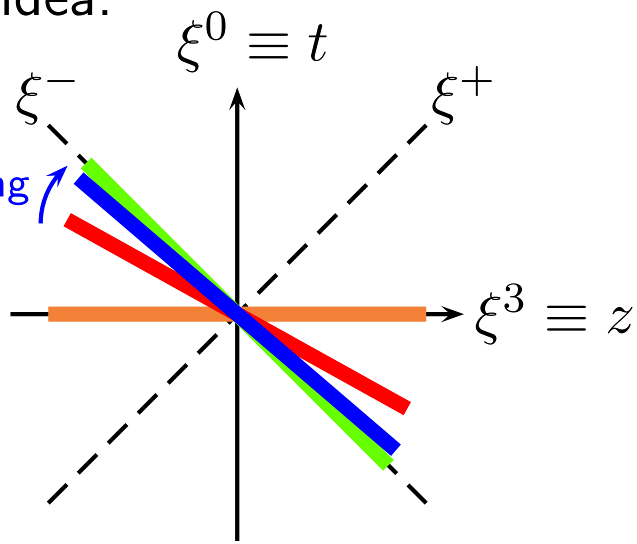
quasi-PDF PDF



Quasi-distribution approach:

X. Ji, *Parton Physics on a Euclidean Lattice*, Phys. Rev. Lett. **110** (2013) 262002

Main idea:



Correlation along the ξ^--direction:

$$q(x) = \frac{1}{2\pi} \int d\xi^- e^{-ixp^+\xi^-} \langle N|\overline{\psi}(\xi^-)\Gamma\mathcal{A}(\xi^-,0)\psi(0)|N\rangle$$

$|N\rangle$ – nucleon at rest in the light-cone frame

Correlation along the $\xi^3 \equiv z$-direction:

$$\tilde{q}(x) = \frac{1}{2\pi} \int dz\, e^{ixP_3 z} \langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle$$

$|N\rangle$ – nucleon at rest in the standard frame

Correlation along the ξ^3-direction:

$$\tilde{q}(x) = \frac{1}{2\pi} \int dz\, e^{ixP_3 z} \langle P|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|P\rangle$$

$|P\rangle$ – boosted nucleon

Matching (Large Momentum Effective Theory (LaMET)

X. Ji, *Parton Physics from Large-Momentum Effective Field Theory*, Sci.China Phys.Mech.Astron. 57 (2014) 1407

$\rightarrow$ brings quasi-distribution to the light-cone distribution, up to power-suppressed effects:

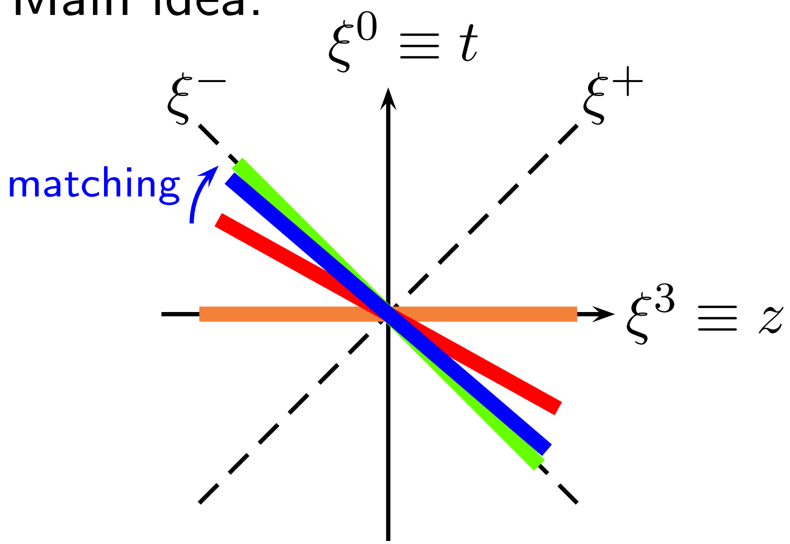
$$\underbrace{\tilde{q}(x,\mu,P_3)}_{\text{quasi-PDF}} = \int_{-1}^{1} \frac{dy}{|y|} \underbrace{C\left(\frac{x}{y}, \frac{\mu}{P_3}\right)}_{\text{pert.kernel}} \underbrace{q(y,\mu)}_{\text{PDF}} + \mathcal{O}\left(\Lambda^2_{\rm QCD}/P_3^2, M_N^2/P_3^2\right)$$

Quasi-PDFs

Quasi-distribution approach:

X. Ji, *Parton Physics on a Euclidean Lattice*, Phys. Rev. Lett. **110** (2013) 262002

Main idea:



Correlation along the ξ^--direction:

$$q(x) = \frac{1}{2\pi}\int d\xi^- e^{-ixp^+\xi^-}\langle N|\overline{\psi}(\xi^-)\Gamma\mathcal{A}(\xi^-,0)\psi(0)|N\rangle$$

$|N\rangle$ – nucleon at rest in the light-cone frame

Correlation along the $\xi^3 \equiv z$-direction:

$$\tilde{q}(x) = \frac{1}{2\pi}\int dz\, e^{ixP_3 z}\langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle$$

$|N\rangle$ – nucleon at rest in the standard frame

Correlation along the ξ^3-direction:

$$\tilde{q}(x) = \frac{1}{2\pi}\int dz\, e^{ixP_3 z}\langle P|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|P\rangle$$

$|P\rangle$ – boosted nucleon

Matching (Large Momentum Effective Theory (LaMET)

X. Ji, *Parton Physics from Large-Momentum Effective Field Theory*, Sci.China Phys.Mech.Astron. 57 (2014) 1407

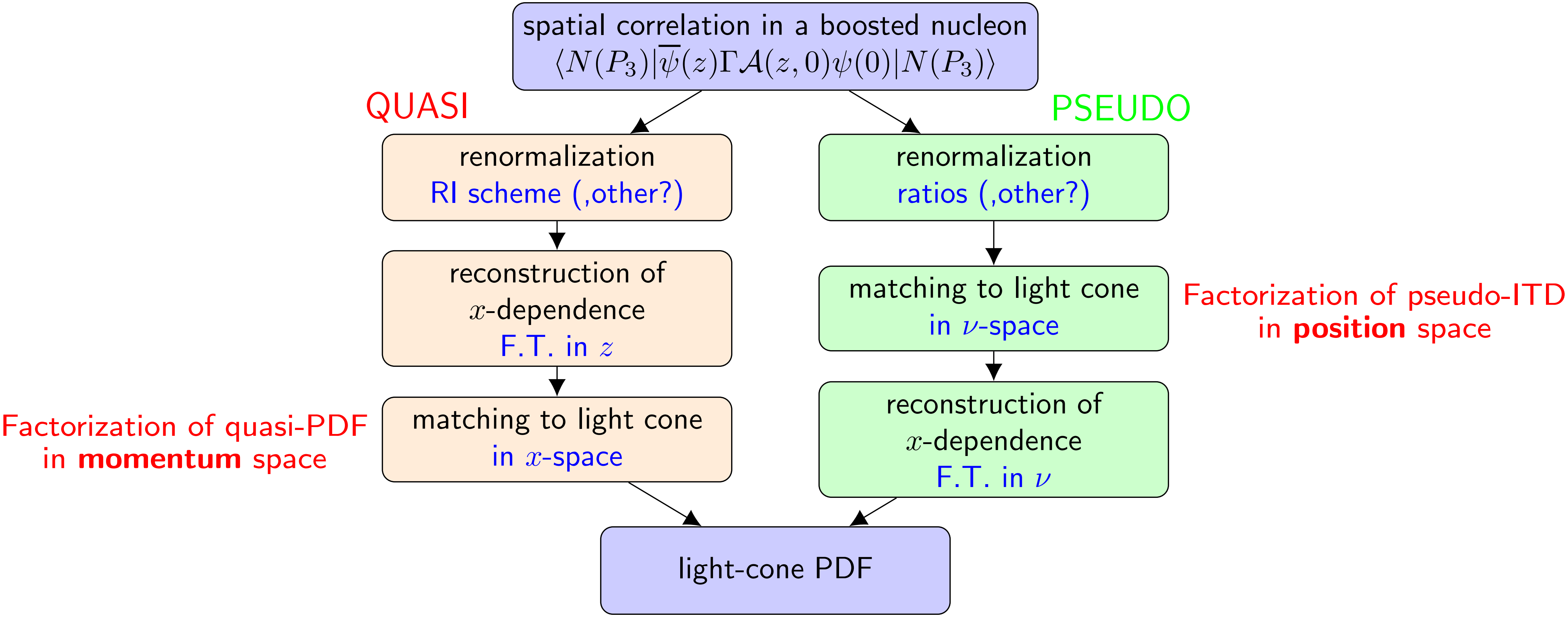
$\rightarrow$ brings quasi-distribution to the light-cone distribution, up to power-suppressed effects:

$$\underbrace{\tilde{q}(x,\mu,P_3)}_{\text{quasi-PDF}} = \int_{-1}^{1}\frac{dy}{|y|}\underbrace{C\left(\frac{x}{y},\frac{\mu}{P_3}\right)}_{\text{pert.kernel}}\underbrace{q(y,\mu)}_{\text{PDF}} + \underbrace{\mathcal{O}\left(\Lambda^2_{\text{QCD}}/P_3^2, M_N^2/P_3^2\right)}_{\text{higher-twist effects}}$$

Quasi-PDFs and pseudo-PDFs

The same matrix elements that define **quasi-distributions** can also be used to construct **pseudo-distributions** A. Radyushkin, Phys. Rev. D96 (2017) 034025

Review: A. Radyushkin, "Theory and applications of parton pseudodistributions", Int. J. Mod. Phys. A35 (2020) 2030002



Recent state-of-the-art unpolarized PDFs @ phys.pt.

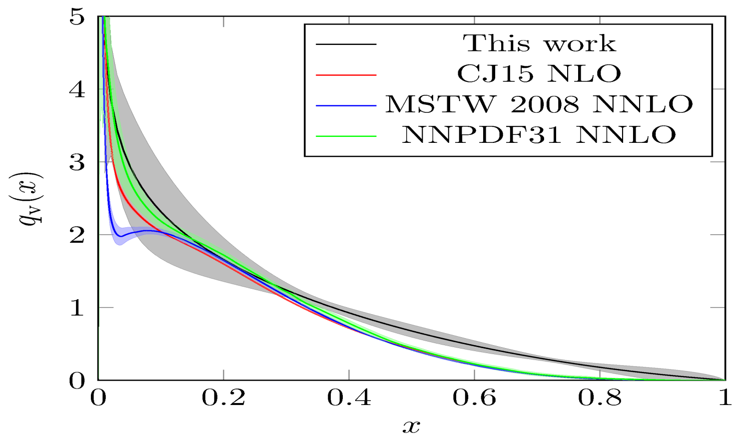
B. Joó et al. (HadStruc)
Phys. Rev. Lett. 125 (2020) 232003

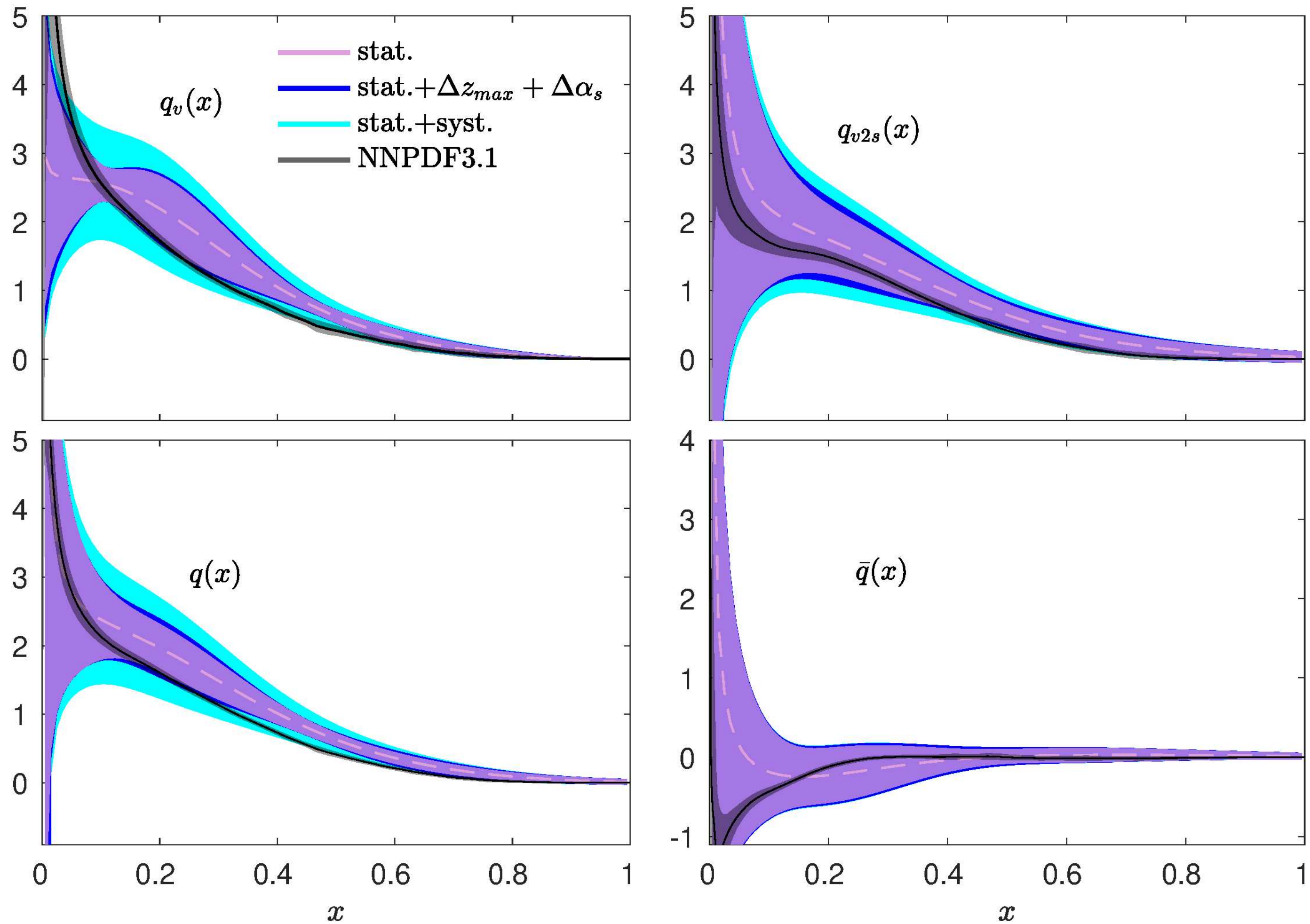
$\overline{\mathrm{MS}}(2\mathrm{GeV})$
$u-d$

M. Bhat et al. (ETMC)
Phys. Rev. D103 (2021) 034510

PSEUDO	clover	$m_\pi = 358$, 278,172 MeV	$a = 0.094$ fm

PSEUDO	TMF	$m_\pi = 130$ MeV	$a = 0.094$ fm





Qualitative agreement with pheno

Quantitative agreement if including "plausible systematics"

True systematics to be investigated

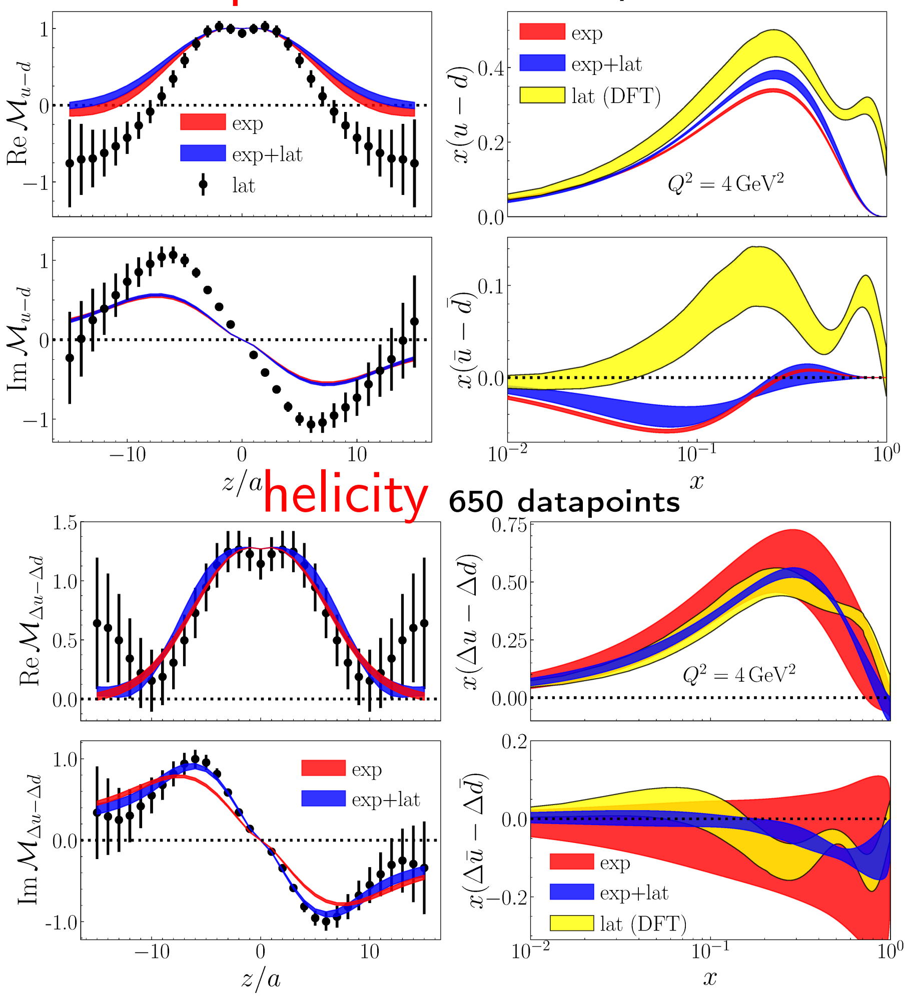
PDFs reconstruction from actual lattice data

JAM framework

J. Bringewatt, N. Sato, W. Melnitchouk, J.-W. Qiu, F. Steffens, M. Constantinou, PRD103(2021)016003

unpolarized 2930 datapoints

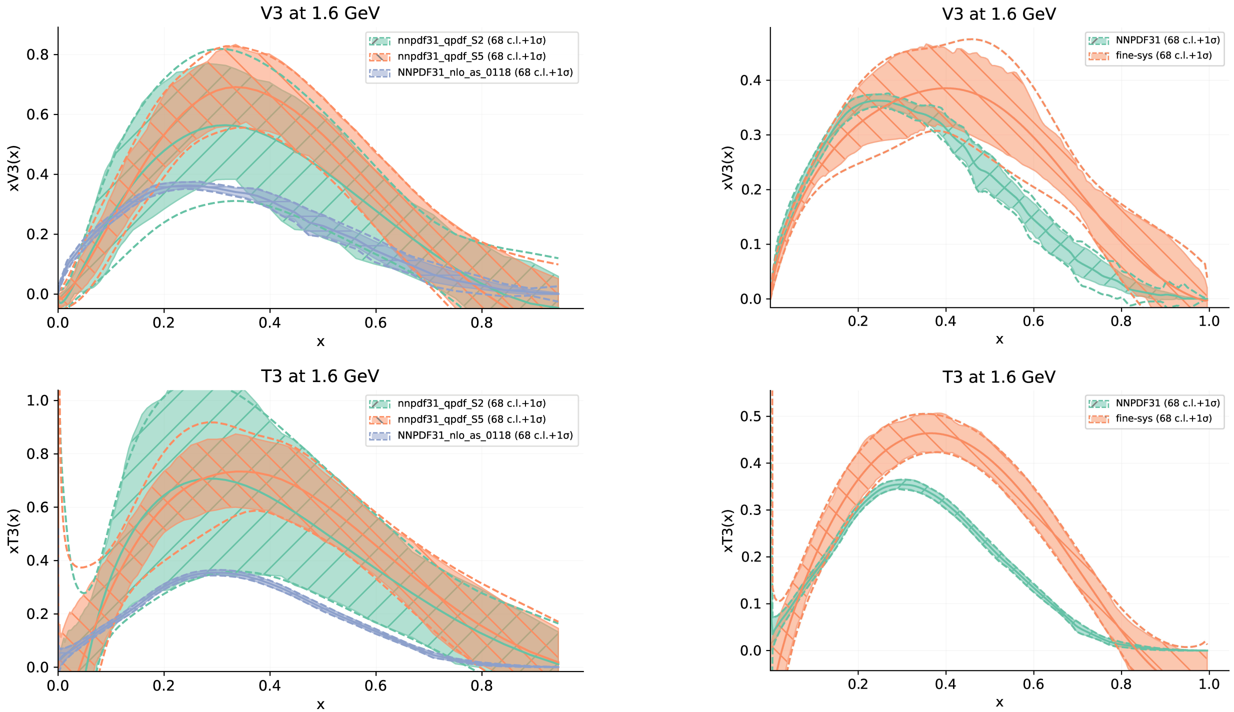
helicity 650 datapoints



NNPDF framework

NNPDF (L. Del Debbio, T. Giani) +K.C., JHEP10(2019)137

NNPDF (L. Del Debbio, T. Giani) + JLab (J. Karpie et al.), JHEP02(2021)138



unpolarized: significant tension **lat**↔**exp**
much improved precision of **lat** needed for any impact
(rather benchmark case)
helicity: promising agreement **lat**↔**exp**
current precision of **lat** provides significant constraints

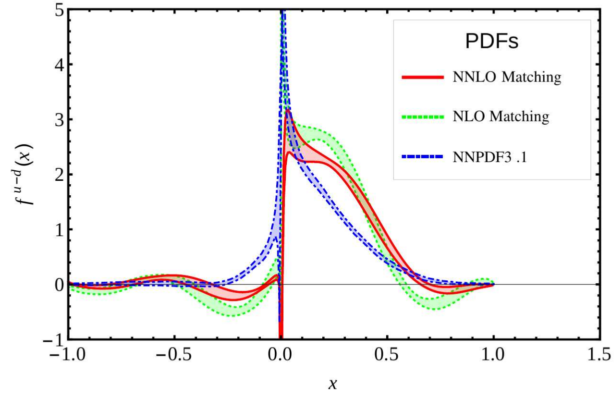
QUASI	TMF	$m_\pi = 130$ MeV	$a = 0.094$ fm

PSEUDO	clover	$m_\pi = 415, 358,$ 278,172 MeV	a=0.091,0.094, 0.127 fm

Other works for isovector nucleon PDFs

- continuum limit
  theoretical framework for $\mathcal{O}(a)$-improvement J. Green, K. Jansen, F. Steffens, PRD101(2020)074509
  lattice results C. Alexandrou et al. (ETMC), Phys. Rev. D103 (2021) 094512 (quasi)
  J. Karpie, K. Orginos, A. Radyushkin, S. Zafeiropoulos (HadStruc), 2105.13313 (pseudo)
  H.-W. Lin, J.-W. Chen, R. Zhang, 2011.14971 (quasi)
- superfine lattice with $a = 0.042$ fm Z. Fan et al. (BNL+MSULat), Phys. Rev. D102 (2020) 074504
- parametrization of systematic uncertainties with Jacobi polynomials + Bayesian fits
  J. Karpie, K. Orginos, A. Radyushkin, S. Zafeiropoulos (HadStruc), 2105.13313
- distillation with momentum smearing
  C. Egerer et al. (HadStruc), Phys. Rev. D103 (2021) 034502
  C. Egerer et al. (HadStruc), 2107.05199
- forward Compton amplitude – extraction of 4 lowest moments of nucleon's structure function F_1 using Feynman-Hellmann method
  K. U. Can et al. (QCDSF/UKQCD/CSSM), Phys. Rev. D102 (2020) 114505
- PDFs of the Δ^+ baryon – can shed light on the sea quark asymmetry of the nucleon
  Y. Chai et al. (Beijing+ETMC), PRD102(2020)014508

Theoretical/methodological developments



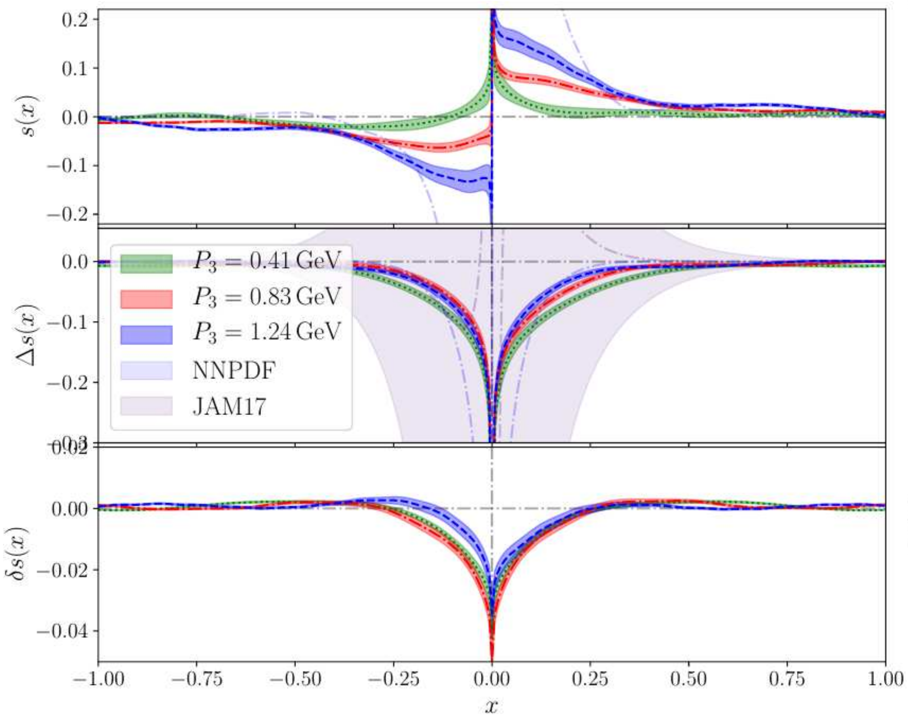
- 2-loop matching
  V. Braun, K. Chetyrkin, B. Kniehl, JHEP 07 (2020) 161
  Z.-Y. Li, Y.-Q. Ma, J.-W. Qiu, Phys. Rev.Lett. 126 (2021) 072001
  L.-B. Chen, W. Wang, R. Zhu, Phys. Rev. D102 (2020) 011503
  L.-B. Chen, W. Wang, R. Zhu, JHEP 10 (2020) 079
  L.-B. Chen, W. Wang, R. Zhu, Phys. Rev.Lett. 126 (2021) 072002
- Developments in non-perturbative renormalization
  hybrid scheme X. Ji et al., Nucl. Phys. B964 (2021) 115311
  residual power divergence K. Zhang et al. (χQCD), 2012.05448
  self-renormalization Y.-K. Huo et al. (LPC), Nucl. Phys. B969 (2021) 115443
- Origin and resummation of threshold logarithms X. Gao et al., Phys. Rev. D103 (2021) 094504
- Renormalon effects in quasi- and pseudo-distributions
  V. Braun, A. Vladimirov, J.-H. Zhang, Phys. Rev. D99 (2019) 014013; W.-Y. Liu, J.-W. Chen, 2010.06623
- Chiral perturbation theory for LaMET W.-Y. Liu, J.-W. Chen, 2011.13536
- FVE for non-local current-current operators
  R. Briceño, J. Guerrero, M. Hansen, C. Monahan, Phys. Rev. D98 (2018) 014511
  R. Briceño, C. Monahan, Phys. Rev. D103 (2021) 094521
- Parton distributions in nongauge theories L. Del Debbio, T. Giani, C. Monahan, JHEP 09(2020)021
- Bayesian determination of OPE Wilson coefficients from lattice and pheno data
  N. Karthik, R. Sufian, 2106.03875
- Bayes-Gauss-Fourier transform for PDF reconstruction
  C. Alexandrou et al. (ETMC), Phys. Rev. D102 (2020) 094508
- Pion (pseudo-)PDFs QCD_3 with 0,2,4,8 flavors N. Karthik, Phys. Rev. D103 (2021) 074512

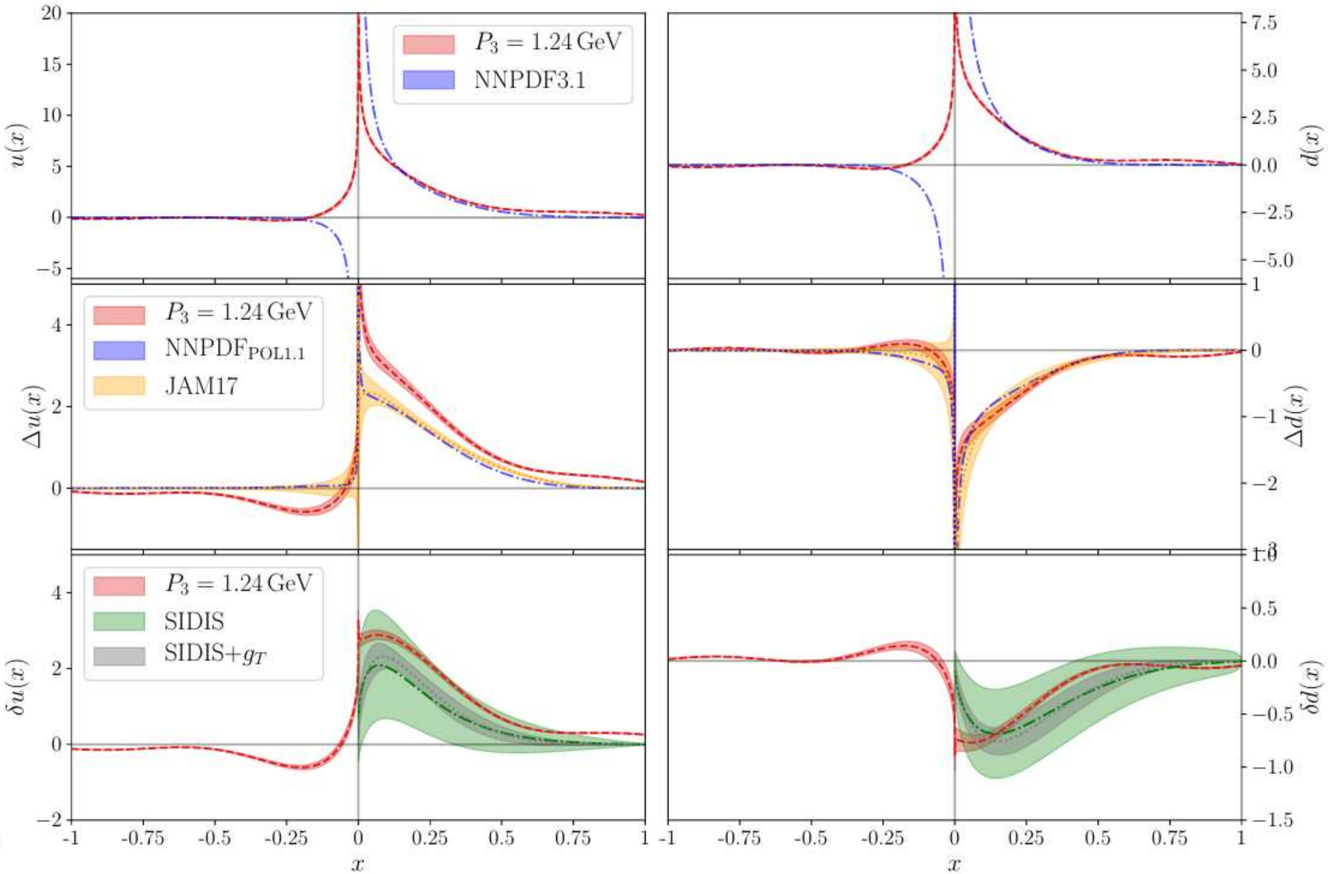
Flavor decomposition

Most studies up to date were for the flavor non-singlet $u-d$ combination.
Important direction: flavor decomposition.
C. Alexandrou et al. (ETMC), Phys. Rev. Lett. 126 (2021) 102003; 2106.16065

QUASI	TMF	$m_\pi = 260$ MeV	$a=0.093$ fm

- disconnected diagrams (hierarchical probing, one-end trick)
- mixing with gluon PDFs neglected





Strange and charm contributions also in: R. Zhang, H.-W. Lin, B. Yoon, 2005.01124

Gluon PDFs

Recent computation of gluon PDFs with crucial role of distillation

T. Khan et al. (HadStruc), 2107.08960

PSEUDO	clover	$m_\pi = 358$ MeV	$a = 0.094$ fm

Key aspects of the calculation:

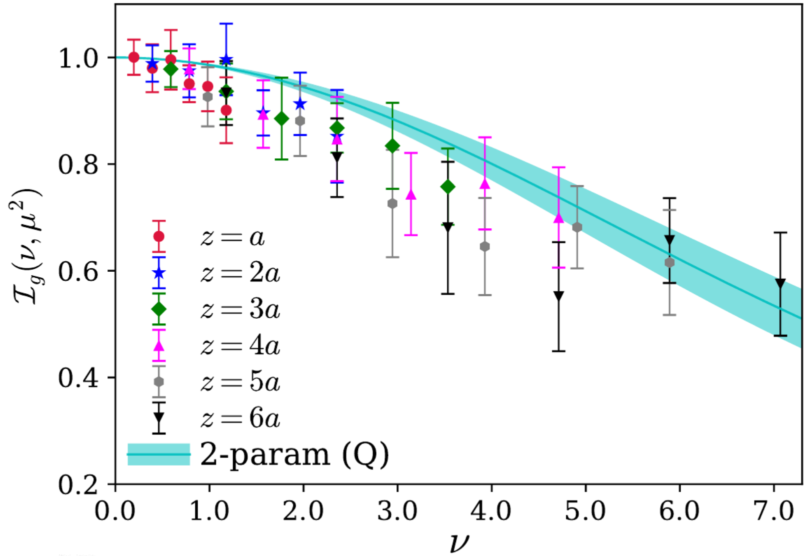
- distillation combined with momentum smearing
- summed GEVP to access smaller temporal separations
- gradient flow to improve signal (extrapolate to $\tau = 0$)

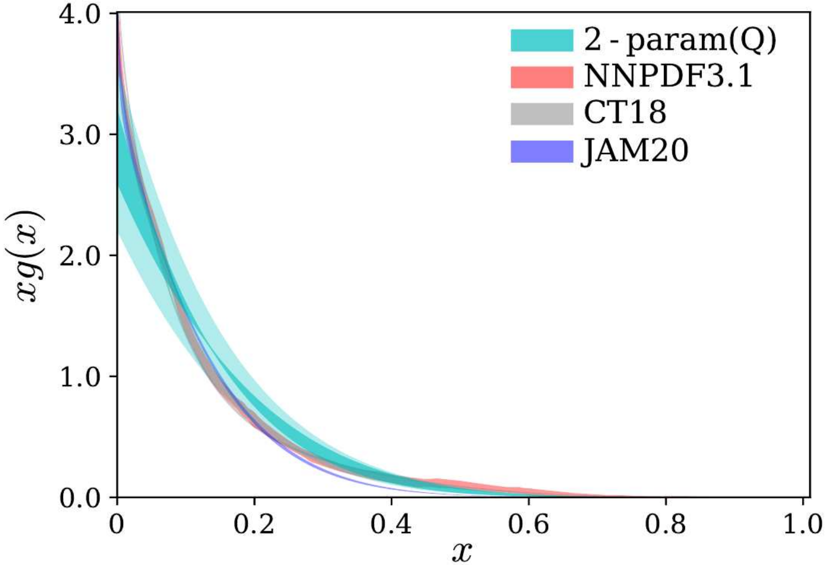
Matching – take only gg part
(mixing with singlet quark neglected)

Fit to pheno-inspired ansatz
(2 or 3-param., incl./excl. cutoff effects term)

Gluon PDFs/ITDs model motivated by counting rules based on pQCD analyses at large-x + pheno. behavior at low-x
R. Sufian, T. Liu, A. Paul, Phys. Rev. D103 (2021) 036007

Other work for gluon pseudo-PDFs:
Z. Fan et al. (MSULat), Int. J. Mod. Phys. A36(2021)13 (nucleon)
Z. Fan et al. (MSULat), 2104.06372 (pion)





First studies also for GPDs

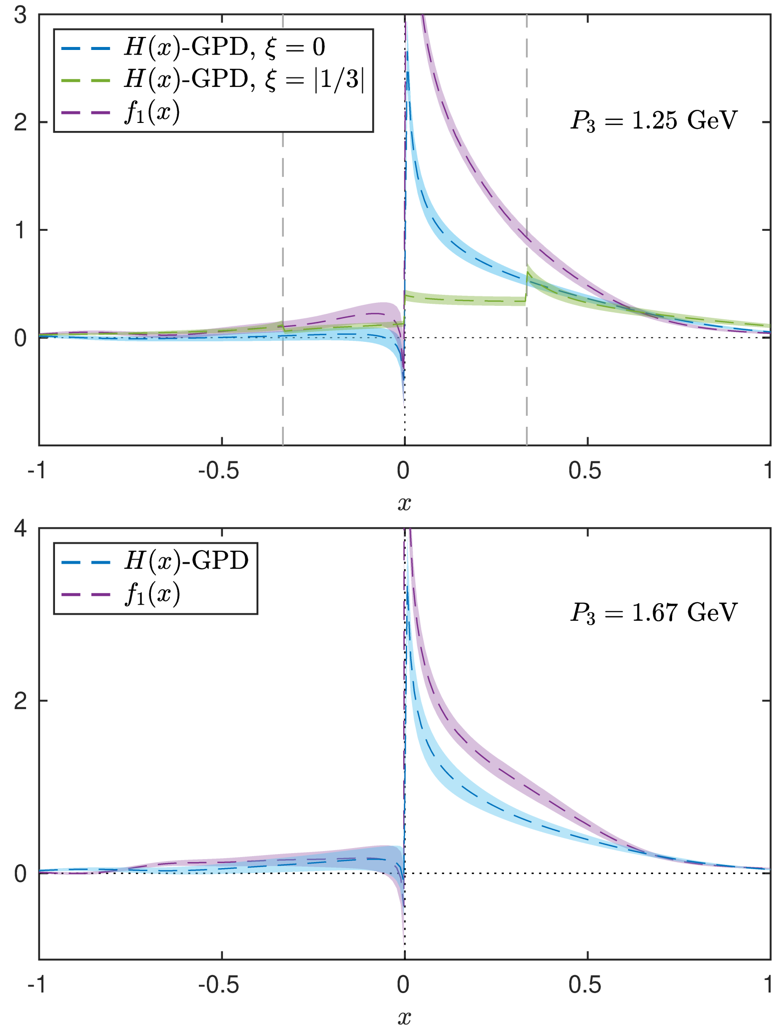
C. Alexandrou et al. (ETMC), Phys. Rev. Lett. 125 (2020) 262001

QUASI	TMF	$m_\pi = 260$ MeV	$a = 0.093$ fm

- nucleon boosts up to 1.67 GeV

Challenges:

- momentum transfer lowers the signal-to-noise ratio
- 2 or 4 GPDs $(H, E, \tilde{H}, \tilde{E})$ contribute to MEs at $Q^2 \neq 0$
  $\Rightarrow$ need to disentangle them using different projectors
- standard GPDs need Breit frame: $P_\perp^i = -P_\perp^f$
- needs optimization of momentum smearing for each $\vec{Q}$



Important insights from models:

S. Bhattacharya, C. Cocuzza, A. Metz, Phys. Lett. B788 (2019) 453
S. Bhattacharya, C. Cocuzza, A. Metz, Phys. Rev. D102 (2020) 054201

Independent work towards GPDs: H.-W. Lin, 2008.12474

Twist-3 PDFs

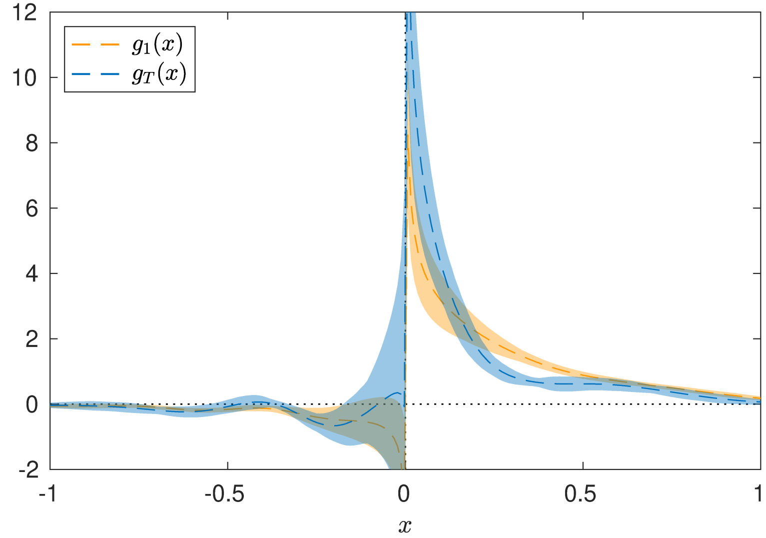
PDFs can be classified according to their twist, which describes the order in $1/Q$ at which they appear in the factorization of structure functions.

LT: **twist-2** – probability densities for finding partons carrying fraction x of the hadron momentum.

Twist-3:

QUASI	TMF	$m_\pi = 260$ MeV	$a = 0.093$ fm

- no density interpretation,
- contain important information about qgq correlations,
- appear in QCD factorization theorems for a variety of hard scattering processes,
- have interesting connections with TMDs,
- important for JLab's 12 GeV program + for EIC,
- however, measurements very difficult.

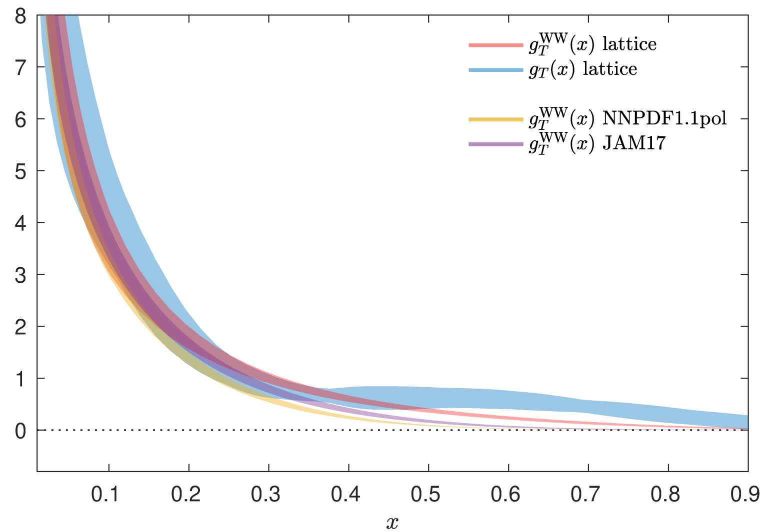


Exploratory studies:

- matching for twist-3 PDFs: g_T, h_L, e
  S. Bhattacharya et al., Phys. Rev. D102 (2020) 034005
  S. Bhattacharya et al., Phys. Rev. D102 (2020) 114025

  BC-type sum rules S. Bhattacharya, A. Metz, 2105.07282

  Note: neglected qgq correlations
  see also: V. Braun, Y. Ji, A. Vladimirov, 2103.12105
- lattice extraction of $g_T^{u-d}(x)$ and $h_L^{u-d}(x)$
  + test of Wandzura-Wilczek approximation
  S. Bhattacharya et al., Phys. Rev. D102 (2020) 111501(R)
  S. Bhattacharya et al., 2107.02574



Twist-3 PDFs

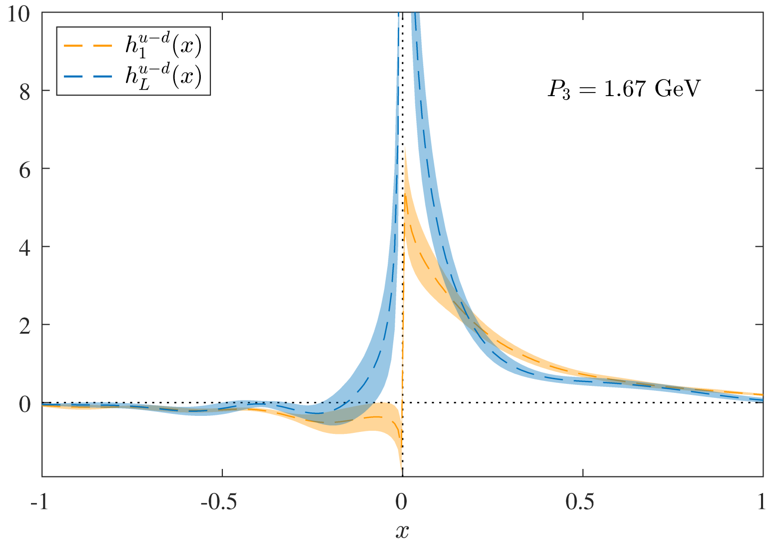
PDFs can be classified according to their twist, which describes the order in $1/Q$ at which they appear in the factorization of structure functions.

LT: **twist-2** – probability densities for finding partons carrying fraction x of the hadron momentum.

Twist-3:

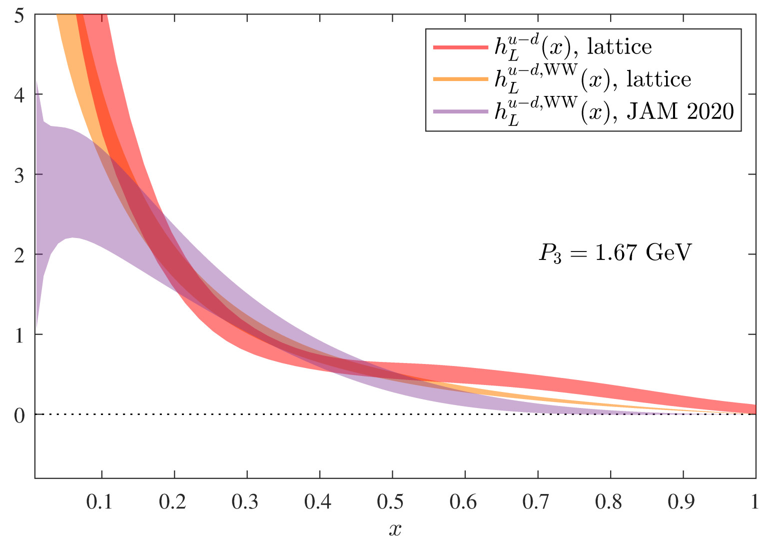
QUASI	TMF	$m_\pi = 260$ MeV	$a = 0.093$ fm

- no density interpretation,
- contain important information about qgq correlations,
- appear in QCD factorization theorems for a variety of hard scattering processes,
- have interesting connections with TMDs,
- important for JLab's 12 GeV program + for EIC,
- however, measurements very difficult.



Exploratory studies:

- matching for twist-3 PDFs: g_T, h_L, e
  S. Bhattacharya et al., Phys. Rev. D102 (2020) 034005
  S. Bhattacharya et al., Phys. Rev. D102 (2020) 114025
  BC-type sum rules S. Bhattacharya, A. Metz, 2105.07282
  Note: neglected qgq correlations
  see also: V. Braun, Y. Ji, A. Vladimirov, 2103.12105
- lattice extraction of $g_T^{u-d}(x)$ and $h_L^{u-d}(x)$
  + test of Wandzura-Wilczek approximation
  S. Bhattacharya et al., Phys. Rev. D102 (2020) 111501(R)
  S. Bhattacharya et al., 2107.02574
- first exploration of twist-3 GPDs
  S. Bhattacharya et al., 2107.12818



Pion PDFs

Interest also in pion PDFs, using several approaches.
Question: large-x behavior $(1-x)^{-1}$ vs. $(1-x)^{-2}$ decay.

C. Alexandrou et al. (ETMC), 2104.02247

moments	TMF	$m_\pi = 260$ MeV	$a = 0.093$ fm

B. Joo et al., Phys. Rev. D100 (2019) 114512

PSEUDO	clover	$m_\pi = 415$ MeV	$a = 0.127$ fm

R. Sufian et al., Phys. Rev. D102 (2020) 054508

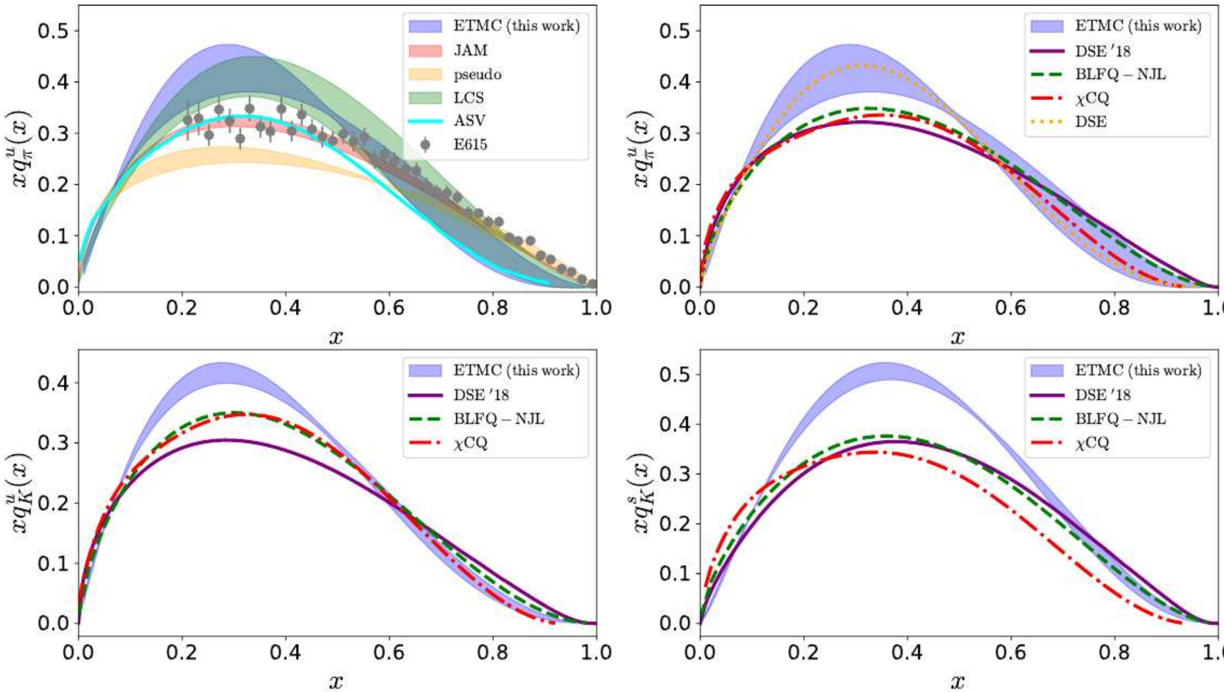
current-current (LCS)	clover	$m_\pi = 415$, 358,278 MeV	$a = 0.127$, 0.094 fm

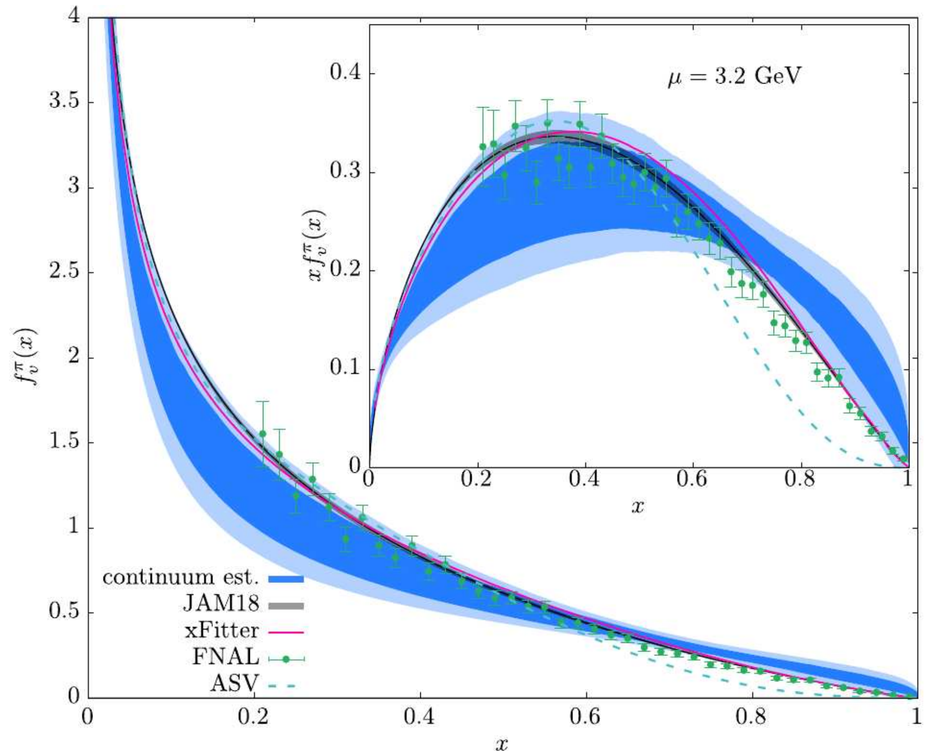
BNL – different analyses:
β from $0.66(34)(22)$ to $1.55(34)(27)$

Pheno analyses:
FNAL E615: $\beta \approx 1$ ASV: $\beta \approx 2$
JAM: $\beta \approx 1.2$ DSE: $\beta \approx 2$
xFitter: $\beta \approx 1$ NJL: $\beta \approx 1$

X. Gao et al. (BNL), Phys. Rev. D102 (2020) 094513

PSEUDO (SDF)	clover on HISQ	$m_\pi^{\rm sea} = 160$ MeV, $m_\pi^{\rm val} = 300$ MeV	$a = 0.06$, 0.04 fm





ETMC: $\beta \approx 2$, pseudo: $\beta = 1.1(4)$, LCS: 2-param.: $\beta = 1.24(22)(7)$, 3-param.: $\beta = 2.12(56)(14)$

Pion PDFs (hybrid renormalization)

The standard procedure of quasi-PDF MEs renormalization:

$$O_{\overline{\mathrm{MS}}}(z,\mu) = Z_{\overline{\mathrm{MS}}}(z,-p^2,\mu)\,\frac{O(z,a)}{Z(z,-p^2,a)}$$

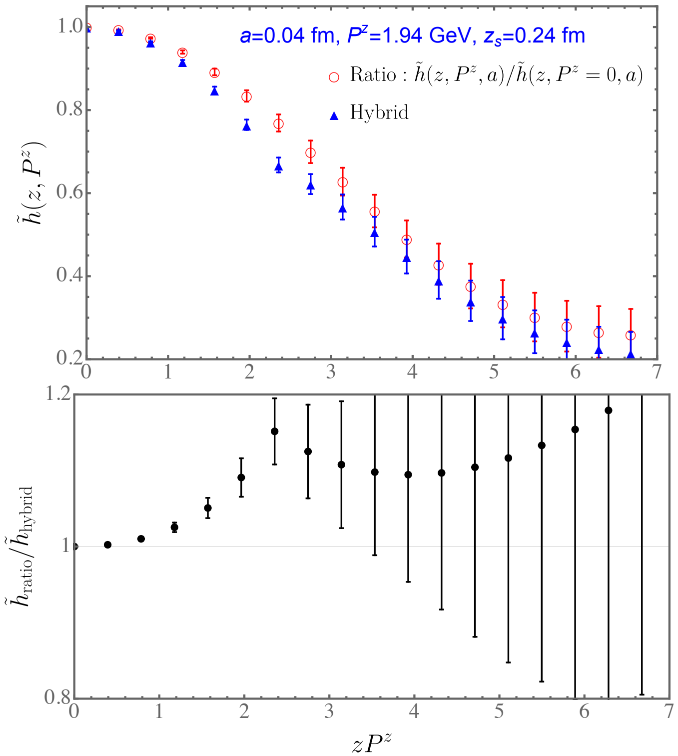
is argued to contain non-perturbative effects at large-z.
X. Ji et al., Nucl. Phys. B964 (2021) 115311

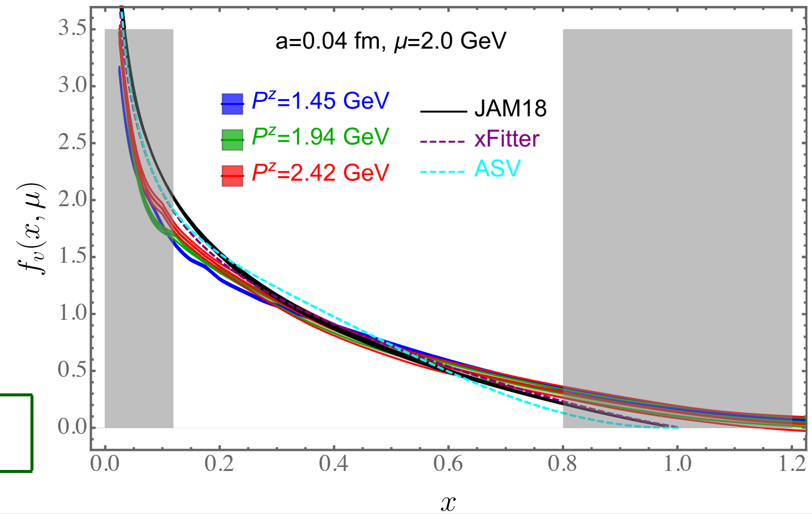
Proposed way out: **hybrid renormalization**.

- short distance $z \leq z_S \approx 0.3$ fm
  – ratio scheme / RI-MOM,
- intermediate distance 0.3 fm $\approx z_S \geq z \leq z_L \approx \Lambda_{\mathrm{QCD}}^{-1}$
  – separate renormalization of log and linear divergences:
  $Z(a,\mu)\exp(-\delta m|z|)O(z,a)$,
  δm – Wilson line mass renormalization, e.g. from the static potential or from fitting MEs at large-z
- large distance $(z_L \approx \Lambda_{\mathrm{QCD}}^{-1} \approx z_L \geq z)$
  – exponential/algebraic extrapolation (Regge-based),
- matching the different procedures at z_S and z_L.

Used recently for extraction of pion PDFs.

QUASI	clover on HISQ	$m_\pi = 300$ MeV	$a = 0.04$ fm





Meson distribution amplitudes (DAs) important for many exclusive decays.

$\longrightarrow$ represent momentum distribution of quarks/antiquarks in the leading $q\bar{q}$ Fock state of the meson's wave function.

Recent work:

- K^* and ϕ mesons with quasi (physical point, continuum limit) J. Hua et al. (LPC), 2011.09788
- π and K mesons with quasi (continuum limit) R. Zhang et al., Phys. Rev. D102 (2020) 094519
- B meson DA formalism with
  pseudo: S. Zhao, A. Radyushkin, Phys. Rev. D103 (2021) 054022
  quasi: W. Wang, Y.-M. Wang, J. Xu, S. Zhao, Phys. Rev. D102 (2020) 011502

Theoretical progress in another method, recently dubbed HOPE (heavy OPE)
Compton tensor with an auxiliary heavy quark + OPE to relate to Mellin/Gegenbauer moments of PDFs/LCDAs W. Detmold, C.-J. D. Lin, Phys. Rev. D73 (2006) 014501
W. Detmold, A. Grebe, I. Kanamori, C.-J. D. Lin, R. Perry, Y. Zhao (HOPE), 2103.09529

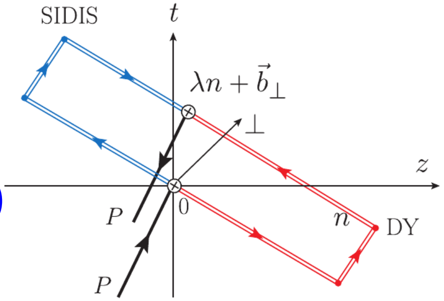
- uses flavor-changing axial vector current: $J_A^\mu = \bar{\Psi}\gamma^\mu\gamma^5\psi + \bar{\psi}\gamma^\mu\gamma^5\Psi$, $\Psi\,(\psi)$ – heavy (light) quark
- all effects of the heavy quark in Wilson coefficients, no power-divergent mixings, suppressed HTE
- recent: shown relation to other approaches, analytic structure of HOPE amplitudes, convergence radius, calculation of 1-loop Wilson coefficients for unpolarized/helicity PDFs and LCDA

Transverse momentum dependent PDFs

PDFs provide information only on the longitudinal momentum distributions, while in many cases important effects also from transverse momentum.

- Important for wide kinematical ranges in Drell-Yan, e^+e^- annihilation, SIDIS
- Example: unpolarized

$$f(x,\vec{k}_\perp) = \frac{1}{2P^+}\int \frac{d\lambda}{2\pi}\frac{d^2\vec{b}_\perp}{(2\pi)^2} e^{-i\lambda x + i\vec{k}_perp\cdot\vec{b}_\perp}\langle P|\bar{\psi}(\lambda n/2+\vec{b}_\perp)\gamma^+\mathcal{W}_n(\lambda n/2+\vec{b}_\perp)\psi(-\lambda n/2)|P\rangle$$



From: X. Ji et al., 2004.03543

- Crucial new aspect: rapidity divergences from soft gluon radiation
  $\Rightarrow$ rapidity regulator δ + UV renormalization scale μ
- Rapidity divergences can be incorporated in the soft function $S(b_\perp,\mu,\delta^+,\delta^-)$
  represents soft gluon radiation effects of a fast-moving charged particle
- Physical renormalized TMD: $f^{\rm TMD} = f/\sqrt{S}$
- Soft function:
  - ⋆ intrinsic part (rapidity-independent)
  - ⋆ rapidity-dependent part defining Collins-Soper kernel $K(b_\perp,\mu)$ – log-derivative of $f^{\rm TMD}$.
- $f^{\rm TMD}(x,b_\perp,\mu,\zeta)$ – final desired object with evolution in the 2 last arguments governed by:
  - ⋆ CS kernel for rapidity ζ
  - ⋆ γ_μ anomalous dimension (consisting of cusp and hard anomalous dimension) for renormalization scale μ
- also: single transverse-spin asymmetry & Sivers Function from LaMET
  X. Ji, Y. Liu, A. Schäfer, F. Yuan, Phys. Rev. D103 (2021) 074005
  light-front wave functions from LaMET   X. Ji, Y. Liu, 2106.05310

Intrinsic soft function

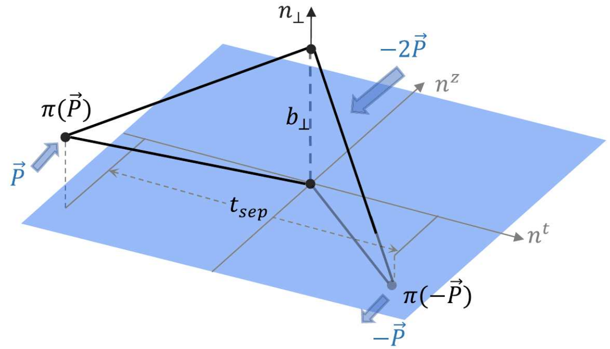
The soft function can be extracted from a pseudoscalar meson form factor

X. Ji, Y. Liu, Y.-S. Liu, Nucl. Phys. B955 (2020) 115054, Phys. Lett. B811 (2020) 135946

$$F_\Gamma(b_\perp, P^z) = \langle \pi(-P^z)|\bar{u}\Gamma u(t, b_\perp)\, \bar{d}\Gamma d(t, 0)|\pi(P^z)\rangle$$

$F_\Gamma(b_\perp, P^z)$ can be factorized into:

- intrinsic soft function
- quasi-TMDWF ≈ pion LCDA with a staple-shaped operator



From: LPC, PRL125(2020)192001

2 groups followed this strategy

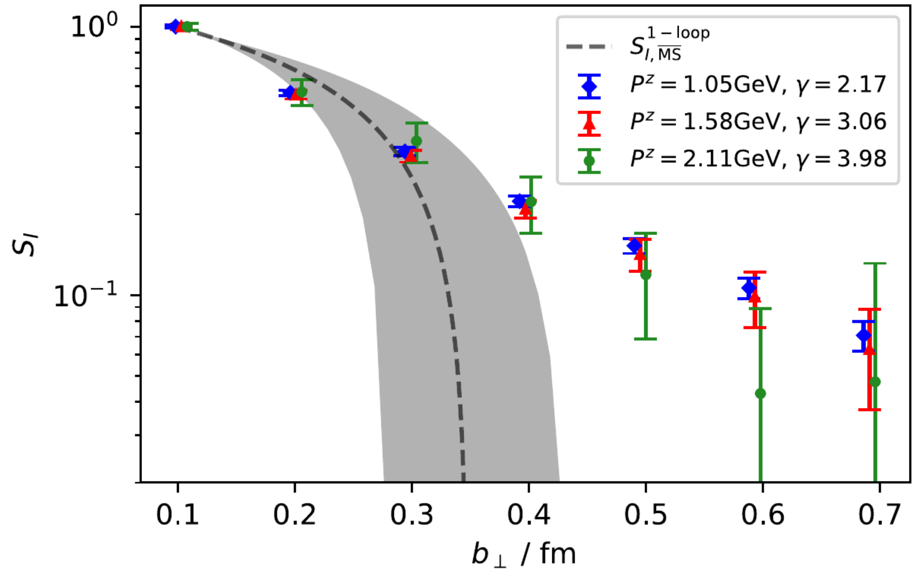
Q.-A. Zhang et al. (LPC), Phys. Rev. Lett. 125 (2020) 192001

Y. Li, S.-C. Xia et al. (Beijing+ETMC), 2106.13027

QUASI	clover	$m_\pi^{\rm sea} = 333$ MeV $m_\pi^{\rm val} = 547$ MeV	$a = 0.098$ fm

LPC calculation:

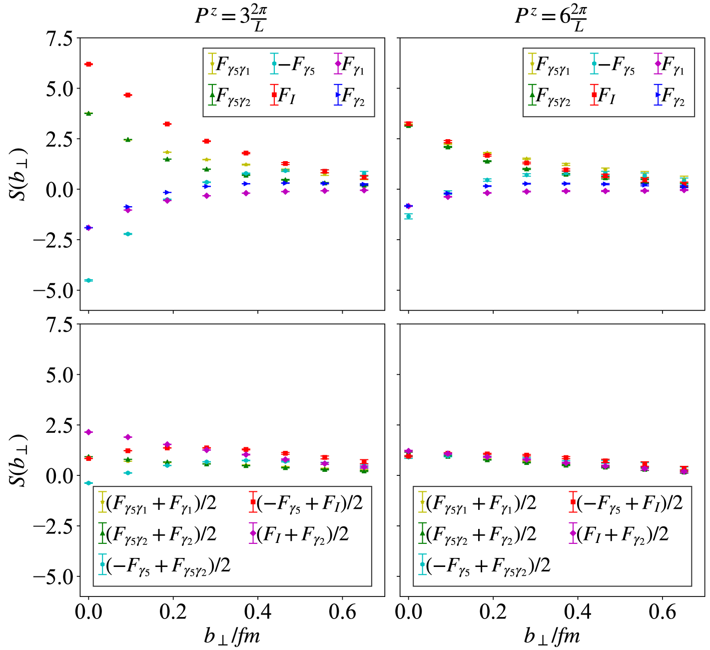
- $\Gamma = I$ – best signal, leading-twist
- renormalization of bare $S_I(b_\perp, 1/a)$: $S_I^{\overline{\rm MS}}(b_\perp, 1/a) = \frac{S_I(b_\perp, 1/a)}{S_I(b_{\perp,0}, 1/a)} S_I^{\overline{\rm MS}}(b_{\perp,0}, \mu)$ ($S_I^{\overline{\rm MS}}(b_{\perp,0}, \mu)$ from 1-loop PT)
- leading-order matching: $1/2N_c + O(\alpha_s)$



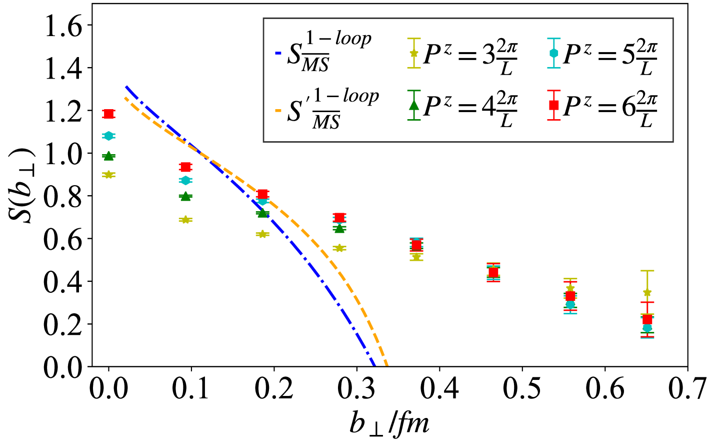
Intrinsic soft function

Beijing+ETMC calculation Y. Li, S.-C. Xia et al. (Beijing+ETMC), 2106.13027

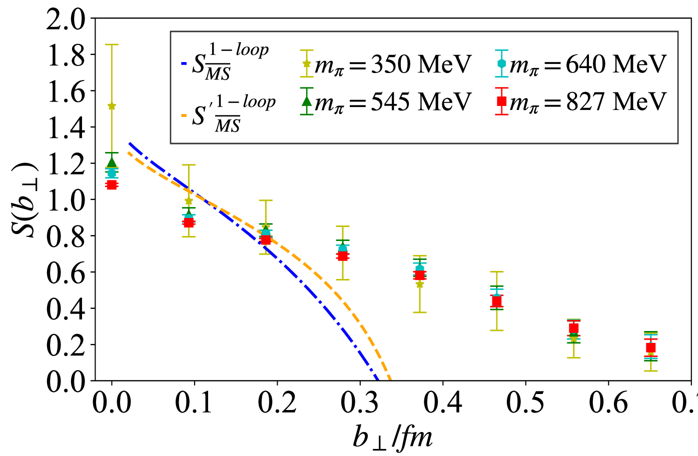
QUASI	TMF	$m_\pi^{\text{sea}}=350$ MeV $m_\pi^{\text{val}}=350$-827 MeV	$a=0.093$ fm



- $\Gamma = I, \gamma_1, \gamma_2, \gamma_5, \gamma_5\gamma_1, \gamma_5\gamma_2$
  found significant higher-twist contamination!
- considered combinations to reduce HTE
  using Fierz identities
- ratio scheme renormalization:
  $C^{\text{ratio}}(b_\perp, l, P_3) = \frac{C(b_\perp,l,P_3)}{C(b_\perp,l,0)} C^{\overline{\text{MS}}}(0,0,0)$
  ($C^{\overline{\text{MS}}}(0,0,0)$ – standard local RI' renormalization)
- leading-order matching: $1/2N_c + O(\alpha_s)$
- test of convergence in hadron boost P_3



test of pion mass effects



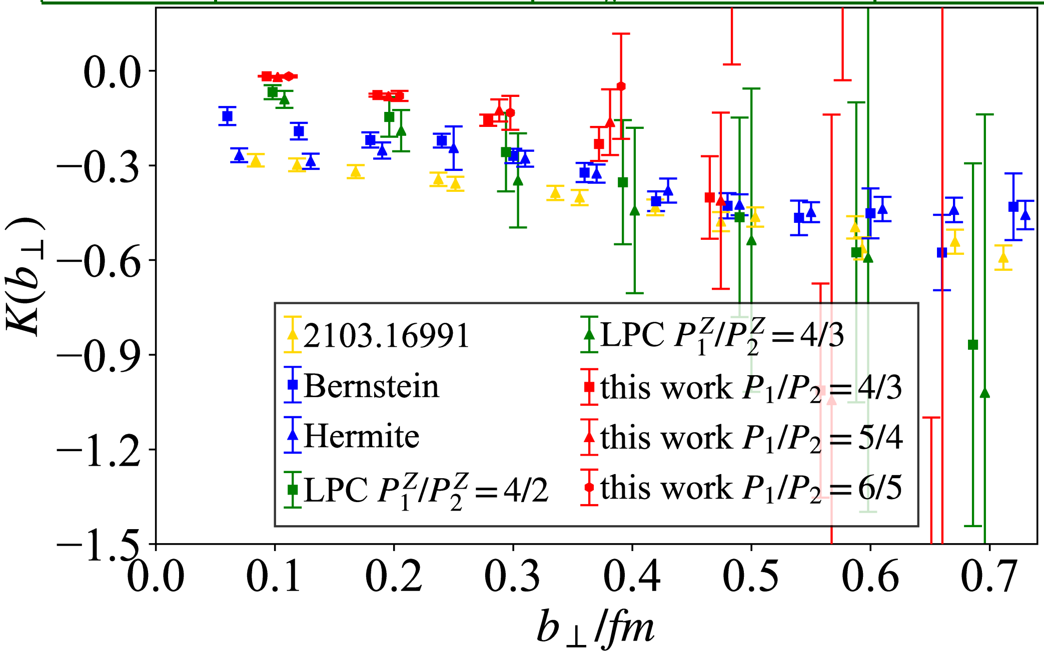
The CS kernel governs the rapidity evolution of TMDs
Two approaches:

- ratio of TMDs at different rapidities
  M. Ebert, I. Stewart, Y. Zhao, PRD99(2019)034505
- ratios of first Mellin moments of TMDs
  M. Schlemmer et al., 2103.16991

MOMENTS	clover	$m_\pi = 422$ MeV	$a = 0.085$ fm

P. Shanahan, M. Wagman, Y. Zhao, PRD102(2020)014511; 2107.11930

QUASI	clover (quench.) clover on HISQ	$m_\pi^{\rm val} = 1.2$ GeV $m_\pi^{\rm val} = 538$ MeV	$a = 0.06$ fm $a = 0.12$ fm

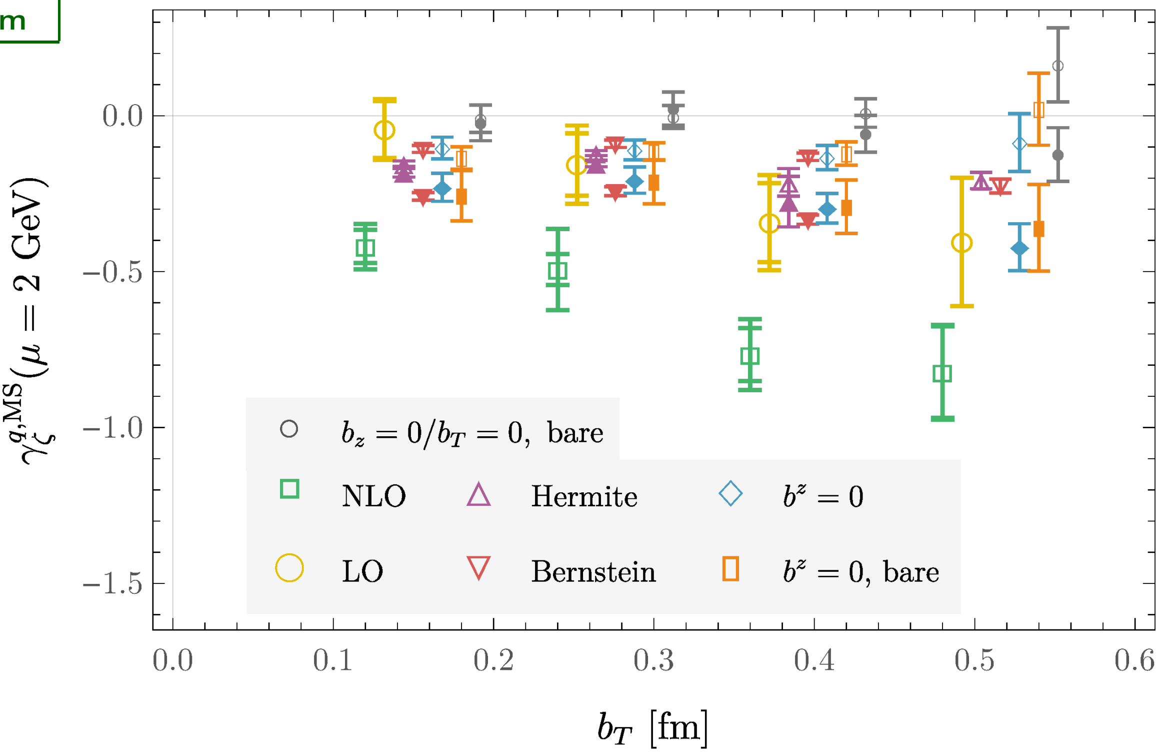


Q.-A. Zhang et al. (LPC), Phys. Rev. Lett. 125 (2020) 192001

QUASI	clover	$m_\pi^{\rm sea} = 333$ MeV $m_\pi^{\rm val} = 547$ MeV	$a = 0.098$ fm

Y. Li, S.-C. Xia et al. (Beijing+ETMC), 2106.13027

QUASI	TMF	$m_\pi^{\rm sea} = 350$ MeV $m_\pi^{\rm val} = 350$-827 MeV	$a = 0.093$ fm



Key prospects for the future

1. Robustness and reliability of the lattice extraction of x-dependent distributions
   $\Rightarrow$ **towards precision studies**
   improvements of lattice techniques
   study and removal of systematic effects

2. Exploration of new directions
   new kinds of distributions higher-twist, GPDs, TMDs, LFWFs
   other hadrons?
   can be phenomenologically relevant, e.g. K^*, ϕ
   can shed light on the nucleon, e.g. Δ^+

3. Synergy between lattice and phenomenology
   unpolarized PDFs – benchmark
   other distributions – potentially crucial impact

Robustness/reliability of lattice extraction

- Lattice-specific systematics:
  - ⋆ isolation of the ground state hadron
  - ⋆ discretization effects
  - ⋆ finite volume effects
  - ⋆ pion mass dependence (if not working at the physical point)

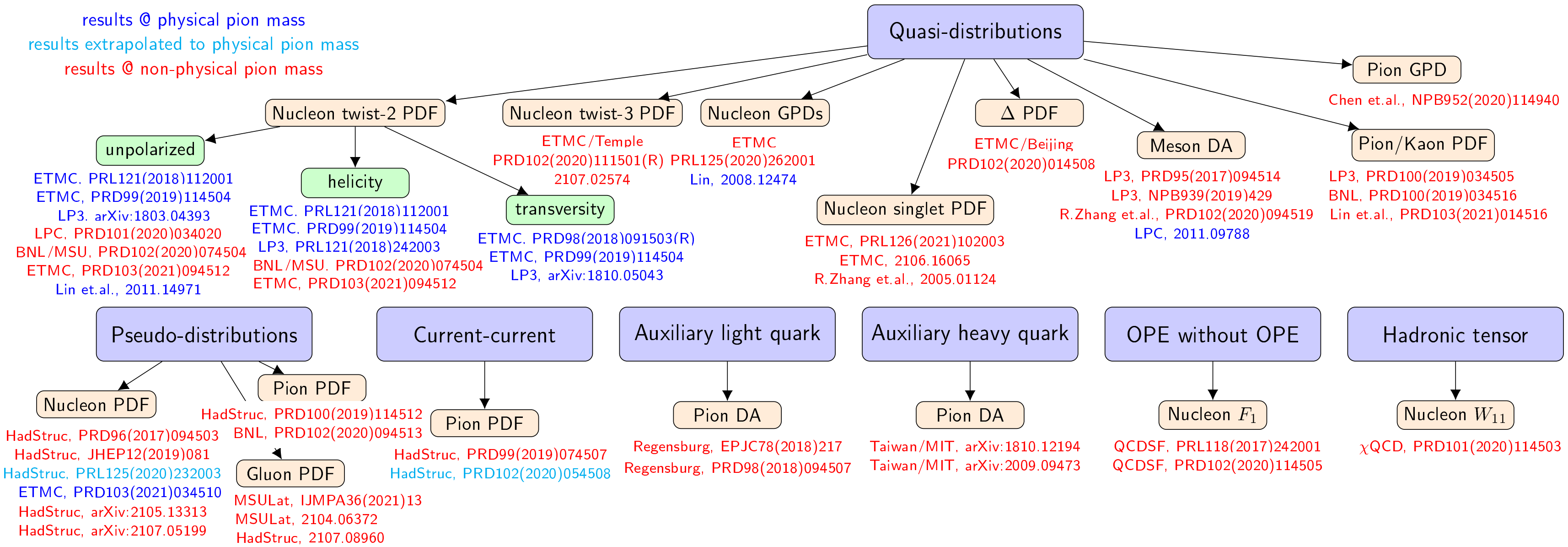
  Note: hierarchy of systematics needs to be observed
- Broader systematics of the lattice calculation:
  - ⋆ reconstruction of the x-dependence
  - ⋆ non-perturbative renormalization
  - ⋆ truncation effects: conversion, evolution, matching
  - ⋆ higher-twist effects

Key challenges:

- lattice: **reliably reach large hadron boosts**
- lattice: **control all lattice-specific systematics**
- pheno: **insights into HTE?**

Conclusions

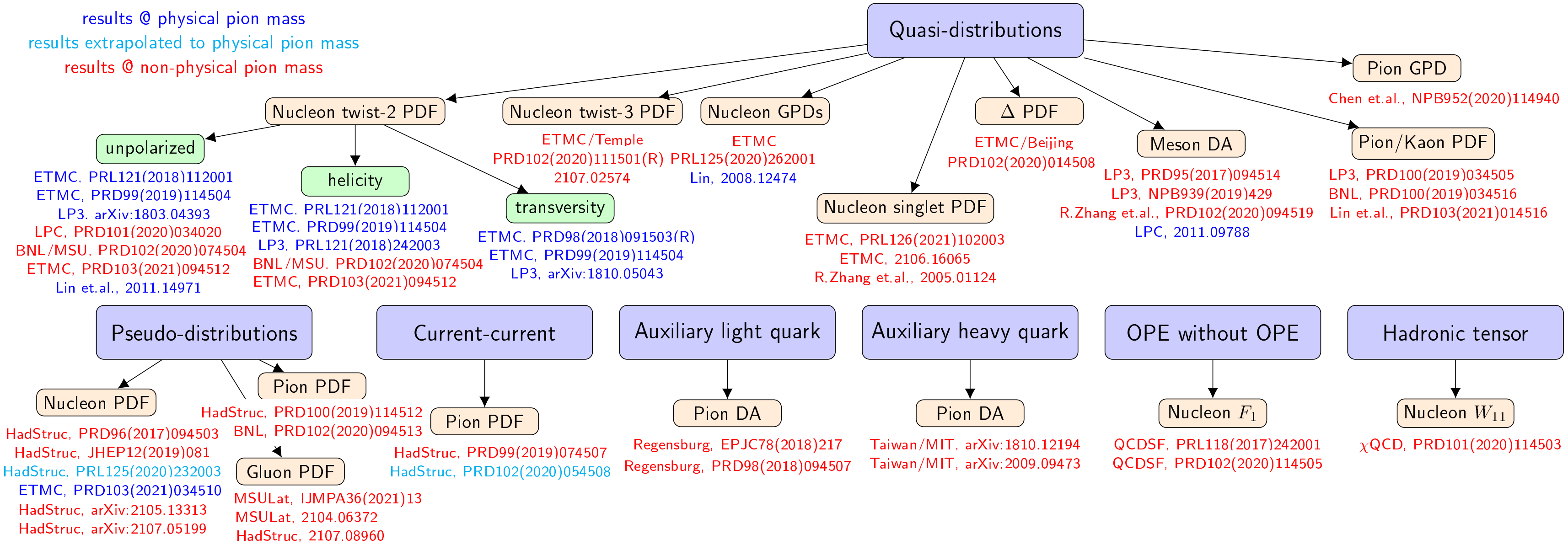
- Message of the talk: **enormous progress in lattice calculations of x-dependent distributions with very encouraging results!**



- Increasing number of distribution types accessible for lattice.
- However, there are still major challenges related to control of several sources of systematics.
- Expect:
  - ⋆ slow, but consistent progress,
  - ⋆ complementary role of LQCD and phenomenology.

Conclusions

- Message of the talk: **enormous progress in lattice calculations of x-dependent distributions with very encouraging results!**



- Increasing number of distribution types accessible for lattice.
- However, there are still major challenges related to control of several sources of systematics.
- Expect:
  - ⋆ slow, but consistent progress,
  - ⋆ complementary role of LQCD and phenomenology.

Thank you for your attention!