Ivan Vitev

Heavy Flavor at the EIC



A Virtual Tribute to Quark Confinement and Hadron Spectra, August 2-6, 2021



Outline of the talk





- Open heavy flavor hadron production
- Heavy flavor jets and jet substructure
- Quarkonium production and exotics



H. Li, Z. Liu, I.V., Phys. Lett. B 816, 136261 (2021)
H. Li, I.V., Phys. Rev. Lett. 126, 252001 (2021)
H. Li, Z. Liu, I.V., in preparation
I. Olivant, I.V., in preparation

For expert-level comprehensive discussion of HF@EICtopics seehttps://indico.bnl.gov/event/9273/





Confinement and hadronization

- Does QCD predict confinement? Yes
- Do we fully understand the static properties of confinement? Not fully Do we understand dynamically how confinement occurs? No
- - Perturbative hadronization
- J. Collins et al. (1982)

Hadronization in • event generators

PYTHIA et al.

String fragmentation



C. McNeile et al. (2013)

10 FOUR-MOMENTTRANSFER Q (GeV)

Deepinelasticscatt

attice gaugetheor , bbdecays

scaling violation

eventshapes

100

ppOW+jets

ep,ppOjets

bbthreshold pp,pp'04+X

STRONGCOUPLING

0.



Hadronization in the nuclear environment - in can provide insights into the dynamics and the space-time picture of hadronization

Total HF cross sections and lessons from HERA data

Realistic evaluations combine FFNS and VFNS, subtraction of double counting is required

$$\begin{split} F_{2,h}^{FFN} &= \sum_{k=1}^{\infty} a_s^k(n_f) \sum_{i=q,g} H_{2,i}^{(k)}(n_f) \otimes f_i(n_f) \,, \\ F_{2,h}^{ZMVFN} &= \sum_{k=1}^{\infty} a_s^k(n_f+1) \sum_{i=q,g} C_{2,i}^{(k)}(n_f+1) \otimes f_i(n_f+1) \,. \end{split}$$

$$\sum_{k=0}^{a} a_{s}(n_{f}+1) \sum_{i=q,g,h} C_{2,i}(n_{f}+1)$$



Confronting HF schemes with HERA data

- Transitions between FFN and VFS
- Depending on order/prescription,
 Q2 good description of DIS data

S. Alekhin et al . (2020)

Hybrid variable flavor number scheme

 Different choices of matching mass values. It allows the user, for example to use Nf=4 above Mb scale for example



Heavy flavor in polarized reactions at the EIC

The EIC is a polarized machine – will constrain precisely the spin content of nucleons and nuclei and their 3D landscape

Hadrons (DD-bar)

L. Zheng et al. 2018

$$\frac{d\sigma^{UT}(\boldsymbol{S}_{T})}{dQ^{2}dyd^{2}\boldsymbol{q}_{T}dy_{J}d^{2}\boldsymbol{p}_{T}} = \sin(\phi_{q} - \phi_{s}) H(Q, y, p_{T}, y_{J}, \mu_{h}) \int_{0}^{\infty} \frac{b^{2}db}{4\pi} J_{1}(b q_{T}) f_{1T,g/N}^{\perp,f}(x, \mu_{h*}) \\ \times \exp\left[-\int_{\mu_{h*}}^{\mu_{h}} \frac{d\mu}{\mu} \Gamma^{h}(\alpha_{s}) - 2\int_{\mu_{h*}}^{\mu_{j}} \frac{d\mu}{\mu} \Gamma^{j_{Q}}(\alpha_{s}) - \int_{\mu_{h*}}^{\mu_{cs}} \frac{d\mu}{\mu} \left(\bar{\Gamma}^{cs_{Q}}(\alpha_{s}) + \bar{\Gamma}^{cs_{\bar{Q}}}(\alpha_{s})\right)\right] \\ \times \exp\left[-S_{NP}^{\perp}(b, Q_{0}, n \cdot p_{g})\right] \cdot \quad \text{Jets} \qquad \text{Z. Kang et al. 2020}$$









See talk by E. Aschenauer

Rather significant asymmetries. Hadronization reduced the asymmetry. Experimental feasibility studies have also been performed (on charm meson)

Production of semi-inclusive hadrons and jets



Based on QCD / SCET factorization. Calculations at next-to-leading order (and resummation where applicable) are standard. Calculations at NNLO also exist but still time consuming

$$E_h \frac{d^3 \sigma^{\ell N \to hX}}{d^3 P_h} = \frac{1}{S} \sum_{i,f} \int_0^1 \frac{dx}{x} \int_0^1 \frac{dz}{z^2} f^{i/N}(x,\mu)$$
$$\times D^{h/f}(z,\mu) \Big[\hat{\sigma}^{i \to f} + f_{\text{ren}}^{\gamma/\ell} \Big(\frac{-t}{s+\mu}, \mu \Big) \hat{\sigma}^{\gamma i \to f} \Big].$$
$$E_J \frac{d^3 \sigma^{\ell N \to jX}}{d^3 P_J} = \frac{1}{S} \sum_{i,f} \int_0^1 \frac{dx}{x} \int_0^1 \frac{dz}{z^2} f_{i/N}(x,\mu)$$

 $\times \hat{\sigma}^{i \to f}(s, t, u, \mu) J_f(z, p_T R, \mu) ,$

$SCET_{(M),G}$ and LCWF

G. Ovanesyan et al . (2012)

Z. Kang et al . (2016)

In-medium splitting functions necessary for higher order and resumed calculations

Develop specific EFTs for particle propagation in matter



Often used in saturation calculations. Can get on one shot massless and massive splitting functions

$$\begin{split} \frac{dN}{xd^2\mathbf{k}_{\perp}} \Big)_{q \to qg} &= \frac{\alpha_s}{2\pi^2} C_F \frac{1 + (1 - x)^2}{x} \int \frac{d\Delta z}{\lambda_g(z)} \int d^2 \mathbf{q}_{\perp} \frac{1}{\sigma_{el}} \frac{d\sigma_{el}^{\text{medium}}}{d^2 \mathbf{q}_{\perp}} \left[-\left(\frac{A_{\perp}}{A_{\perp}^2}\right)^2 + \frac{B_{\perp}}{B_{\perp}^2} \cdot \left(\frac{B_{\perp}}{B_{\perp}^2} - \frac{C_{\perp}}{C_{\perp}^2}\right) \right] \\ &\times \left(1 - \cos[(\Omega_1 - \Omega_2)\Delta z]\right) + \frac{C_{\perp}}{C_{\perp}^2} \cdot \left(2\frac{C_{\perp}}{C_{\perp}^2} - \frac{A_{\perp}}{A_{\perp}^2} - \frac{B_{\perp}}{B_{\perp}^2}\right) \left(1 - \cos[(\Omega_1 - \Omega_3)\Delta z]\right) \\ &+ \frac{B_{\perp}}{B_{\perp}^2} \cdot \frac{C_{\perp}}{C_{\perp}^2} \left(1 - \cos[(\Omega_2 - \Omega_3)\Delta z]\right) + \frac{A_{\perp}}{A_{\perp}^2} \cdot \left(\frac{A_{\perp}}{A_{\perp}^2} - \frac{D_{\perp}}{D_{\perp}^2}\right) \cos[\Omega_4\Delta z] \\ &+ \frac{A_{\perp}}{A_{\perp}^2} \cdot \frac{D_{\perp}}{D_{\perp}^2} \cos[\Omega_5\Delta z] + \frac{1}{N_c^2} \frac{B_{\perp}}{B_{\perp}^2} \cdot \left(\frac{A_{\perp}}{A_{\perp}^2} - \frac{B_{\perp}}{B_{\perp}^2}\right) \left(1 - \cos[(\Omega_1 - \Omega_2)\Delta z]\right) \right]. \end{split}$$

- Factorize form the hard part
- Gauge-invariant
- Depend on the properties of the medium
- Can be expressed as proportional to Altarelli-Parisi



M. Sievert et al . (2018)

Differences between AA and eA

 AA and eA collisions are very different. Due to the LPM effect the "energy loss" decreases rapidly. The kinematics to look for in-medium interactions / effects on hadronization very different



- Jets at any rapidity roughly in the co-moving plasma frame (Only~ transverse motion at any rapidity)
- Largest effects at midrapidity
- Higher C.M. energies correspond to larger plasma densities



- Jets are on the nuclear rest frame.
 Longitudinal momentum matters
- Largest effects are at forward rapidities
- Smaller C.M. energies (larger only increase the rapidity gap)

Heavy flavor hadrons at the EIC

Multiple uses of heavy flavor

 Constrain gluon and c/b distributions.
 Look for intrinsic charm





- Constrain the transport properties of cold nuclear matter
- Shed light on the picture of hadronization, differentiate between energy loss and hadron absorption
- Go beyond energy loss phenomenology at the EIC



X. Li et al. (2020)

Modification of FFs

Vacuum splitting functions provide correction to vacuum showers and correspondingly modification to DGLAP evolution for FFs

$$\frac{dD_{q}(z,Q)}{d\ln Q} = \frac{\alpha_{s}(Q^{2})}{\pi} \int_{z}^{1} \frac{dz'}{z'} \left\{ P_{q \to qg}(z',Q) D_{q}\left(\frac{z}{z'},Q\right) + P_{q \to gq}(z',Q) D_{g}\left(\frac{z}{z'},Q\right) - P_{q \to gq}\left(\frac{z}{z'},Q\right) - P_{q \to gq}\left(\frac{z}{z'$$



Always enhancement at small z but for pions (light hadrons) at very small values – mostly suppression

 Very pronounced differences between light and heavy flavor fragmentation

Light and heavy flavor suppression at the EIC

2.0 $\dots \pi^+$ $18 \text{ GeV}(e) \times 275 \text{ GeV}(A)$ $- D^{0}$ $-2 < \eta < 0$ 1.5 $-B^{0}$ $R_{\rm eA}(p_T)$ 0.5 Backward rapidity, large C.M. energy 0.0 2 4 6 8 10 p_T [GeV] $5 \text{ GeV}(e) \times 40 \text{ GeV}(A)$ π^{\neg} D^0 $2 < \eta < 4$ 1.5 R^0 Forward rapidity, small C.M. energy $R_{\rm eA}(p_T)$ 0 0.0 3 5 7 6 8 p_T [GeV]

Given the much larger C.M. energy that at HERMES this is the picture to study first



Light pions show the largest nuclear suppression at the EIC. However to differentiate models of hadronization heavy flavor mesons are necessary



Modification of heavy flavor cross sections

A more differential ratio vs the momentum fraction of the hadron

The difference in the suppression pattern of pions and D, B mesons is characteristic of the in-medium evolution/energy loss approach

Detailed and constrained predictions for the EIC

$$R_{eA}^{h}(p_T, \eta, z) = \frac{\frac{N^{h}(p_T, \eta, z)}{N^{\text{inc}}(p_T, \eta)}\Big|_{e+Au}}{\frac{N^{h}(p_T, \eta, z)}{N^{\text{inc}}(p_T, \eta)}\Big|_{e+p}}$$

Normalized by inclusive large radius jet production. To LO equivalent inclusive normalization



Jet production

Z. Kang et al. (2016)

L. Dai et al. (2016)

A useful modern way (though not unique) to calculate jet cross sections

Factorization formula

$$E_J \frac{d^3 \sigma^{lN \to jX}}{d^3 P_J} = \frac{1}{S} \sum_{i,f} \int_0^1 \frac{dx}{x} \int_0^1 \frac{dz}{z^2} f_{i/N}(x,\mu)$$

 $\times \hat{\sigma}^{i \to f}(s,t,u,\mu) J_f(z,p_T R,\mu) ,$
 $\mu_J = \omega_J \tan \frac{\mathcal{R}}{2} = (2p_T \cosh \eta) \tan \left(\frac{R}{2 \cosh \eta}\right) \approx p_T R$
In-medium jet functions
 $J_q^{\text{med},(1)}(z,\omega R,\mu) = \left[\int_{z(1-z)\omega \tan(R/2)}^{\mu} dq_\perp P_{qq}(z,q_\perp)\right]$
 $+ \int_{z(1-z)\omega \tan(R/2)}^{\mu} dq_\perp P_{gq}(z,q_\perp) .$

- Stable in numerical implementation
- Similarly for gluon jets

H. Li et al. (2020)



Cross section contribution

+





The physics of in-medium jet modification

Jet results at the EIC

H. Li et al. (2020)

$$R_{\rm eA}(R) = \frac{1}{A} \frac{\int_{\eta_1}^{\eta_2} d\sigma / d\eta dp_T \big|_{e+A}}{\int_{\eta_1}^{\eta_2} d\sigma / d\eta dp_T \big|_{e+p}}$$



Two types of nuclear effect play a role

- Initial-state effects parametrized in nuclear parton distribution functions or nPDFs
- Final-state effects from the interaction of the jet and the nuclear medium – in-medium parton showers and jet energy loss

How to separate them? Define the ratio of modifications for 2 radii (it is a double ratio)

$$R_R = R_{eA}(R) / R_{eA}(R=1)$$



- Jet energy loss effects are larger at smaller C.M. energies
- Remarkably, effects can be almost a factor of 2!

Inclusive heavy jet production



Furthermore, there can be a significant Weiszacker-Williams photon contribution

Heavy flavor jets at EIC



Z. Liu et al. (2021)



- The modification of heavy flavor jets in eA is significant
- There is much larger sensitivity to the gluon distributions but initial-state and final-state effects can still be separated
- There is a pronounced rapidity dependence of the heavy flavor jet suppression



We have also done the calculation for b-jets and slightly smaller, but still significant modification

Heavy flavor jets substructure in DIS



Z. Liu et al. (2021)



Related to the modification of jet cross sections is the modification of jet substructure. Example - Soft dropped momentum sharing distributions

- Modification of both c-jets and b-jets substructure in eA is relatively small
- It is dominated by limited phase space



Kinematically not possible in DIS but illustrates very well the difference with HIC

Intrinsic charm and strangeness at the EIC

EIC will finally have the precision to answer long standing questions about large-x structure – strangeness and intrinsic charm

- Intrinsic charm genuine non-perturbative contribution to the proton wave function – can affect HQ schemes, masses, global fits
 Strangeness – can be accessed via CC reactions.
- Strangeness can be accessed via CC reactions. Requires high statistics, can look for enhanced strangeness







Double charm jet NC event

Reconstructed Jet p_T [GeV]

Production of quarkonia



• NRQCD factorization formula. Short distance cross sections (perturbatively calculable) and long distance matrix elements (fit to data, scaling relations)

$$d\sigma(a+b\to Q+X) = \sum_{n} d\sigma(a+b\to Q\overline{Q}(n)+X) \langle \mathcal{O}_n^{\mathcal{Q}} \rangle$$

Results in DIS and EIC specifics

Data and photoproduction theory at HERA

M. Butenchoen *et al.* (2010)



• Good description of cross sections can can be achieved but still some tensions remain in understanding polarization, especially at low p_T

• At EIC we have lower CM energies than HERA. Production at low p_T can probe gluon TMDs and universal Shape functions

$$\frac{d\sigma}{d^2\mathbf{q}} = \sigma_0([n]) H(2m_Q, \mu; [n]) \int d^2\mathbf{k} \, \mathbf{F_{g/P}}(\mathbf{x}, \mathbf{k}) \, \mathbf{Sh}(\mathbf{q} - \mathbf{k}; [\mathbf{n}])$$

Low transverse momentum gluon production

S. Fleming *et al.* (2019)

NRQCD with Glauber Gluons

1

• At the level of the Lagrangian

 $p_Q \sim (\lambda^2, \lambda, \lambda, \lambda) \qquad \qquad \ell_Q \sim (\lambda^2, \lambda, \lambda, \lambda)$

Possible scaling for the virtual gluons interacting

with the heavy quarks $\begin{array}{cccc}
0 & 1 & 2 & 3 & + & - & \bot \\
q_G \sim (\lambda^2, \lambda^1, \lambda^1, \lambda^2) \sim (\lambda^2, \lambda^2, \lambda_{\perp})_n
\end{array}$

2) $q_C \sim (\lambda^2, \lambda^1, \lambda^1, \lambda^1) \sim (\lambda^1, \lambda^1, \lambda_{\perp})_n$

 $\mathcal{L}_{\mathrm{NRQCD}_G} = \mathcal{L}_{\mathrm{NRQCD}} + \mathcal{L}_{Q-G/C}(\psi, A_{G/C}^{\mu,a})$

$$+ \mathcal{L}_{g-G/C}(A_s^{\mu,b}, A_{G/C}^{\mu,a}) + \psi \longleftrightarrow \chi$$

Y. Makris *et al.* (2019)

- Calculated the leading power and next to leading power contributions 3 different ways
- Glauber gluons transverse to the direction of propagation contribution \mathcal{L}_{0}^{t}
- Coulomb gluons isotropic momentum distribution

Background field method	Perform a shift in the gluon field in the NRQCD Lagrangian then perform the power-counting	
Hybrid method	From the full QCD diagrams for single effective Glauber/Coulomb gluon perform the corresponding power-counting, read the Feynman rules	
Matching method	Full QCD diagrams describing the forward cattering of incoming heavy quark and a ight quark or a gluon. We also derive the cree level expressions of the effective fields in terms of the QCD ingredients	

$$^{(0)}_{Q-G/C}(\psi, A^{\mu,a}_{G/C}) = \sum_{\mathbf{p},\mathbf{q}_T} \psi^{\dagger}_{\mathbf{p}+\mathbf{q}_T} \left(-gA^0_{G/C} \right) \psi_{\mathbf{p}} \quad (collinear/static/soft).$$

$$\mathcal{L}_{Q-G}^{(1)}(\psi, A_G^{\mu, a}) = g \sum_{\mathbf{p}, \mathbf{q}_T} \psi_{\mathbf{p}+\mathbf{q}_T}^{\dagger} \Big(\frac{2A_G^{\mathbf{n}}(\mathbf{n} \cdot \boldsymbol{\mathcal{P}}) - i \Big[(\boldsymbol{\mathcal{P}}_{\perp} \times \mathbf{n}) A_G^{\mathbf{n}} \Big] \cdot \boldsymbol{\sigma}}{2m} \Big) \psi_{\mathbf{p}} \quad (collinear)$$
$$\mathcal{L}_{Q-C}^{(1)}(\psi, A_C^{\mu, a}) = 0 \quad (static)$$

$$\mathcal{L}_{Q-C}^{(1)}(\psi, A_C^{\mu, a}) = g \sum_{\mathbf{p}, \mathbf{q}_T} \psi_{\mathbf{p}+\mathbf{q}_T}^{\dagger} \Big(\frac{2\mathbf{A}_C \cdot \boldsymbol{\mathcal{P}} + [\boldsymbol{\mathcal{P}} \cdot \mathbf{A}_C] - i \Big[\boldsymbol{\mathcal{P}} \times \mathbf{A}_C\Big] \cdot \boldsymbol{\sigma}}{2m} \Big) \psi_{\mathbf{p}} \quad (soft)$$
_{Quarkonium Formation}

Phenomenological applications



See talks by Y. Akamatsu & A. Viaro

Phenomenology with NRQCD_G

At the moment data exists in heavy ion collisions (as far as nucleus A is involved)

 Correct hierarchy of excited and ground state suppression

2.00

- QGP thermal effect on the wavefunction and collisional dissociation
- EIC only CNM collisional effects

Collisional interactions



I. Olivant et al. in progress

Y. Makris *et al.* (2019)



Work to extend this to e+A is under way. Cold nuclear matter vs QGP, lower transverse momenta

Quarkonia and small-x physics



(a) Dependence on $x_{\mathbb{P}}$.

(b) MV model compared to the LCPT dipole.

Conclusions

- Heavy flavor production is an essential part of the EIC science. For expert level discussion of various experimental and theoretical aspects see <u>https://indico.bnl.gov/event/9273/</u> HF can shed light on the dynamical aspects of confinement
- Heavy flavor provides complementary probe of the TMD stricture of nucleons/nuclei, small-x saturation physics, parton distributions, and transport properties of cold nuclear matter
- Open heavy flavor production has been calculated at NLO, in-medium evolution of fragmentation understood. We have detailed and differential predictions of D and B meson cross section modification, which can differentiate between energy loss and hadron absorption
- Calculations of open heavy flavor jets in e+A collisions are near complete. They
 require more careful treatment of energy/mass scales. Preliminary results are
 promising large suppression of c-jets. Heavy jet substructure in reactions with nuclei
 quite different from HIC, exhibits new features that could be studied at the EIC
- Quarkonium production, as described by NRQCD, can be further constrained at EIC. New developments are theories of quarkonium production in matter (NRQCD_G), low P_T and gluon TMDs at the EIC. e+A collisions can shed light on the structure of heavy exotics

Confinement and hadronization

Do we understand dynamically how confinement occurs? No



$$\mathcal{P}_{A/I}(z, P_{\rm T}) = \frac{1}{2z(2\pi)^3} \int dx^- d^2 x_{\rm T} e^{ik^+x^- -ik_{\rm T}x_{\rm T}}$$

 Hadronization in event generators

 $\times \frac{1}{3} \operatorname{tr}_{\operatorname{color}} \frac{1}{2} \operatorname{tr}_{\operatorname{Dirac}} \{ \gamma^+ \langle 0 | \psi(x) a_A(P^+, 0) a_A(P^+, 0) \overline{\psi}(0) | 0 \rangle \}$

String fragmentation PYTHIA et al. $f(z) \propto \frac{1}{z} (1-z)^{a} \exp\left(-\frac{bm_{\perp}^{2}}{z}\right)$

Cluster hadronization



J. Collins et al. (1982)

program model	PYTHIA string	HERWIG cluster
energy-momentum picture	powerful	simple
	predictive	unpredictive
parameters	few	many
flavour composition	messy	simple
	unpredictive	in-between
parameters	many	few

Hadronization in the nuclear environment in can provide insights into the dynamics and the space-time picture of hadronization

Differential branching spectra



Most importantly – additional medium-induced contribution to factorization formulas (final-state) – Additional scaling violation due to the medium-induced shower. Additional component to jet functions

- Production of hadrons and jets can be understood from the broader and softer splitting functions
- Holds to higher orders in opacity



Use of LHC data to constrain heavy favor PDFs – PROSA collaboration

O. Zenaiev et al . (2018)

 Adding LHCb and ALICE data to the heavy flavor data from HERA

Inclusion of heavy flavor reduces PDF uncertainties for sea quarks and gluons at small x, especially FFNS









Important to do combined analysis at the EIC

Exotics states at the EIC



 $\sigma_{\gamma n \to X^+ n}(W, Q^2 = 0) f(M_X),$

- Photoproduction through photon-Pomeron fusion lead predominantly to J^{PC}=1⁻⁻ states like the J/ψ, etc., so is only sensitive to exotic with those quantum numbers
- Photon-Reggeon fusion leads to states with a wider range of spin, parity and even charge



Heavy exotic states at the EIC

See talk by E. Braaten

Many exotic states – mesons (tetraquarks) and baryons (pentaquarks) being observed



S. Olsen *et al., R*MP(2018)

Use the nucleus as a "filter" for the heavy states

New physics observables

Structure and formation process of new exotic hadrons, e.g. X(3872) can be explored by measuring their suppression in e+A collisions.

Relative modification of X(3872)/ $\psi(2S)$ projection at \sqrt{s} = 63.2GeV



Can this be measured?



Resummation

- Jet production is one of the cornerstonoe processes of QCD. Light jets have been studied for a long time.
- Recent advances are based in SCET precision theory for small radius jets and heavy flavor jets

The SiJFs Evolve according to DGLAP-like equations

$$\frac{d}{d\ln\mu^2} \left(\begin{array}{c} J_{J_Q/s}(x,\mu) \\ J_{J_s/g}(x,\mu) \end{array} \right) = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dz}{z} \left(\begin{array}{cc} P_{qq}(z) & 2P_{gq(z)} \\ P_{qg}(z) & P_{gg}(z) \end{array} \right) \left(\begin{array}{c} J_{J_Q/s}(x/z,\mu) \\ J_{J_s/g}(x/z,\mu) \end{array} \right)$$

We use the Mellin moment space approach to solve this equation

Resums $ln \mu/p_T R$

scales

 $\ln R$

m

 $p_T R$

 m_Q

 $\Lambda_{
m QCD}$

ln -

$$\mathcal{M}_{g \to Q\bar{Q}}^{\mathrm{in-jet}}(p_T R, m) = 2 \sum_{l=g,Q} \bar{K}_{l/g}(p_T R, m, \mu_F) \bar{D}_{Q/l}(m, \mu_F)$$
Resums ln p_TR/m
The integrated perturbative the jet typical scale The integrated parton fragmentation function from parton *l* to parton *Q*
Bauer, Mereghetti 2013, Dai, Kim, Leibovich 2016, 2018

Modification of light hadrons at HERMES

 Account for nuclear geometry, i.e. the production point and the path length of propagation of the hard parton, NLO



 $Q^{2}, z) = \frac{\frac{N^{\pi}(v, Q^{2}, z)}{N^{e}(v, Q^{2})}\Big|_{A}}{\frac{N^{\pi}(v, Q^{2}, z)}{N^{e}(v, Q^{2})}\Big|_{D}}$



Transport properties:

$$q - hat(g) = 0.12 \frac{GeV^2}{fm} (vary \times 2,/2)$$
$$q - hat(q) = 0.05 \frac{GeV^2}{fm} (vary \times 2,/2)$$



N. Chang et al. (2014)

NB: this is our extraction of transport properties - others vary up/down by a factor of 2