Outline of the talk

- Theory and heavy flavor probes of QCD dynamics
- Open heavy flavor hadron production
- Heavy flavor jets and jet substructure
- Quarkonium production and exotics

H. Li, Z. Liu, I.V., in preparation
I. Olivant, I.V., in preparation

For expert-level comprehensive discussion of HF@EIC topics see
https://indico.bnl.gov/event/9273/
Confinement and hadronization

- Does QCD predict confinement? Yes
- Do we fully understand the static properties of confinement? Not fully
- Do we understand dynamically how confinement occurs? No

- Perturbative hadronization  
  J. Collins et al. (1982)

- Hadronization in event generators

- Hadronization in the nuclear environment - in can provide insights into the dynamics and the space-time picture of hadronization
Total HF cross sections and lessons from HERA data

Realistic evaluations combine FFNS and VFNS, subtraction of double counting is required

1. Transitions between FFN and VFS
2. Depending on order/prescription, $Q_2$ – good description of DIS data

S. Alekhin et al. (2020)

Hybrid variable flavor number scheme

- Different choices of matching mass values. It allows the user, for example, to use $N_f=4$ above $M_b$ scale for example

Confronting HF schemes with HERA data

V. Bertone et al. (2018)
Heavy flavor in polarized reactions at the EIC

The EIC is a polarized machine – will constrain precisely the spin content of nucleons and nuclei and their 3D landscape

\[
\frac{d\sigma^{UT}(S_T)}{dQ^2dyd^2q_Tdy_Jd^2p_T} = \sin(\phi_q - \phi_b) H(Q, y, p_T, y_J, \mu_b) \int_0^\infty \frac{b^2 db}{4\pi} J_1(b q_T) f_{1T,q/N}(x, \mu_b) \times \exp \left[ - \int_{\mu_b^s}^{\mu_h} \frac{d\mu}{\mu} \Gamma^h(\alpha_s) - 2 \int_{\mu_b^s}^{\mu_j} \frac{d\mu}{\mu} \Gamma^j(\alpha_s) - \int_{\mu_b^s}^{\mu_{cs}} \frac{d\mu}{\mu} (\tilde{\Gamma}^{cs,\phi}(\alpha_s) + \tilde{\Gamma}^{cs,\phi}(\alpha_s)) \right] \times \exp \left[ - S_{NP}^h(b, Q_0, n \cdot p_g) \right].
\]

L. Zheng et al. 2018

Hadrons (DD-bar)

Jets Z. Kang et al. 2020

See talk by E. Aschenauer

EIC 20 GeV × 250 GeV, charm jets

EIC 20 GeV × 250 GeV, bottom jets

Rather significant asymmetries. Hadronization reduced the asymmetry. Experimental feasibility studies have also been performed (on charm meson)
Production of semi-inclusive hadrons and jets

Based on QCD / SCET factorization. Calculations at next-to-leading order (and resummation where applicable) are standard. Calculations at NNLO also exist but still time consuming.

In the medium: factorization, with modified J (jet), B (beam), S (soft) functions

\[ E_h \frac{d^3 \sigma^{N \rightarrow hX}}{d^3 P_h} = \frac{1}{S} \sum_{i,f} \int_0^1 \frac{dx}{x} \int_0^1 \frac{dz}{z^2} f_{\text{i/N}}(x, \mu) \times D^{h/f}(z, \mu) [\hat{\sigma}^{i \rightarrow f} + \hat{f}_{\text{ren}} \left( \frac{-t}{s + u}, \mu \right) \hat{\sigma}^{\gamma i \rightarrow f}] . \]

\[ E_J \frac{d^3 \sigma^{N \rightarrow jX}}{d^3 P_J} = \frac{1}{S} \sum_{i,f} \int_0^1 \frac{dx}{x} \int_0^1 \frac{dz}{z^2} f_{i/N}(x, \mu) \times \hat{\sigma}^{i \rightarrow f}(s, t, u, \mu) J_f(z, p_T R, \mu) , \]

In the medium: factorization, with modified J (jet), B (beam), S (soft) functions


In the medium we still have a multi-scale problem

A. Idilbi et al. (2008)  G. Ovanesyan et al. (2011)
In-medium splitting functions necessary for higher order and resumed calculations

\[ \frac{dN(\text{tot.})}{dx d^2 k_\perp} = \frac{dN(\text{vac.})}{dx d^2 k_\perp} + \frac{dN(\text{med.})}{dx d^2 k_\perp} \]

Often used in saturation calculations. Can get on one shot massless and massive splitting functions

\[
\left( \frac{dN}{dx d^2 k_\perp} \right)_{q \rightarrow qg} = \frac{\alpha_s}{2\pi^2} C_F \frac{1 + (1 - x)^2}{x} \int d^2 q_\perp \frac{1}{\sigma_{el}} \frac{d\sigma_{\text{medium}}}{d^2 q_\perp} \left[ - \left( \frac{A_1}{A_2} \right)^2 + \frac{B_1}{B_2} \left( \frac{B_1}{B_2} - \frac{C_1}{C_2} \right) \times \left( 1 - \cos[(\Omega_1 - \Omega_2)\Delta z] \right) \right]
\]

- Factorize form the hard part
- Gauge-invariant
- Depend on the properties of the medium
- Can be expressed as proportional to Altarelli-Parisi
Differences between AA and eA

- AA and eA collisions are very different. Due to the LPM effect the “energy loss” decreases rapidly. The kinematics to look for in-medium interactions / effects on hadronization very different

- Jets at any rapidity roughly in the co-moving plasma frame (Only~ transverse motion at any rapidity)
- Largest effects at midrapidity
- Higher C.M. energies correspond to larger plasma densities

- Jets are on the nuclear rest frame. Longitudinal momentum matters
- Largest effects are at forward rapidities
- Smaller C.M. energies (larger only increase the rapidity gap)
Constrain the transport properties of cold nuclear matter

Shed light on the picture of hadronization, differentiate between energy loss and hadron absorption

Go beyond energy loss phenomenology at the EIC

Multiple uses of heavy flavor

- Constrain gluon and c/b distributions. Look for intrinsic charm

- Constrain the transport properties of cold nuclear matter

- Shed light on the picture of hadronization, differentiate between energy loss and hadron absorption

- Go beyond energy loss phenomenology at the EIC
The coupled Altarelli-Parisi evolution equations Eq. (45)-Eq. (47) simplify tremendously for cross section formulas from initial state to the final state. To check if there are additional factors from reversing A-P equations and the energies are higher than RHIC.

The evolution equations are given by standard Altarelli-Parisi equations:

The complete medium-induced splitting functions look like:

The evolution equations with the initial conditions for parton densities where the individual terms with all the plus prescriptions and virtual pieces are summarized in so-called PDF's at the hard scattering scale interpretation: probability of the parton

As a result of solving the A-P evolution equations we get the full LL series resummed by:

The medium-induced splitting functions look like:

As a result of solving the A-P evolution equations we get the full LL series resummed by:

$$\frac{dD_{q}(z,Q)}{d\ln Q} = \frac{\alpha_s(Q^2)}{\pi} \int_{z}^{1} \frac{dz'}{z'} \left\{ P_{q\rightarrow qg}(z',Q)D_{q}\left(\frac{z}{z'},Q\right) + P_{q\rightarrow gg}(z',Q)D_{g}\left(\frac{z}{z'},Q\right) \right\} ,$$

$$\frac{dD_{q}(z,Q)}{d\ln Q} = \frac{\alpha_s(Q^2)}{\pi} \int_{z}^{1} \frac{dz'}{z'} \left\{ P_{g\rightarrow qg}(z',Q)D_{q}\left(\frac{z}{z'},Q\right) + P_{g\rightarrow gg}(z',Q)D_{g}\left(\frac{z}{z'},Q\right) \right\} ,$$

$$\frac{dD_{g}(z,Q)}{d\ln Q} = \frac{\alpha_s(Q^2)}{\pi} \int_{z}^{1} \frac{dz'}{z'} \left\{ P_{g\rightarrow qg}(z',Q)D_{g}\left(\frac{z}{z'},Q\right) \right\} ,$$

$$R_{eA}^{\pi}(v,Q^2,z) = \frac{N^\pi(v,Q^2,z)}{N^e(v,Q^2)} \left[ \frac{A}{D} + P_{g\rightarrow qg}(z',Q)\left(D_q\left(\frac{z}{z'},Q\right) + f_q\left(\frac{z}{z'},Q\right)\right) \right] .$$

**Modification of FFs**

Vacuum splitting functions provide correction to vacuum showers and correspondingly modification to DGLAP evolution for FFs.

Z. Liu et al. (2020)

- Always enhancement at small $z$ but for pions (light hadrons) at very small values – mostly suppression
- Very pronounced differences between light and heavy flavor fragmentation
Light and heavy flavor suppression at the EIC

Given the much larger C.M. energy that at HERMES this is the picture to study first.

Light pions show the largest nuclear suppression at the EIC. However to differentiate models of hadronization heavy flavor mesons are necessary.
Modification of heavy flavor cross sections

A more differential ratio vs the momentum fraction of the hadron

The difference in the suppression pattern of pions and D, B mesons is characteristic of the in-medium evolution/energy loss approach

Detailed and constrained predictions for the EIC

\[
R_{eA}^h(p_T, \eta, z) = \frac{N^h(p_T, \eta, z)_{e+Au}}{N^h(p_T, \eta, z)_{e+p}}
\]

Normalized by inclusive large radius jet production. To LO equivalent inclusive normalization

Z. Liu et al. (2020)
Jet production

A useful modern way (though not unique) to calculate jet cross sections

**Factorization formula**

\[
E_J \frac{d^3\sigma^{1N\to jX}}{d^3 P_J} = \frac{1}{S} \sum_{i,f} \int_0^1 \frac{dx}{x} \int_0^1 \frac{dz}{z^2} f_{i/N}(x, \mu) \\
\times \hat{\sigma}^{i\to f}(s, t, u, \mu) J_f(z, p_T R, \mu), \\
\mu_J = \omega_J \tan \frac{R}{2} = (2p_T \cosh \eta) \tan \left( \frac{R}{2 \cosh \eta} \right) \approx p_T R.
\]

**In-medium jet functions**

\[
J^{\text{med,}(1)}_q(z, \omega R, \mu) = \left[ \int_{z(1-z)\omega \tan(R/2)}^\mu dq_\perp P_{qq}(z, q_\perp) \right] + \\
+ \int_{z(1-z)\omega \tan(R/2)}^\mu dq_\perp P_{gg}(z, q_\perp).
\]

- Stable in numerical implementation
- Similarly for gluon jets

**Cross section contribution**

\[
d\sigma_{\text{jet,med}}^{\text{PbPb}} = \sum_{i=q,\bar{q},g} \sigma_i^{(0)} \otimes J^\text{med}_i
\]

The physics of in-medium jet modification

Z. Kang et al. (2016)  
L. Dai et al. (2016)  
H. Li et al. (2020)
Jet results at the EIC

H. Li et al. (2020)

Two types of nuclear effect play a role

- Initial-state effects parametrized in nuclear parton distribution functions or nPDFs
- Final-state effects from the interaction of the jet and the nuclear medium – in-medium parton showers and jet energy loss

How to separate them? Define the ratio of modifications for 2 radii (it is a double ratio)

- Jet energy loss effects are larger at smaller C.M. energies
- Remarkably, effects can be almost a factor of 2!
Heavy flavor jets introduce a new mass scale that requires careful treatment SiJF.

In eA collisions there is an in-medium contribution to the SiJFs.

Furthermore, there can be a significant Weiszacker-Williams photon contribution.
Heavy flavor jets at EIC

- The modification of heavy flavor jets in eA is significant
- There is much larger sensitivity to the gluon distributions but initial-state and final-state effects can still be separated
- There is a pronounced rapidity dependence of the heavy flavor jet suppression

We have also done the calculation for b-jets and slightly smaller, but still significant modification
Heavy flavor jets substructure in DIS

Related to the modification of jet cross sections is the modification of jet substructure. Example - Soft dropped momentum sharing distributions

- Modification of both c-jets and b-jets substructure in eA is relatively small
- It is dominated by limited phase space

Kinematically not possible in DIS but illustrates very well the difference with HIC

\[ z_g = \min(\frac{pT_1, pT_2}{pT_1 + pT_2}) > z_{cut} (\frac{\Delta R_{12}}{R_0})^\delta \]

\[ p(\theta_g, z_g) |_j = \frac{\frac{dN_{\text{vac, MLL}}}{dz_g d\theta_g}}{\int_0^1 d\theta \int_{z_{cut}}^{1/2} dz \sum_i \left( \frac{dN_{\text{vac}}}{dz d\theta} \right)_{j \rightarrow i}} \]

\[ \Delta R_{12} \]

\[ r_g = \Delta R_{12} \]

\[ p_{T_1}, p_{T_2} \]

\[ \eta = 2.4, R = 1 \]

\[ p_T = 10 \text{ GeV} \]

\[ E = 94 \text{ GeV} \]

\[ R = 0.4 \]
EIC will finally have the precision to answer long standing questions about large-x structure – strangeness and intrinsic charm

- Intrinsic charm – genuine non-perturbative contribution to the proton wave function – can affect HQ schemes, masses, global fits
- Strangeness – can be accessed via CC reactions. Requires high statistics, can look for enhanced strangeness

M. Arratia et al. (2020)

T. Hobbs et al. (2017)
Production of quarkonia

- Non-Relativistic QCD (NRQCD) - a particular type of effective theory (EFT)

$\mathcal{L}_{\text{NRQCD}} = \mathcal{L}_{\text{light}} + \psi^\dagger \left( iD_0 + \frac{D^2}{2M} \right) \psi + \chi^\dagger \left( iD_0 - \frac{D^2}{2M} \right) \chi$

- Ultra-soft
  \[ p^\mu_s \sim m_Q v(1, 1, 1, 1) \]
  \[ p^\mu_{us} \sim m_Q v^2 (1, 1, 1, 1) \]

- + heavy - soft interactions at NLO

- Typical momentum if heavy quark:
  \[ |p_Q| \sim m_Q v \]

- Typical kinetic energy if heavy quark:
  \[ K_Q \sim m_Q v^2 \]

- NRQCD factorization formula. Short distance cross sections (perturbatively calculable) and long distance matrix elements (fit to data, scaling relations)

\[
d\sigma(a + b \rightarrow Q + X) = \sum_n d\sigma(a + b \rightarrow Q\bar{Q}(n) + X) \langle \mathcal{O}_n^Q \rangle
\]
Results in DIS and EIC specifics

- Data and photoproduction theory at HERA

- Good description of cross sections can can be achieved but still some tensions remain in understanding polarization, especially at low $p_T$

- At EIC we have lower CM energies than HERA. Production at low $p_T$ can probe gluon TMDs and universal Shape functions

$$\frac{d\sigma}{d^2q} = \sigma_0([n]) H(2m_Q, \mu; [n]) \int d^2k F_{g/P}(x, k) S_h(q - k; [n])$$

Low transverse momentum gluon production

M. Butenchoen et al. (2010)

S. Fleming et al. (2019)
NRQCD with Glauber Gluons

- At the level of the Lagrangian

\[ \mathcal{L}_{\text{NRQCD}_G} = \mathcal{L}_{\text{NRQCD}} + \mathcal{L}_{Q-G/C}(\psi, A_{G/C}^{\mu,a}) + \mathcal{L}_{g-G/C}(A_{s}^{\mu,b}, A_{G/C}^{\mu,a}) + \psi \leftrightarrow \chi \]

Y. Makris et al. (2019)

- Glauber gluons - transverse to the direction of propagation contribution
- Coulomb gluons - isotropic momentum distribution

Background field method
Perform a shift in the gluon field in the NRQCD Lagrangian then perform the power-counting

Hybrid method
From the full QCD diagrams for single effective Glauber/Coulomb gluon perform the corresponding power-counting, read the Feynman rules

Matching method
Full QCD diagrams describing the forward scattering of incoming heavy quark and a light quark or a gluon. We also derive the tree level expressions of the effective fields in terms of the QCD ingredients

Phenomenological applications
See talks by Y. Akamatsu & A. Viaro
Phenomenology with NRQCD$_G$

At the moment data exists in heavy ion collisions (as far as nucleus A is involved)

- Correct hierarchy of excited and ground state suppression

Work to extend this to e+A is under way. Cold nuclear matter vs QGP, lower transverse momenta

- QGP – thermal effect on the wavefunction and collisional dissociation
- EIC – only CNM collisional effects

Collisional interactions

Y. Makris et al. (2019)

I. Olivant et al. in progress
Quarkonia and small-\(x\) physics

The EIC will probe the small-\(x\) gluon saturation regime. The connection between CGC and TMD physics is actively explored. It is first important to unambiguously establish the saturation regime.

**Abdulkhalek et al. 2021**

Coherent and incoherent J/\(\psi\) photo production

More differential observables such as the asymmetries of exclusive J/\(\psi\) production

**A. Dumitru et al. 2021**

\[
\frac{d\sigma_{eA+eA\rightarrow eA}}{dxF_{p}dQ^2dyd\phi_{k\Delta}} = \frac{\alpha_{em}}{32\pi^3Q^2xF_{p}} \sum_{x=0,\pm 1} \left\{ (1-y) \left| \left\langle \tilde{M}_{0,\lambda'}^{*A,V}_{Y} \right\rangle_{Y} \right|^2 
+ \frac{1}{4} \left[ 1 + (1-y)^2 \right] \sum_{\lambda'=\pm 1} \left| \left\langle \tilde{M}_{\lambda,\lambda'}^{*A,V}_{Y} \right\rangle_{Y} \right|^2 
- \frac{\sqrt{2}}{2} (2-y) \sqrt{1-y} \sum_{\lambda'=\pm 1} \text{Re} \left( \left\langle \tilde{M}_{0,\lambda'}^{*A,V}_{Y} \right\rangle_{Y} \left\langle \tilde{M}_{\lambda,\lambda'}^{*A,V}_{Y} \right\rangle_{Y}^* \right) \cos(\phi_{k\Delta}) 
+ (1-y) \text{Re} \left( \left\langle \tilde{M}_{0,\lambda'}^{*A,V}_{Y} \right\rangle_{Y} \left\langle \tilde{M}_{\lambda,\lambda'}^{*A,V}_{Y} \right\rangle_{Y}^* \right) \cos(2\phi_{k\Delta}) \right\}
\]

\(v_n = \frac{\int^{2\pi}_{0} d\phi_{k\Delta} e^{i\phi_{k\Delta}} d\sigma_{e+p \rightarrow e+p+V}/dtd\phi_{k\Delta} dQ^2 dxF_{p}}{\int^{2\pi}_{0} d\phi_{k\Delta} d\sigma_{e+p \rightarrow e+p+V}/dtd\phi_{k\Delta} dQ^2 dxF_{p}}\)

(a) Dependence on \(x_p\).

(b) MV model compared to the LCPT dipole.
Heavy flavor production is an essential part of the EIC science. For expert level discussion of various experimental and theoretical aspects see https://indico.bnl.gov/event/9273/ HF can shed light on the dynamical aspects of confinement

Heavy flavor provides complementary probe of the TMD stricture of nucleons/nuclei, small-x saturation physics, parton distributions, and transport properties of cold nuclear matter

Open heavy flavor production has been calculated at NLO, in-medium evolution of fragmentation understood. We have detailed and differential predictions of D and B meson cross section modification, which can differentiate between energy loss and hadron absorption

Calculations of open heavy flavor jets in e+A collisions are near complete. They require more careful treatment of energy/mass scales. Preliminary results are promising – large suppression of c-jets. Heavy jet substructure in reactions with nuclei quite different from HIC, exhibits new features that could be studied at the EIC

Quarkonium production, as described by NRQCD, can be further constrained at EIC. New developments are theories of quarkonium production in matter (NRQCD$_C$), low $P_T$ and gluon TMDs at the EIC. e+A collisions can shed light on the structure of heavy exotics
Confinement and hadronization

- Do we understand dynamically how confinement occurs? No
- Perturbative hadronization
- Hadronization in event generators

\[ \mathcal{P}_{A/1}(z, P_T) = \frac{1}{2z(2\pi)^3} \int dx^- d^2x_T e^{ik\cdot x - ik_T \cdot x_T} \times \frac{1}{3} \text{tr}_{\text{color}} \frac{1}{2} \text{tr}_{\text{Dirac}} \{\gamma^+\langle 0|\psi(x)A^+(P^+, 0)a_A(P^+, 0)|\psi(0)\rangle 0}\]

\[ f(z) \propto \frac{1}{Z} (1 - z)^a \exp \left(-\frac{bm^2}{Z}\right) \]

• Hadronization in the nuclear environment - in can provide insights into the dynamics and the space-time picture of hadronization

String fragmentation

Cluster hadronization

PYTHIA et al.

HERWIG et al.

J. Collins et al. (1982)
Production of hadrons and jets can be understood from the broader and softer splitting functions. The additional medium-induced contribution to factorization formulas (final-state) can lead to additional scaling violation due to the medium-induced shower. This results in an additional component to jet functions.
Adding LHCb and ALICE data to the heavy flavor data from HERA

Inclusion of heavy flavor reduces PDF uncertainties for sea quarks and gluons at small x, especially FFNS

Important to do combined analysis at the EIC
In addition to quarkonia, we have a large number of heavy quark exotic hadrons:

- Photoproduction through photon-Pomeron fusion lead predominantly to $J^{PC}=1^{--}$ states like the $J/\psi$, etc., so is only sensitive to exotic with those quantum numbers.
- Photon-Reggeon fusion leads to states with a wider range of spin, parity and even charge.

\[
\sigma(ep \to eX^{+n}) = \int \frac{dQ^2}{k} d^2N_{(k, Q^2)} dkdQ^2 \\
\times \sigma_{\gamma^*p \to X^{+n}(W, Q^2)} \\
\sigma_{\gamma^*p \to X^{+n}(W, Q^2)} = \left( \frac{M_X^2}{M_X^2 + Q^2} \right)^\eta \\
\sigma_{\gamma p \to X^{+n}(W, Q^2 = 0)} f(M_X),
\]
Many exotic states – mesons (tetraquarks) and baryons (pentaquarks) being observed

S. Olsen et al., RMP(2018)

New physics observables
Structure and formation process of new exotic hadrons, e.g. X(3872) can be explored by measuring their suppression in e+A collisions.

Relative modification of \( \frac{X(3872)}{\psi(2S)} \) projection at \( \sqrt{s} = 63.2 \text{GeV} \)

Use the nucleus as a “filter” for the heavy states

Arleo et al., PRC, 61 054906 (2000)
Can this be measured?

Yes it can. Quite accurately on top of that

Differentially vs rapidity and momentum fraction

- Can constrain the transport properties to about 30%
- Based on realistic detector concepts,
- Full Fun4All framework

Geant

X. Li et al. (2021)  R. Abhdul Khalek et al. (2021)
The SiJFs Evolve according to DGLAP-like equations

\[ \frac{d}{d \ln \mu^2} \left( \frac{J_{JQ/s}(x, \mu)}{J_{J/s}(x, \mu)} \right) = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dz}{z} \begin{pmatrix} P_{qq}(z) & 2P_{gq}(z) \\ P_{qg}(z) & P_{gg}(z) \end{pmatrix} \begin{pmatrix} J_{JQ/s}(x/z, \mu) \\ J_{J/s}(x/z, \mu) \end{pmatrix} \]

We use the Mellin moment space approach to solve this equation

\[ M_{g \rightarrow Q\bar{Q}}^{\text{in-jet}}(p_{TR}, m) = 2 \sum_{l=g,Q} \tilde{K}_{l/g}(p_{TR}, m, \mu_F) \tilde{D}_{Q/l}(m, \mu_F) \]

The integrated perturbative kernel at the jet typical scale

Bauer, Mereghetti 2013, Dai, Kim, Leibovich 2016, 2018
Account for nuclear geometry, i.e. the production point and the path length of propagation of the hard parton, NLO

We constrain a range of transport properties to explore from HERMES

\[ R_{eA}^\pi (v, Q^2, z) = \left. \frac{N^\pi(v, Q^2, z)}{N^e(v, Q^2, z)} \right|_A \left. \frac{N^\pi(v, Q^2, z)}{N^e(v, Q^2, z)} \right|_D \]

We have

\[ q - \hat{h}(g) = 0.12 \frac{GeV^2}{fm} \] (vary \(\times 2, /2\))

\[ q - \hat{h}(q) = 0.05 \frac{GeV^2}{fm} \] (vary \(\times 2, /2\))

N. Chang et al. (2014)

NB: this is our extraction of transport properties - others vary up/down by a factor of 2