

QCD at high temperature and density

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Virtual Confinement 2021

Overview



Outline

Transition(s) at $\mu = 0$

Chiral transition line

Large density

Hadron properties and transport

Summary

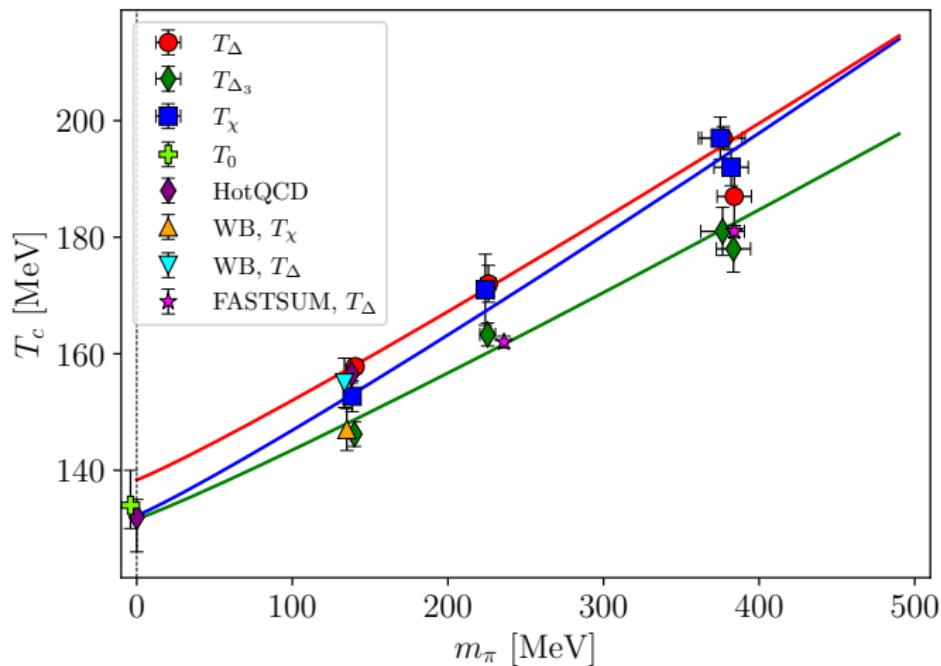
Omissions

I will not talk about

- Equation of state
- Nonzero magnetic field
- Critical endpoint(!)
- Non-equilibrium
- and much more

Pseudocritical temperature

Good agreement between different groups at physical point



$U_A(1)$ restoration

- Chiral transition can be 1st order if $U_A(1)$ is restored
- $O(4)$ or $U(2)_L \otimes U(2)_R / U(2)_V$ universality class
- $U_A(1)$ restoration $\leftrightarrow \pi - \delta$ degeneracy
- $U_A(1)$ restoration $\leftrightarrow \chi_c^{\text{disc}} = 0$ in chirally restored phase

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$U_A(1)$ broken	$U_A(1)$ restored
hotQCD, DWF (2012)	Cossu et al, overlap (2013)
LLNL/RBC, DWF (2014)	Brandt et al, Wilson–Clover (2016)
Dick et al, HISQ (2015)	JLQCD, DWF/overlap (2017)
Mazur et al, Overlap/HISQ (2021)	JLQCD, DWF/overlap (2021)
Ding et al, HISQ cont (2021)	

Thanks to Anirban Lahiri for the compilation of results

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- No consensus as yet
- Only one continuum extrapolated result

$U_A(1)$ restoration

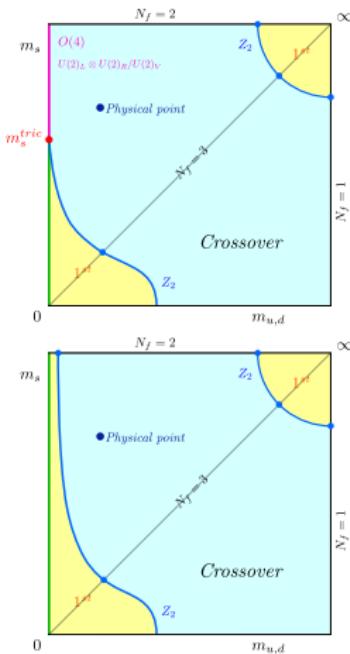
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- No consensus as yet
- Only one continuum extrapolated result
- FRG [Braun et al (2020)] consistent with restoration and $O(4)$

Chiral limit



- 1st order expected for $N_f = 3, m_u = m_d = m_s = 0$
- 1st or 2nd for $N_f = 2 + 1, m_s^{\text{phys}}$?
- If 2nd, what is m_π^c ?

[Cuteri et al, 2107.12739]

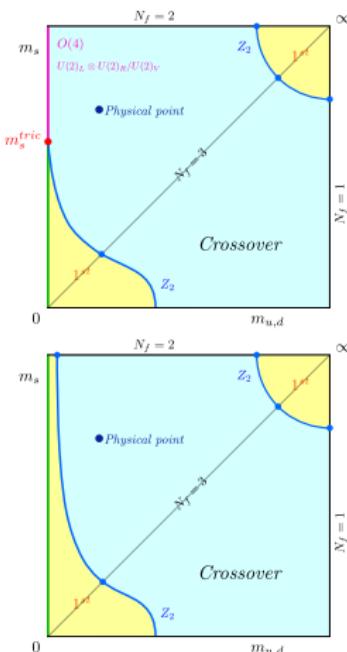
Chiral limit

$$N_f = 2 + 1$$

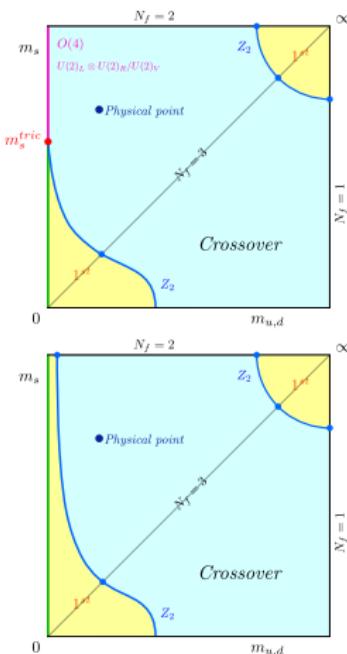
- Bazavov et al (2017), HISQ: $m_\pi^c \lesssim 50$ MeV
- Kuramashi et al (2020), Wilson–Clover: $m_\pi^c \lesssim 110$ MeV
- Ohno et al (2021), Wilson–Clover: Evidence for 1st order
- Sharma et al (2021), HISQ: No evidence for 1st order

First order region shrinks towards continuum limit

[Cuteri et al, 2107.12739]



Chiral limit

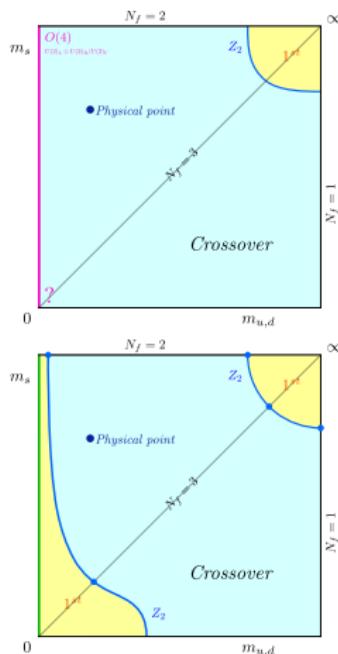


$$N_f = 3, 4, 5, \dots$$

- Ohno et al (2021), Wilson–Clover:
Evidence for 1st order for $N_f = 4$
- Cuteri et al (2021), staggered, tricritical scaling analysis
Consistent with 2nd order
- Reanalysis of Wilson–Clover data gives same result

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Chiral limit



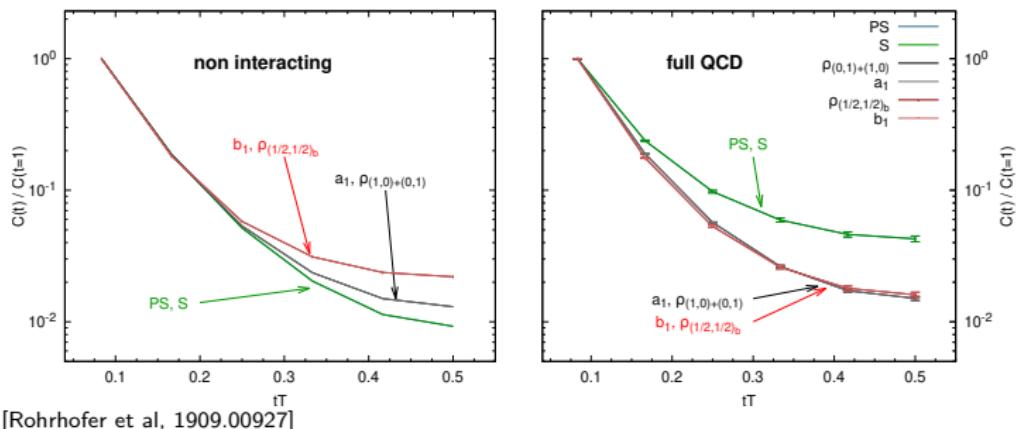
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Additional phases at high T ?

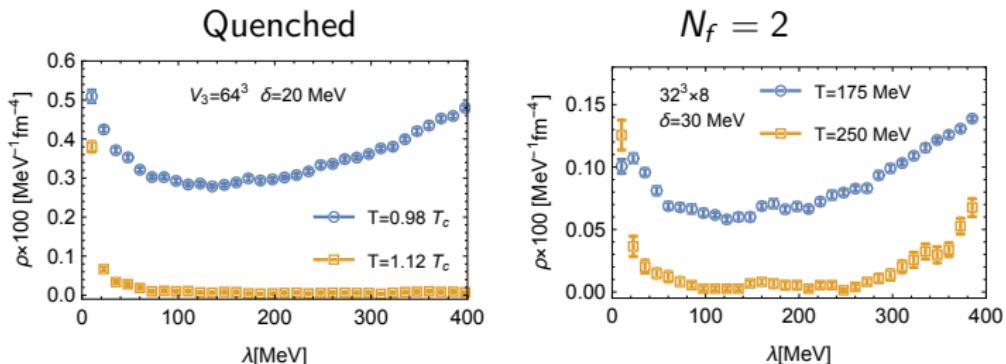
Surprise 1: Correlator symmetries [Glozman et al]



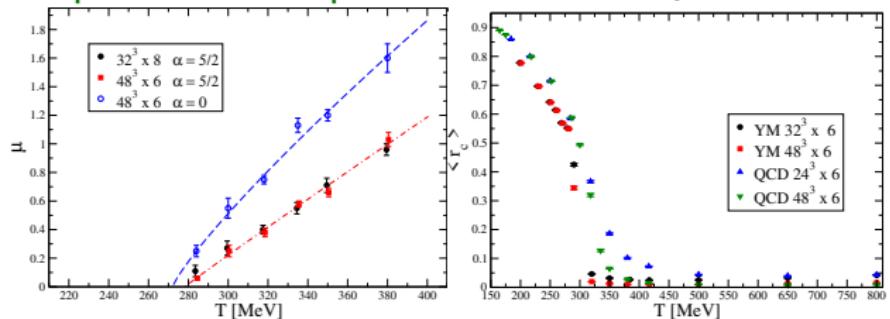
- Indicative of chiral–spin $SU(2)_{CS}, SU(2N_f)$ symmetries
- Symmetries of **chromoelectric** interaction
- Broken by **chromomagnetic** interaction
- **Conjecture:** intermediate phase, “stringy fluid”

Additional phases at high T ?

Surprise 2: Dirac zero modes [Alexandru, Horvath: 1906.04087, 2103.05607]



Surprise 3: Monopole condensation [Cardinali, D'Elia, Pasqui: 2107.02745]



Chiral transition line

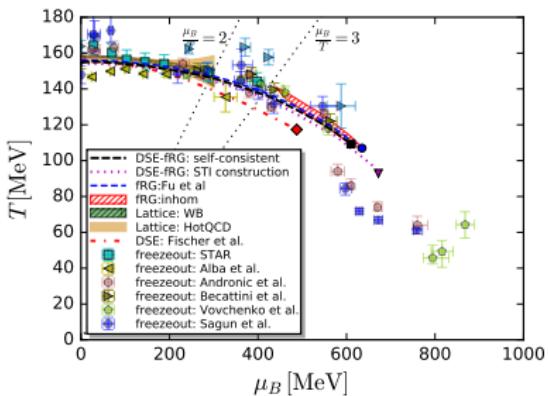
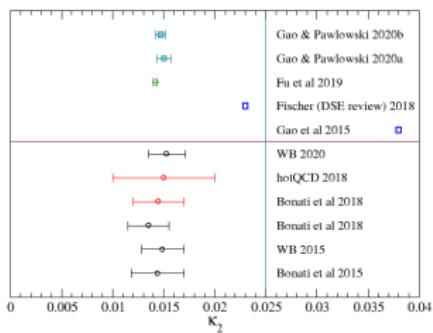
Methods:

- Continuation from imaginary μ (different expansion schemes)
- Taylor expansion
- Reweighting
- DSE/FRG methods

Shape of transition line

$$\frac{T_c(\mu_B)}{T_c(0)} = 1 - \kappa_2 \left(\frac{\mu_B}{T_c} \right)^2 - \kappa_4 \left(\frac{\mu_B}{T_c} \right)^4 + \mathcal{O}(\mu_B^6)$$

$\kappa_4 \approx 0$ within errors.



[Gao&Pawlowski: 2010.13075]

Fluctuations

$$\chi_{i,j,k}^{B,Q,S} = \frac{\partial^{i+j+k}(p/T64)}{(\partial\hat{\mu}_B)^i(\partial\hat{\mu}_Q)^j(\partial\hat{\mu}_S)^k}, \quad \hat{\mu}_a \equiv \frac{\mu_a}{T}$$

Baryon number cumulants and observables

Mean

$$M_B = \chi_1^B$$

Variance

$$\sigma_B^2 = \chi_2^B$$

Skewness

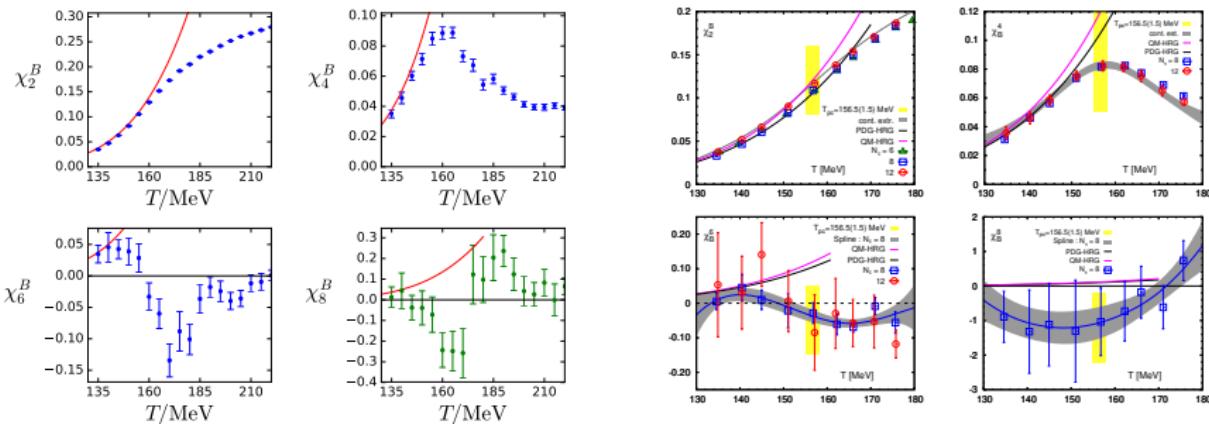
$$S_B = \frac{\chi_3^B}{(\chi_2^B)^{3/2}}$$

Kurtosis

$$\kappa_B = \frac{\chi_4^B}{(\chi_2^B)^2}$$

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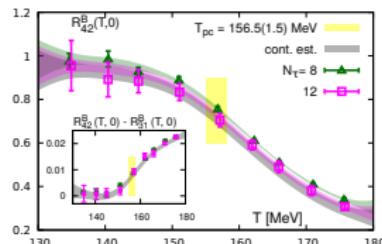
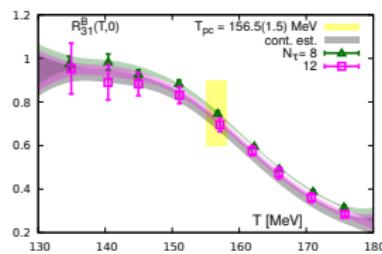
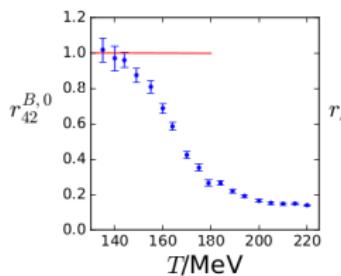
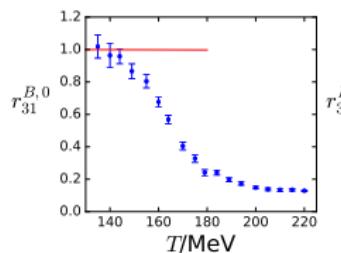


[hotQCD collaboration, 2001.08530]

[WB collaboration, 1805.04445]

Skewness and kurtosis

$$R_{31} = S_B \sigma_B^3 / M_B, \quad R_{42} = \kappa_B \sigma_B^2$$

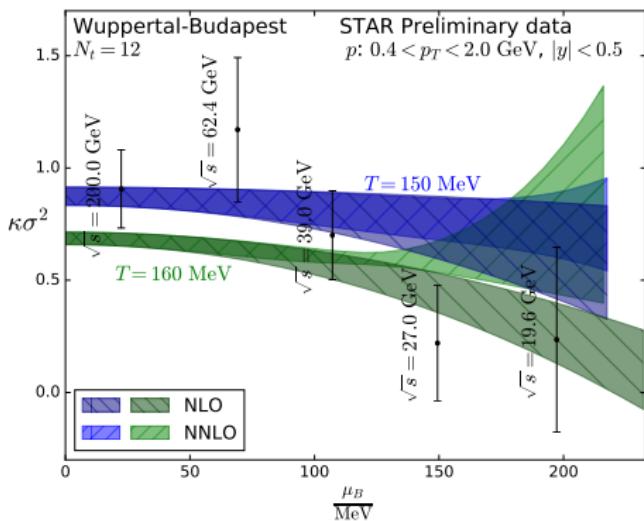


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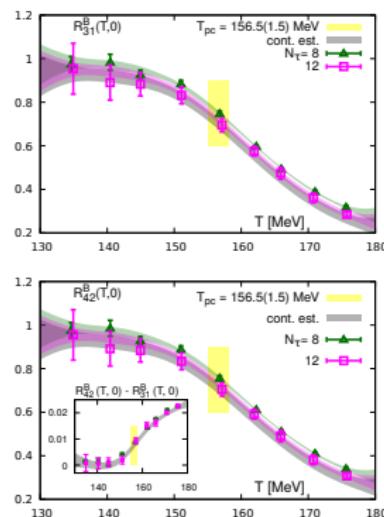
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Large density



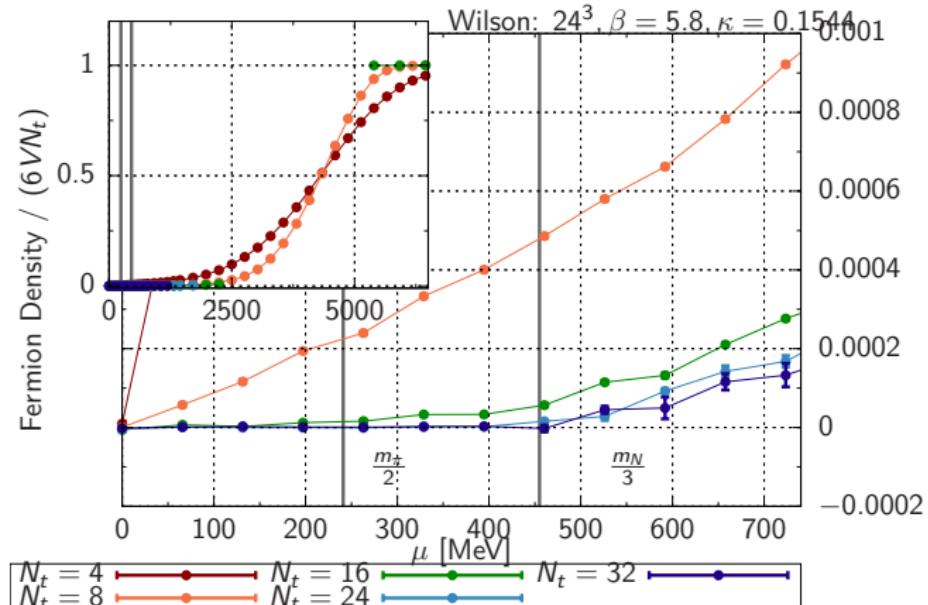
Large density

- **Sign problem:** see Plenary: Wiese, Wed 1230
- **Complex Langevin**
- Lefschetz thimble-based methods
- Density of states
- Canonical formulation
- ...
- Strong coupling expansion
- QCD-like theories
- Perturbation theory

Complex Langevin

$N_f = 2$ Wilson fermions, $m_\pi = 500$ MeV, $a = 0.06$ fm

[Attanasio, Jäger, Ziegler]



See also Ito et al, 2007.08778; Tsutsui, Lat21

Strong coupling expansion

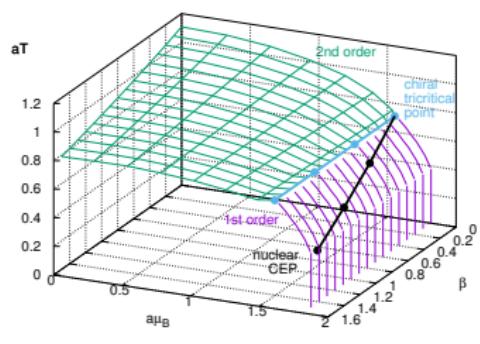
[Review by Philipsen, 2104.03696]

- Expand in characters $\chi_r(U_p) = \text{Tr } D_r(U_p)$
- Hopping parameter expansion for heavy quarks
- Dual formulation for light quarks [Gagliardi, Kim, Unger: 1710.07564, 1911.08389]
- Continuous-time formulation [Kleegrewe, Unger: 2005.10813]

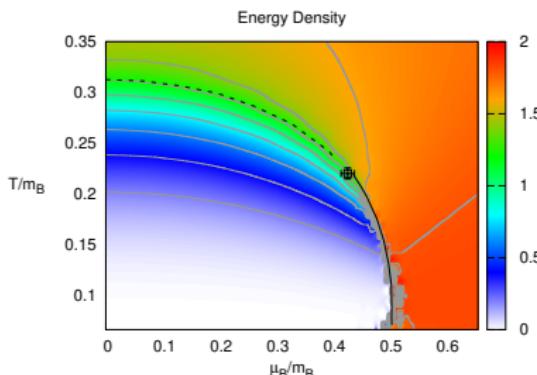
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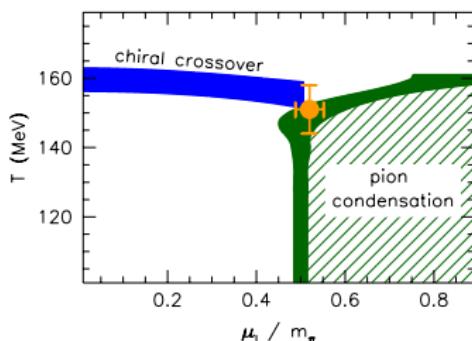


[2005.10813]

QCD-like theories

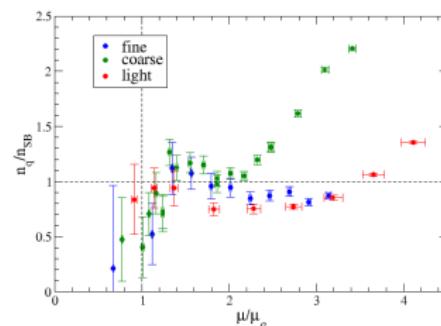
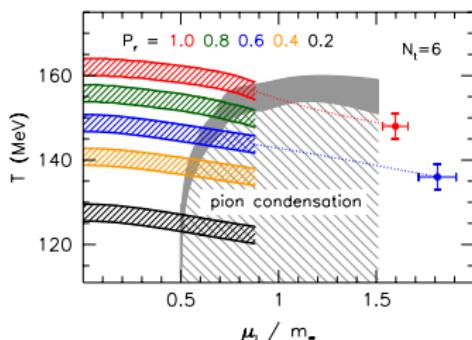
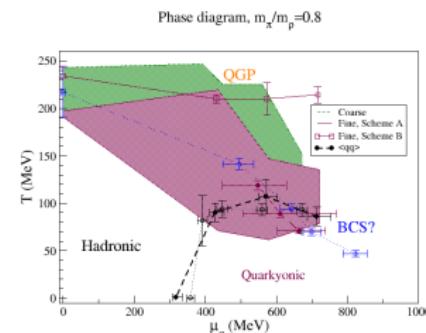
Isospin QCD

[Brandt et al: 1712.08190, 1912.07451]



Q₂CD

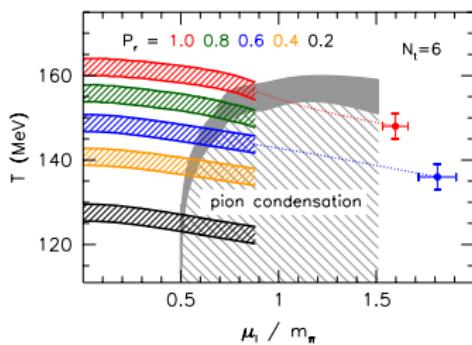
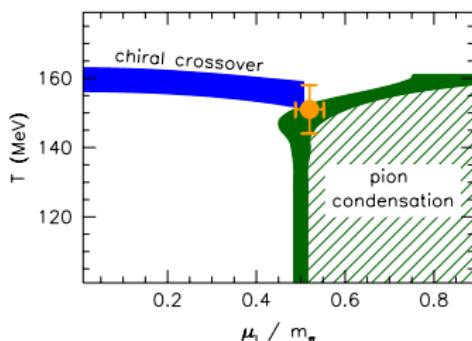
[Boz et al: 1912.01975]



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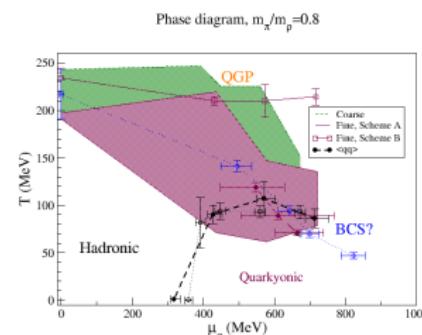
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Q₂CD

[Boz et al: 1912.01975]



- Consistent results from [Astrakhantsev et al, Buividovich et al, Ito et al]
- Deconfinement at low T , large μ not settled
- Quarkyonic phase in large- N_c

[Philipsen, Scheunert: 1908.03136]

Perturbation theory and compact star constraints

Kurkela et al, 1903.09121, 2103.05658, 2105.05132, talk by Tyler Gorda

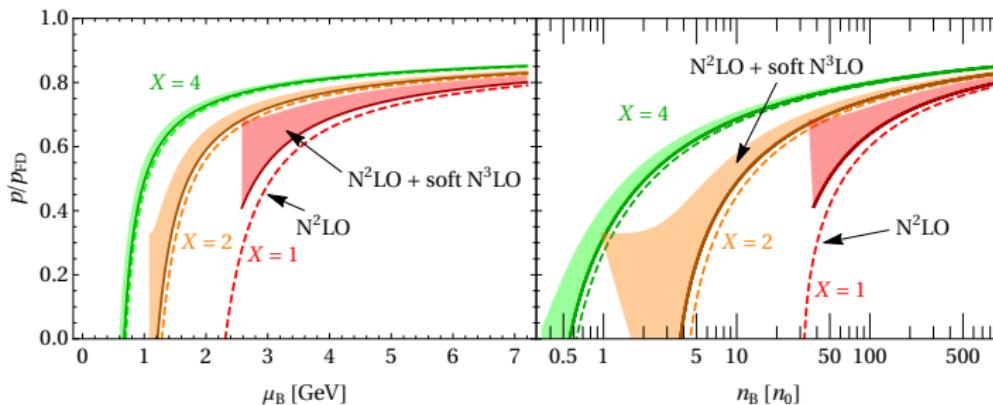
(see also Hebeler et al 1303.4662, 2005.14164, 2105.06981)

- Low-energy EoS from nuclear physics, chiral EFT
- High energy from perturbation theory
- Interpolate with piecewise polytropes $p \propto n^\gamma$
- Nuclear matter: $\gamma \sim 2.5$, Perturbative: $\gamma \in [0.5, 0.8]$
- Constrain with compact star observations

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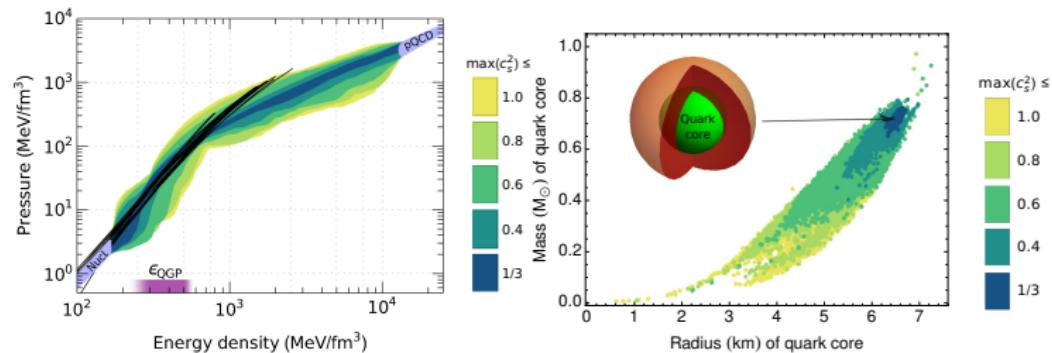


[Gorda et al, 2103.051658]

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Hadron properties and transport

- Baryons
- Heavy quarkonium
 - Charmonium?
 - Mass shift and width from NRQCD
 - Real-time static quark potential
 - Real-time evolution: talks by Akamatsu, Vairo
- Transport
 - Heavy quark diffusion
 - Conductivity
 - Jet quenching coefficient \hat{q}

Light (and charmed) baryons

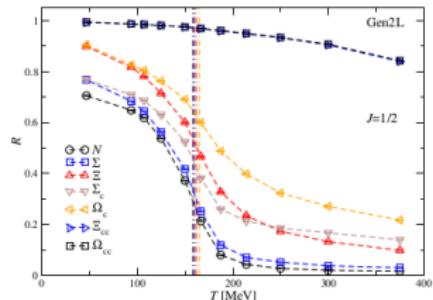
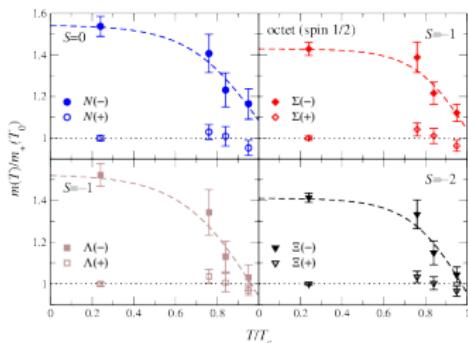
[FASTSUM, 1502.03603, 1703.09246, 1812.07393, 2007.04188]

- Positive and negative parity states encoded in same correlator
- Measure of parity restoration:

$$R(\tau) = \frac{G(\tau) - G\beta - \tau}{G(\tau) + G(\beta + \tau)}$$

$$R = \frac{\sum_{n=0}^{\beta/2-1} R(\tau_n)/\sigma^2(\tau_n)}{\sum_{n=0}^{\beta/2-1} 1/\sigma^2(\tau_n)}$$

- Impact on HRG model?



Heavy quarkonium

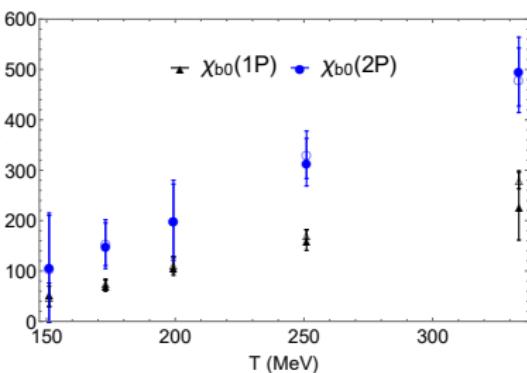
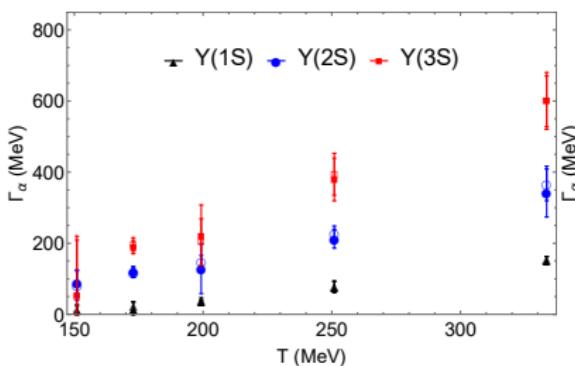
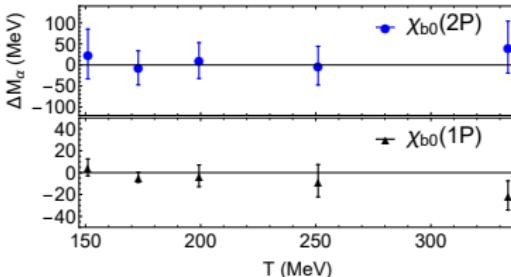
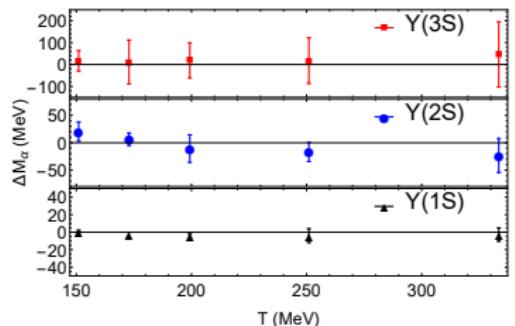
Progress towards quantitative understanding of mass shifts and widths

Larsen et al: 1908.08437, 1910.07374, 2008.00100, talk Tue 0700

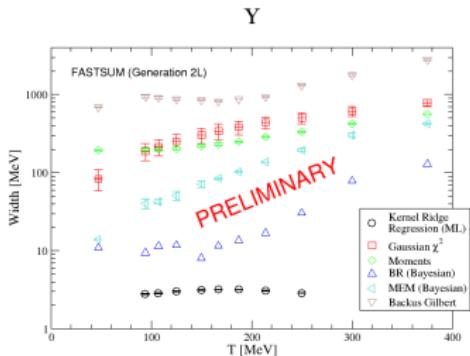
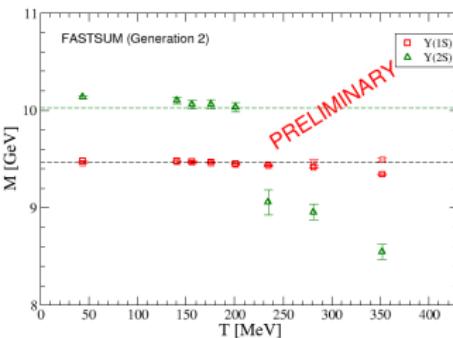
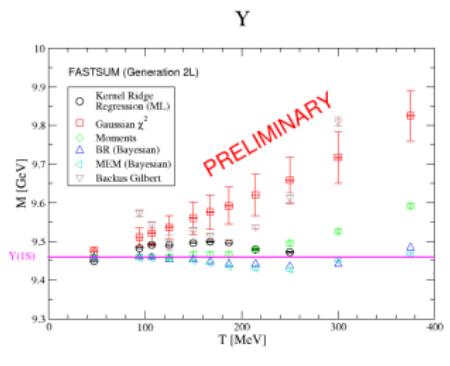
- NRQCD on hotQCD (HISQ $N_f = 2 + 1$) ensembles,
 $m_\pi = 160$ MeV
- Optimised sources for ground and excited states
- Fit to gaussian + low/high energy tail

Υ and χ_{b0} masses and widths

[Larsen et al, 1910.07374, Tue 0700]



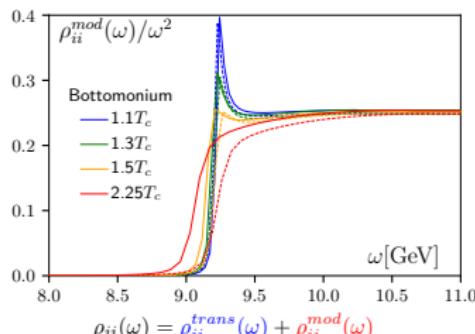
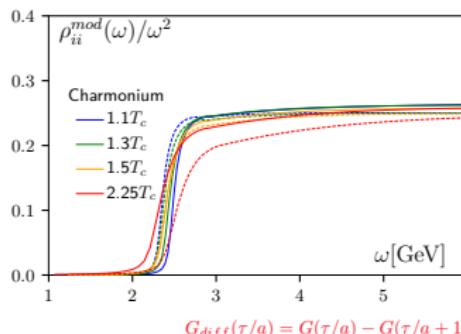
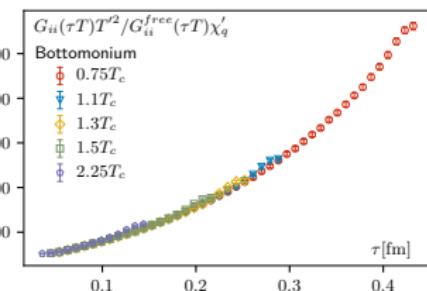
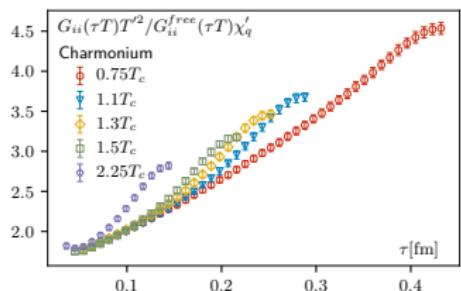
Υ mass and width (FASTSUM)



- Talk by Spriggs, Tue 0745
- Better understanding of systematic effects needed
- Small negative mass shift also observed by
Kim et al: 1808.08781;
Kelly et al: 1802.00667 (charm)

Heavy quarkonium (Bielefeld)

[Shu 2021]



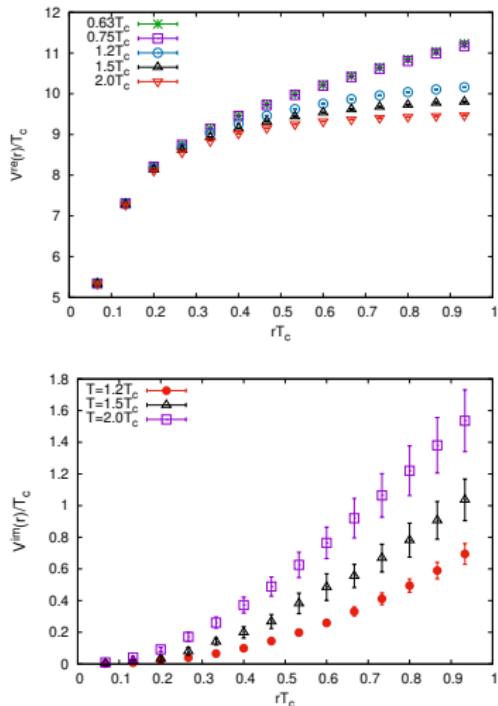
$$G_{diff}(\tau/a) = G(\tau/a) - G(\tau/a+1)$$

$$\rho_{ii}(\omega) = \rho_{ii}^{trans}(\omega) + \rho_{ii}^{mod}(\omega)$$

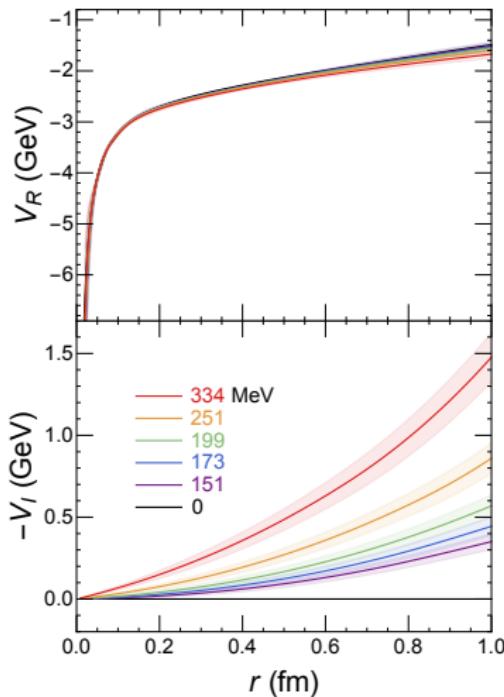
- Fit large frequency part of SFP (model as dotted line) to the difference correlator
- For J/\psi no resonance peak is needed to describe the lattice data even at $1.1T_c$
- For Upsilon the resonance peak persists to $1.5T_c$

Heavy quark potential

Bala and Datta, 1909.10548:
Direct from Wilson loops



Shi et al, 2105.07862:
Machine learning reconstruction



Heavy quark diffusion

Effective theory

- Electric field insertion in Polyakov loop correlator
- Benefit: smooth approach as $\omega \rightarrow 0$
- Drawback: noisy — use noise reduction techniques (multilevel algorithm, Wilson flow)

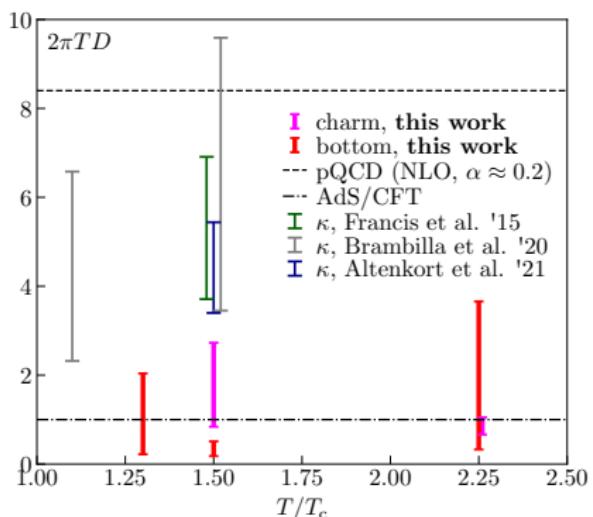
[Banerjee et al 2011, Francis et al 2015, Brambilla et al 2020, Altenkort et al 2021]

Fermionic correlators

- Transport peak of $\langle V_i V_j \rangle$ spectral function
- Benefit: can directly include quark mass effects
- Drawback: hard to extract narrow transport peak

[Bielefeld 2012–2021]

Heavy quark diffusion summary



[from Shu 2021]

Conductivity

Methods

Vector current correlator

$$G_{ij}(\tau, \vec{p}) = \int d^3x e^{i\vec{p}\cdot\vec{x}} \left\langle V_i(\tau, \vec{x}) V_j(0, \vec{0}) \right\rangle = \frac{1}{2\pi} \int_0^\infty K(\omega, \tau; T) \rho_{ij}(\omega, \vec{p}) d\omega$$

$$\sigma = \lim_{\omega \rightarrow 0} \frac{\rho_{ii}^{em}(\omega, \vec{0})}{6\omega}$$

Determine $\rho(\omega)$ from $G(\tau)$

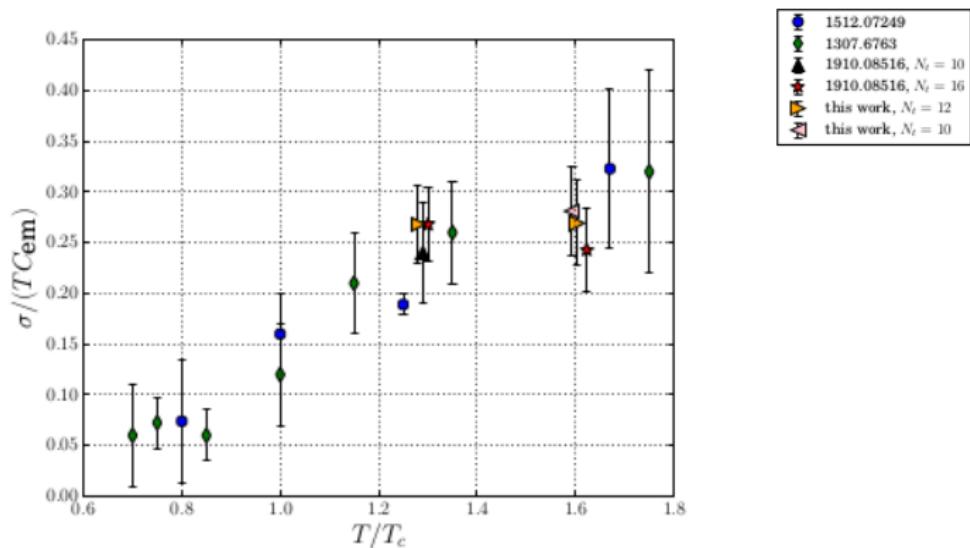
Lattice calculations

Collaboration	arXiv	N_f	action	m_π / MeV	Method
FASTSUM	1307.6763	2+1	Clover	390	MEM
Mainz	1512.07249	2	Clover	290	Ansatz + BG
Moscow/Pisa ¹	1912.08616	2+1	Staggered	135	Tikhonov + BG
Moscow/Pisa ²	Trunin 2021	2+1	Staggered	135	Tikhonov + BG

¹Also with magnetic field

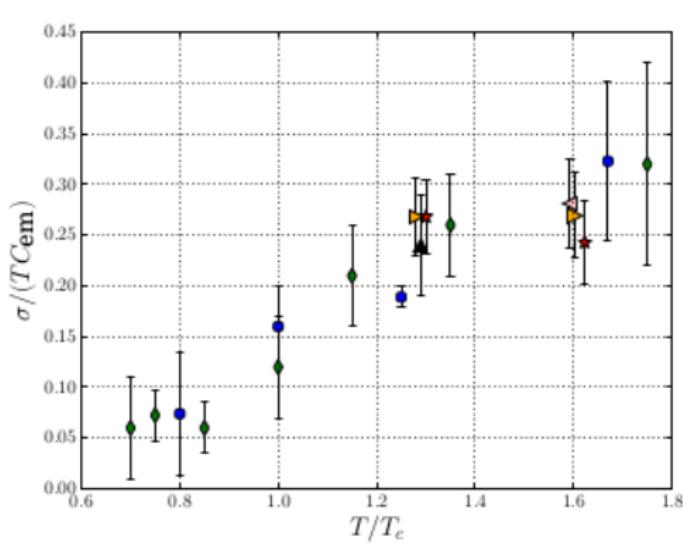
²Also with chemical potential

Conductivity: summary of lattice results

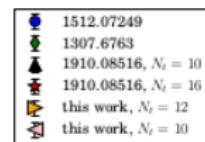


[from Trunin 2021]

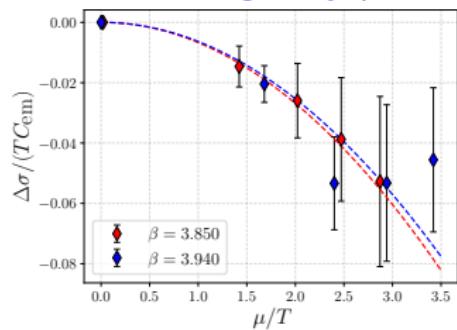
Conductivity: summary of lattice results



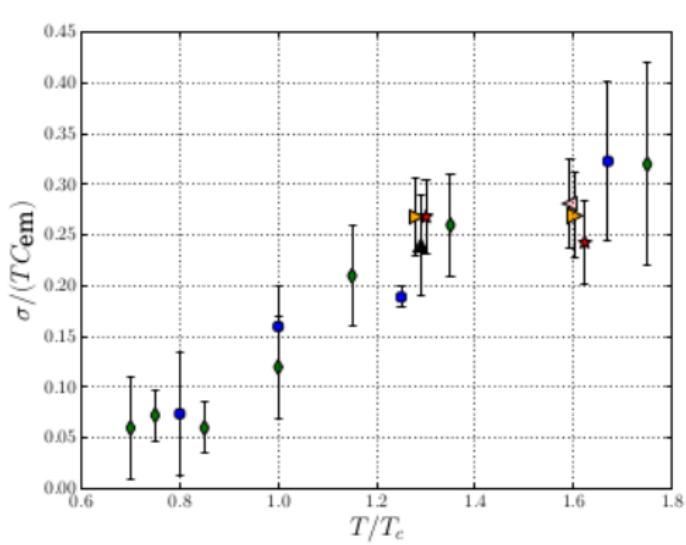
[from Trunin 2021]



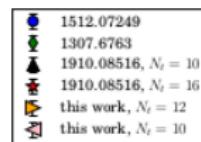
Nonzero imaginary μ



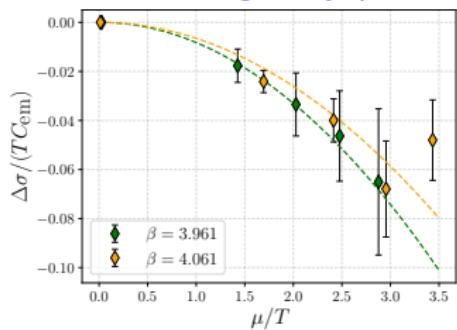
Conductivity: summary of lattice results



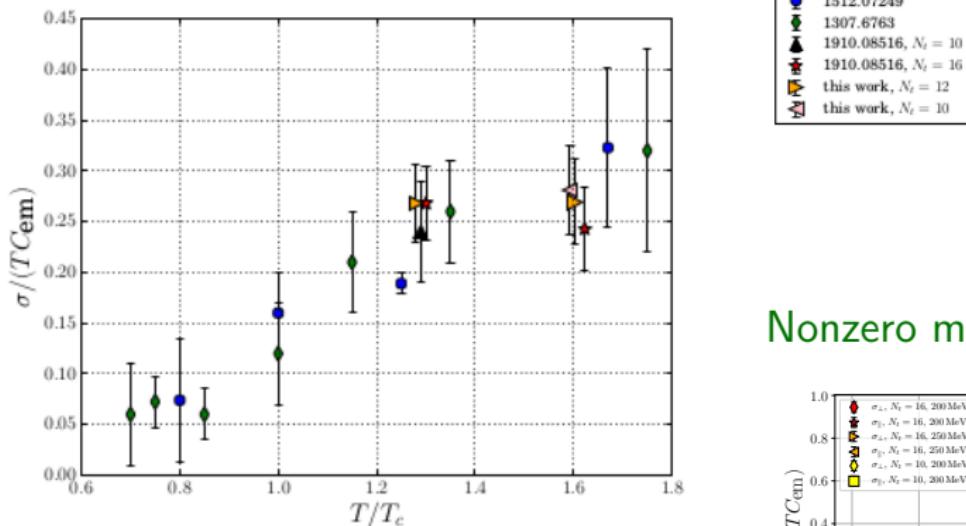
[from Trunin 2021]



Nonzero imaginary μ

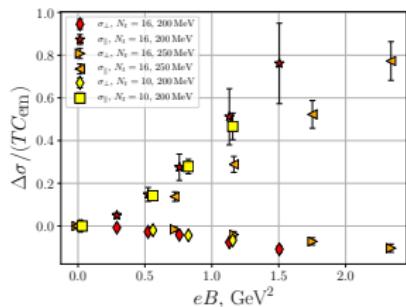


Conductivity: summary of lattice results



[from Trunin 2021]

Nonzero magnetic field



Summary

- Precision reached for T_c , EoS
- Precision within reach for fluctuations
- Approaching quantitative understanding of hadron and (some) transport properties
- Large μ : a lot done, more to do
- More surprises around the corner?

Outlook

Thanks

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THANK YOU FOR YOUR ATTENTION