Nonequilibrium evolution of quarkonium in medium

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A Virtual Tribute to Quark Confinement and the Hadron Spectrum 2021

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Shiori Kajimoto, Takahiro Miura, Yukana Kaida, Masayuki Asakawa (Osaka)

Refs:
Quarkonia in heavy-ion collisions

What we are getting to know

- Non-equilibrium evolution of quarkonia in static and homogeneous medium
Quarkonia in heavy-ion collisions

What we ignore in this talk

- Interaction between initially uncorrelated pairs (justified for bottoms)
- Effects of non-static and inhomogeneous medium (for simplicity)
- Heavy quark pair creation in medium (suppressed by $e^{-M/T}$)
- Heavy quark pair annihilation in medium (suppressed by $1/M^2$)
Quarkonia in heavy-ion collisions

What we do not know

- Initial condition of quarkonia
  - assume singlet/octet wave packets, vacuum states, etc
- How quarkonia hadronize
  - assume evolution freezes at $T = T_f$
Quarkonia in a static and uniform medium \((T > T_c)\)

Key quantities: self-energy of a static quarkonium

- Non-local in NRQCD description:
  Complex potential
- Local in pNRQCD description (→ Antonio’s talk):
  HQ momentum diffusion constant, thermal dipole self-energy coeff.

How do they determine quarkonium evolution?
↔ What can we learn from experiment, in principle?

I will explain how one can model the in-medium dynamics of quarkonia with complex potential

Contents

1. Open quantum system: basics and its application to quarkonia in QGP
2. Simulation of Lindblad equation: decoherence, dissipation, and thermalization
Minimum basics of open quantum system

\[
\begin{align*}
H_S & \quad S \quad H_I \quad E \\
H_E & \quad \text{trace out } E \\
S & \quad L_k
\end{align*}
\]

Lindblad equation: evolution of reduced density matrix \( \rho_S(t) \equiv \text{Tr}_E \rho_{\text{tot}}(t) \)

\[
\frac{d}{dt} \rho_S(t) = -i [H'_S, \rho_S] + \sum_k \left( L_k \rho_S L_k^\dagger - \frac{1}{2} L_k^\dagger L_k \rho_S - \frac{1}{2} \rho_S L_k^\dagger L_k \right) = \mathcal{L}(\rho_S)
\]

dissipator \( \mathcal{D}(\rho_S) \)

\[
= H_{\text{eff}} \rho_S - \rho_S H_{\text{eff}}^\dagger + \frac{i}{2} \sum_k L_k \rho_S L_k^\dagger \\
= H_{\text{eff}} - \frac{i}{2} \sum_k L_k^\dagger L_k
\]

if the evolution is Markovian, preserves probability and (complete) positivity

[Gorini-Kossakowski-Sudarshan (76), Lindblad (76)]
Lindblad equation for weak system-environment coupling

Born-Markov approximation for $H_I = V_S \otimes V_E$ (interaction picture)

$$\frac{d}{dt} \rho_S(t) = \int_0^\infty ds \langle V_E(s)V_E(0) \rangle \left[ V_S(t-s)\rho_S(t)V_S(t) - V_S(t)V_S(t-s)\rho_S(t) \right] + h.c. + \mathcal{O}(V^3)$$

Quantum Brownian regime$^1$

- Slow system time scale $\rightarrow$ derivative expansion

\[ V_S(t-s) \approx V_S(t) - s\dot{V}_S(t) + \cdots = V_S(t) - is[H_S, V_S(t)] + \cdots \]

\[ \rightarrow \quad L \propto V_S - \frac{i}{4T} \dot{V}_S + \cdots \]

- Condition for derivative expansion ($\tau_{S/E} = \text{system/env. timescale}$)

\[ \tau_S \sim 4/M \alpha^2 \sim 1/0.11\text{GeV} \gg \tau_E \sim 1/T \sim 1/0.3\text{GeV} \]

$^1$There is another regime “Quantum optical limit,” where $H_I(t)$ has discrete spectra
Lindblad equation from NRQCD \[ \text{[Akamatsu (15, 20)]} \]

1. **NRQCD Lagrangian** \((1/M\text{-expansion } + v\text{-counting})\)

\[
\mathcal{L}_{\text{NRQCD}} = \mathcal{L}_{q+A} + \psi^\dagger \left[ iD_t + \frac{\vec{D}^2}{2M} \right] \psi + \chi^\dagger \left[ iD_t - \frac{\vec{D}^2}{2M} \right] \chi + \cdots ,
\]

light sector

2. **Quantum mechanics of a heavy quark pair** \((\vec{\nabla}_Q \sim Mv \gg g\vec{A} \sim Mv^3)\)

\[
H = \mathcal{H}_{q+A} + \frac{p_Q^2}{2M} + \frac{p_{Qc}^2}{2M} + gA_0^a(\vec{x}_Q^t) t_Q^a - gA_0^a(\vec{x}_{Qc}^t) t_{Qc}^a
\]

environment system interaction \(H_I\)

- System \(\otimes\) Environment interaction

\[
H_I = \int_k \left( e^{ikx_Q^t} t_Q^a - e^{ikx_{Qc}^t} t_{Qc}^a \right) \otimes g\tilde{A}_0(k)
\]

= \(V_S(k)\)

= \(V_E(k)\)

3. **Lindblad operators** \((\sum_k \to \int_k)\)

\[
L_k = \sqrt{\tilde{D}(k)} \left[ e^{ikx_Q^t} t_Q^a - e^{ikx_{Qc}^t} t_{Qc}^a + \mathcal{O}(\dot{x}_Q, \dot{x}_{Qc}) \right] + \mathcal{O}(g^2)
\]

rate\(^{1/2}\) \(\propto g\) scattering with transfer \(k\)

derivative exp.

perturbative exp.
Diagrammatic representation of Lindblad kernels

Gluon propagators with hard-thermal loop self-energies

Self-energy = complex potential \((r \sim 1/gT)\)

\[
\Delta H - \frac{i}{2} \int L_k^\dagger L_k = V(r)[t_Q^a t_{Q_c}^{a*}] + i \left( D(r)[t_Q^a t_{Q_c}^{a*}] - C_F D(0) \right) + \cdots
\]

\[
V(r) = -\frac{\alpha}{r} e^{-m_D r}, \quad D(r) = \int_k e^{i k \cdot r} \tilde{D}(k), \quad \tilde{D}(k) = g^2 T \frac{\pi m_D^2}{k(k^2 + m_D^2)^2}
\]
Modeling the Lindblad equation

Singlet complex potential in perturbation theory \([\text{Laine+}(07), \text{Beraudo+}(08), \text{Brambilla+}(08)]\)

\[
V^{(\text{singlet})}(r) = C_F \left[ V(r) - i(D(0) - D(r)) \right]
\]

Complex potential from non-perturbative thermal Wilson loop \([\text{Rothkopf+} (12,15)]\)

- Plateau \(\text{Im} V(r \to \infty)\) yet to be seen
Numerical methods for solving the Lindblad equation

Stochastic unravelling of $\rho_S$: give a mixed-state wave-function ensemble

$$\rho_S(t) = \left\langle \psi(t) \right| \left\langle \psi(t) \right\rangle = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} \left| \psi_i(t) \right\rangle \left\langle \psi_i(t) \right|$$

**Method 1: Quantum State Diffusion** [Gisin-Percival (92)]

- Nonlinear stochastic equation with complex white noises ($d\xi_k^* d\xi_\ell = 2\delta_{k\ell} dt$)

$$|d\tilde{\psi}\rangle = |\tilde{\psi}(t + dt)\rangle - |\psi(t)\rangle$$

$$= \left[ \mathcal{L}(|\psi\rangle \langle \psi|) - \langle \mathcal{L}(|\psi\rangle \langle \psi|) \psi \rangle |\psi(t)\rangle \right] dt + \frac{1}{\sqrt{2}} \sum_k L_k |\psi(t)\rangle d\xi_k$$

$\rightarrow$ closest pure state to Lindblad evolution

$\rightarrow$ mixed state

$|\psi(t + dt)\rangle = \text{normalize } |\tilde{\psi}(t + dt)\rangle \quad \rightarrow \quad \text{repeat}$

**Method 2: Quantum Jump** [Plenio-Knight (98)] $\rightarrow$ Antonio's talk
### Numerical simulations

Most simulations use stochastic unravelling

<table>
<thead>
<tr>
<th>Inter-quark distance $r$</th>
<th>NRQCD</th>
<th>pNRQCD</th>
</tr>
</thead>
<tbody>
<tr>
<td>can be long</td>
<td></td>
<td>short</td>
</tr>
<tr>
<td>weak</td>
<td></td>
<td>can be large</td>
</tr>
<tr>
<td>heavier</td>
<td></td>
<td>lighter</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>NRQCD</th>
<th>Dissipation</th>
<th>Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>1D, U(1)</td>
<td>no</td>
<td>Stochastic Potential [Akamatsu-Rothkopf (12), Kajimoto+ (18)]</td>
</tr>
<tr>
<td>3D, U(1)</td>
<td>no</td>
<td>Stochastic Potential [Rothkopf (14)]</td>
</tr>
<tr>
<td>1D, SU(3)</td>
<td>no</td>
<td>Stochastic Potential [Sharma-Tiwari (20), Kajimoto+ (in prep.)]</td>
</tr>
<tr>
<td>1D, U(1)</td>
<td>yes</td>
<td>Quantum State Diffusion [Akamatsu+ (19), Miura+ (20)]</td>
</tr>
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<td>yes</td>
<td>Quantum State Diffusion [Miura+ (in prep.)]</td>
</tr>
<tr>
<td>1D, U(1)</td>
<td>yes</td>
<td>Direct evolution [Alund+ (21)]</td>
</tr>
</tbody>
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<table>
<thead>
<tr>
<th>pNRQCD</th>
<th>Dissipation</th>
<th>Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>1+D, SU(3)</td>
<td>no</td>
<td>Direct evolution for S and P waves [Brambilla+ (17, 18)]</td>
</tr>
<tr>
<td>3D, SU(3)</td>
<td>no</td>
<td>Quantum Jump [Brambilla+ (20, 21)]</td>
</tr>
<tr>
<td>1D, SU(3)</td>
<td>yes</td>
<td>Quantum State Diffusion [Miura-Kaida+ (in prog.)]</td>
</tr>
</tbody>
</table>
QSD simulation: solitonic wave functions of an event [Akamatsu+ (18)]

Solitonic wave functions for U(1) single HQ case (← only in this slide)

- Nonlinear terms (localization) v.s. Kinetic term (“diffusion”)
- Similar nonlinear equation (w/o noise) is used to find pointer states

[Busse-Hornberger (09)]
QSD simulation: density matrix [Miura+ (in prep.)]

Model complex potential: \( V^{(\text{singlet})}_{\text{complex}}(r) = C_F V(r) - iC_F(D(0) - D(r)) \)

\[
C_F V(r) = -\frac{0.3}{r} e^{-2Tr}, \quad C_F D(r) = \frac{T}{\pi} e^{-(Tr)^2}, \quad T = 0.1M
\]

\( \to \) Color resolution scale of QGP \( \ell \sim 1/T = 10/M \)

**Singlet** \( |\rho_s(x, y)|^2 \)

**Octet** \( |\rho_o(x, y)|^2 \)

Singlet ground state
QSD simulation: density matrix [Miura+ (in prep.)]

Model complex potential: \[ V^{\text{singlet}}_{\text{complex}}(r) = C_F V(r) - i C_F (D(0) - D(r)) \]

\[ C_F V(r) = -\frac{0.3}{r} e^{-2Tr}, \quad C_F D(r) = \frac{T}{\pi} e^{-(Tr)^2}, \quad T = 0.1M \]

→ Color resolution scale of QGP \( \ell \sim 1/T = 10/M \)

Singlet \( |\rho_s(x, y)|^2 \)

Octet \( |\rho_o(x, y)|^2 \)

Dipole excitation to octet
QSD simulation: density matrix [Miura+ (in prep.)]

Model complex potential: $V^{\text{singlet}}_{\text{complex}}(r) = C_F V(r) - iC_F(D(0) - D(r))$

$$C_F V(r) = -\frac{0.3}{r} e^{-2Tr}, \quad C_F D(r) = \frac{T}{\pi} e^{-(Tr)^2}, \quad T = 0.1M$$

→ Color resolution scale of QGP $\ell \sim 1/T = 10/M$

Singlet $|\rho_s(x, y)|^2$

Octet $|\rho_o(x, y)|^2$

Decoherence in octet
QSD simulation: density matrix [Miura+ (in prep.)]

Model complex potential: $V_{\text{complex}}^{(\text{singlet})}(r) = C_F V(r) - i C_F (D(0) - D(r))$

$$C_F V(r) = -\frac{0.3}{r} e^{-2 T r}, \quad C_F D(r) = \frac{T}{\pi} e^{-(T r)^2}, \quad T = 0.1 M$$

→ Color resolution scale of QGP $\ell \sim 1/T = 10/M$

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Decoherence in octet
QSD simulation: density matrix [Miura+ (in prep.)]

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→ Color resolution scale of QGP $\ell \sim 1/T = 10/M$

Singlet $|\rho_s(x, y)|^2$

Octet $|\rho_o(x, y)|^2$

De-excitation to singlet
QSD simulation: density matrix [Miura+ (in prep.)]

Model complex potential: \( V_{\text{complex}}^{(\text{singlet})}(r) = C_F V(r) - i C_F (D(0) - D(r)) \)

\[
C_F V(r) = -\frac{0.3}{r} e^{-2Tr}, \quad C_F D(r) = \frac{T}{\pi} e^{-(Tr)^2}, \quad T = 0.1M
\]

→ Color resolution scale of QGP \( \ell \sim 1/T = 10/M \)

Singlet \( |\rho_s(x, y)|^2 \)

Octet \( |\rho_o(x, y)|^2 \)

De-excitation to singlet → equilibrated?
QSD simulation: equilibration [Miura+ (in prep.)]

Evolution of eigenstate occupation

Eigenstate occupation in the steady state

Steady state is independent of initial conditions

Approach to the Boltzmann distribution with environment temperature
QSD simulation: role of dissipation [Miura+ (in prep.)]

Evolution of eigenstate occupation

Without dissipation, all states get equally occupied
Dissipation is non-negligible from early time

Decoherence is not effective for a localized bound state
→ Need to take account of heavy quark’s motion during decoherence
(=dissipation)
Summary

Quarkonium Lindblad equations carry information of QGP

- NRQCD: complex potential
- pNRQCD: local coefficients $\kappa$ and $\gamma$

Quarkonium Lindblad equation is yet to be complete

- NRQCD: valid in weak-coupling regime and can model for any size
- pNRQCD: valid in non-perturbative regime and in the dipole limit
- For $T \lesssim 200\text{MeV}$, quantum Brownian regime may cease to hold [Yao+ (19)]

$$\text{QGP corr. time} \sim \frac{1}{T} \ll \text{quarkonium period} \sim \frac{1}{\Delta E} \sim \frac{1}{110\text{MeV}}$$

Simulation of Lindblad equation

- NRQCD: equilibration achieved by balancing decoherence and dissipation
- pNRQCD: phenomenological application has started
- Need to check the validity of dipole approximation for pNRQCD by comparing with NRQCD simulation [Miura-Kaida+ (in prog.)]
- Quantum simulation? [Hu-Zia-Kais (20), de Jong et al (20)]
Appendix
Physical picture: decoherence + dissipation