Nonequilibrium evolution of quarkonium in medium

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A Virtual Tribute to Quark Confinement and the Hadron Spectrum 2021

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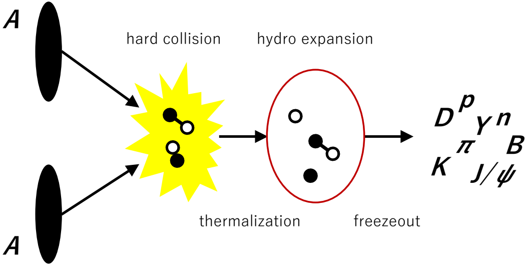
Refs:
Miura et al, PRD101 (2020) 034011 and in prep., & Akamatsu, 2009.10559

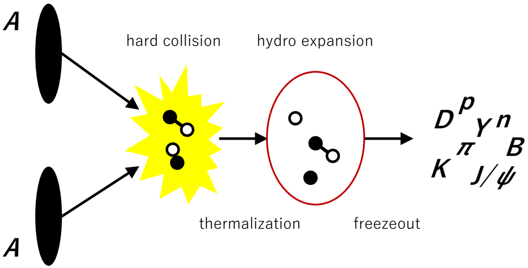
Quarkonia in heavy-ion collisions



What we are getting to know

- Non-equilibrium evolution of quarkonia in static and homogeneous medium

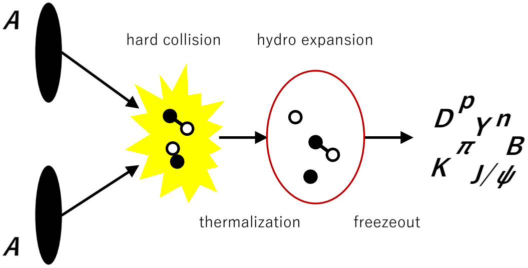
Quarkonia in heavy-ion collisions



What we ignore in this talk

- Interaction between initially uncorrelated pairs (justified for bottoms)
- Effects of non-static and inhomogeneous medium (for simplicity)
- Heavy quark pair creation in medium (suppressed by $e^{-M/T}$)
- Heavy quark pair annihilation in medium (suppressed by $1/M^2$)

Quarkonia in heavy-ion collisions



What we do not know

- Initial condition of quarkonia
  - assume singlet/octet wave packets, vacuum states, etc
- How quarkonia hadronize
  - assume evolution freezes at $T = T_f$

Quarkonia in a static and uniform medium $(T > T_c)$

Key quantities: self-energy of a static quarkonium

- Non-local in NRQCD description:
  Complex potential
- Local in pNRQCD description ($\rightarrow$ Antonio's talk):
  HQ momentum diffusion constant, thermal dipole self-energy coeff.

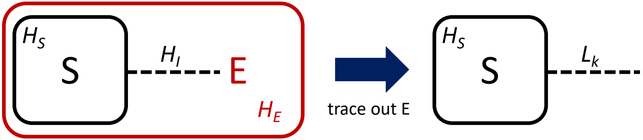
How do they determine quarkonium evolution?
$\leftrightarrow$ What can we learn from experiment, in principle?

I will explain how one can model the in-medium dynamics of quarkonia with complex potential

Contents

1. Open quantum system: basics and its application to quarkonia in QGP
2. Simulation of Lindblad equation: decoherence, dissipation, and thermalization

Minimum basics of open quantum system



Lindblad equation: evolution of reduced density matrix $\rho_S(t) \equiv \mathrm{Tr}_E \rho_{\mathrm{tot}}(t)$

$$\frac{d}{dt}\rho_S(t) = -i\left[H'_S, \rho_S\right] + \underbrace{\sum_k \left(L_k \rho_S L_k^\dagger - \frac{1}{2} L_k^\dagger L_k \rho_S - \frac{1}{2} \rho_S L_k^\dagger L_k \right)}_{\text{dissipator } \mathcal{D}(\rho_S)} = \mathcal{L}(\rho_S)$$

$$= -i\left(H_{\mathrm{eff}}\rho_S - \rho_S H_{\mathrm{eff}}^\dagger\right) + \underbrace{\sum_k L_k \rho_S L_k^\dagger}_{\text{transitions/scatterings}} \quad , \quad H_{\mathrm{eff}} = \underbrace{H'_S - \frac{i}{2}\sum_k L_k^\dagger L_k}_{H_S \,+\, \text{self-energy}}$$

if the evolution is Markovian, preserves probability and (complete) positivity

[Gorini-Kossakowski-Sudarshan (76), Lindblad (76)]

Lindblad equation for weak system-environment coupling

Born-Markov approximation for $H_I = V_S \otimes V_E$ (interaction picture)

$$\frac{d}{dt}\rho_S(t) = \int_0^\infty ds \underbrace{\langle V_E(s)V_E(0)\rangle}_{\text{environment correlator}} \begin{bmatrix} V_S(t-s)\rho_S(t)V_S(t) \\ -V_S(t)V_S(t-s)\rho_S(t) \end{bmatrix} + h.c. + \mathcal{O}(V^3)$$

Quantum Brownian regime[1]

- Slow system time scale $\to$ derivative expansion

$$V_S(t-s) \approx V_S(t) - s\dot{V}_S(t) + \cdots = V_S(t) - is[H_S, V_S(t)] + \cdots$$
$$\to \quad L \propto V_S + \frac{i}{4T}\dot{V}_S + \cdots$$

- Condition for derivative expansion ($\tau_{S/E}$ = system/env. timescale)

$$\tau_S \sim \underbrace{4/M\alpha^2}_{\text{Coulombic}} \sim 1/0.11\text{GeV} \gg \tau_E \sim 1/T \sim 1/0.3\text{GeV}$$

[1] There is another regime "Quantum optical limit," where $H_I(t)$ has discrete spectra

Lindblad equation from NRQCD [Akamatsu (15, 20)]

1. NRQCD Lagrangian ($1/M$-expansion + v-counting)

$$\mathcal{L}_{\rm NRQCD} = \underbrace{\mathcal{L}_{q+A}}_{\text{light sector}} + \psi^\dagger \left[iD_t + \frac{\vec{D}^2}{2M} \right] \psi + \chi^\dagger \left[iD_t - \frac{\vec{D}^2}{2M} \right] \chi + \cdots ,$$

2. Quantum mechanics of a heavy quark pair ($\vec{\nabla}_Q \sim Mv \gg g\vec{A} \sim Mv^3$)

$$H = \underbrace{H_{q+A}}_{\text{environment}} + \underbrace{\frac{p_Q^2}{2M} + \frac{p_{Q_c}^2}{2M}}_{\text{system}} + \underbrace{gA_0^a(\vec{x}_Q)t_Q^a - gA_0^a(\vec{x}_{Q_c})t_{Q_c}^{a*}}_{\text{interaction } H_I}$$

- System $\otimes$ Environment interaction

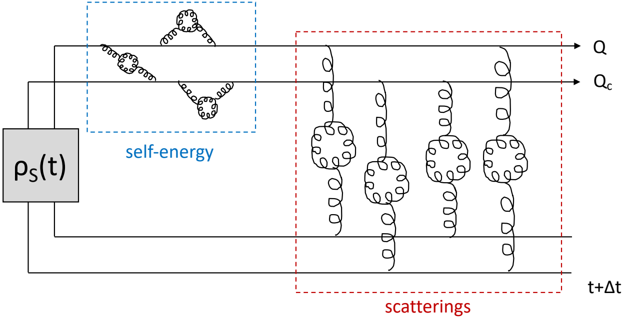
$$H_I = \int_k \underbrace{\left(e^{ikx_Q} t_Q^a - e^{ikx_{Q_c}} t_{Q_c}^{a*} \right)}_{= V_S(k)} \otimes \underbrace{g\tilde{A}_0(k)}_{= V_E(k)}$$

3. Lindblad operators ($\sum_k \to \int_k$)

$$L_k = \underbrace{\sqrt{\tilde{D}(k)}}_{\text{rate}^{1/2} \propto g} \left[\underbrace{e^{ikx_Q} t_Q^a - e^{ikx_{Q_c}} t_{Q_c}^{a*}}_{\text{scattering with transfer } k} + \underbrace{\mathcal{O}(\dot{x}_Q, \dot{x}_{Q_c})}_{\text{derivative exp.}} \right] + \underbrace{\mathcal{O}(g^2)}_{\text{perturbative exp.}}$$

Diagrammatic representation of Lindblad kernels

Gluon propagators with hard-thermal loop self-energies



Self-energy = complex potential ($r \sim 1/gT$)

$$\Delta H - \frac{i}{2}\int_k L_k^\dagger L_k = V(r)[t^a_Q t^{a*}_{Q_c}] + i\Big(D(r)[t^a_Q t^{a*}_{Q_c}] - C_F D(0)\Big) + \underbrace{\cdots}_{\text{expansions}}$$

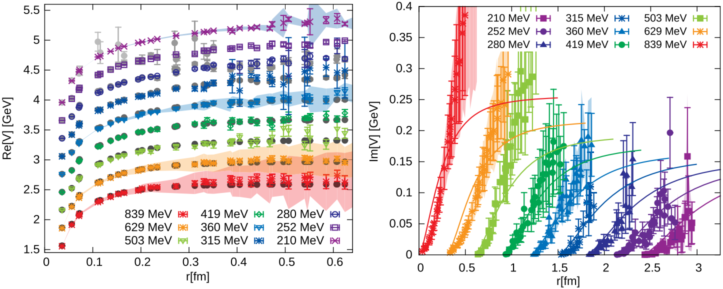
$$V(r) = -\frac{\alpha}{r}e^{-m_D r}, \quad D(r) = \int_k e^{ik\cdot r}\tilde{D}(k), \quad \tilde{D}(k) = g^2 T \frac{\pi m_D^2}{k(k^2+m_D^2)^2}$$

Modeling the Lindblad equation

Singlet complex potential in perturbation theory [Laine+(07),Beraudo+(08),Brambilla+(08)]

$$V^{(\mathrm{singlet})}_{\mathrm{complex}}(r) = C_F\left[V(r) - i(D(0) - D(r))\right]$$

Complex potential from non-perturbative thermal Wilson loop [Rothkopf+ (12,15)]



- Plateau $\mathrm{Im}V(r \to \infty)$ yet to be seen

Numerical methods for solving the Lindblad equation

Stochastic unravelling of ρ_S: give a mixed-state wave-function ensemble

$$\rho_S(t) = \underbrace{\overline{|\psi(t)\rangle\langle\psi(t)|}}_{\text{ensemble average}} = \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} |\psi_i(t)\rangle\langle\psi_i(t)|$$

Method 1: Quantum State Diffusion [Gisin-Percival (92)]

- Nonlinear stochastic equation with complex white noises ($\overline{d\xi_k^* d\xi_\ell} = 2\delta_{k\ell} dt$)

$$\begin{aligned} |d\tilde{\psi}\rangle &= |\tilde{\psi}(t+dt)\rangle - |\psi(t)\rangle \\ &= \underbrace{\left[\mathcal{L}(|\psi\rangle\langle\psi|) - \langle\mathcal{L}(|\psi\rangle\langle\psi|)\rangle_\psi\right] |\psi(t)\rangle dt}_{\to \text{ closest pure state to Lindblad evolution}} + \underbrace{\frac{1}{\sqrt{2}} \sum_k L_k |\psi(t)\rangle d\xi_k}_{\to \text{ mixed state}} \end{aligned}$$

$|\psi(t+dt)\rangle =$ normalize $|\tilde{\psi}(t+dt)\rangle \quad \to \quad$ repeat

Method 2: Quantum Jump [Plenio-Knight (98)] $\to$ Antonio's talk

Numerical simulations

Most simulations use stochastic unravelling

	NRQCD	pNRQCD
Inter-quark distance r	can be long	short
Coupling g	weak	can be large
Simulation cost	heavier	lighter

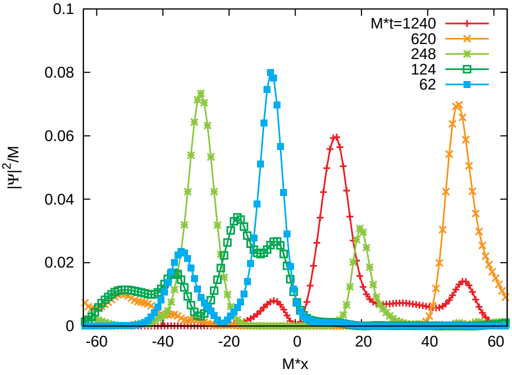
NRQCD	Dissipation	Method
1D, U(1)	no	Stochastic Potential [Akamatsu-Rothkopf (12), Kajimoto+ (18)]
3D, U(1)	no	Stochastic Potential [Rothkopf (14)]
1D, SU(3)	no	Stochastic Potential [Sharma-Tiwari (20), Kajimoto+ (in prep.)]
1D, U(1)	yes	Quantum State Diffusion [Akamatsu+ (19), Miura+ (20)]
1D, SU(3)	yes	Quantum State Diffusion [Miura+ (in prep.)]
1D, U(1)	yes	Direct evolution [Alund+ (21)]

pNRQCD	Dissipation	Method
1_+D, SU(3)	no	Direct evolution for S and P waves [Brambilla+ (17, 18)]
3D, SU(3)	no	Quantum Jump [Brambilla+ (20, 21)]
1D, SU(3)	yes	Quantum State Diffusion [Miura-Kaida+ (in prog.)]

QSD simulation: solitonic wave functions of an event [Akamatsu+ (18)]

Solitonic wave functions for U(1) single HQ case (← only in this slide)

- Nonlinear terms (localization) v.s. Kinetic term ("diffusion")
- Similar nonlinear equation (w/o noise) is used to find pointer states [Busse-Hornberger (09)]



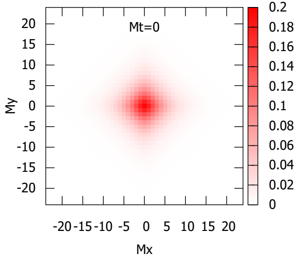
QSD simulation: density matrix [Miura+ (in prep.)]

Model complex potential: $V^{(\mathrm{singlet})}_{\mathrm{complex}}(r) = C_F V(r) - iC_F(D(0) - D(r))$

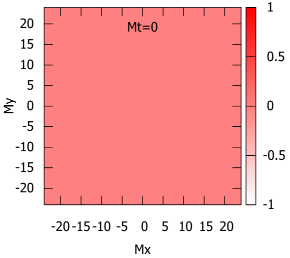
$$C_F V(r) = -\frac{0.3}{r} e^{-2Tr}, \quad C_F D(r) = \frac{T}{\pi} e^{-(Tr)^2}, \quad T = 0.1M$$

$\rightarrow$ Color resolution scale of QGP $\ell \sim 1/T = 10/M$

Singlet $|\rho_s(x,y)|^2$



Octet $|\rho_o(x,y)|^2$



Singlet ground state

QSD simulation: density matrix [Miura+ (in prep.)]

Model complex potential: $V^{(\mathrm{singlet})}_{\mathrm{complex}}(r) = C_F V(r) - iC_F(D(0) - D(r))$

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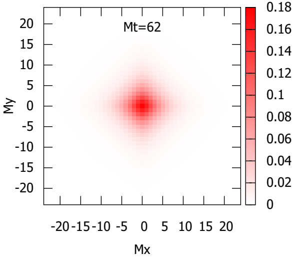

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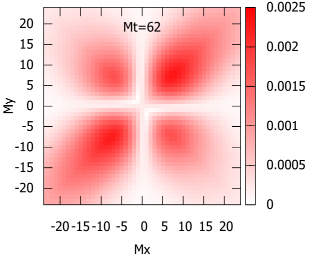

Dipole excitation to octet

QSD simulation: density matrix [Miura+ (in prep.)]

Model complex potential: $V^{\text{(singlet)}}_{\text{complex}}(r) = C_F V(r) - iC_F(D(0) - D(r))$

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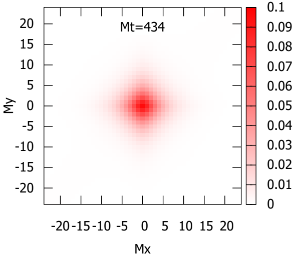
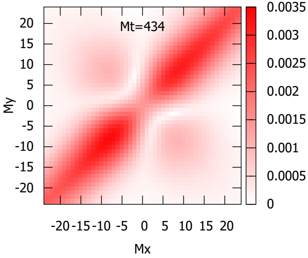


Octet $|\rho_o(x,y)|^2$



Decoherence in octet

QSD simulation: density matrix [Miura+ (in prep.)]

Model complex potential: $V^{(\mathrm{singlet})}_{\mathrm{complex}}(r) = C_F V(r) - iC_F(D(0) - D(r))$

$$C_F V(r) = -\frac{0.3}{r} e^{-2Tr}, \quad C_F D(r) = \frac{T}{\pi} e^{-(Tr)^2}, \quad T = 0.1M$$

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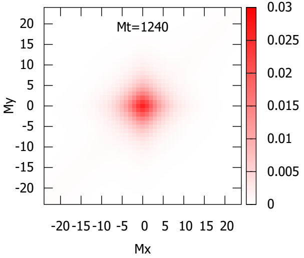
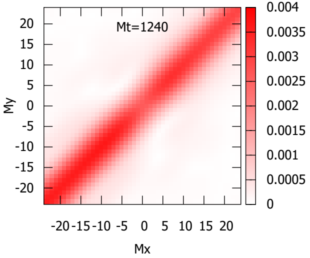


Octet $|\rho_o(x,y)|^2$



Decoherence in octet

QSD simulation: density matrix [Miura+ (in prep.)]

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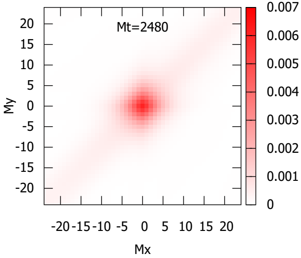
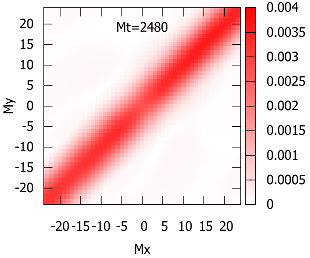
Singlet $|\rho_s(x,y)|^2$



Octet $|\rho_o(x,y)|^2$



De-excitation to singlet

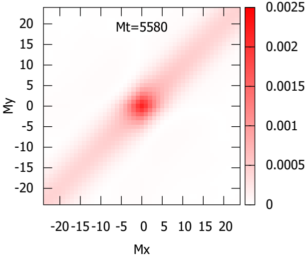
QSD simulation: density matrix [Miura+ (in prep.)]

Model complex potential: $V^{(\mathrm{singlet})}_{\mathrm{complex}}(r) = C_F V(r) - iC_F(D(0) - D(r))$

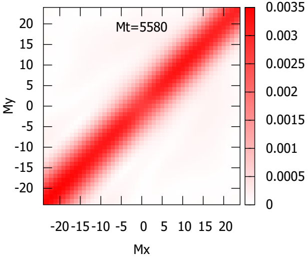
$$C_F V(r) = -\frac{0.3}{r} e^{-2Tr}, \quad C_F D(r) = \frac{T}{\pi} e^{-(Tr)^2}, \quad T = 0.1M$$

$\rightarrow$ Color resolution scale of QGP $\ell \sim 1/T = 10/M$

Singlet $|\rho_s(x,y)|^2$



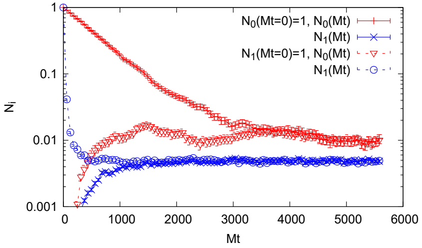
Octet $|\rho_o(x,y)|^2$



De-excitation to singlet $\rightarrow$ equilibrated?

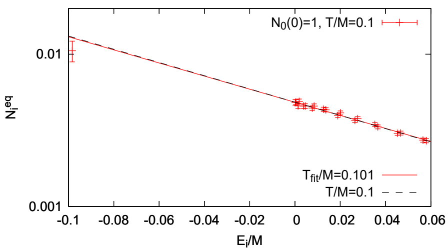
QSD simulation: equilibration [Miura+ (in prep.)]

Evolution of eigenstate occupation



Steady state is independent of initial conditions

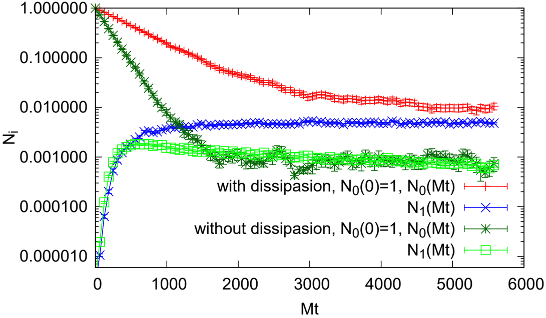
Eigenstate occupation in the steady state



Approach to the Boltzmann distribution with environment temperature

QSD simulation: role of dissipation [Miura+ (in prep.)]

Evolution of eigenstate occupation



Without dissipation, all states get equally occupied
Dissipation is non-negligible from early time

Decoherence is not effective for a localized bound state
→ Need to take account of heavy quark's motion during decoherence (=dissipation)

Summary

Quarkonium Lindblad equations carry information of QGP

- NRQCD: complex potential
- pNRQCD: local coefficients κ and γ

Quarkonium Lindblad equation is yet to be complete

- NRQCD: valid in weak-coupling regime and can model for any size
- pNRQCD: valid in non-perturbative regime and in the dipole limit
- For $T \lesssim 200\mathrm{MeV}$, quantum Brownian regime may cease to hold [Yao+ (19)]

$$\text{QGP corr. time} \sim \frac{1}{T} \ll \text{quarkonium period} \sim \frac{1}{\Delta E} \sim \frac{1}{110\mathrm{MeV}}$$

Simulation of Lindblad equation

- NRQCD: equilibration achieved by balancing decoherence and dissipation
- pNRQCD: phenomenological application has started
- Need to check the validity of dipole approximation for pNRQCD by comparing with NRQCD simulation [Miura-Kaida+ (in prog.)]
- Quantum simulation? [Hu-Zia-Kais (20), de Jong et al (20)]

Appendix

Physical picture: decoherence + dissipation