Quarkonium in medium

an EFT and open quantum system description

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Based on

(1) N. Brambilla, M.A. Escobedo, M. Strickland, A. Vairo, P. Vander Griend and J.H. Weber
Bottomonium production in heavy-ion collisions using quantum trajectories: Differential observables and momentum anisotropy
arXiv:2107.06222

(2) N. Brambilla, M.A. Escobedo, M. Strickland, A. Vairo, P. Vander Griend and J.H. Weber
Bottomonium suppression in an open quantum system using the quantum trajectories method
JHEP 05 (2021) 136 arXiv:2012.01240

(3) N. Brambilla, M.A. Escobedo, J. Soto and A. Vairo
Heavy quarkonium suppression in a fireball
Phys. Rev. **D 97** (2018) 074009 arXiv:1711.04515

(4) N. Brambilla, M.A. Escobedo, J. Soto and A. Vairo
Quarkonium suppression in heavy-ion collisions: an open quantum system approach
Phys. Rev. **D 96** (2017) 034021 arXiv:1612.07248

Energy scales

Quarkonium being a composite system is characterized by several energy scales:

- the scales of a non-relativistic bound state
  (v is the relative heavy-quark velocity; $v \sim \alpha_{\rm s}$ for a Coulombic bound state):
  M (mass),
  Mv (momentum transfer, inverse distance),
  Mv^2 (kinetic energy, binding energy, potential V), ...

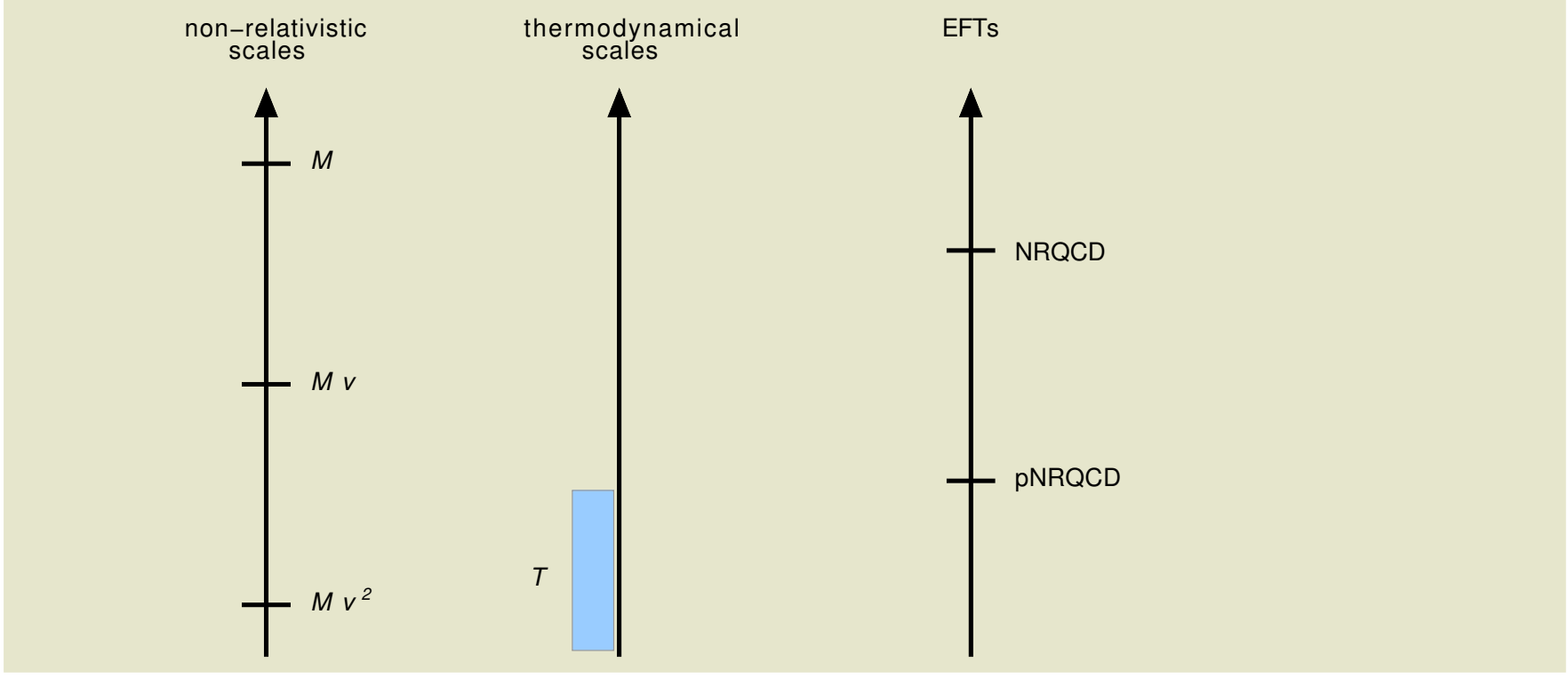
- the thermodynamical scales:
  T (temperature), ...

  T stands for a generic inverse correlation length characterizing the medium.
  For definiteness we will assume that the system is locally in thermal equilibrium so that a slowly varying time-dependent temperature can be defined.

The non-relativistic scales are hierarchically ordered: $M \gg Mv \gg Mv^2$

Non-relativistic EFTs of QCD

The existence of a hierarchy of energy scales calls for a description of the system in terms of a hierarchy of EFTs. We assume T (~ 400 MeV) $< Mv$ (~ 1.5 GeV, for Υ).

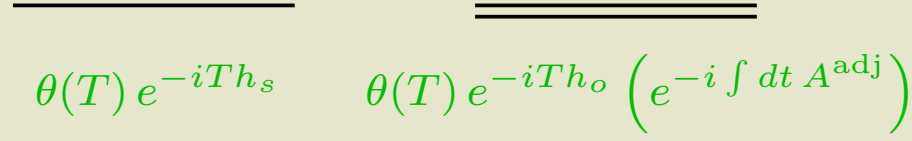


- Brambilla Pineda Soto Vairo RMP 77 (2005) 1423
  Brambilla Ghiglieri Petreczky Vairo PRD 78 (2008) 014017

pNRQCD

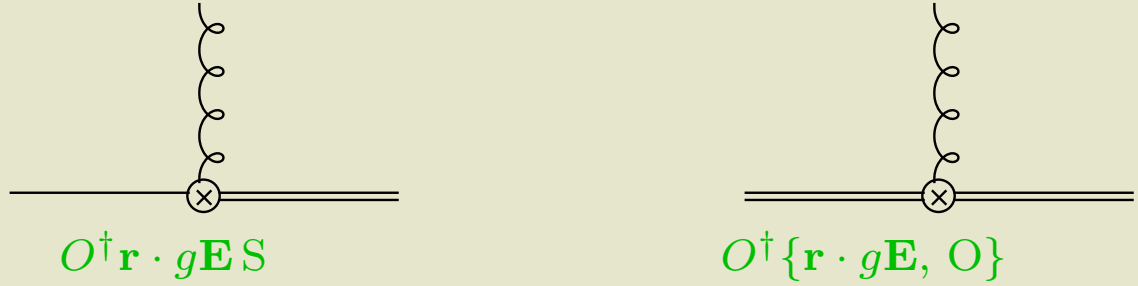
$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu\, a} + \sum_{i=1}^{n_f} \bar{q}_i \, i \not{D} q_i + \int d^3 r \, \mathrm{Tr} \left\{ \mathrm{S}^\dagger \left(i\partial_0 - h_s \right) \mathrm{S} + \mathrm{O}^\dagger \left(i D_0 - h_o \right) \mathrm{O} \right\}$$

- LO in r



$$+V_A \, \mathrm{Tr} \left\{ \mathrm{O}^\dagger \mathbf{r} \cdot g\mathbf{E}\, \mathrm{S} + \mathrm{S}^\dagger \mathbf{r} \cdot g\mathbf{E}\, \mathrm{O} \right\} + \frac{V_B}{2} \mathrm{Tr} \left\{ \mathrm{O}^\dagger \mathbf{r} \cdot g\mathbf{E}\, \mathrm{O} + \mathrm{O}^\dagger \mathrm{O} \mathbf{r} \cdot g\mathbf{E} \right\}$$

- NLO in r



$+ \cdots$

pNRQCD: fields and potentials

Fields:

- $S^\dagger$ creates a quark-antiquark pair in a color singlet configuration.
- $O^\dagger$ creates a (unbound) quark-antiquark pair in a color octet configuration.

Potentials:

- quark-antiquark color singlet Hamiltonian $= h_s = \dfrac{\mathbf{p}^2}{M} - \dfrac{4}{3}\dfrac{\alpha_{\rm s}}{r} + \dots$
- quark-antiquark color octet Hamiltonian $= h_o = \dfrac{\mathbf{p}^2}{M} + \dfrac{\alpha_{\rm s}}{6r} + \dots$
- $V_A = V_B = 1$ up to higher corrections in $\alpha_{\rm s}$.

Open quantum system

- System: heavy quarks/quarkonium
- Environment: quark gluon plasma

We may define a (reduced) density matrix in pNRQCD for the heavy quark-antiquark pair in a color singlet and octet configuration:

$$\begin{aligned} \langle \mathbf{r}', \mathbf{R}' | \rho_s(t'; t) | \mathbf{r}, \mathbf{R} \rangle &\equiv \mathrm{Tr}\{\rho_{\mathrm{full}}(t_0)\, S^\dagger(t, \mathbf{r}, \mathbf{R}) S(t', \mathbf{r}', \mathbf{R}')\} \\ \langle \mathbf{r}', \mathbf{R}' | \rho_o(t'; t) | \mathbf{r}, \mathbf{R} \rangle \frac{\delta^{ab}}{8} &\equiv \mathrm{Tr}\{\rho_{\mathrm{full}}(t_0)\, O^{a\dagger}(t, \mathbf{r}, \mathbf{R}) O^b(t', \mathbf{r}', \mathbf{R}')\} \end{aligned}$$

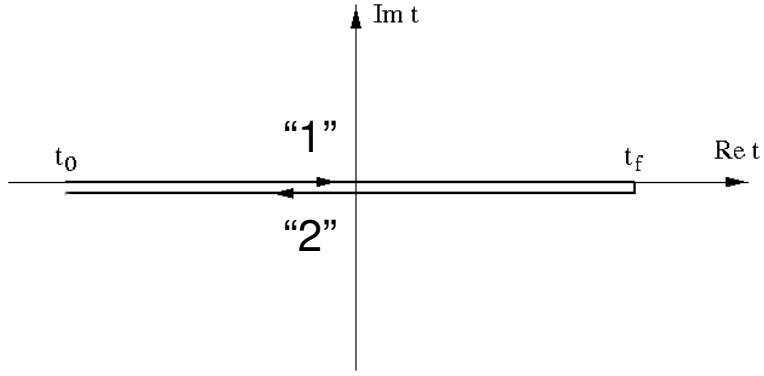
$t_0 \approx 0.6$ fm is the time formation of the plasma.

The system is in non-equilibrium because through interaction with the environment (quark gluon plasma) singlet and octet quark-antiquark states continuously transform in each other although the number of heavy quarks is conserved: $\mathrm{Tr}\{\rho_s\} + \mathrm{Tr}\{\rho_o\} = 1$.

Closed-time path formalism

In the closed-time path formalism we can represent the density matrices as 12 propagators on a closed time path:

$$\begin{aligned} \langle \mathbf{r}', \mathbf{R}' | \rho_s(t'; t) | \mathbf{r}, \mathbf{R} \rangle &= \langle S_1(t', \mathbf{r}', \mathbf{R}') S_2^\dagger(t, \mathbf{r}, \mathbf{R}) \rangle \\ \langle \mathbf{r}', \mathbf{R}' | \rho_o(t'; t) | \mathbf{r}, \mathbf{R} \rangle \frac{\delta^{ab}}{8} &= \langle O_1^b(t', \mathbf{r}', \mathbf{R}') O_2^{a\dagger}(t, \mathbf{r}, \mathbf{R}) \rangle \end{aligned}$$



Differently from the thermal equilibrium case 12 propagators are relevant (in thermal equilibrium they are exponentially suppressed).

12 propagators are not time ordered, while 11 and 22 operators select the forward time direction $\propto \theta(t - t')$, $\theta(t' - t)$.

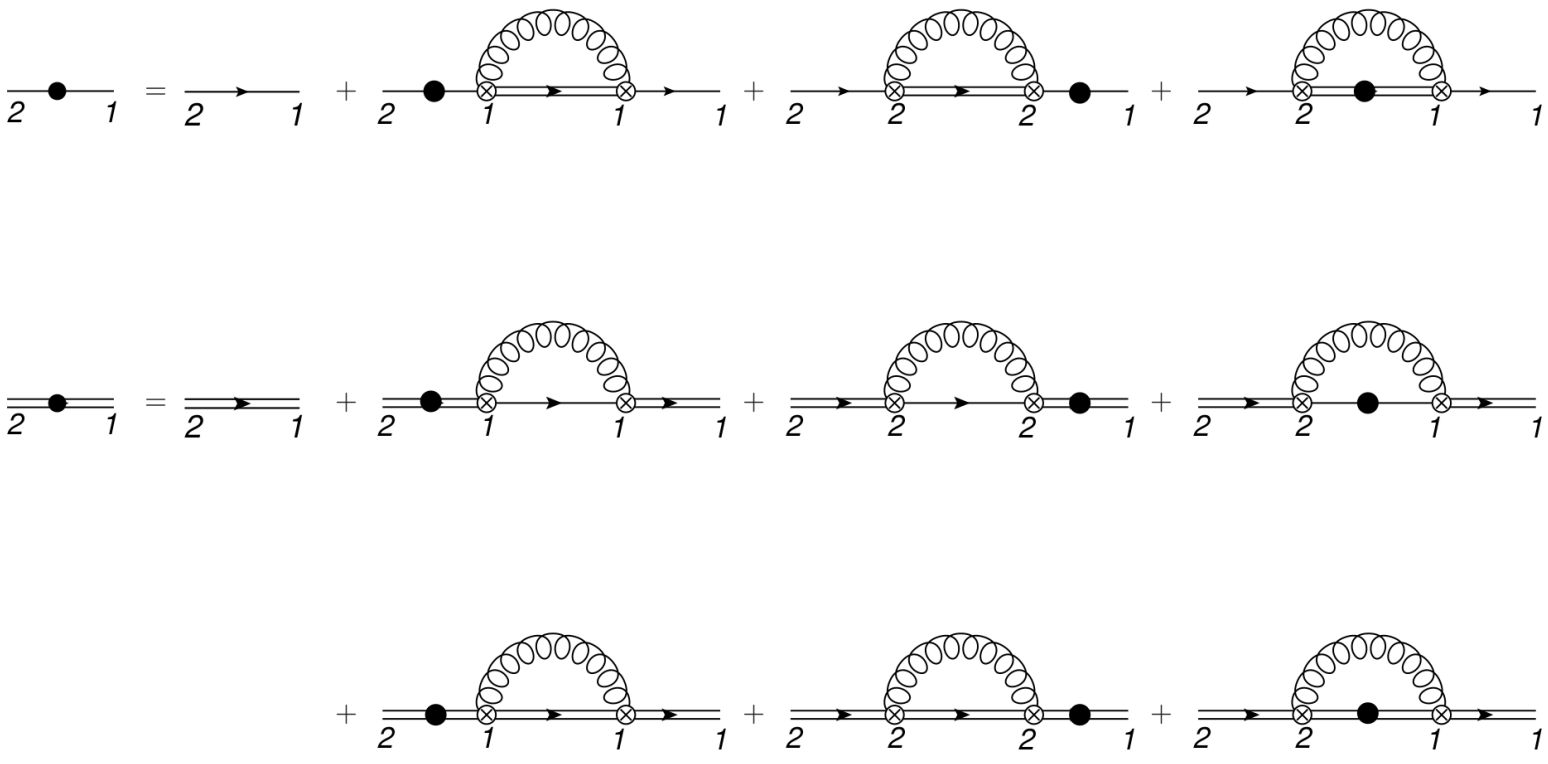
Expansions

- The density of heavy quarks is much smaller than the one of the light d.o.f.: we expand at first order in the heavy quark-antiquark density.
- We consider T much smaller than the inverse Bohr radius of the quarkonium: we expand up to order r^2 in the multipole expansion.

The evolution depends on the density at initial time: non Markovian evolution.

Resummation

Resumming $(t - t_0) \times$ self-energy contributions ...



(Resummation is accurate at order r^2 and consistent with unitary evolution in the absence of dissipation.)

Evolution equations I

... and differentiating over time we obtain the coupled evolution equations:

$$\frac{d\rho_s(t;t)}{dt} = -i[h_s, \rho_s(t;t)] - \Sigma_s(t)\rho_s(t;t) - \rho_s(t;t)\Sigma_s^\dagger(t) + \Xi_{so}(\rho_o(t;t), t)$$

$$\frac{d\rho_o(t;t)}{dt} = -i[h_o, \rho_o(t;t)] - \Sigma_o(t)\rho_o(t;t) - \rho_o(t;t)\Sigma_o^\dagger(t) + \Xi_{os}(\rho_s(t;t), t) + \Xi_{oo}(\rho_o(t;t), t)$$

- The evolution equations are now valid for large time.
- The evolution equations are Markovian.

Interpretation

- The self energies Σ_s and Σ_o provide the in-medium induced mass shifts, $\delta m_{s,o}$, and widths, $\Gamma_{s,o}$, for the color-singlet and color-octet heavy quark-antiquark pairs respectively:

$$\begin{aligned} -i\Sigma_{s,o}(t) + i\Sigma_{s,o}^{\dagger}(t) &= 2\,\mathrm{Re}\left(-i\Sigma_{s,o}(t)\right) = 2\delta m_{s,o}(t) \\ \Sigma_{s,o}(t) + \Sigma_{s,o}^{\dagger}(t) &= -2\,\mathrm{Im}\left(-i\Sigma_{s,o}(t)\right) = \Gamma_{s,o}(t) \end{aligned}$$

- Ξ_{so} accounts for the production of singlets through the decay of octets, and Ξ_{os} and Ξ_{oo} account for the production of octets through the decays of singlets and octets respectively. There are two octet production mechanisms/octet chromoelectric dipole vertices in the pNRQCD Lagrangian.
- The conservation of the trace of the sum of the densities, i.e., the conservation of the number of heavy quarks, follows from

$$\begin{aligned} \mathrm{Tr}\left\{\rho_s(t;t)\left(\Sigma_s(t) + \Sigma_s^{\dagger}(t)\right)\right\} &= \mathrm{Tr}\left\{\Xi_{os}(\rho_s(t;t),t)\right\} \\ \mathrm{Tr}\left\{\rho_o(t;t)\left(\Sigma_o(t) + \Sigma_o^{\dagger}(t)\right)\right\} &= \mathrm{Tr}\left\{\Xi_{so}(\rho_o(t;t),t) + \Xi_{oo}(\rho_o(t;t),t)\right\} \end{aligned}$$

Evolution equations II

An alternative way of writing the evolution equations is

$$\frac{d\rho}{dt} = -i[H,\rho] + \sum_{nm} h_{nm} \left(L_i^n \rho L_i^{m\,\dagger} - \frac{1}{2}\{L_i^{m\,\dagger} L_i^n, \rho\} \right)$$

$$\rho = \begin{pmatrix} \rho_s & 0 \\ 0 & \rho_o \end{pmatrix} \qquad H = \begin{pmatrix} h_s + \frac{\Sigma_s - \Sigma_s^\dagger}{2i} & 0 \\ 0 & h_o + \frac{\Sigma_o - \Sigma_o^\dagger}{2i} \end{pmatrix}$$

$$\Sigma_s(t) = r^i \, A_i^{so\,\dagger}(t) \qquad \Sigma_o(t) = \frac{r^i \, A_i^{os\,\dagger}(t)}{8} + \frac{5}{16} r^i \, A_i^{oo\,\dagger}(t)$$

$$L_i^0 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} r^i \qquad L_i^1 = \begin{pmatrix} 0 & 0 \\ 0 & \frac{5}{16} A_i^{oo\,\dagger} \end{pmatrix}$$

$$L_i^2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} r^i \qquad L_i^3 = \begin{pmatrix} 0 & \frac{1}{8} A_i^{os\,\dagger} \\ A_i^{so\,\dagger} & 0 \end{pmatrix}$$

with $A_i^{so}(t) = \frac{g^2}{6} \int_{t_0}^{t} dt_2 \, e^{i h_s (t_2 - t)} \, r^j \, e^{i h_o (t - t_2)} \, \langle E^{a,j}(t_2, \mathbf{0}) E^{a,i}(t, \mathbf{0}) \rangle$

Positivity

The matrix h_{nm} is

$$h = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

If h were a positive definite matrix then it would always be possible to redefine the operators L_i^n in such a way that the evolution equation would be of the Lindblad form.

Since, however, h is not a positive definite matrix, the Lindblad theorem does not guarantee that the equations may be brought into a Lindblad form.

A special case is the strongly-coupled case. There $L_i^1 \propto L_i^0$ and $L_i^3 \propto L_i^2$, which allows to set to zero, after a redefinition of L_i^n, the eigenvectors of h associated with negative eigenvalues, eventually leading to an evolution equation of the Lindblad form.

Time scales

Environment correlation time: $\tau_E \sim \dfrac{1}{T}$

System intrinsic time scale: $\tau_S \sim \dfrac{1}{E}$

System relaxation time: $\tau_R \sim \dfrac{1}{\text{self-energy}} \sim \dfrac{1}{\alpha_s a_0^2 \Lambda^3}$ $\quad a_0 =$ Bohr radius, $\Lambda = T, E$

- Because we have assumed $1/a_0 \gg \Lambda$, it follows $\tau_R \gg \tau_S, \tau_E$
  which, after resummation ($t - t_0 \gg \tau_R$), qualifies the system as Markovian.

- If $T \gg E$ then $\tau_S \gg \tau_E$
  which qualifies the motion of the system as quantum Brownian.

◦ `Akamatsu PRD 91 (2015) 056002`

From the evolution equations to the Lindblad equation

Under the Markovian

$$\tau_R \gg \tau_S, \tau_E \quad \text{or} \quad \frac{1}{a_0} \gg E, T$$

and quantum Brownian motion condition

$$\tau_S \gg \tau_E \quad \text{or} \quad T \gg E$$

at (N)LO in E/T the evolution equations can be written in the Lindblad form.

Heavy quark-antiquarks in a strongly coupled medium

If $E \ll T$ the Lindblad equation for a strongly coupled plasma reads

$$\frac{d\rho}{dt} = -i[H,\rho] + \sum_i (C_i \rho C_i^\dagger - \frac{1}{2}\{C_i^\dagger C_i, \rho\})$$

$$\rho = \begin{pmatrix} \rho_s & 0 \\ 0 & \rho_o \end{pmatrix}$$

$$H = \begin{pmatrix} h_s & 0 \\ 0 & h_o \end{pmatrix} + \frac{r^2}{2}\gamma(t) \begin{pmatrix} 1 & 0 \\ 0 & \frac{7}{16} \end{pmatrix}$$

$$C_i^0 = \sqrt{\frac{\kappa(t)}{8}}\, r^i \begin{pmatrix} 0 & 1 \\ \sqrt{8} & 0 \end{pmatrix}, \qquad C_i^1 = \sqrt{\frac{5\kappa(t)}{16}}\, r^i \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

Thermal width and mass shift

The quantity κ is related to the thermal decay width of the heavy quarkonium.
In particular for $1S$ states, we have

$$\Gamma(1S) = -2\langle \mathrm{Im}\,(-i\Sigma_s)\rangle = 3a_0^2\,\kappa$$

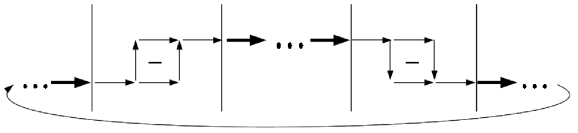
The quantity γ is related to the thermal mass shift of the heavy quarkonium.
In particular for $1S$ states, we have

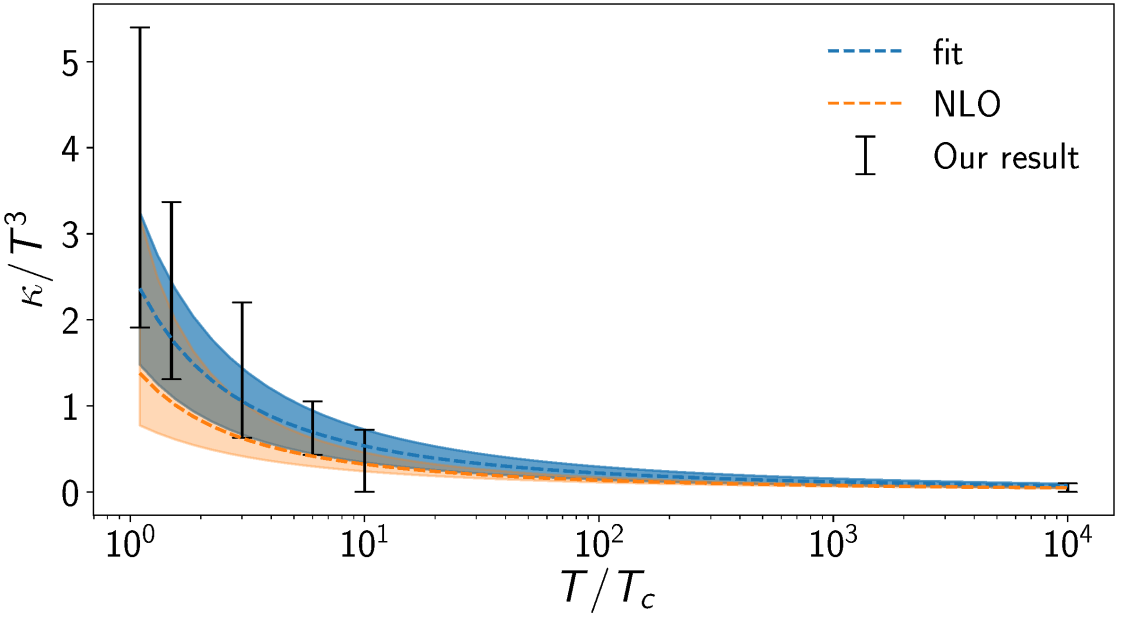
$$\delta M(1S) = \langle \mathrm{Re}\,(-i\Sigma_s)\rangle = \frac{3}{2}a_0^2\,\gamma$$

κ

Low energy parameters may be determined by numerical calculations in lattice QCD.
κ is the heavy-quark momentum diffusion coefficient:

$$\kappa = \frac{g^2}{18}\,\mathrm{Re}\int_{-\infty}^{+\infty} ds\,\langle\,\mathrm{T}\,E^{a,i}(s,\mathbf{0})\,\phi^{ab}(s,0)\,E^{b,i}(0,\mathbf{0})\rangle\,=$$

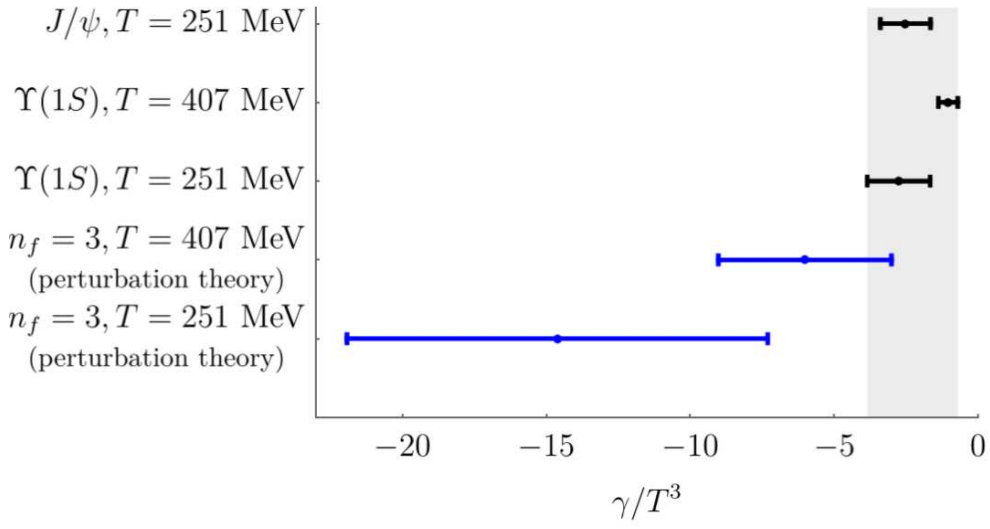




- Brambilla Leino Petreczky Vairo PRD 102 (2020) 074503

γ

$$\gamma = \frac{g^2}{18} \, \mathrm{Im} \int_{-\infty}^{+\infty} ds \, \langle \, \mathrm{T} \, E^{a,i}(s, \mathbf{0}) \, \phi^{ab}(s, 0) \, E^{b,i}(0, \mathbf{0}) \rangle$$



- Brambilla Escobedo Vairo Vander Griend PRD 100 (2019) 054025
  from the lattice data of Kim Petreczky Rothkopf JHEP 11 (2018) 088

  for an Euclidean version Eller Ghiglieri Moore PRD 99 (2019) 094042

Evolution set up

- After heavy-ion collision, heavy quark-antiquarks propagate freely up to 0.6 fm.
- From 0.6 fm to the freeze-out time t_F they propagate in medium.
- We assume the medium to be locally in thermal equilibrium.
- We use a 3+1D dissipative relativistic hydrodynamics code that makes use of the quasiparticle anisotropic hydrodynamics (aHydroQP) framework. The code uses a realistic equation of state fit to lattice QCD measurements and is tuned to soft hadronic data collected in 5.02 TeV collisions using smooth optical Glauber initial conditions.
  - `Alqahtani Nopoush Strickland PRC 92 (2015) 054910, 95 (2017) 034906`
    `Alqahtani Nopoush Strickland PPNP 101 (2018) 204`

Quantum trajectories algorithm

The `QTraj` code implements the quantum trajectories algorithm and the waiting time approach as follows.

1. Initialize a wave function $|\psi(t_0)\rangle$ at initial time t_0, which corresponds to the initial quantum state of the particle given by $\rho(t_0) = |\psi(t_0)\rangle\langle\psi(t_0)|$.
2. Generate a random number $0 < r_1 < 1$ and evolve the wave function forward in time with H_{eff} until $|| \, e^{-i \int_{t_0}^{t} dt' \, H_{\text{eff}}(t')} |\psi(t_0)\rangle \, ||^2 \leq r_1$ where $H_{\text{eff}} = H - i\Gamma/2$, $\Gamma = \sum \Gamma_n$ and $\Gamma_n = C_n^\dagger C_n$. Denote the first time step fulfilling the inequality as the jump time t_j. If the jump time is greater than the simulation run time t_F, end the simulation at time t_F; otherwise, proceed to step 3.
3. At time t_j, initiate a quantum jump:
   (a) If the system is in a singlet configuration, jump to octet. If the system is in an octet configuration, generate a random number $0 < r_2 < 1$ and jump to singlet if $r_2 < 2/7$; otherwise, remain in the octet configuration.
   (b) Generate a random number $0 < r_3 < 1$; if $r_3 < l/(2l+1)$, take $l \to l - 1$; otherwise, take $l \to l + 1$.
   (c) Multiply the wavefunction by r and normalize.
4. Continue from step 2.

◦ `Ba Omar Escobedo Islam Strickland Thapa Vander Griend Weber arXiv:2107.06147`

Jumps and probabilities

The probabilities in step 3 correspond to the branching fractions into a state of different color and/or angular momentum:

$$p_n = \frac{\langle\psi(t)|\Gamma_n|\psi(t)\rangle}{\langle\psi(t)|\Gamma|\psi(t)\rangle}$$

Each evolution of the wave function from time t_0 to t_F is called a quantum trajectory.
In practice, a large number of quantum trajectories must be generated.
As the number of trajectories considered increases, the average converges to the solution of the Lindblad equation.

- `Dalibard Castin Molmer PRL 68 (1992) 580`
  `Daley AP 63 (2014) 77`

Simulation set up

We employ a radial lattice of NUM$= 4096$ lattice sites and a radial length of L$= 80\,\mathrm{GeV}^{-1}$, corresponding to a radial lattice spacing of $a \approx 0.0195\,\mathrm{GeV}^{-1}$. The real time integration is discretized with a time step of dt$= 0.001\,\mathrm{GeV}^{-1}$.

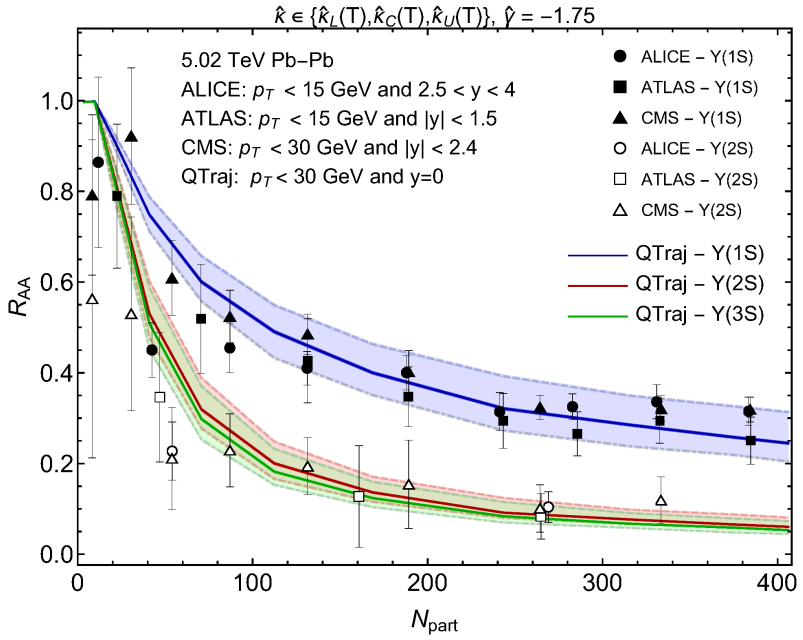
We sample approximately 7 - 9 $\times 10^5$ independent physical trajectories for each choice of $\kappa(T)/T^3$ and γ/T^3, with approximately 50-100 quantum trajectories per physical trajectory. To generate each physical trajectory, we sample the bottomonium production point in the transverse plane using the nuclear binary collision overlap profile $N_{AA}^{\mathrm{bin}}(x, y, b)$, the initial transverse momentum of the state p_T from an E_T^{-4} spectrum, and the initial azimuthal angle ϕ of the state's momentum uniformly in $[0, 2\pi)$. We bin the results for the survival probability as a function of centrality, p_T, and ϕ. This allows us to make predictions for differential observables such as R_{AA} as a function of p_T and elliptic flow.

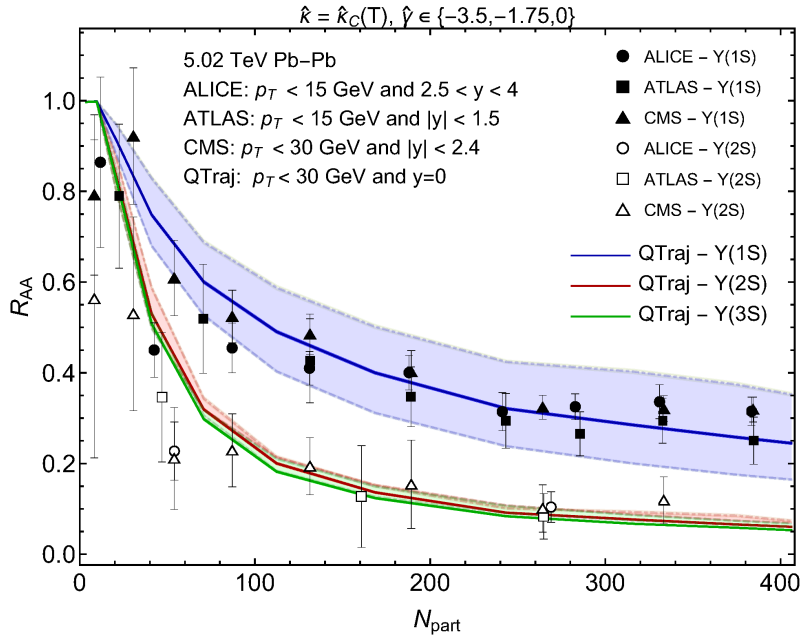
To ensure that the hierarchy of energy scales of the EFT is fulfilled, we evolve the state in the vacuum when the temperature falls below $T_F = 250$ MeV.

Bottomonium nuclear modification factor

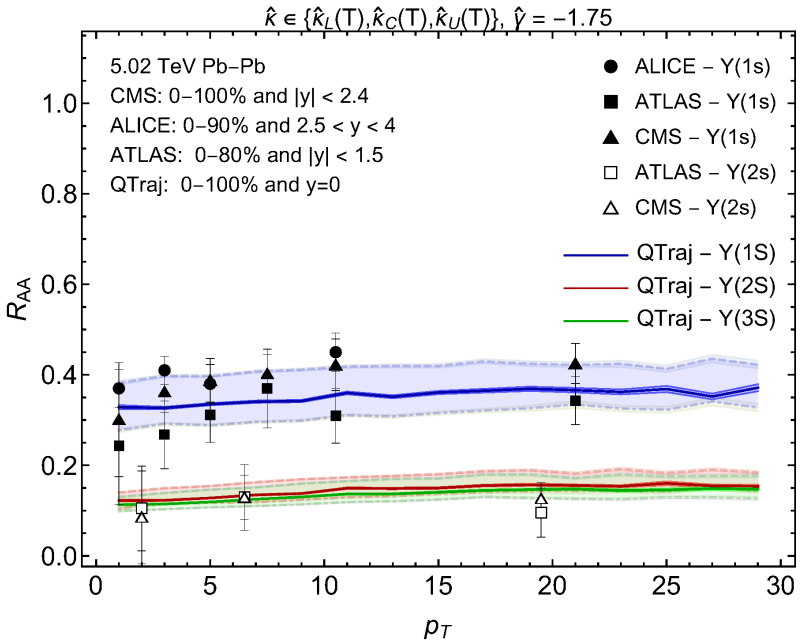
We compute the nuclear modification factor R_{AA} from

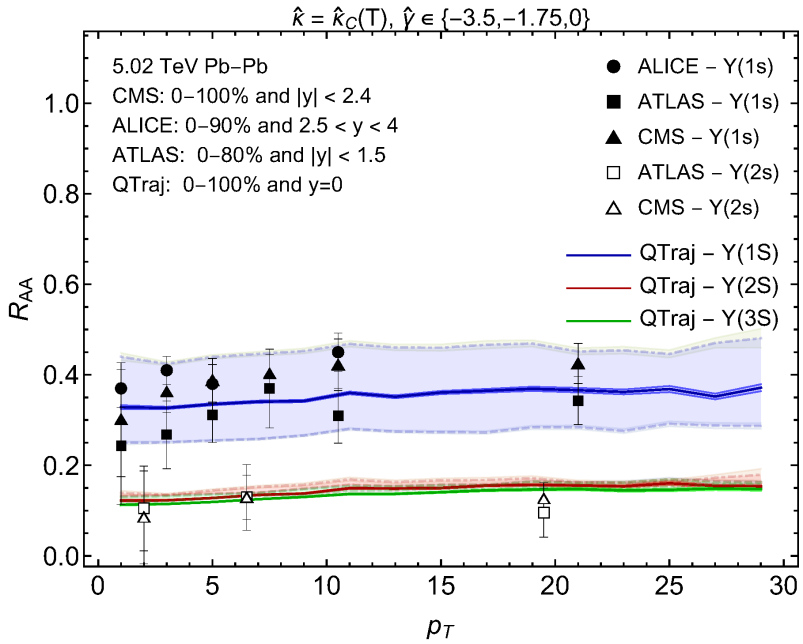
$$R_{AA}(nS) = \frac{\langle n, \mathbf{q}|\rho_s(t_F; t_F)|n, \mathbf{q}\rangle}{\langle n, \mathbf{q}|\rho_s(0; 0)|n, \mathbf{q}\rangle}$$



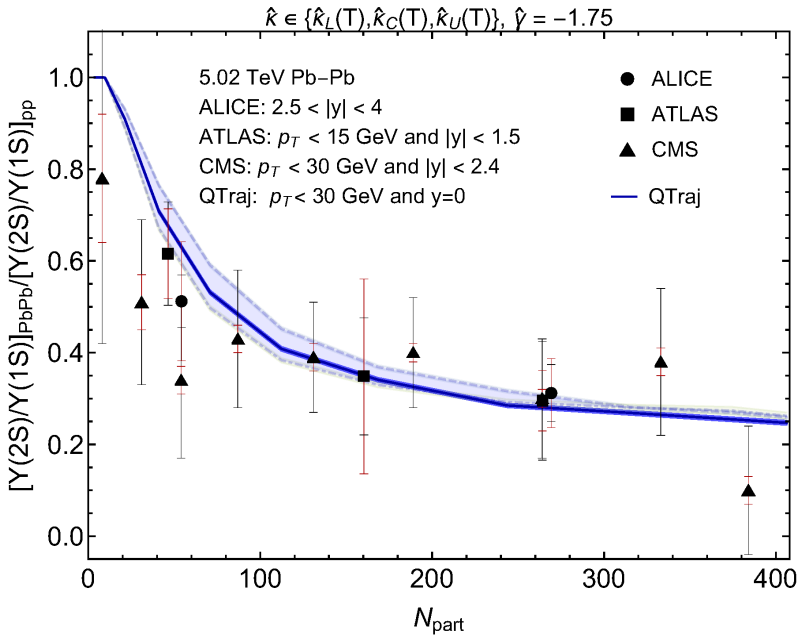


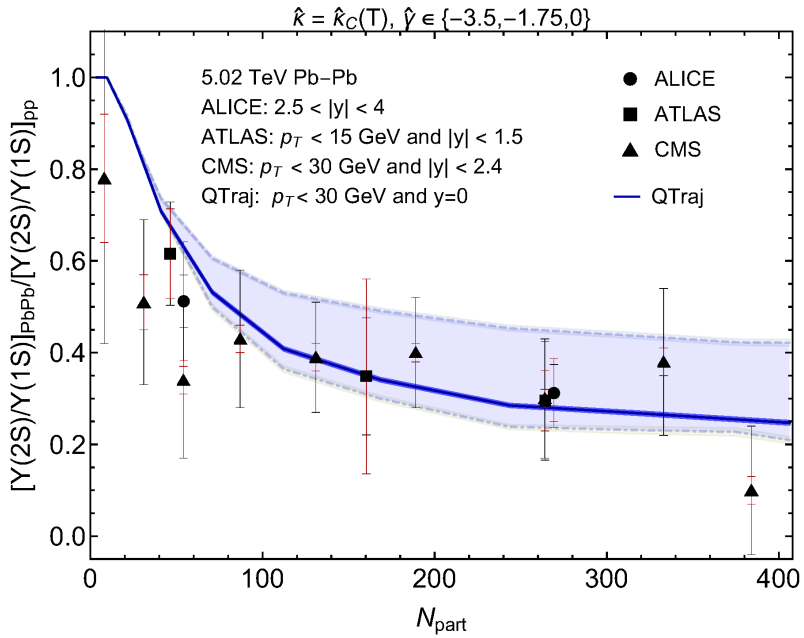
Bottomonium nuclear modification factor vs p_T



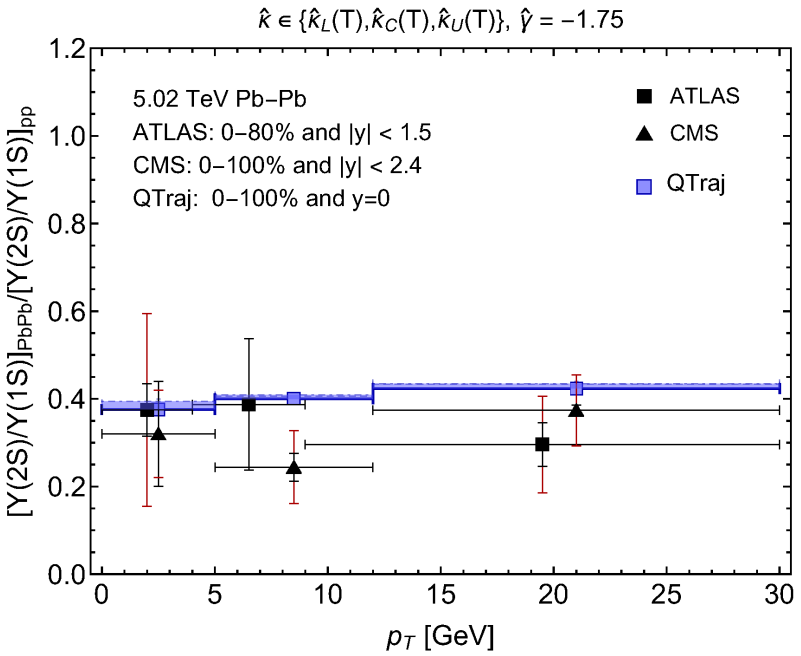


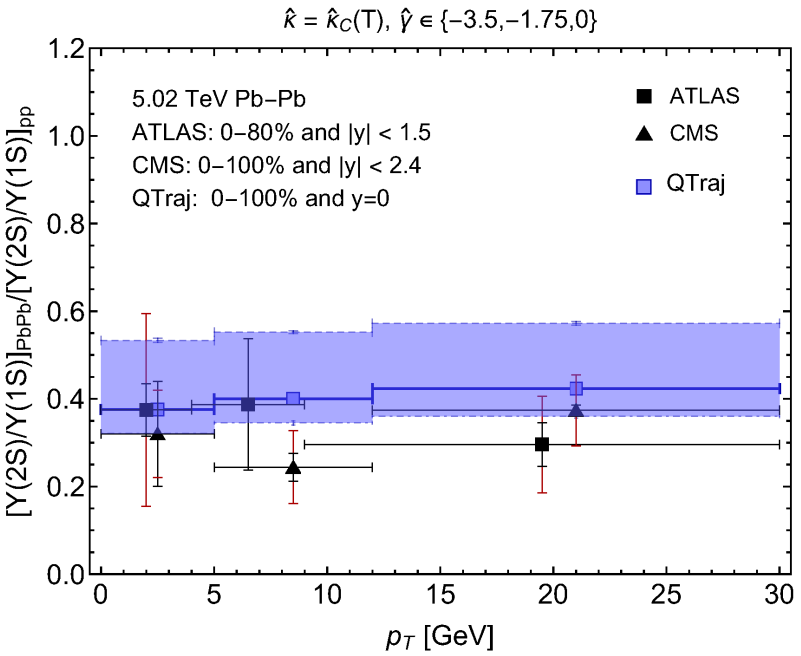
Double ratio $R_{AA}[\Upsilon(2S)]$ to $R_{AA}[\Upsilon(1S)]$



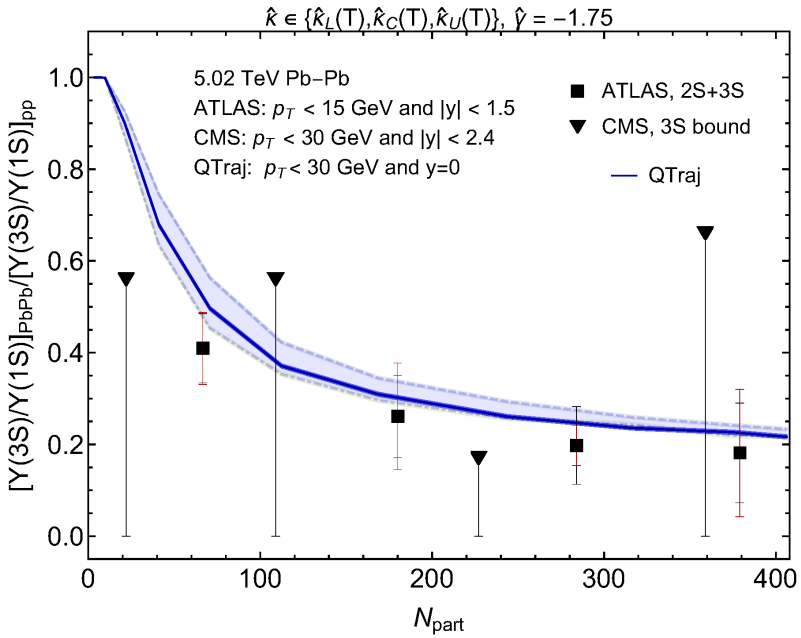


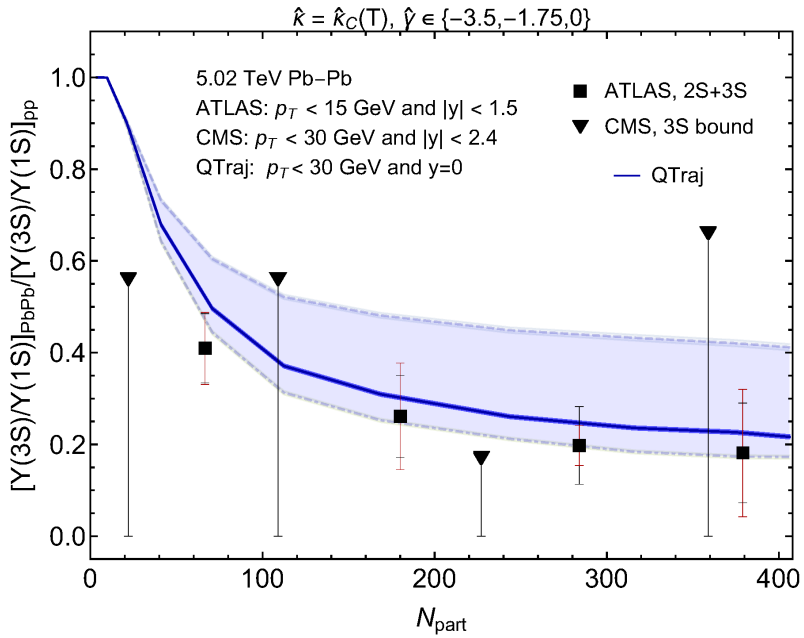
Double ratio $R_{AA}[\Upsilon(2S)]$ to $R_{AA}[\Upsilon(1S)]$ vs p_T



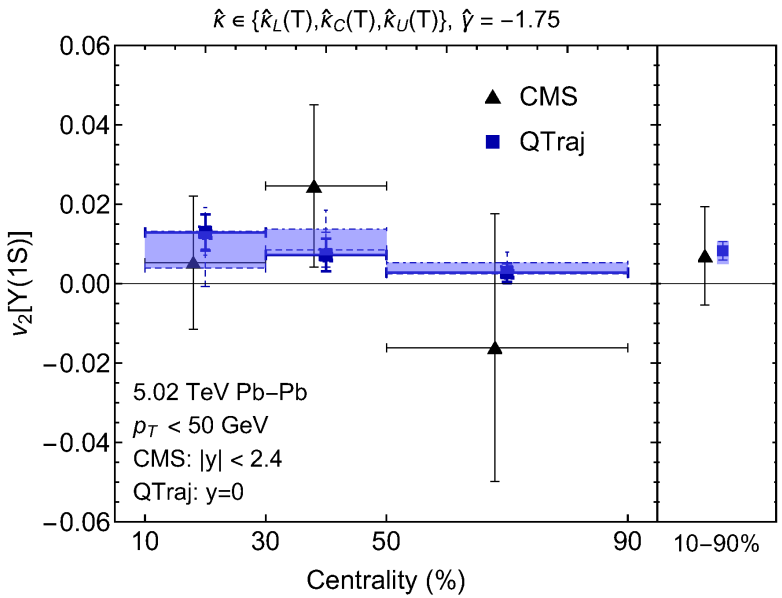


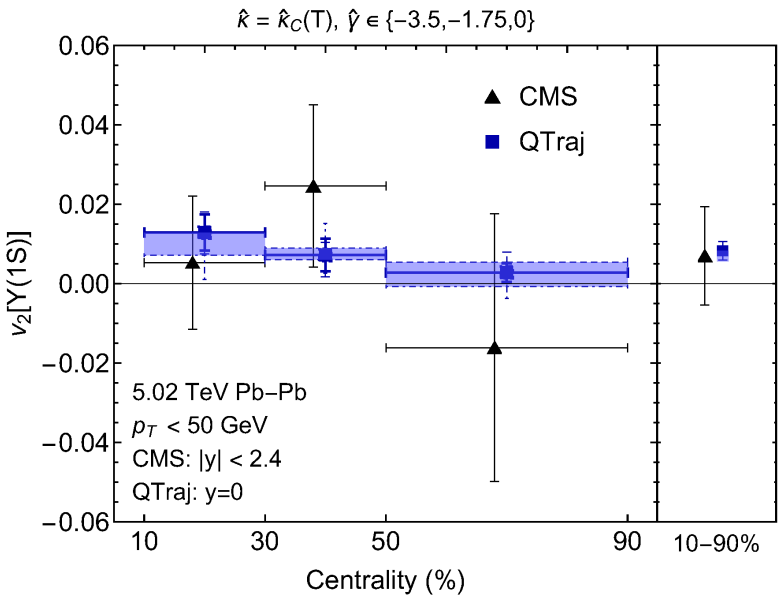
Double ratio $R_{AA}[\Upsilon(3S)]$ to $R_{AA}[\Upsilon(1S)]$



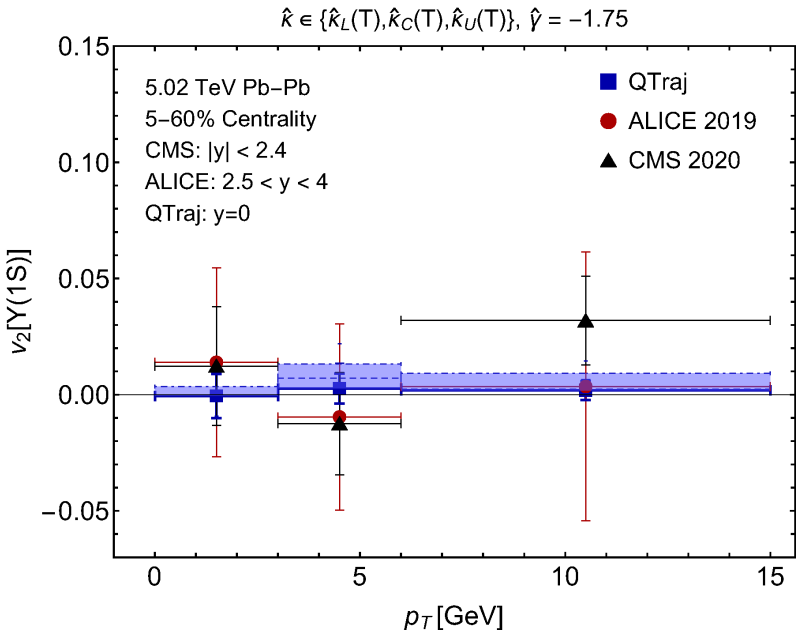


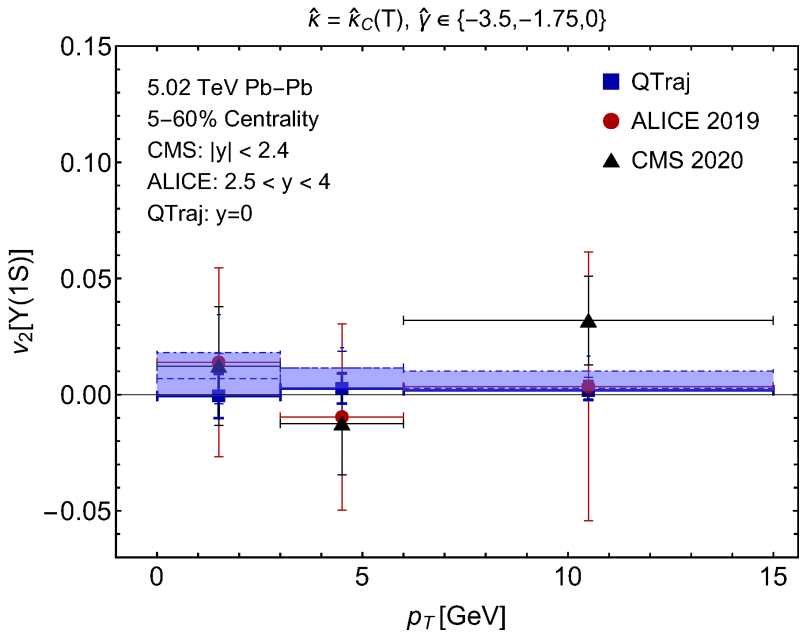
Elliptic flow v_2 of the $\Upsilon(1S)$



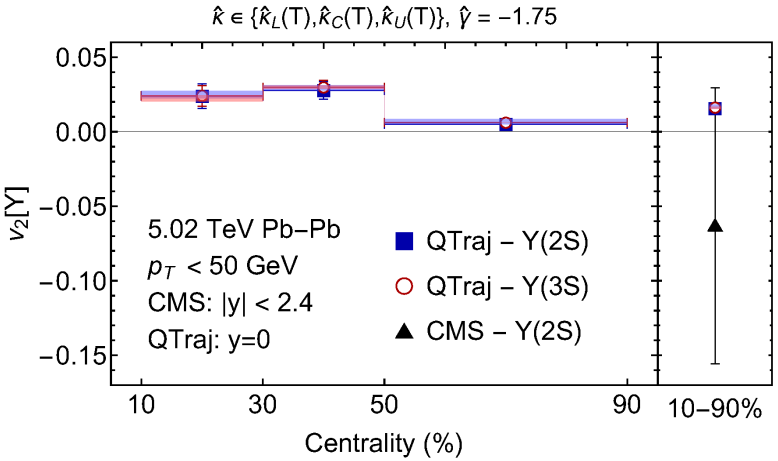


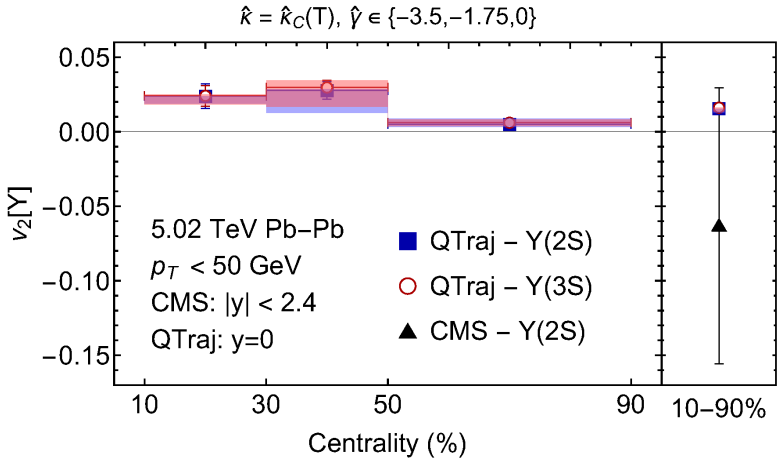
Elliptic flow v_2 of the $\Upsilon(1S)$ vs p_T





Elliptic flow v_2 of the $\Upsilon(2S)$ and $\Upsilon(3S)$





Conclusions

We have shown how the heavy quark-antiquark pair out-of-equilibrium evolution can be treated in the framework of pNRQCD. With respect to previous determinations:

- the medium may be a strongly-coupled plasma (not necessarily a quark-gluon plasma) whose characteristics are determined by lattice calculations;
- the total number of heavy quarks, i.e., $\mathrm{Tr}\{\rho_s\} + \mathrm{Tr}\{\rho_o\}$, is preserved by the evolution equations;
- the non-abelian nature of QCD is fully accounted for;
- the treatment does not rely on classical approximations.

The evolution equations follow from assuming the inverse size of the quark-antiquark system to be larger than any other scale of the medium and from being accurate at first non-trivial order in the multipole expansion and at first order in the heavy-quark density.

Under some conditions (large time, quasistatic evolution, quantum Brownian motion) the evolution equations are of the Lindblad form. Their numerical solution provides $R_{AA}[\Upsilon(nS)]$ and differential observables in good agreement with LHC data.