Status of the Sign Problem in QCD and Perspectives on Simulating Lattice Gauge Theories with Quantum Technology

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A Virtual Tribute to Quark Confinement and the Hadron Spectrum 2021
August 4, 2021

UNIVERSITÄT BERN

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Outline

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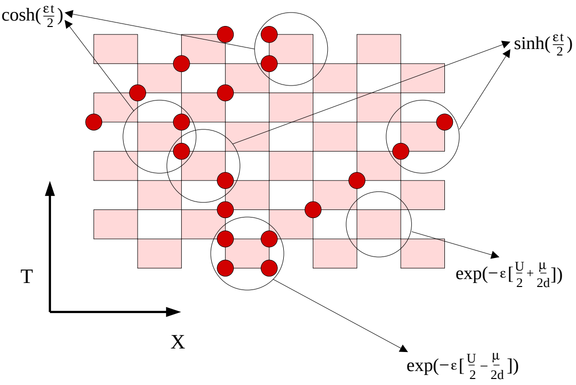
Path integral

$$Z_f = \mathrm{Tr}[\exp(-\varepsilon H_1)\exp(-\varepsilon H_2)\ldots\exp(-\varepsilon H_M)]^N$$
$$= \sum_{[n]} \mathrm{Sign}[n]\exp(-S[n])$$



Sign problem of fermionic path integrals

$$Z_f = \mathrm{Tr}\exp(-\beta H) = \sum_{[n]} \mathrm{Sign}[n]\exp(-S[n]) \;, \quad \mathrm{Sign}[n] = \pm 1$$

Average sign is exponentially small

$$\langle \mathrm{Sign} \rangle = \frac{\sum_{[n]} \mathrm{Sign}[n]\exp(-S[n])}{\sum_{[n]} \exp(-S[n])} = \frac{Z_f}{Z_b} = \exp(-\beta V \Delta f)$$

The statistical error is exponentially large

$$\frac{\sigma_{\mathrm{Sign}}}{\langle \mathrm{Sign} \rangle} = \frac{\sqrt{\langle \mathrm{Sign}^2 \rangle - \langle \mathrm{Sign} \rangle^2}}{\sqrt{N}\langle \mathrm{Sign} \rangle} = \frac{\exp(\beta V \Delta f)}{\sqrt{N}} \;.$$

Some very hard sign problems are NP complete

M. Troyer, UJW, Phys. Rev. Lett. 94 (2005) 170201.

Some specific exponentially hard sign problems were solved completely using meron-cluster algorithms

S. Chandrasekharan, UJW, Phys. Rev. Lett. 83 (1999) 3116.

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But several groups are working (exponentially?) hard on it.

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Questions related to complex Langevin (stochastic quantization):

- How to guarantee correct results?
- How to avoid excursions away from the $SU(3)$ group manifold?
- How to deal with boundary terms at poles of the drift force?

- G. Aarts, E. Seiler, I.-O. Stamatescu, Phys. Rev. D81 (2010) 054508.
- E. Seiler, D. Sexty, I.-O. Stamatescu, Phys. Lett. 723 (2013) 213.
- J. B. Kogut, D. K. Sinclair, Phys. Rev. D100 (2019) 054512.
- M. Scherzer, E. Seiler, D. Sexty, I.-O. Stamatescu, Phys. Rev. D101 (2020) 014501.
- Y. Ito, H. Matsufuru, Y. Namekawa, J. Nishimura, S. Shimasaki, A. Tsuchiya, S. Tsutsui, JHEP 2020 (2020) 144.
- F. Attanasio, B. Jäger, F. P. G. Ziegler, EPJ A56 (2020) 251.

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Questions related to Lefshetz thimbles (complex saddle points):

- How to flow close to the thimble?
- How to efficiently deal with the Jacobian?
- How to sample multiple thimbles?

- A. Alexandru, G. Basar, P. F. Bedaque, N. C. Warrington, arXiv:2007.05436.
- F. Di Renzo, S. Singh, K. Zambello, Phys. Rev. D103 (2021) 034513.

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Questions related to Taylor expansion around $\mu = 0$
and analytic continuation from imaginary μ:

- How to solve signal-to-noise problems in expansion coefficients?
- How to truncate or resum the expansion?
- How to determine the radius of convergence?

- P. de Forcrand, O. Philipsen, Nucl. Phys. B642 (2002) 290.
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- C. R. Allton, M. Doring, S. Ejiri, S. J. Hands, O. Kaczmarek, F. Karsch, E. Laermann, K. Redlich, Phys. Rev. D71 (2005) 054508.
- S. Borsanyi, G. Endrodi, Z. Fodor, S. D. Katz, S. Krieg, C. Ratti, K. K. Szabo, JHEP 08 (2012) 053.
- A. Bazavov, H.-T. Ding, P. Hegde, O. Kaczmarek, F. Karsch, N. Karthik, E. Laermann, A. Lahiri, R. Larsen, S.-T. Li, S. Mukherjee, H. Ohno, P. Petreczky, H. Sandmeyer, C. Schmidt, S. Sharma, P. Steinbrecher, Phys. Lett. B795 (2019) 15.
- S. Borsanyi, Z. Fodor, J. N. Guenther, R. Kara, S. D. Katz, P. Parotto, A. Pasztor, C. Ratti, K. K. Szabo, Phys. Rev. Lett. 126 (2021) 232001.

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It might be interesting to explore:

- Other bases of the Hilbert space (includes dualization)
- Real-time evolution with or without dissipation
- New stratagies for completely solving exponentially hard sign problems

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- Real-time evolution with or without dissipation
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Keep searching: many things remain to be explored.

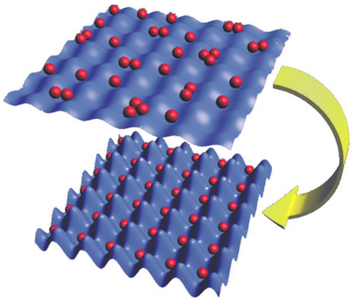
Outline

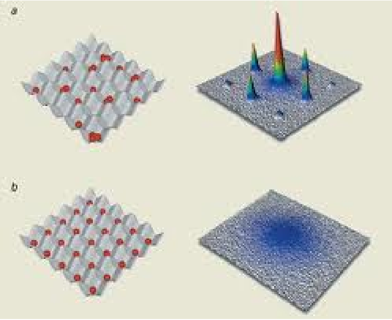
Richard Feynman's vision from 1982 (Int. J. Theor. Phys. 21)

"I'm not happy with all the analyses that go with just the classical theory, because nature isn't classical, dammit, and if you want to make a simulation of nature, you'd better make it quantum mechanical, and by golly it's a wonderful problem, because it doesn't look so easy."

"It does seem to be true that all the various field theories have the same kind of behavior, and can be simulated in every way, apparently, with little latticeworks of spins and other things."

Ultra-cold atoms in optical lattice as analog quantum simulator





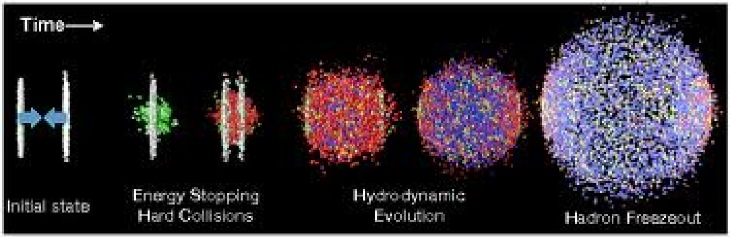
Transition from a superfluid to a Mott insulator

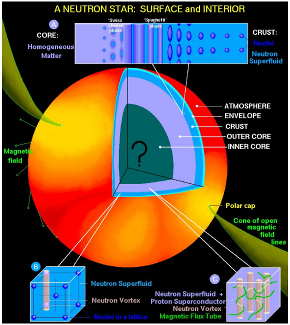
M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, I. Bloch,
Nature 415 (2002) 39.

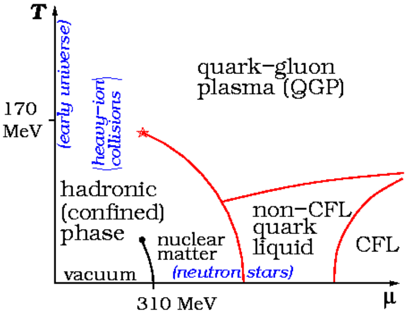
Quantum simulations have been accurately verified by classical simulations

S. Trotzky, L. Pollet, F. Gerbier, U. Schnorrberger, I. Bloch, N. Prokovéf, B. Svistunov, M. Troyer, Nat. Phys. 6 (2010) 998.

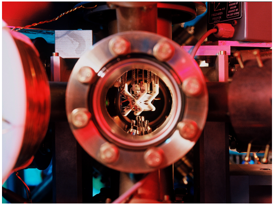
Can heavy-ion collision physics or nuclear astrophysics benefit from quantum simulations in the long run?

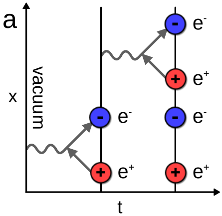


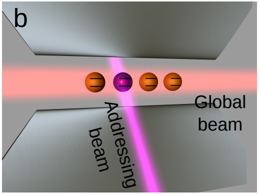




Digital 4-qubit ion-trap quantum computation of pair creation







- Quantum computer consisting of four trapped Ca ions that act as four qubits, which are manipulated by external laser beams.
- Precisely controllable many-body quantum device, executing a prescribed sequence of quantum gate operations.
- State of simulated system is encoded as quantum information.
- Dynamics is represented by a sequence of quantum gates, following a stroboscopic Trotter decomposition.

E. A. Martinez, C. A. Muschik, P. Schindler, D. Nigg, A. Erhard, M. Heyl, P. Hauke, M. Dalmonte, T. Monz, P. Zoller, R. Blatt, Nature 534 (2016) 516.

Outline

Different descriptions of dynamical Abelian gauge fields:

Maxwell's classical electromagnetic gauge fields

$$\vec{\nabla}\cdot\vec{E}(\vec{x},t)=\rho(\vec{x},t),\quad \vec{\nabla}\cdot\vec{B}(\vec{x},t)=0,\quad \vec{B}(\vec{x},t)=\vec{\nabla}\times\vec{A}(\vec{x},t)$$

Quantum Electrodynamics (QED) for perturbative treatment

$$E_i=-i\frac{\partial}{\partial A_i},\quad [E_i(\vec{x}),A_j(\vec{x}\,')]=i\delta_{ij}\delta(\vec{x}-\vec{x}\,'),\quad \left[\vec{\nabla}\cdot\vec{E}-\rho\right]|\Psi[A]\rangle=0$$

Wilson's $U(1)$ lattice gauge theory for classical simulation

$$U_{xy}=\exp\left(ie\int_x^y d\vec{l}\cdot\vec{A}\right)=\exp(i\varphi_{xy})\in U(1),\quad E_{xy}=-i\frac{\partial}{\partial\varphi_{xy}},$$

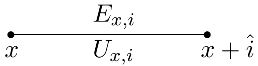
$$[E_{xy},U_{xy}]=U_{xy},\quad \left[\sum_i(E_{x,x+\hat{i}}-E_{x-\hat{i},x})-\rho\right]|\Psi[U]\rangle=0$$

$U(1)$ quantum link models for quantum simulation

$$U_{xy}=S^+_{xy},\quad U^\dagger_{xy}=S^-_{xy},\quad E_{xy}=S^3_{xy},$$

$$[E_{xy},U_{xy}]=U_{xy},\quad [E_{xy},U^\dagger_{xy}]=-U^\dagger_{xy},\quad [U_{xy},U^\dagger_{xy}]=2E_{xy}$$

Hamiltonian formulation of Wilson's $U(1)$ lattice gauge theory



$$U = \exp(i\varphi), \ U^\dagger = \exp(-i\varphi) \in U(1)$$

Electric field operator E

$$E = -i\partial_\varphi, \ [E, U] = U, \ [E, U^\dagger] = -U^\dagger, \ [U, U^\dagger] = 0$$

Generator of $U(1)$ gauge transformations

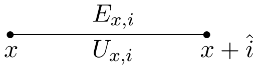
$$G_x = \sum_i (E_{x-\hat{i},i} - E_{x,i}), \quad [H, G_x] = 0$$

$U(1)$ gauge invariant Hamiltonian

$$H = \frac{e^2}{2} \sum_{x,i} E_{x,i}^2 - \frac{1}{2e^2} \sum_{x,i\neq j} (U_{x,i} U_{x+\hat{i},j} U^\dagger_{x+\hat{j},i} U^\dagger_{x,j} + \text{h.c.})$$

operates in an infinite-dimensional Hilbert space per link

Quantum link formulation of $U(1)$ lattice gauge theory



$U = S^+, \; U^\dagger = S^-$

Electric field operator E

$$E = S^3, \; [E, U] = U, \; [E, U^\dagger] = -U^\dagger, \; [U, U^\dagger] = 2E$$

Generator of $U(1)$ gauge transformations

$$G_x = \sum_i (E_{x-\hat{i},i} - E_{x,i}), \quad [H, G_x] = 0$$

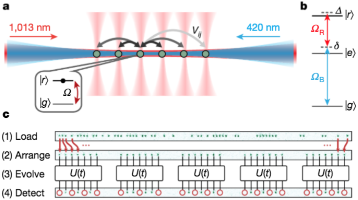
$U(1)$ gauge invariant Hamiltonian

$$H = \frac{e^2}{2} \sum_{x,i} E_{x,i}^2 - \frac{1}{2e^2} \sum_{x,i\neq j} (U_{x,i} U_{x+\hat{i},j} U^\dagger_{x+\hat{j},i} U^\dagger_{x,j} + \text{h.c.})$$

operates in a finite-dimensional Hilbert space per link

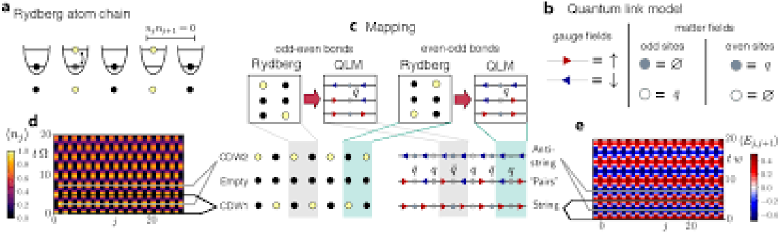
D. Horn, Phys. Lett. B100 (1981) 149.
P. Orland, D. Rohrlich, Nucl. Phys. B338 (1990) 647.
S. Chandrasekharan, UJW, Nucl. Phys. B492 (1997) 455.

A quantum simulator with 51 Rb atoms in optical tweezers



H. Bernien, S. Schwartz, A. Keesling, H. Levine, A. Omran, H. Pichler, S. Choi, A. S. Zibrov, M. Endres, M. Greiner, V. Vuletic, M. D. Lukin, Nature 551 (2017) 579.

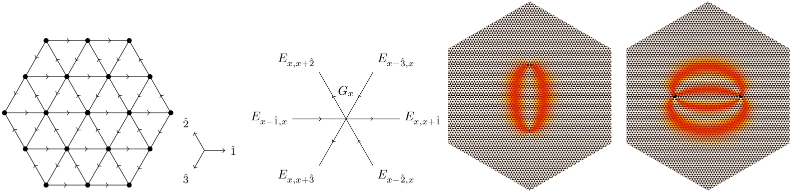
realizes the dynamics of spin $\frac{1}{2}$ quantum link models



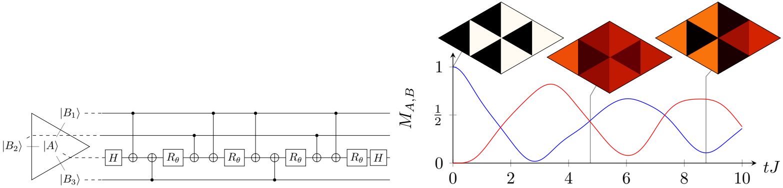
F. M. Surace, P. P. Mazza, G. Giudici, A. Lerose, A. Gambassi, M. Dalmonte, Phys. Rev. X10 (2020) 021041.

A. Celi, B. Vermersch, O. Viyuela, H. Pichler, M. D. Lukin, P. Zoller, Phys. Rev. X10 (2020) 021057.

The spin $\frac{1}{2}$ $U(1)$ quantum link model on a triangular lattice displays novel “nematic” confined phases.
Energy distribution for strings connecting charges ± 1 and ± 2:



Quantum circuit allows the resource-efficient quantum simulation of the time-evolution of the order parameter

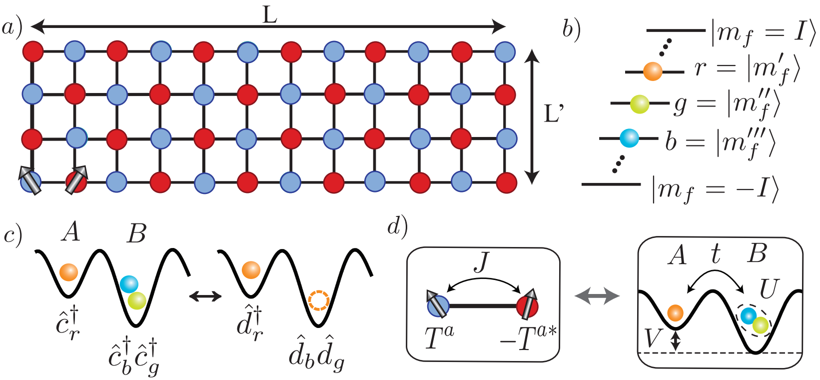


D. Banerjee, S. Caspar, F.-J. Jiang, J.-H. Peng, UJW, arXiv:2107.01283

Outline

Ladder of $SU(N)$ quantum spins embodied with alkaline-earth atoms

$$H = -J \sum_{\langle xy \rangle} T_x^a T_y^{a*}, \quad [T_x^a, T_y^b] = i\delta_{xy} f_{abc} T_x^c$$



C. Laflamme, W. Evans, M. Dalmonte, U. Gerber, H. Mejia-Diaz, W. Bietenholz, UJW, P. Zoller, Annals Phys. 360 (2016) 117.

Goldstone boson fields in $\mathbb{C}P(N-1) = SU(N)/U(N-1)$

$$P(x)^\dagger = P(x), \quad \mathrm{Tr}P(x) = 1, \quad P(x)^2 = P(x)$$

Low-energy effective action

$$\begin{aligned} S[P] &= \int_0^\beta dt \int_0^L dx \int_0^{L'} dy \ \mathrm{Tr}\left\{\rho_s' \partial_y P \partial_y P \right. \\ &+ \left. \rho_s \left[\partial_x P \partial_x P + \frac{1}{c^2} \partial_t P \partial_t P\right]\right\} \end{aligned}$$

Very large correlation length

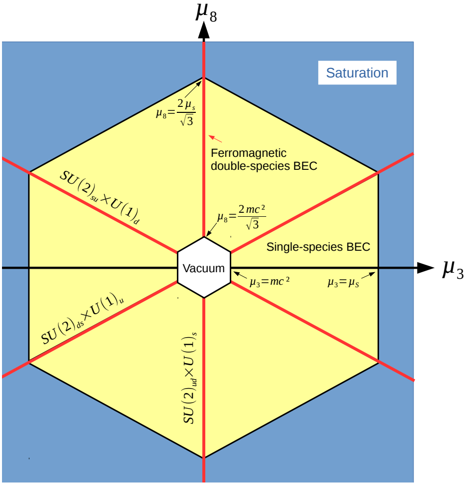
$$\xi \propto \exp(4\pi L' \rho_s / cN) \gg L', \quad \frac{1}{g^2} = L' \rho_s$$

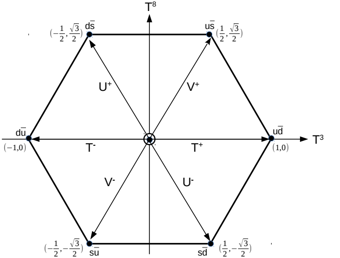
Dimensional reduction to the $(1+1)$-d $\mathbb{C}P(N-1)$ model

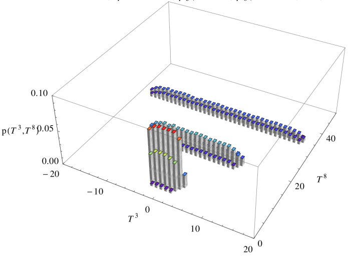
$$S[P] = \int_0^\beta dt \int_0^L dx \ \frac{1}{g^2} \mathrm{Tr}\left[\partial_x P \partial_x P + \frac{1}{c^2} \partial_t P \partial_t P\right]$$

K. Harada, N. Kawashima, M. Troyer, PRL 90 (2003) 117203.
B. B. Beard, M. Pepe, S. Riederer, UJW, PRL 94 (2005) 010603.

Ferromagnetic Double-Species BEC in the $\mathbb{C}P(2)$ Model







W. Evans, U. Gerber, M. Hornung, UJW, Annals Phys. 398 (2018) 92.

Outline

$U(N)$ quantum link operators

$$U^{ij} = S_1^{ij} + iS_2^{ij},\ U^{ij\dagger} = S_1^{ij} - iS_2^{ij},\ i,j \in \{1,2,\dots,N\},\ [U^{ij},(U^\dagger)^{kl}] \neq 0$$

$SU(N)_L \times SU(N)_R$ gauge transformations of a quantum link

$$[L^a, L^b] = if_{abc}L^c,\ [R^a, R^b] = if_{abc}R^c,\ a,b,c \in \{1,2,\dots,N^2-1\}$$

$$[L^a, R^b] = [L^a, E] = [R^a, E] = 0$$

Infinitesimal gauge transformations of a quantum link

$$[L^a, U] = -\lambda^a U,\ [R^a, U] = U\lambda^a,\ [E, U] = U$$

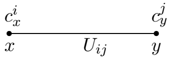
Algebraic structures of different quantum link models

$U(N)$: $U^{ij},\ L^a,\ R^a, E,\ 2N^2 + 2(N^2-1) + 1 = 4N^2 - 1$ $SU(2N)$ generators

$SO(N)$: $O^{ij},\ L^a,\ R^a,\ N^2 + 2\frac{N(N-1)}{2} = N(2N-1)$ $SO(2N)$ generators

$Sp(N)$: $U^{ij},\ L^a,\ R^a,\ 4N^2 + 2N(2N+1) = 2N(4N+1)$ $Sp(2N)$ generators

R. Brower, S. Chandrasekharan, UJW, Phys. Rev. D60 (1999) 094502



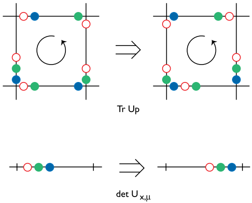
Fermionic rishons at the two ends of a link

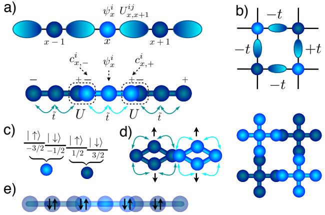
$$\{c_x^i, c_y^{j\dagger}\} = \delta_{xy}\delta_{ij}, \ \{c_x^i, c_y^j\} = \{c_x^{i\dagger}, c_y^{j\dagger}\} = 0$$

Rishon representation of link algebra

$$U_{xy}^{ij} = c_x^i c_y^{j\dagger}, \ L_{xy}^a = c_x^{i\dagger}\lambda_{ij}^a c_x^j, \ R_{xy}^a = c_y^{i\dagger}\lambda_{ij}^a c_y^j, \ E_{xy} = \frac{1}{2}(c_y^{i\dagger}c_y^i - c_x^{i\dagger}c_x^i)$$

"Rishon abacus" implemented with ultra-cold alkaline-earth atoms (^{87}Sr or ^{173}Yb) in an optical super-lattice with color encoded in nuclear spin





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Some analog quantum simulator constructions

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Outline

Conclusions

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• The path towards quantum simulation of QCD will be a long one.

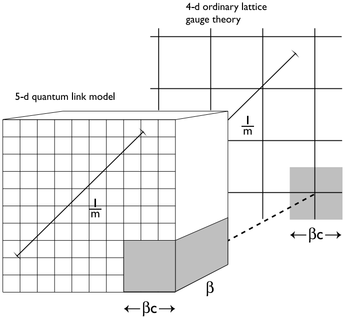
However, with a lot of interesting physics along the way.

Low-energy effective action for a quantum link model in a $(4+1)$-d massless non-Abelian Coulomb phase

$$S[G_\mu] = \int_0^\beta dx_5 \int d^4x \; \frac{1}{2e^2} \left(\mathrm{Tr}\; G_{\mu\nu} G_{\mu\nu} + \frac{1}{c^2} \mathrm{Tr}\; G_{\mu 5} G_{\mu 5} \right),$$

undergoes dimensional reduction from $4+1$ to 4 dimensions

$$S[G_\mu] \rightarrow \int d^4x \; \frac{1}{2g^2} \mathrm{Tr}\; G_{\mu\nu} G_{\mu\nu}, \;\; \frac{1}{g^2} = \frac{\beta}{e^2}, \;\; \frac{1}{m} \sim \exp\left(\frac{24\pi^2 \beta}{11 N e^2} \right)$$



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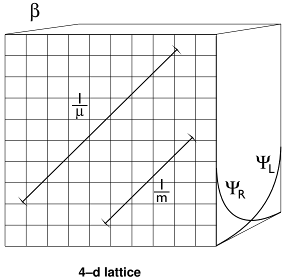
Quarks as Domain Wall Fermions

$$
\begin{aligned}
H &= J \sum_{x,\mu\neq\nu} \mathrm{Tr}[U_{x,\mu} U_{x+\hat{\mu},\nu} U^\dagger_{x+\hat{\nu},\mu} U^\dagger_{x,\nu}] + J' \sum_{x,\mu} [\det U_{x,\mu} + \det U^\dagger_{x,\mu}] \\
&+ \frac{1}{2} \sum_{x,\mu} [\Psi^\dagger_x \gamma_0 \gamma_\mu U_{x,\mu} \Psi_{x+\hat{\mu}} - \Psi^\dagger_{x+\hat{\mu}} \gamma_0 \gamma_\mu U^\dagger_{x,\mu} \Psi_x] + M \sum_x \Psi^\dagger_x \gamma_0 \Psi_x \\
&+ \frac{r}{2} \sum_{x,\mu} [2\Psi^\dagger_x \gamma_0 \Psi_x - \Psi^\dagger_x \gamma_0 U_{x,\mu} \Psi_{x+\hat{\mu}} - \Psi^\dagger_{x+\hat{\mu}} \gamma_0 U^\dagger_{x,\mu} \Psi_x].
\end{aligned}
$$



$$
\mu = 2M \exp(-M\beta), \ \frac{1}{m} \propto \exp(\frac{24\pi^2 \beta}{(11N - 2N_f)e^2}), \ M > \frac{24\pi^2}{(11N - 2N_f)e^2}
$$