Quark Masses and the Strong Coupling

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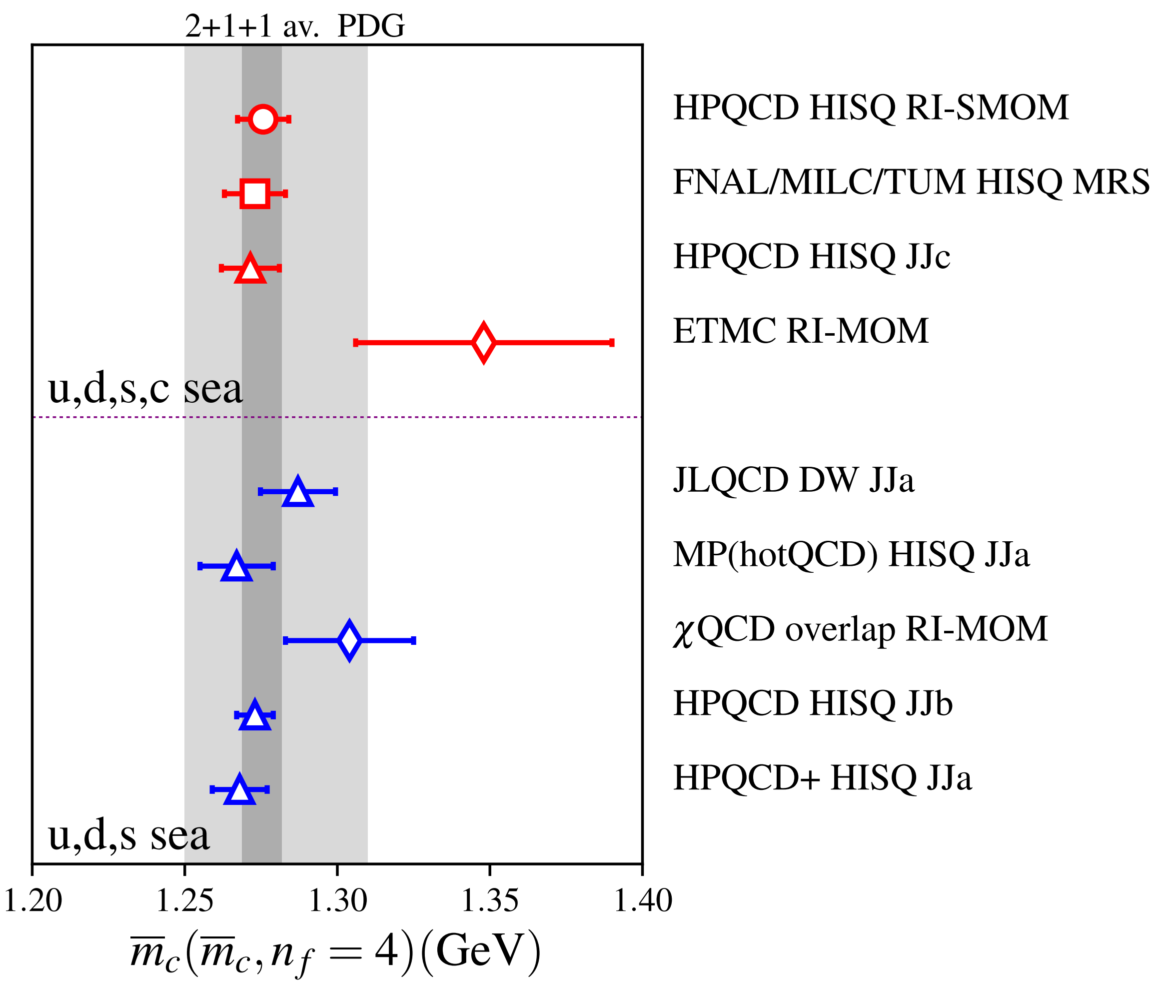
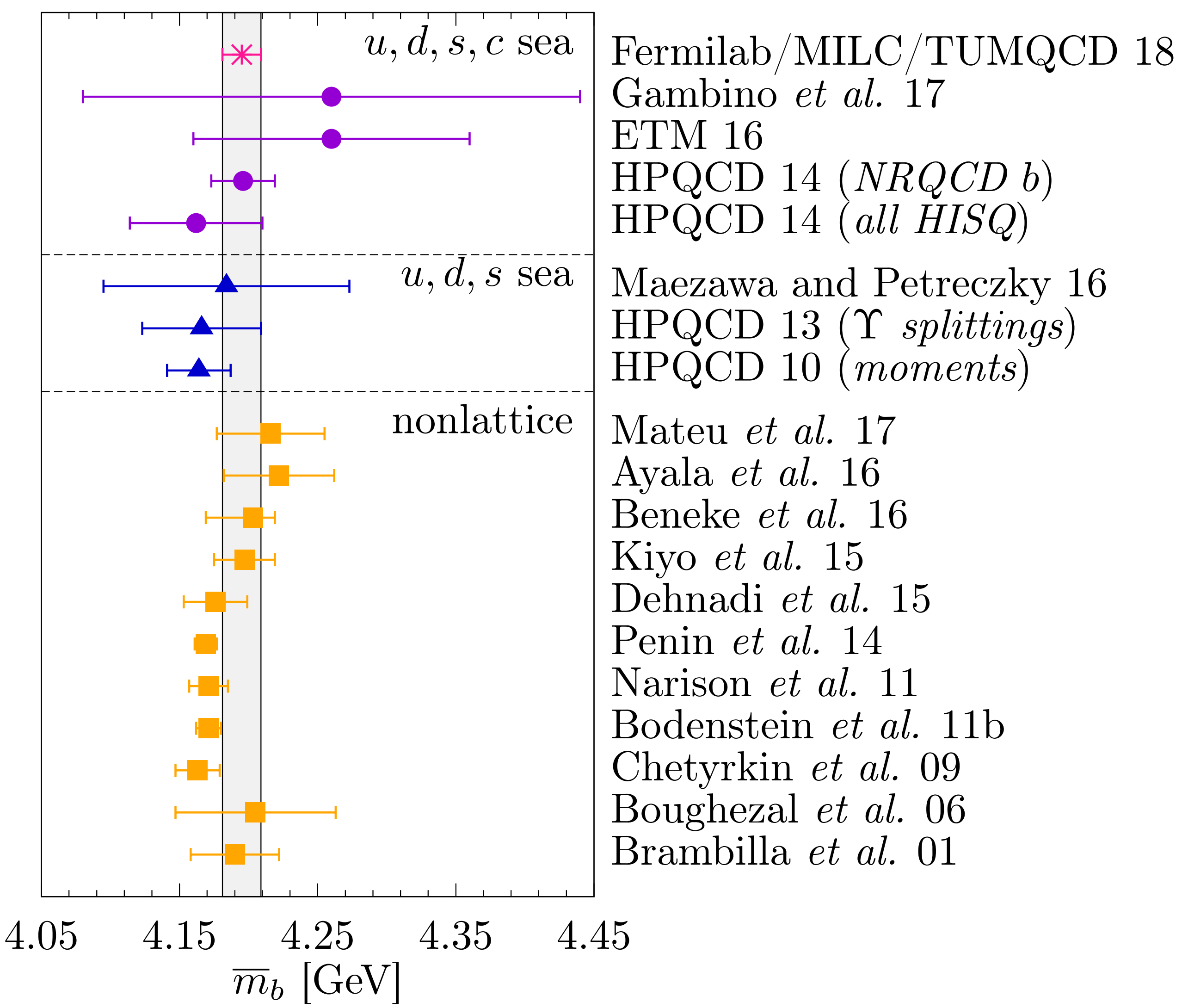
Virtual Tribute to Quark Confinement and the Hadron Spectrum
gather.town & zoom | August 2–6, 2021

QCD Parameters

- $1 + n_f + 1$ free parameters, which must be fixed from experiment:
  - α_s, $\{m_q\}$, θ—
  - in Standard Model, $m_q = y_q v/\sqrt{2}$, $\bar{\theta} = \theta - \arg\det\mathbf{y}$
- Strong CP problem: why is $\bar{\theta}$ so small?
  - Limit on $\bar{\theta}$ via $d_n = F_n(0)\bar{\theta}$ marred by all *ab initio* calculations $F_n(0)$ being consistent with 0.
- Because of confinement, all parameters deduced from properties of hadrons!

Heavy Comparisons

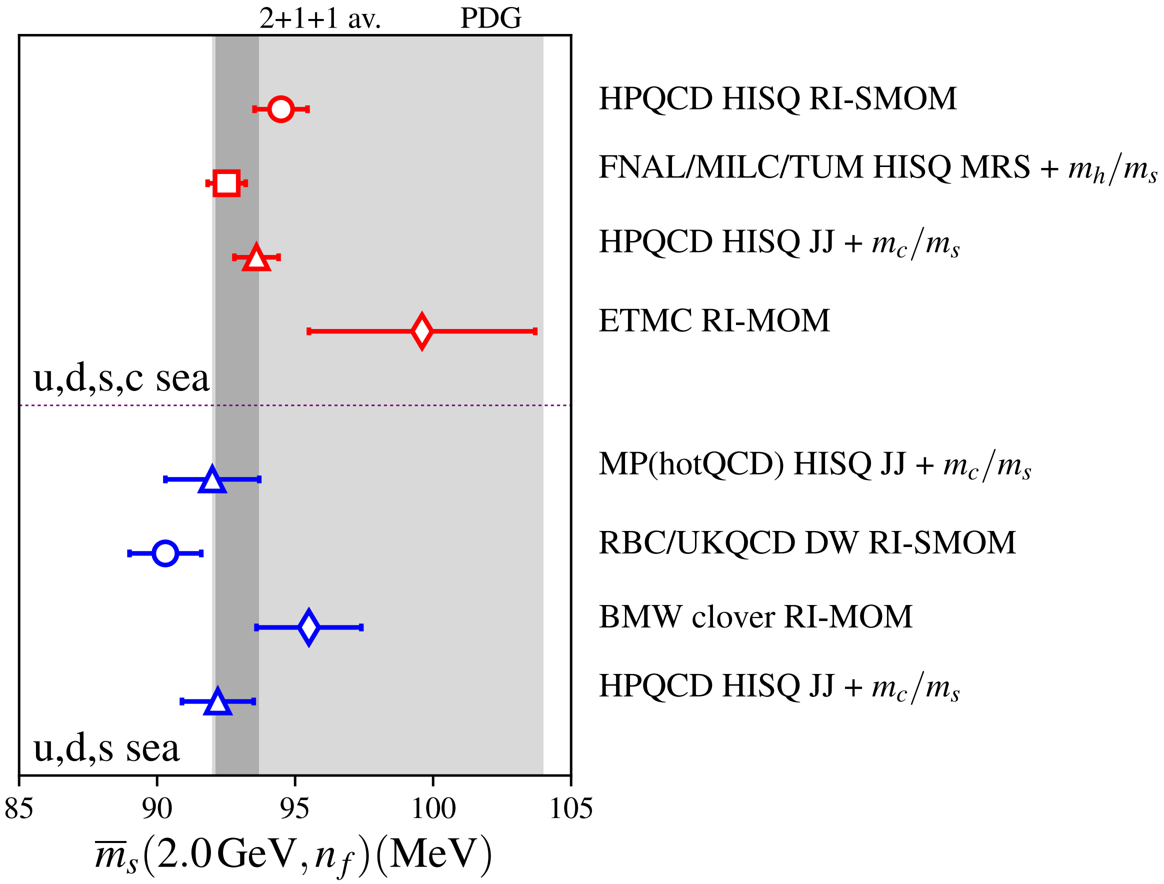


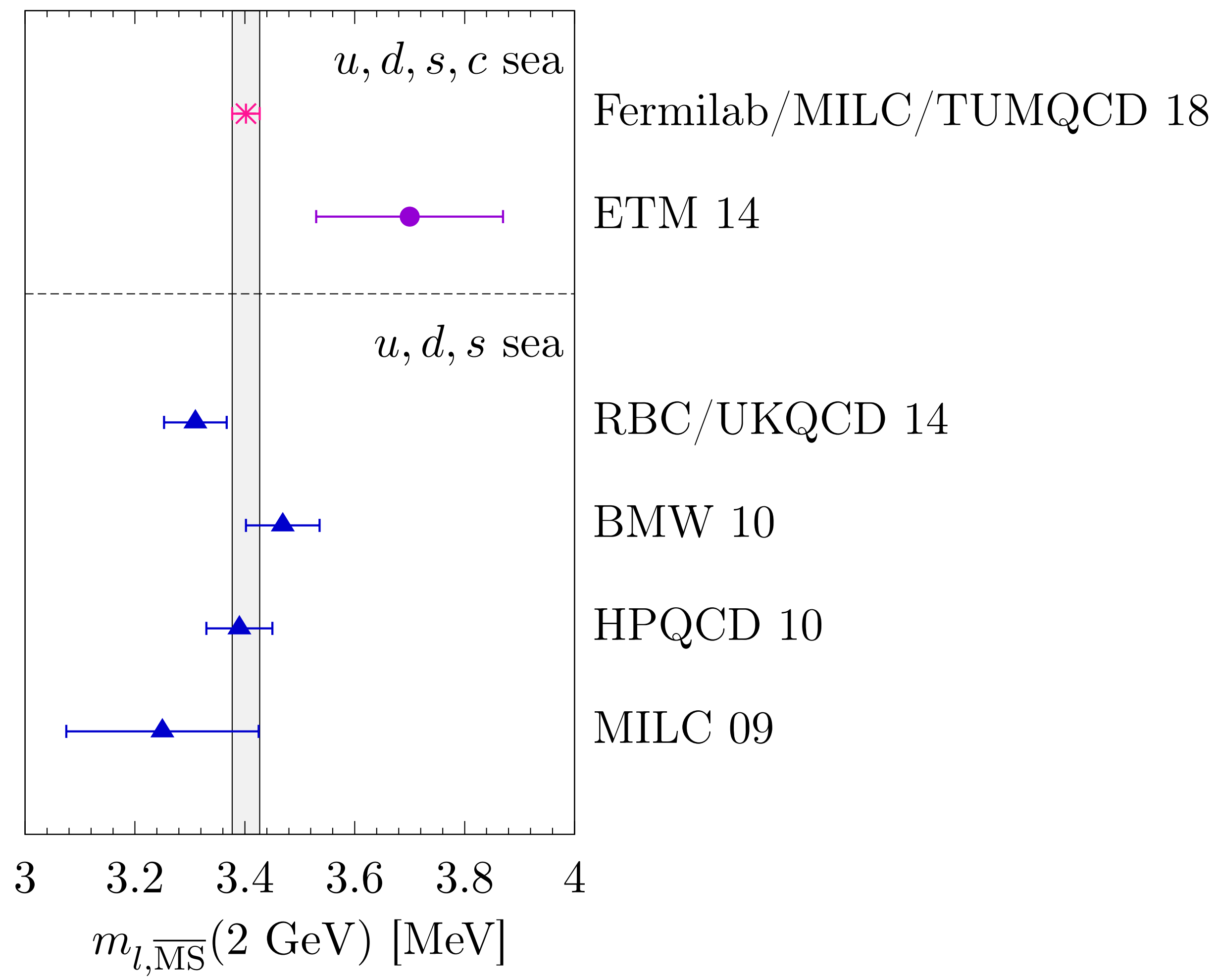


- Precision: 0.3% for bottom to 0.5% for charm.

plots from arXiv:1802.04248, arXiv:1805.06225

Light Comparisons





- 0.75% for strange quark.
- 2% for up quark.

plots from arXiv:1802.04248, arXiv:1805.06225

Precise Results from Lattice QCD

- Numerical results [quoting arXiv:1802.04248]:

α_s parametric not PT truncation

  - Masses:

$$m_{l,\overline{\mathrm{MS}}}(2\ \mathrm{GeV}) = 3.404(14)_{\mathrm{stat}}(08)_{\mathrm{syst}}(19)_{\alpha_s}(04)_{f_{\pi,\mathrm{PDG}}}\ \mathrm{MeV}$$
$$m_{u,\overline{\mathrm{MS}}}(2\ \mathrm{GeV}) = 2.118(17)_{\mathrm{stat}}(32)_{\mathrm{syst}}(12)_{\alpha_s}(03)_{f_{\pi,\mathrm{PDG}}}\ \mathrm{MeV}$$
$$m_{d,\overline{\mathrm{MS}}}(2\ \mathrm{GeV}) = 4.690(30)_{\mathrm{stat}}(36)_{\mathrm{syst}}(26)_{\alpha_s}(06)_{f_{\pi,\mathrm{PDG}}}\ \mathrm{MeV}$$
$$m_{s,\overline{\mathrm{MS}}}(2\ \mathrm{GeV}) = 92.52(40)_{\mathrm{stat}}(18)_{\mathrm{syst}}(52)_{\alpha_s}(12)_{f_{\pi,\mathrm{PDG}}}\ \mathrm{MeV}$$
$$m_{c,\overline{\mathrm{MS}}}(3\ \mathrm{GeV}) = 984.3(4.2)_{\mathrm{stat}}(1.6)_{\mathrm{syst}}(3.2)_{\alpha_s}(0.6)_{f_{\pi,\mathrm{PDG}}}\ \mathrm{MeV}$$
$$m_{b,\overline{\mathrm{MS}}}(m_{b,\overline{\mathrm{MS}}}) = 4203(12)_{\mathrm{stat}}(1)_{\mathrm{syst}}(8)_{\alpha_s}(1)_{f_{\pi,\mathrm{PDG}}}\ \mathrm{MeV}$$

  - Mass ratios:

$$m_c/m_s = 11.784(11)_{\mathrm{stat}}(17)_{\mathrm{syst}}(00)_{\alpha_s}(08)_{f_{\pi,\mathrm{PDG}}}$$
$$m_b/m_s = 53.93(7)_{\mathrm{stat}}(8)_{\mathrm{syst}}(1)_{\alpha_s}(5)_{f_{\pi,\mathrm{PDG}}}$$
$$m_b/m_c = 4.577(5)_{\mathrm{stat}}(7)_{\mathrm{syst}}(0)_{\alpha_s}(1)_{f_{\pi,\mathrm{PDG}}}$$

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,PDG MeV

$$m_u = 2\ \mathrm{MeV} \approx 4 m_e,\ m_u = 50\sigma_{m_u}$$

$)f_{\pi,\mathrm{PDG}}$ MeV

$$m_{b,\overline{\mathrm{MS}}}(m_{b,\overline{\mathrm{MS}}}) = 4203(12)_{\mathrm{stat}}(1)_{\mathrm{syst}}(8)_{\alpha_s}(1)_{f_{\pi,\mathrm{PDG}}}\ \mathrm{MeV}$$

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Synopsis

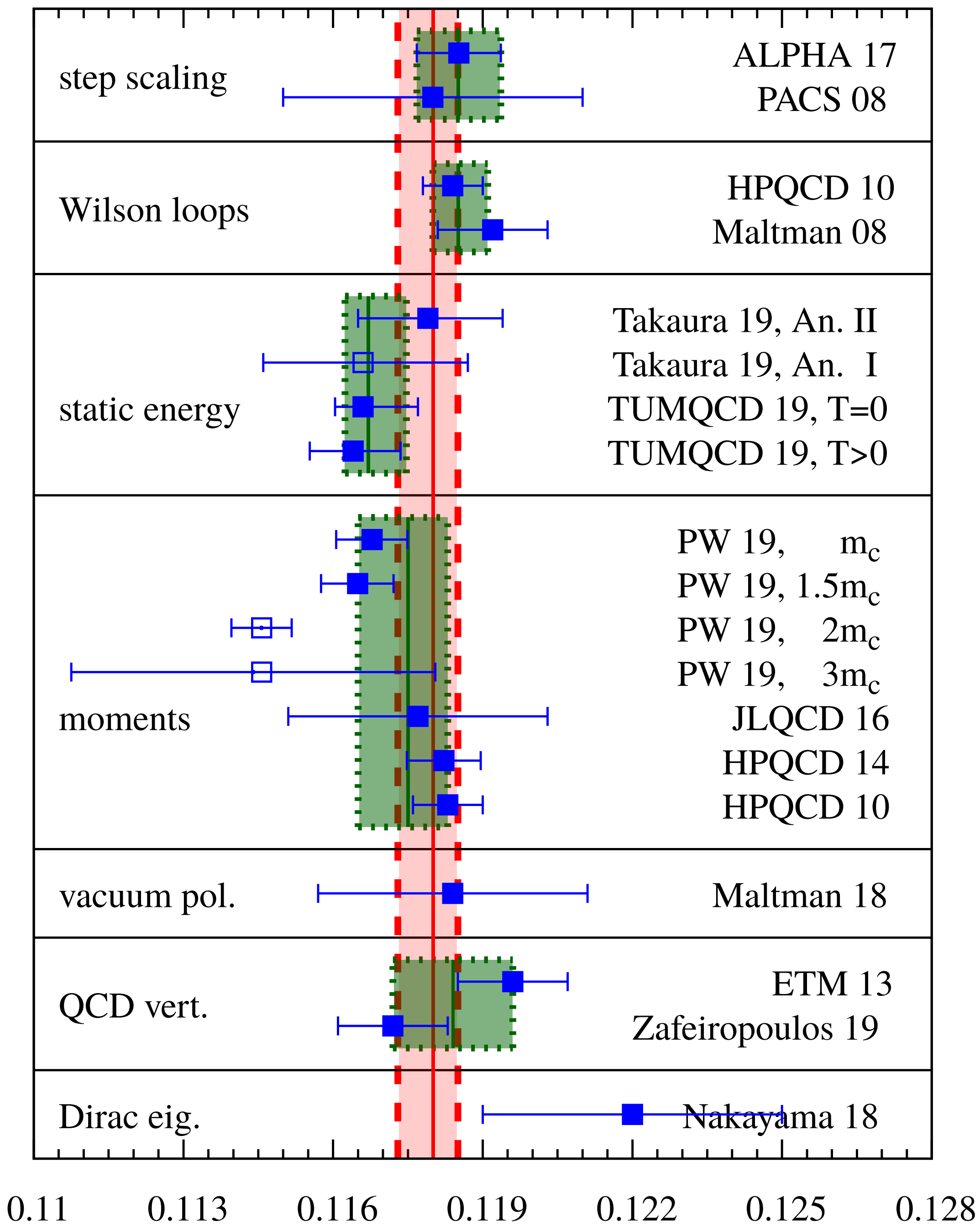
- Precise results for all quarks but top (use pQCD + Tevatron, LHC, ILC).
- Good agreement—plots not from FLAG 2019, but results shown are all highly rated.
- Several different methods: RI-SMOM, correlator moments, HQET+MRS.
- Common features:
  - adjust bare lattice mass until chosen hadron mass agrees with PDG;
  - compute a regulator-independent renormalized mass;
  - convert this mass to $\overline{\mathrm{MS}}$ with (multi-loop) perturbation theory.
- Precise results for charm & bottom ⇒ light-quark masses via mass ratios.

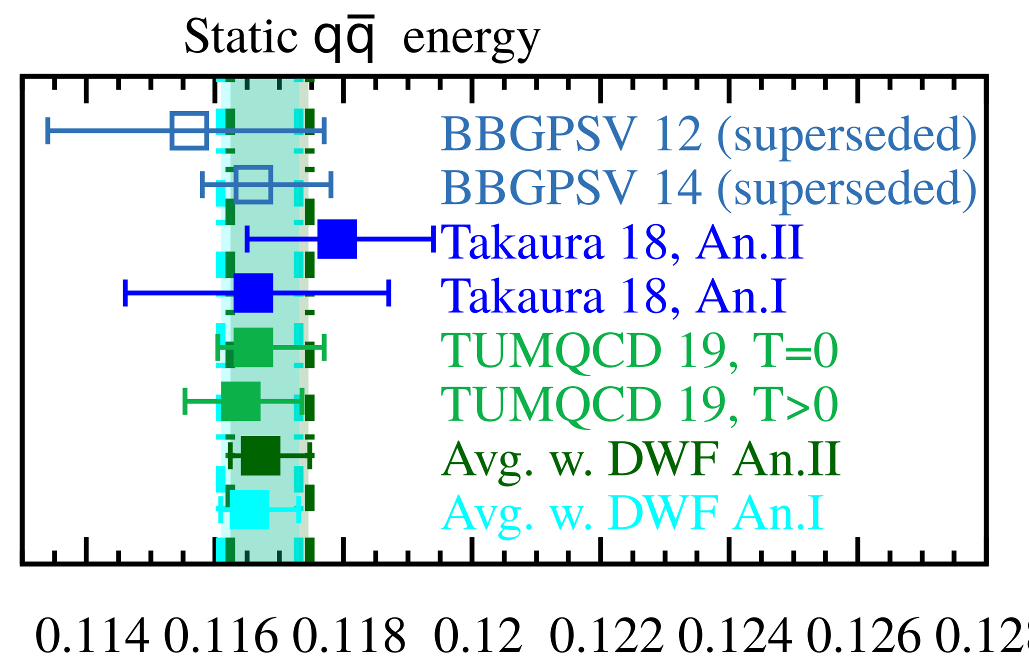
What’s the Strong Coupling?

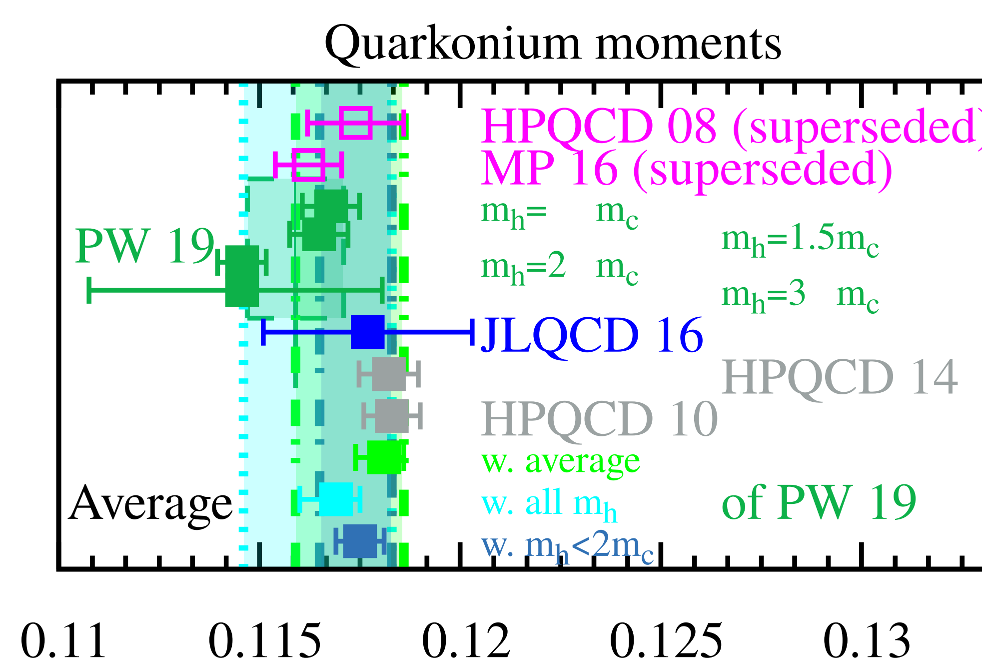
- You can’t scatter quark off antiquarks and measure the cross section.
- Need definition, preferably regularization-independent, in QFT.
- Natural candidates stem from energy between static quark and static antiquark:
    - gauge invariant;
    - related to a scattering amplitude.
- Alas, ambiguous in coordinate space: $V(r)$ has a renormalon (continuum) or linear UV divergence (lattice):
    - canceled by quark-antiquark self-energy.

Lattice Comparisons

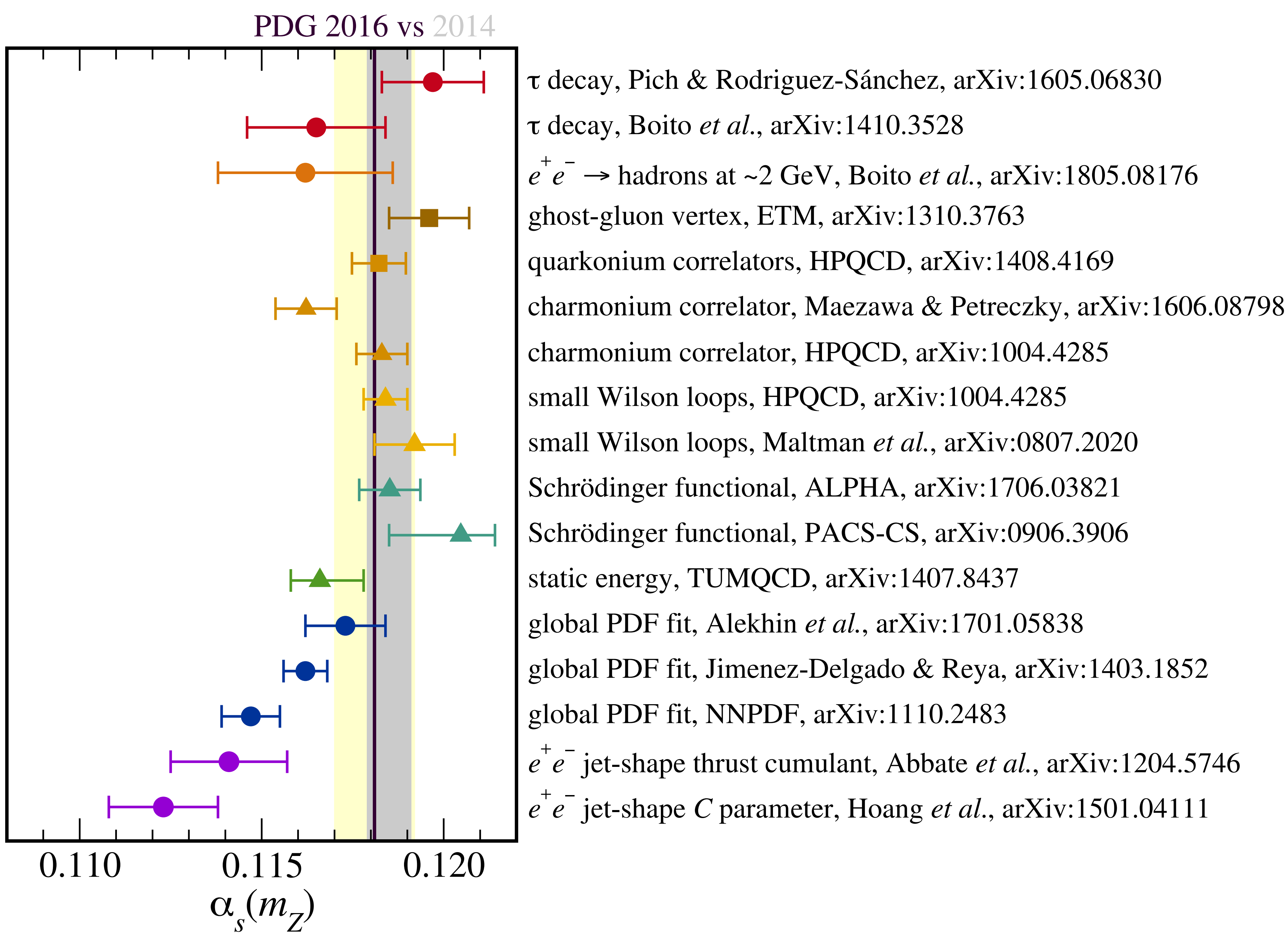
plots from Komijani, Petreczky, Weber [arXiv:2003.11703]







Scattering, Decay, and Lattice Comparisons



Outline

- Introduction: precise results for all quarks but top.
- What's a quark mass?
- What does it mean that the quoted up-quark mass has a 2% uncertainty? Or what does “50 sigma from zero” say about the strong CP problem?
    - Requires discussion of renormalization:
        - most of which you know;
        - pay attention to additional additive effects lying beyond perturbation theory.

What's a Quark Mass?

What’s a Quark Mass?

- You can’t put a quark on a scale and weigh it.
- Need definition, preferably regularization-independent, in QFT.
- Natural candidate is the “perturbative pole mass.” Alas, ambiguous:
  - physics—infrared gluons need to find a sink;
  - mathematics—obstruction to Borel summation of perturbative series;
  - numbers: $m_{\text{b,pole}}/\bar{m}_b = (1, 1.093, 1.143, 1.183, 1.224)$,
    $\bar{m}_h \equiv m_{h,\overline{\text{MS}}}(\bar{m}_h)$;
  - nonsense: pole mass makes little sense for a light quark, m_l, because the natural scale for self-energy contributions is m_l itself.

Unambiguous Definitions

- All come from quantum field theory:
  - bare mass of a cutoff Lagrangian, *e.g.*, lattice gauge theory;
  - renormalized masses—
    - based on a simple physical observable, *e.g.*, correlator moment, quarkonium mass as computed in perturbation theory, ...;
    - Ward identities;
    - regulator-independent via momentum-space subtraction;
    - computationally simple, *e.g.*, (modified) minimal subtraction in dimensional regularization.

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    - ba [illegible] oment, qu [illegible];
    - W [illegible];
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unambiguous but not unique

Quark Propagator

- Consider quark propagator $\text{FT}\left[q(x)\bar{q}(0)\right]$.

- The quark field is a **3**, so have to choose a (covariant) gauge. Then,

$$\text{FT}\left[q(x)\bar{q}(0)\right] = \frac{i}{\not{p} - m_0 - \Sigma(p^2; m_0)}$$

$$\Sigma(p^2; m_0) = \not{p}A(p^2; m_0) - C(p^2; m_0)$$

where m_0 is chosen to absorb UV divergences not compensated w/ Z_q.

- The second term could have additive renormalization:

$$C(p^2; m_0) = m_0^* + (m_0 - m_0^*)B(p^2; m_0) + \mathrm{O}(\Lambda_{\text{QCD}}) + \mathrm{O}(m_d m_s/\Lambda_{\text{QCD}})$$

linear UV

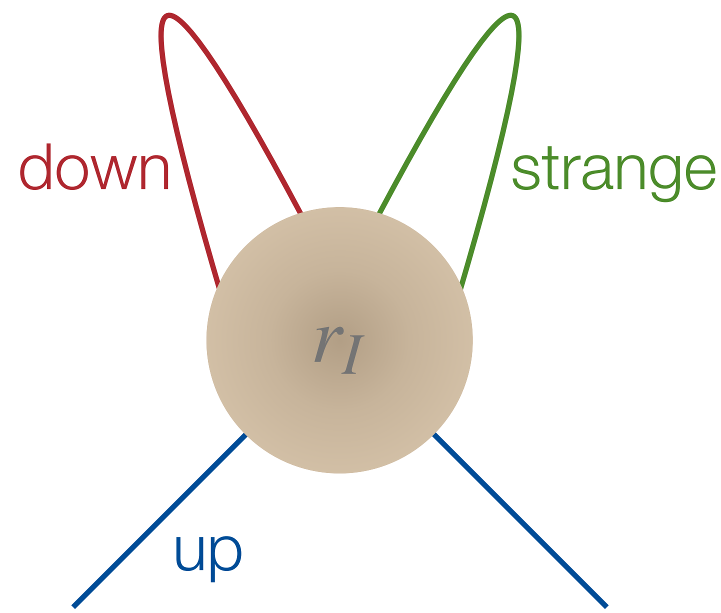
condensates, renormalons

instantons

Instantons?

- Here "instanton" is any gauge-field configuration with nonzero topological charge Q.

- Consider $Q = 1$:



Georgi, McArthur (1981)
Choi, Kim, Sze
Kaplan, Manohar
Banks, Nir, Seiberg
Cohen, Kaplan, Nelson
Creutz
Srednicki
Bardeen (2018)

- Zero mode: $\left(\mathrm{Det} \times S_{\mathrm{up}}\right) r_I e^{-S_I} = \dfrac{(m_u+\lambda)(m_d+\lambda)(m_s+\lambda)}{(m_u+\lambda)} r_I e^{-S_I}$

$$= \frac{m_u m_d m_s}{m_u} r_I \exp\left[-\frac{2\pi}{\alpha_s(1/r_I)}\right]$$

$$\sim \frac{m_d m_s}{\Lambda_{\mathrm{QCD}}} \quad \text{when } r_I \Lambda_{\mathrm{QCD}} \sim 1$$

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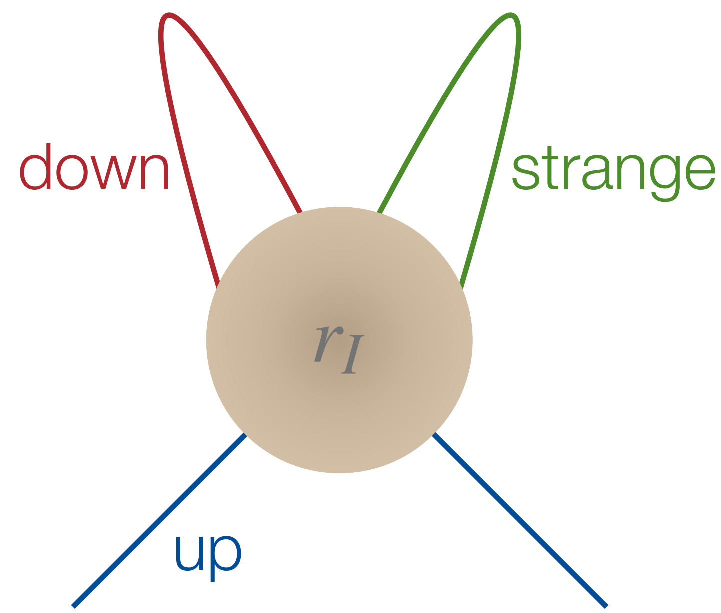

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$$= \frac{m_u m_d m_s}{m_u} r_I \boxed{(r_I \Lambda_{\mathrm{QCD}})^{18}}$$

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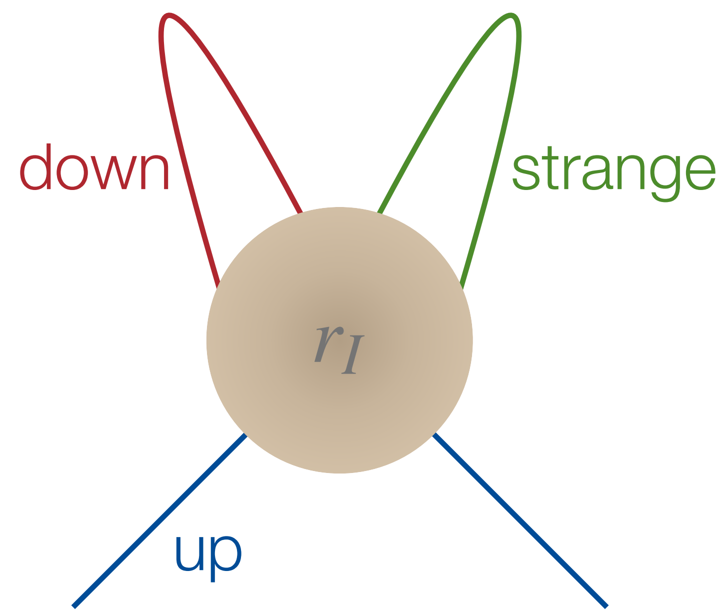

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$$= \frac{m_u m_d m_s}{m_u} r_I \left(r_I \Lambda_{\mathrm{QCD}}\right)^{18}$$

$$\sim \frac{m_d m_s}{\Lambda_{\mathrm{QCD}}} \quad \text{when } r_I \Lambda_{\mathrm{QCD}} \sim 1$$

2 MeV?

Pole Mass

- The pole mass is defined via

$$m_{\rm pole} = \lim_{p^2 \to m_{\rm pole}^2} \frac{m_0 - C(p^2; m_0)}{1 - A(p^2; m_0)}$$

- If we "knew" $m_{\rm pole}$, m_0 would be chosen to absorb UV divergences and any additive terms.

- Perturbation theory with a chirally symmetric UV regulator:

  - $C(p^2;m_0) = m_0 B(p^2;m_0)$ and the "nonperturbative" terms are lost;

  - develop asymptotic expansion in α_s for A & B and obtain $m_{\rm pole}$ order-by-order using iteration;

  - infrared finite & gauge independent at every order of perturbation theory.

Pole Mass II

- The natural scale for perturbation theory is of order m_{pole}:
  - fine for heavy quarks with $m \gg \Lambda_{\text{QCD}}$, modulo renormalons;
  - for light quarks with $m \lesssim \Lambda_{\text{QCD}}$, even the perturbative loops are long-distance contributions.
- Self energy could be calculated in lattice gauge theory (Landau gauge), establishing a curve on the (m_{pole}, m_0) plane, but with no prospect of converting m_{pole} to anything useful.
- Renormalization is not just supposed to make quantities UV finite:
  - to gain the full power of the renormalization group, one wants to separate long- and short-distance quantities.

Mass-independent Renormalization

- For light quarks, the conceptually (and computationally) cleanest schemes are mass independent.
- (Modified) minimal subtraction MS ($\overline{\text{MS}}$) is the best known example; limited to dimensional regularization and, thus, perturbation theory.
- Regulator-independent momentum-subtraction:

$$m(\mu) = \frac{m_0 - C(-\mu^2; m_0)}{1 - A(-\mu^2; m_0)} \qquad Z_q(\mu) = 1 - A(-\mu^2)$$

- Renormalized mass $m(\mu)$ vanishes when the numerator vanishes, *i.e.*, at the self-consistent of $m_0^*(\mu) = C(-\mu^2; m_0^*(\mu))$.
- Additive contribution to C (at $p^2 = -\mu^2$) is absorbed into $m_0^*(\mu)$!?!

Additive Corrections

- With a large, space-like momentum routed through the quark, only short-distance contributions matter:
  - renormalons disappear;
  - instanton effects now of order $m_d m_s \Lambda^{18}\mu^{-19}$;
  - but now the OPE tells us that condensates appear [Politzer, 1976; Pascual, de Rafael, 1982], *e.g.*, $\langle \bar{q}q \rangle / \mu^2$—
    - because of gauge fixing, icky condensates like $\langle A^2 \rangle$ can also arise.
- It makes more sense to omit these contributions from the renormalized mass: then a purely perturbative conversion to MS is well-defined.

Mass Ratios

- In particular, removing (i.e., fitting away) the condensates means

$$\frac{m_{Rb}(\mu_R)}{m_{Rc}(\mu_R)} = \frac{m_{\overline{\text{MS}}b}(\mu_{\overline{\text{MS}}})}{m_{\overline{\text{MS}}c}(\mu_{\overline{\text{MS}}})} = \frac{m_{0b}}{m_{0c}} + \mathrm{O}(a^2)$$

  i.e., “mass independent” schemes.

- Last equality holds if the lattice fermions have some chiral symmetry (staggered, overlap)—
  - more work for other lattice fermions (Wilson, domain wall) needed.
- The precise results use staggered fermions.
- Perturbative conversion under best control for heavy quarks; use the ratios to get the up, down, and strange masses.

Methods for Heavy Quarks

RI/SMOM

hep-lat/9411010, arXiv:0712.1061, arXiv:1306.3881, arXiv:1805.06225

- Here, the bare charm mass is fixed to a meson mass.
- The renormalization constant is computed via $Z_m Z_S = 1$, renormalizing the scalar density in a scheme similar to that outlined above:
  - fit away (milder) condensates by varying μ;
  - extrapolating valence mass to zero to make Z_m mass independent;
  - convert to $\overline{\text{MS}}$ with perturbation theory at ~5 GeV.
- Adjust light bare masses to further meson masses (one-to-one).
- Use ratios of bare masses to obtain light $\overline{\text{MS}}$ masses.

Quarkonium Moments

hep-lat/9505025, arXiv:0805.2999, arXiv:1408.4169, arXiv:1901.06424

- Here, the bare charm (bottom) mass is fixed to a meson mass.
- A physical renormalized charm mass is defined via time moments of the Euclidean correlation function:
  - natural scale is $2m_c$ ($2m_b$), so perturbation theory should work;
  - fit away (mild) condensates by varying m_h in $m_c < m_h < m_b$;
  - analyze moments with $\overline{\text{MS}}$ perturbation theory at ~3–10 GeV.
- Adjust light bare masses to further meson masses (one-to-one).
- Use ratios of bare masses to obtain light $\overline{\text{MS}}$ masses.

HQET ⊕ MRS

arXiv:1701.00347, arXiv:1712.04983, arXiv:1802.04248

- Here, the bare charm (bottom) mass is fixed to a heavy-light meson mass.

- The leading renormalon is removed from the pole mass, called minimal renormalon subtraction (MRS):

$$m_{\text{MRS}} \equiv \bar{m}\left(1+\sum_{n=0}^{\infty}\left[r_n - R_n\right]\alpha_{\text{g}}^{n+1}(\bar{m})\right) + \mathcal{J}_{\text{MRS}}(\bar{m})$$

  - the $r_n - R_n$ are very small; $\mathcal{J}_{\text{MRS}}(\bar{m})$ is known exactly and can be computed via a convergent series in $1/\alpha_s(\bar{m})$.

  - fit the binding energy away by varying m_h in $m_c < m_h < m_b$:

$$M = m_{\text{MRS}} + \bar{\Lambda} + \frac{\mu_\pi^2 - \mu_G^2(\bar{m})}{2m_{\text{MRS}}} + \cdots$$

- Use ratios of bare masses to obtain light $\overline{\text{MS}}$ masses.

Summary, Outlook

Summary

- Precise bottom and charm masses are determined via three distinct methods with very different systematics:
    - heavy-quark scale makes a clean separation of short- and long-distance contributions possible (OPE, EFT);
    - nonperturbative short-distance contributions are fit away or tiny;
    - tests of reliability of conversion to $\overline{\text{MS}}$.
- Precise light masses are obtained from these via mass ratios that are the same in all mass-independent schemes.
- Precise results from MILC’s 2+1+1 HISQ (or 2+1 asqtad) ensembles, *i.e.*, with staggered quarks.

HISQ Ensembles: 2+1+1

MILC, arXiv:1212.4768, arXiv:1712.09262

a (fm)	size	$am_l'/am_s'/am_c'$	# confs	# sources	notes
≈ 0.15	$16^3\times 48$	0.0130/0.065/0.838	1020	4	
≈ 0.15	$24^3\times 48$	0.0064/0.064/0.828	1000	4	
≈ 0.15	$32^3\times 48$	0.00235/0.0647/0.831	1000	4	physical
≈ 0.12	$24^3\times 64$	0.0102/0.0509/0.635	1040	4	
≈ 0.12	$32^3\times 64$	0.00507/0.0507/0.628	1020	4	also 24^3, 40^3
≈ 0.12	$48^3\times 64$	0.00184/0.0507/0.628	999	4	physical
≈ 0.12	$24^3\times 64$	0.0102/0.03054/0.635	1020	4	$m_s' < m_s$
≈ 0.12	$24^3\times 64$	0.01275/0.01275/0.640	1020	4	$m_s' = m_l'$
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≈ 0.12	$32^3\times 64$	0.00507/0.012675/0.628	1020	4	$m_s' \ll m_s$
≈ 0.12	$32^3\times 64$	0.00507/0.00507/0.628	1020	4	$m_s' = m_l'$
≈ 0.12	$32^3\times 64$	0.0088725/0.022815/0.628	1020	4	$m_s' < m_s$
≈ 0.09	$32^3\times 96$	0.0074/0.037/0.440	1005	4	
≈ 0.09	$48^3\times 96$	0.00363/0.0363/0.430	999	4	
≈ 0.09	$64^3\times 96$	0.0012/0.0363/0.432	484	4	physical
≈ 0.06	$48^3\times 144$	0.0048/0.024/0.286	1016	4	
≈ 0.06	$64^3\times 144$	0.0024/0.024/0.286	572	4	
≈ 0.06	$96^3\times 192$	0.0008/0.022/0.260	842	6	physical
≈ 0.042	$64^3\times 192$	0.00316/0.0158/0.188	1167	6	
≈ 0.042	$144^3\times 288$	0.000569/0.01555/0.1827	429	6	physical
≈ 0.03	$96^3\times 288$	0.00223/0.01115/0.1316	724	4	

Remark

- Idea that $m_u = 2\ \mathrm{MeV}$ could come from instantons seems implausible:
  - such contributions to the self energy enter into pole mass of light quarks, which is not portable;
  - thus, not what lattice QCD does;
  - in RI/(S)MOM scheme this contribution is clearly tiny;
  - similarly for the bare lattice mass, which probably somehow includes effects for r_I in $(250\ \mathrm{GeV})^{-1} \ll r_I < a$;
  - starting from heavy-quark masses, large instantons are not probed.
- Hence, $m_u = 2\ \mathrm{MeV}$ stems from Yukawa coupling at $250\ \mathrm{GeV}$.