Direct CP violation and the $\Delta I=1/2$ rule in $K\to\pi\pi$ decay from the Standard Model

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Motivation

- Likely explanation for matter/antimatter asymmetry in Universe, baryogenesis, requires violation of CP.
- Amount of CPV in Standard Model appears too low to describe measured M/AM asymmetry: tantalizing hint of new physics.
- Direct CPV first observed in late 90s at CERN (NA31/NA48) and Fermilab (KTeV) in \( K^0 \rightarrow \pi\pi \):

\[
\eta_{00} = \frac{A(K_L \rightarrow \pi^0\pi^0)}{A(K_S \rightarrow \pi^0\pi^0)}, \quad \eta_{+-} = \frac{A(K_L \rightarrow \pi^+\pi^-)}{A(K_S \rightarrow \pi^+\pi^-)}.
\]

\[
\text{Re}(\epsilon'/\epsilon) \approx \frac{1}{6} \left( 1 - \left| \frac{\eta_{00}}{\eta_{\pm}} \right|^2 \right) = 16.6(2.3) \times 10^{-4} \quad \text{(experiment)}
\]

- Small size of \( \epsilon' \) makes it particularly sensitive to new direct-CPV introduced by many BSM models.
- Looking for deviations from experiment may help shed light on origin of M/AM asymmetry.
• A Standard Model prediction of $\varepsilon'$ also provides a new horizontal band constraint on CKM matrix in $\rho$-$\eta$ plane:

new constraint from this work!

• While underlying weak process occurs at high energies $\sim M_w=80$ GeV, $K \to \pi\pi$ decays receive large corrections from low-energy hadronic physics $O(\Lambda_{QCD})\sim 250$ MeV.

• Lattice QCD is the only known ab initio, systematically improvable technique for studying non-perturbative QCD.
Overview of calculation

Hadronic energy scale $<< M_w$ – use weak effective theory (3 flavors)

\[ A(K^0 \to \pi^+\pi^-) = \sqrt{\frac{2}{3}} A_0 e^{i\delta_0} + \sqrt{\frac{1}{3}} A_2 e^{i\delta_2}, \]
\[ A(K^0 \to \pi^0\pi^0) = \sqrt{\frac{2}{3}} A_0 e^{i\delta_0} - 2\sqrt{\frac{1}{3}} A_2 e^{i\delta_2}. \]

\[ \epsilon' = \frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}} \left( \frac{\text{Im}A_2}{\text{Re}A_2} - \frac{\text{Im}A_0}{\text{Re}A_0} \right) \]

\[ \omega = \frac{\text{Re}A_2}{\text{Re}A_0} \]

\[ I=2 \text{ decay} \]
\[ I=0 \text{ decay} \]

Hadronic energy scale $<< M_w$ – use weak effective theory (3 flavors)

\[ A^I = F \frac{G_F}{\sqrt{2}} V_{ud}^* V_{us} \sum_{i=1}^{10} \sum_{j=1}^{7} \left[ (z_i(\mu) + \tau y_i(\mu)) Z_{i,j}^{\text{lat}} \overrightarrow{\text{MS}} M^I_j, \text{lat} \right] \]

\[ \tau = -\frac{V_{ts}^* V_{td}}{V_{us}^* V_{ud}} = 0.0014606 + 0.00060408i \]

Imaginary part solely responsible for CPV
(everything else is pure-real)

LL finite-volume correction

renormalization matrix (mixing)
Use RI-SMOM
convert to MSbar
perturbatively

\[ M_{j,\text{lat}}^I = \langle (\pi \pi)_I | Q_j | K \rangle \] (lattice)

10 effective four-quark operators
Anatomy of a lattice calculation

- Lattice QCD uses Monte Carlo techniques to sample the discretized (Euclidean) Feynman path integral directly, generating an ensemble of N “gauge configurations”.

- Expectation value of some Green’s function computed over the ensemble converges to path integral value in large N limit.

- Green’s function composed of operators created from quark fields that create/destroy states of interest.

- Operators create all states with same quantum numbers, eg $\bar{u} \gamma^5 d$ creates pions and all excited pion-like states.

- Contributions of each state $i$ decay exponentially in time as $\exp(-E_i t)$ due to Euclidean time.

- Extract contributions of lightest states by fitting large time dependence.

- Challenges:
  - Computationally expensive, requiring months to years of running on the world’s fastest supercomputers.
  - Much like experiment, have both statistical and systematic errors.
  - Systematic errors (e.g. from discretization or from fitting) require careful analysis and treatment.
Lattice QCD for $K \to \pi \pi$

operator with $\pi \pi$ q. numbers.

$\langle 0 | O_{snk}(t_{snk}) H_W(t) O_{src}(t_{src}) | 0 \rangle$

operator with kaon q. numbers.

state contributions exponentially falling according to their energy (Euclidean time!)

$= \sum_{n,m} \langle 0 | O_{snk}(t_{snk}) | n \rangle \langle n | H_W(t) | m \rangle \langle m | O_{src}(t_{src}) | 0 \rangle \times e^{-E_n(t_{snk} - t)} e^{-E_m(t - t_{src})}$

- Extract matrix elements by fitting time dependence in limit of large $(t_{snk} - t)$, $(t_{src} - t)$ at which lower-energy states dominate.

- Series is necessarily truncated for fit: Systematic errors arise if excited state effects not properly taken into account.
$I=2$ calculation

- $A_2$ can be measured very precisely using “standard” lattice techniques.

- Most recent result (2015):
  - Computed with large, $\sim (5.5 \text{ fm})^3$ volumes
  - Physical quark masses
  - Two lattice spacings (2.36 GeV and 1.73 GeV) $\rightarrow$ Continuum limit taken.

- $<1\%$ statistical error!

- 10\% and 12\% total errors on Re($A_2$) and Im($A_2$) resp.

- Dominant sys. errors due to truncation of PT series in computation of renormalization and Wilson coefficients.

\( \Delta I = 1/2 \) rule

- In experiment kaons \( \sim 450 \times (!) \) more likely to decay into \( I=0 \) pi-pi states than \( I=2 \).

\[
\frac{\text{Re}A_0}{\text{Re}A_2} = 22.45(6) \quad (\text{the } \Delta I = 1/2 \text{ rule})
\]

- Perturbative running to charm scale accounts for about a factor of 2. Where does the remaining 10x come from? New Physics?


Strong cancellation between the two dominant contractions not predicted by naive factorization:

\[
\text{Re}(A_2) \sim 1 + 2
\]

find \( 2 \approx -0.7 \) \( 1 \) heavily suppressing \( \text{Re}(A_2) \).


Pure-lattice calculation

\[
\frac{\text{Re}(A_0)}{\text{Re}(A_2)} = 19.9(5.0)
\]

[Re(A_0) agrees with expt.]
**I=0 Calculation**

- $A_0$ is more difficult than $A_2$, primarily because $I=0$ $\pi\pi$ state has *vacuum quantum numbers*.
- “Disconnected diagrams” dominate statistical noise.

![Disconnected Diagrams](image)

**2015 calculation**

- Physical quark masses on single, coarse lattice ($a^{-1}=1.38$ GeV) but with large (4.6 fm)$^3$ physical volume to control FV errors.
- G-parity boundary conditions remove dominant unphysical contribution from stationary $\pi\pi$ state.
- Single $\pi\pi$ operator:
  
  - $21\%$ and $65\%$ stat errors on $\text{Re}(A_0)$ and $\text{Im}(A_0)$ due to disconn. diagrams and, for $\text{Im}(A_0)$ a strong cancellation between $Q_4$ and $Q_6$.
  - Dominant, $15\%$ systematic error due again to PT truncation errors.

2015 calculation: $\varepsilon'$

- $\text{Re}(A_0)$ and $\text{Re}(A_2)$ from expt.
- Lattice values for $\text{Im}(A_0)$, $\text{Im}(A_2)$ and the phase shifts,

\[
\text{Re} \left( \frac{\varepsilon'}{\varepsilon} \right) = \text{Re} \left\{ \frac{i \omega e^{i(\delta_2 - \delta_0)}}{\sqrt{2\varepsilon}} \left[ \frac{\text{Im} A_2}{\text{Re} A_2} - \frac{\text{Im} A_0}{\text{Re} A_0} \right] \right\} \\
= 1.38(5.15)(4.43) \times 10^{-4}, \quad \text{(our result)} \\
16.6(2.3) \times 10^{-4}, \quad \text{(experiment)}
\]

- Result is 2.1σ below experimental value.
- Total error on $\text{Re}(\varepsilon'/\varepsilon)$ is $\sim 3x$ the experimental error.
- “This is now a quantity accessible to lattice QCD”!
- Focus since has been to improve statistics and reduce / improve understanding of systematic errors.
The “$\pi\pi$ puzzle”

- Essential to understand $\pi\pi$ system:
  - Energy needed to extract ground-state matrix element
  - Energy also needed to compute phase-shift (Luscher)
  - Derivative of phase-shift w.r.t. energy is required for Lellouch-Luscher finite-volume correction (F)

- 2015 calculation phase shift $\delta_0 (E_{\pi\pi} \approx m_K) = 23.8(5.0)^\circ$ substantially smaller than prediction obtained by combining dispersion theory with experimental input, $36^\circ$.

- Result was very stable under varying fit range and also with 2-state fits.
- Increasing statistics by almost $7x$ did not resolve ($\delta_0 = 19.1(2.5)^\circ$)
- Nevertheless, most likely explanation is excited-state contamination hidden by rapid reduction in signal/noise.
Resolving the $\pi\pi$ puzzle

- To better resolve the ground-state we have introduced 2 more $\pi\pi$ operators:

  \[
  \pi(311): \quad \rho(3, 1, 1)/L
  \]

- Obtain parameters by simultaneous fitting to matrix of correlation functions, eg for $\pi\pi$ 2pt Green’s function:

  \[
  \Sigma = \frac{1}{\sqrt{2}} (\bar{u}u + \bar{d}d)
  \]

\[
C_{ij}(t) = \langle 0 | O_i^\dagger(t) O_j(0) | 0 \rangle = C + \sum_\alpha A_{i,\alpha} A_{j,\alpha} e^{-E_\alpha t}
\]

- A far more powerful technique than just increasing statistics alone.
- 741 configurations measured with 3 operators.
Effect of multiple operators on $\pi\pi$

Result compatible with dispersive value: $\delta_0(479.5\text{ MeV}) = 32.3(1.0)(1.4)^\circ$

[For more details, cf talk by Tianle Wang, Lattice 2021]
Effect of multiple operators on $K \to \pi\pi$

- Convenient to visualize data by taking “optimal” linear combination of the two most important operators that best projects onto ground-state.

$$O_{\text{opt}} = r_1 O_{\pi\pi(111)} + r_2 O_{\sigma}$$

$$r_1 = 5.24(18) \times 10^{-7} \quad \text{using } \pi\pi \text{ fits}$$

$$r_2 = -2.86(17) \times 10^{-4}$$

**Strong, clear plateau + improved precision**
**$K\rightarrow\pi\pi$ fit results**

- Examine many fit ranges, #states and #operators

  - little indication of exc. stat. cont. for $Q_2$

  - final fit

- Adopt uniform fit $t'_{\text{min}}$ = 5 which is stable for all $Q_i$

- Evidence that excited state error was significantly underestimated in 2015 work
Systematic error budget

- Primary systematic errors of 2015 work:
  - Finite lattice spacing: 12%
  - Wilson coefficients: 12%
  - Renormalization (mostly PT matching): 15%
  - Excited-state: ≤ 5% but now known to be significantly underestimated
  - Lellouch-Luscher factor (derivative of ππ phase shift wrt. energy): 11%

- In our new work we have used step-scaling to raise the renormalization scale from 1.53 \(\rightarrow\) 4.00 GeV: 15% \(\rightarrow\) 5%

- 3 operators have dramatically improved understanding of ππ system: Lellouch-Luscher factor 11% \(\rightarrow\) 1.5%

- Detailed analysis shows no evidence of remaining excited-state contamination: Excited state error now negligible!

- Still single lattice spacing: Discretization error unchanged.

- Evidence that Wilson coefficient systematics are driven by using PT for 3-4f matching, not improved by higher \(\mu\): Wilson coeff error unchanged.
Isospin breaking + EM effects

- Our simulation does not include effects of isospin breaking or EM effects.
- While these effects are typically small $O(1\%)$, heavy suppression of $A_2$ ($\Delta I=1/2$ rule) means relative effect on $A_2$ and $\epsilon'$ could be $O(20\%)$.
- Current best determination of effect uses NLO $\chi$PT and $1/N_c$ expansion predicts 23% correction to our result: Include as separate systematic error.

[Cirigliano et al., JHEP 02 (2020) 032]
Final result for $\varepsilon'$

Combining our new result for $\text{Im}(A_0)$ and our 2015 result for $\text{Im}(A_2)$, and again using expt. for the real parts, we find

$$
\text{Re} \left( \frac{\varepsilon'}{\varepsilon} \right) = \text{Re} \left\{ \frac{i \omega e^{i(\delta_2-\delta_0)}}{\sqrt{2\varepsilon}} \left[ \frac{\text{Im} A_2}{\text{Re} A_2} - \frac{\text{Im} A_0}{\text{Re} A_0} \right] \right\} 
$$

$$
= 0.00217(26)(62)(50) 
$$

Consistent with experimental result:

$$
\text{Re}(\varepsilon'/\varepsilon)_{\text{expt}} = 0.00166(23) 
$$
The road ahead

- Primary pure-lattice systematic is discretization error (12%). Currently estimated using scaling of $l=2$ operators but there may be significant “error on the error”.

- Near-term availability of next-gen supercomputers (Perlmutter, Aurora) opens up opportunity to perform a full continuum extrapolation.

- Current plan is two additional ensembles with following properties:
  - Physical pion and kaon masses.
  - Same gauge action allowing continuum extrapolation with 3 points.
  - Same physical volume such that $\pi\pi$ energy remains the same and the interaction remains physical.

- $40^3\times64, a^{-1}=1.7$ GeV and $48^3\times64, a^{-1}=2.1$ GeV are computationally feasible while providing a good lever arm ($a^2$ scaling).

- Already started generating $16^3$ test ensembles for tuning.

- Measurement code has been ported to NVidia and Intel GPUs, utilizing Grid’s portable GPU kernel API.
The road ahead pt.2

- Independent calculation of $\epsilon'$ using multiple operators to extract on-shell matrix elements as excited-state contributions in a periodic lattice is well under way. [cf. talk by M.Tomii, Lattice 2021]
  - Avoid complications of using G-parity BCs
  - Uses existing MDWF+I ensembles with physical pion masses
  - 2 lattice spacings allowing continuum limit
- We are developing techniques to perform 3-4f matching in the Wilson coefficients non-perturbatively in order to avoid relying on PT at the charm scale. [PoS LATTICE2018 (2019) 216]
- Also working on laying the groundwork for the lattice calculation of EM contributions. [cf. talk by J.Karpie, Lattice 2021]
Conclusions

- Completed update on our 2015 lattice determination of $A_0$ and $\varepsilon'$
  - 3.2x increase in statistics.
  - Improved systematic errors, notably use of multi-operator techniques essentially removes excited-state systematic.
- Reproduce experimental value for $\Delta I=1/2$ rule, demonstrating that QCD sufficient to solve this decades-old puzzle.
- Result for $\varepsilon'$ consistent with experimental value.
- Total error is $\sim$3.6x that of experiment.
- $\varepsilon'$ remains a promising avenue to search for new physics, but greater precision is required.
- The work goes on....
**Excited state contamination**

- Primary concern is residual excited-state contamination
- See excellent consistency and strong plateaus among fits for 2+ operators with $t_{\text{min}} \geq 4$
- Also examine 2 and 3-state fits with 3 ops:
  - 3-state fit with lower $t_{\text{min}} = 4$ describes data well outside fit range.
  - Complete consistency in gnd-state matrix elem of best fit