

# Excited states of Bottomonia in QGP from lattice QCD

Rasmus Larsen

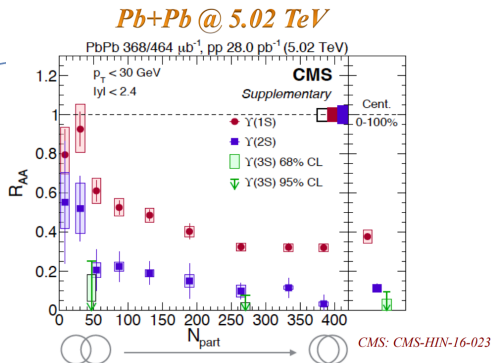
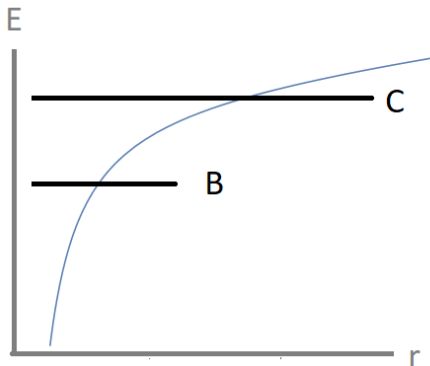
University of Stavanger

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[Rasmus Larsen, Stefan Meinel, Swagato Mukherjee and Peter Petreczky,  
arXiv:1910.07374]

# Motivation

- Motivation:
  - Use Bottomonium states as probe for change in color screening
  - Experimental results show suppression of Bottomonium states at finite temperature
  - We will look for the Bottomonia states at finite temperature using lattice NRQCD



- Approach
  - Non Relativistic QCD (NRQCD) on lattice
    - with  $O(v^4)$  corrections, plus  $O(v^6)$  spin corrections
  - 2+1 flavor HotQCD configurations from  $T = 151\text{MeV}$  to  $T = 334\text{MeV}$
  - Pion mass 160MeV, Kaon mass physical
  - Explore  $\Upsilon$ ,  $\chi_b$
- Main observable: Correlation function  $C(\tau)$ 
  - $C(\tau)$  is the zero momentum of the state of interest

$$\int d^3x \langle O(\tau, \mathbf{x}) O^\dagger(0, 0) \rangle = C(\tau) = \int_0^\infty \rho(\omega) \exp(-\omega\tau) d\omega \quad (1)$$

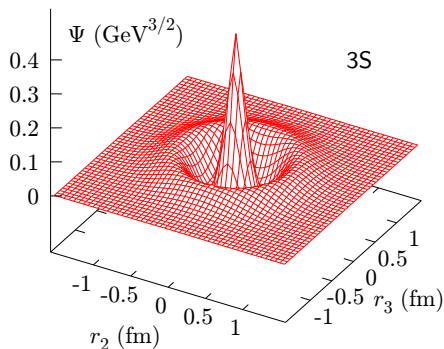
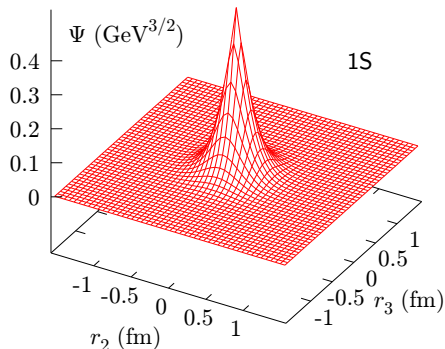
$$O_i(\tau, \mathbf{x}) = \sum_{\mathbf{r}} \Psi_i(\mathbf{r}) \bar{q}(\mathbf{x} + \mathbf{r}, \tau) \Gamma q(\mathbf{x}, \tau) \quad (2)$$

- Invert equation to find spectral function  $\rho(\omega)$

# Extended Sources

- Source calculated from discretized schroedinger equation with confining potential that reproduces zero temperature spectrum

$$O_i(\mathbf{x}, t) = \sum_{\mathbf{r}} \Psi_i(\mathbf{r}) \bar{q}(\mathbf{x} + \mathbf{r}, t) \Gamma q(\mathbf{x}, t) \quad (3)$$

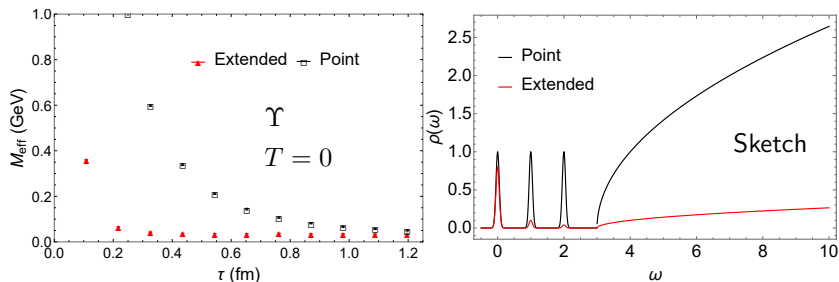


# Spectral Function

- Plateaus of the effective mass  $M_{eff} \rightarrow$  Mass state exists in  $\rho(\omega)$

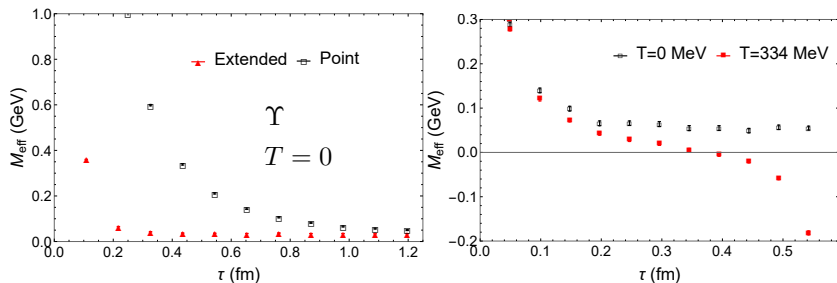
$$M_{eff} = \frac{1}{a} \log[C(\tau)/C(\tau + a)] = -\frac{\partial}{\partial \tau} \log(C(\tau)) \quad (4)$$

- Continuum in  $\rho(\omega)$  with point sources dominates contribution to correlation function
- Solution:
  - Use sources with finite (extended) size  $\rightarrow$  project onto specific region in  $\omega$



# Continuum Subtracted S-wave

- Extended sources greatly reduces continuum contribution

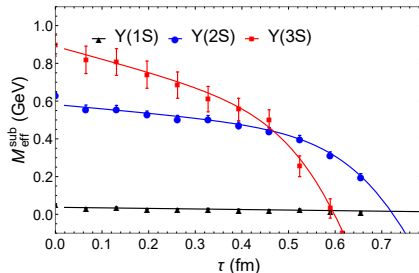


- Small  $\tau$  behavior similar at  $T = 0$  and  $T \neq 0$
- Extract continuum  $C_{\text{high}}(\tau)$  from  $T = 0$  results
- 0 Corresponds to energy of  $\eta_b$  at  $T = 0 \text{ MeV}$ .

$$\begin{aligned} C(\tau) &= Ae^{-M\tau} + C_{\text{high}}(\tau) \\ C_{\text{sub}}(\tau, T) &= C(\tau, T) - C_{\text{high}}(\tau) \end{aligned} \quad (5)$$

# Finite Temperature Subtracted Effective Mass

- Drop in effective mass as  $\tau \rightarrow 1/T$
- Linear behavior at small to mid range  $\tau$

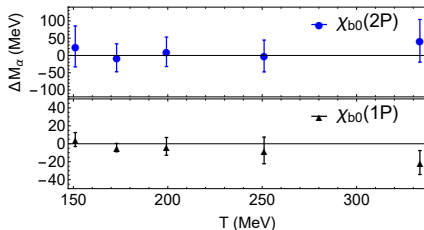
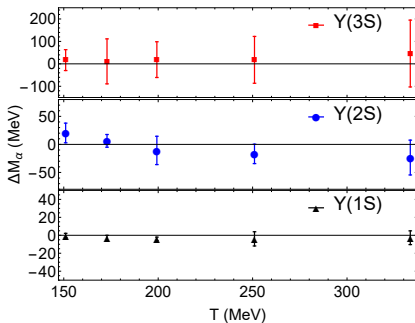


- Information in correlation function is thus

$$C_{\text{sub}}(\tau, T) \sim \exp(-M_{\alpha}\tau + \frac{1}{2}\Gamma_{\alpha}^2\tau^2 + O(\tau^3)) \quad (6)$$

$$\rho_{\alpha}(\omega, T) = A_{\alpha}(T) \exp\left(-\frac{[\omega - M_{\alpha}(T)]^2}{2\Gamma_{\alpha}^2(T)}\right) + A_{\alpha}^{\text{cut}}(T) \delta(\omega - \omega_{\alpha}^{\text{cut}}(T))$$

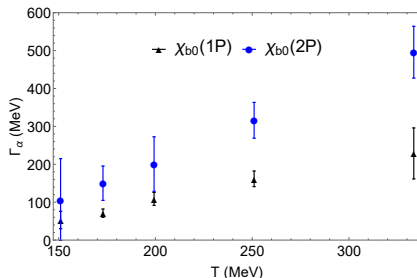
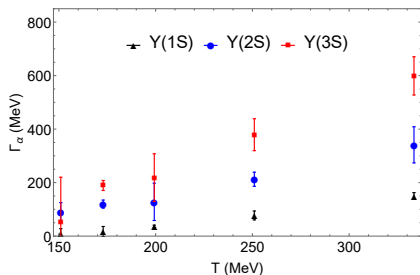
- The mass is found to be consistent with zero temperature results [R. Larsen et al., arXiv:1910.07374]
- $\Delta M_\alpha = M_\alpha(T) - M_\alpha(0)$





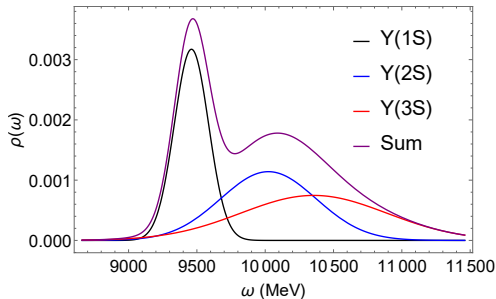
# Spectral Width

- Spectral width grows with temperature like  $T^2$  [R. Larsen et al., arXiv:1910.07374]
- The higher the energy of the state, the wider the spectral function becomes



# Picture at finite temperature

- Our results indicate the following picture
  - No significant change in energy/mass of states
  - Large spectral width, such that states start to overlap
  - Spectral function with equal weight at  $T = 334\text{MeV}$  shown below



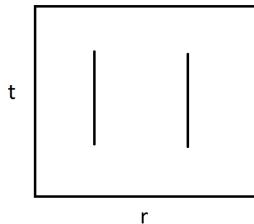
# Wilson Lines Correlator

- Same procedure as for Bottomonia works for Wilson lines Correlator
- Wilson line is a product of links, that transport quarks from 1 point to another

$$W(t, x) = \prod_i^t U_4(i, x) \quad (7)$$

- Infinitely heavy quarks stay fixed at same position
- A quark and anti-quark will interfere with each other, to create different states based on the possible energies

$$\begin{aligned} C(t, x) &= \text{Tr}(W(t, 0)W(t, x)^\dagger) \quad (8) \\ &= \int_0^\infty \rho(\omega) \exp(-\omega\tau) d\omega \end{aligned}$$



- Measurement is not gauge invariant
- We gauge fix to Coulomb gauge

# Measurements with Wilson Lines

- Measurement contain contribution from continuum, just like for Bottomonia
- We remove the continuum in the same procedure as for Bottomonia

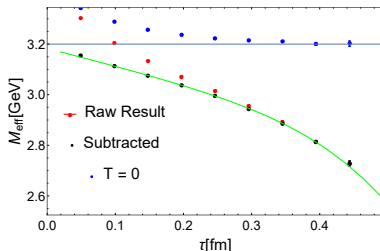


Figure:  $T=334\text{MeV}$ ,  $r=0.44\text{fm}$ .

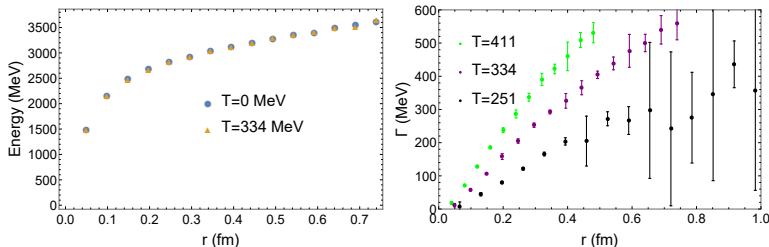
- Information in correlation function (Black points) is thus

$$C_{\text{sub}}(\tau, T) \sim \exp(-M_{\alpha}\tau + \frac{1}{2}\Gamma_{\alpha}^2\tau^2 + O(\tau^3)) \quad (9)$$

$$\rho_{\alpha}(\omega, T) = A_{\alpha}(T) \exp\left(-\frac{[\omega - M_{\alpha}(T)]^2}{2\Gamma_{\alpha}^2(T)}\right) + A_{\alpha}^{\text{cut}}(T) \delta(\omega - \omega_{\alpha}^{\text{cut}}(T))$$

# Energy and Width of Wilson Lines Correlator

- Almost no change in energy, but increasing width



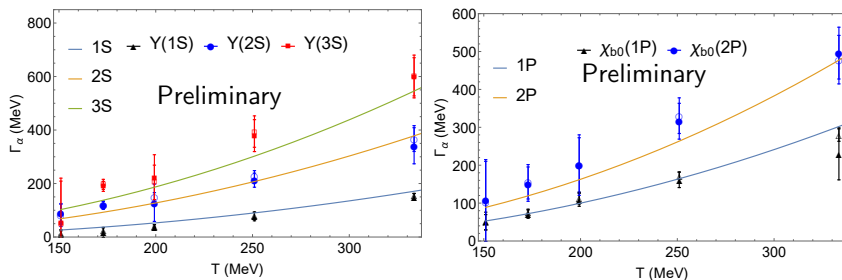
**Figure:** (Left) Energy obtained for  $T = 334$  MeV. (Right) Width at several temperatures around 300 MeV.

- Width obtained from Wilson lines correlator consistent with width of  $\Upsilon$  when looking at distance of  $\Upsilon$ 's average radius.

# Wilson Lines Correlators and Bottomonia States

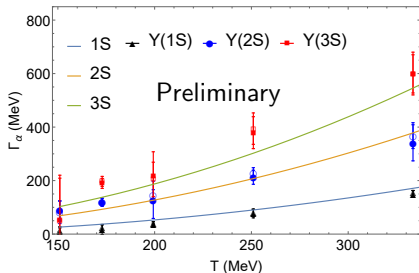
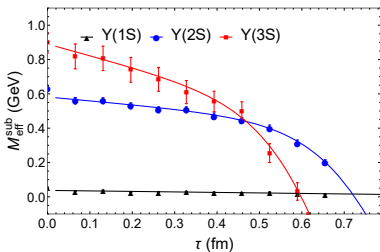
- Calculate spectral width of bottomonia from Wilson lines results
- Wave function from discretized Shroedinger equation

$$\Gamma_\alpha = \int dx^3 |\psi_\alpha(x)|^2 \Gamma_r(|x|) \quad (10)$$

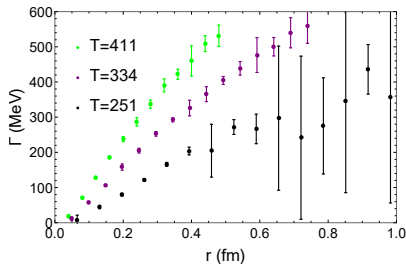


- Width obtained from linear rise in width of Wilson lines correlator
- Wavefunctions calculated with mass 5.5GeV.

# Conclusion



- Linear behavior in effective mass observed
- Behavior explained by large spectral width  $\sim 160 - 600 \text{ MeV}$
- 2S and 3S, 1P and 2P spectral functions overlap strongly above  $T = 200 \text{ MeV}$
- Width from Wilson lines correlator shows same behavior as bottomonia results



# Extra



# Bethe-Salpeter Wavefunction

- Bethe-Salpeter amplitude for bottomonium

$$C_{\alpha}^r(\tau) = \langle O_{qq}^r(\tau) \tilde{O}_{\alpha}^{\dagger}(0) \rangle, \quad (11)$$

- Measure probability of quarks to be separated

$$O_{qq}^r(\tau) = \int d^3x \bar{q}(\mathbf{x}, \tau) \Gamma q(\mathbf{x} + \mathbf{r}, \tau). \quad (12)$$

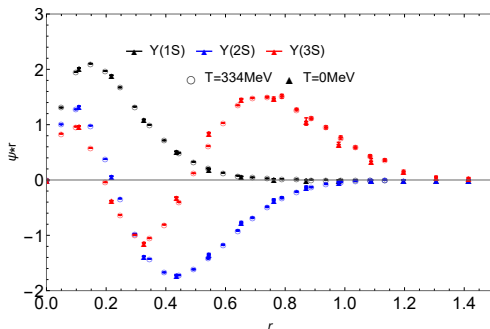
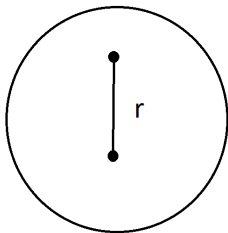
- Remove Energy dependence to get time independent result

$$e^{E_{\alpha}\tau} \tilde{C}_{\alpha}^r(\tau) \quad (13)$$

- or normalize to 1

# Spatial Localization of States

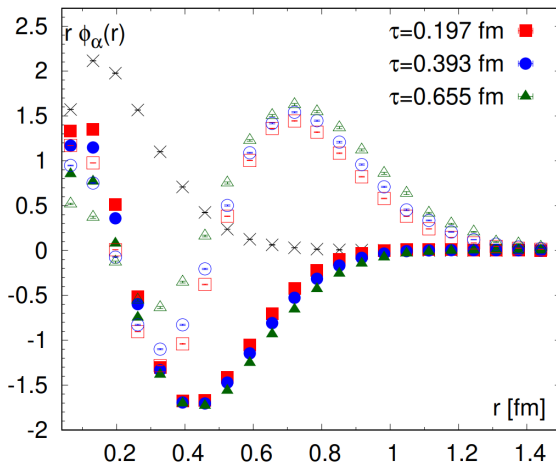
- Arxiv:2008.00100
- Look at the change in shape using the Bethe-Salpeter wavefunction
- Shows the distribution of the state as a function of distance  $r$



- No significant difference in shape is seen at  $\tau \sim 0.4\text{ fm}$

# Bethe-Salpeter Wavefunction 2

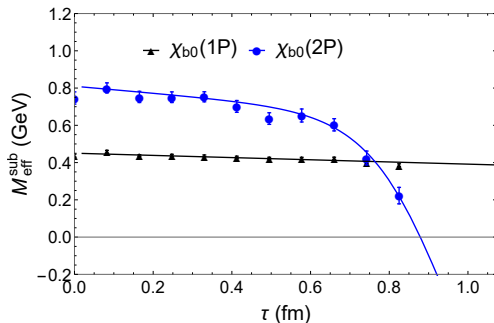
- For large temperature BS wavefunction is seen to move out slightly to larger distances as  $\tau$  increases
- $T=251\text{MeV}$



# Backup

# Continuum Subtracted P-wave

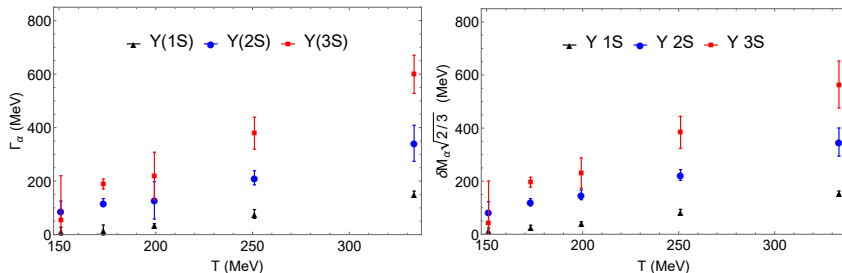
- Same procedure works for  $\chi_b$
- Figure below is for  $T = 199MV$



- $M_{\text{eff}}^{\text{sub}}$  goes to energy of the  $T = 0$  result when extrapolated to  $\tau = 0$

# Ansatz Comparison

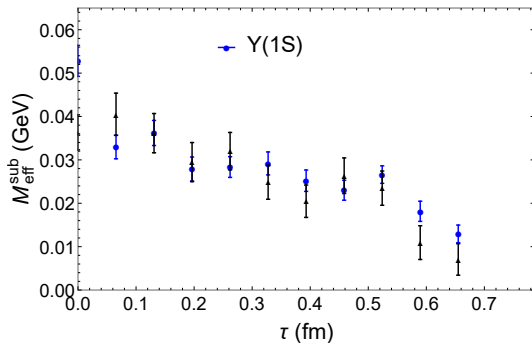
- Ansatz with Gaussian spectral function (left) and 3 delta functions representing a width (right) are consistent
- $\delta M_\alpha$  is difference between middle delta function and the two other delta functions



- Width  $\sqrt{\langle \omega^2 \rangle - \langle \omega \rangle^2}$  is equal to:
  - $\Gamma_\alpha$  for Gaussian ansatz
  - $\delta M_\alpha \sqrt{2/3}$  for 3 delta functions ansatz

# Dependence on Source

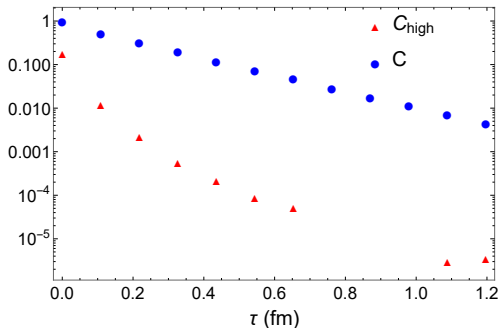
- Slope consistent between smeared sources and wavefunction sources
- Drop off at  $\tau \sim 1/T$  smaller for wavefunctions



# Continuum Subtracted

- Small  $\tau$  behavior same at  $T = 0$  and  $T \neq 0$
- Extract difference from groundstate at  $T = 0$

$$C(\tau) = Ae^{-M\tau} + C_{high}(\tau)$$



- Subtract result from finite temperature result

$$C_{sub}(\tau, T) = C(\tau, T) - C_{high}(\tau). \quad (14)$$