NNLO QCD corrections to the B-meson mixing

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based on 2106.05979, 21XX.XXXXX
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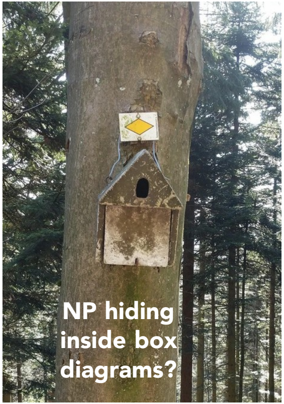
A Virtual Tribute to Quark Confinement and the Hadron Spectrum 2021
2nd of August 2021

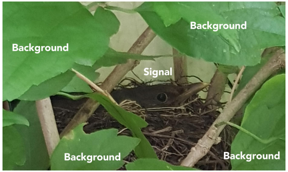
Karlsruhe Institute of Technology

Outline

1. **Flavor physics and the precision frontier**
2. **B-meson mixing**
   - Theory
   - Calculation
   - Phenomenology
3. **Summary and Outlook**

- No new physics in sight at the high-energy frontier
- Growing importance of the precision frontier
- Flavor physics: increasing number of anomalies (LFU violation, muon $g-2$, ...) that challenge the validity of the SM
- New physics hiding around the corner?
- However: Working at the precision frontier is impossible without precision physics
- Precision physics implies precise measurements and precise theoretical predictions
- Theory and experiment must go hand in hand trying to decrease existing uncertainties in SM predictions





- Neutral meson mixing in the SM : Loop-induced FCNC process.
- $B_s^0 - \bar{B}_s^0$ oscillations

$$i\frac{d}{dt}\begin{pmatrix}|B_s^0(t)\rangle \\ |\bar{B}_s^0(t)\rangle\end{pmatrix} = \left(\hat{M} - \frac{i}{2}\hat{\Gamma}\right)\begin{pmatrix}|B_s^0(t)\rangle \\ |\bar{B}_s^0(t)\rangle\end{pmatrix},$$

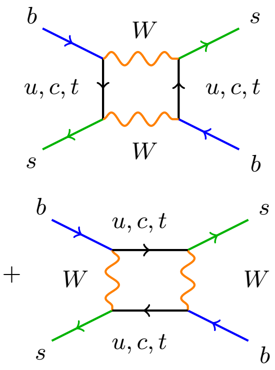
$$\hat{M} = \begin{pmatrix} M_{11} & M_{12} \\ M_{12}^* & M_{22}\end{pmatrix}, \quad \hat{\Gamma} = \begin{pmatrix}\Gamma_{11} & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma_{22}\end{pmatrix}$$

- Diagonalize the matrices

$$|B_{s,L}\rangle = p\,|B_s^0\rangle + q\,|\bar{B}_s^0\rangle$$
$$|B_{s,H}\rangle = p\,|B_s^0\rangle - q\,|\bar{B}_s^0\rangle$$

- Mass eigenstates: $|B_{s,L}\rangle$ (lighter) and $|B_{s,H}\rangle$ (heavier)
- Flavor eigenstates: $|B_s^0\rangle$ and $|\bar{B}_s^0\rangle$

$$M_{12} - \frac{i}{2}\Gamma_{12} \propto$$



- Physical observables depend on: $|M_{12}|$, $|\Gamma_{12}|$, ϕ_s
- ΔM_s: $B_s^0 - \bar{B}_s^0$ oscillation frequency

$$\Delta M_s = M_H - M_L \approx 2|M_{12}|$$

t quark is dominant in SM, sensitivity to NP in the loops

- $\Delta\Gamma_s$: $B_s^0 - \bar{B}_s^0$ width difference

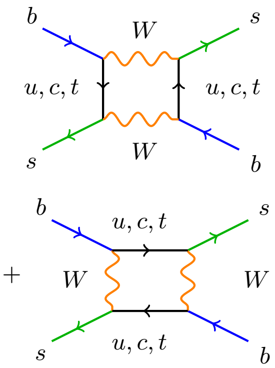
$$\Delta\Gamma_s = \Gamma_L - \Gamma_H \approx 2|\Gamma_{12}| \cos\phi_s$$

only u and c contribute, precision probe of SM, little room for NP

- ϕ_s: CP-asymmetry in the mixing

$$a_{\rm fs} = \mathrm{Im}\left(\frac{\Gamma_{12}}{M_{12}}\right) = \left|\frac{\Gamma_{12}}{M_{12}}\right| \sin\phi_s$$

$$M_{12} - \frac{i}{2}\Gamma_{12} \propto$$



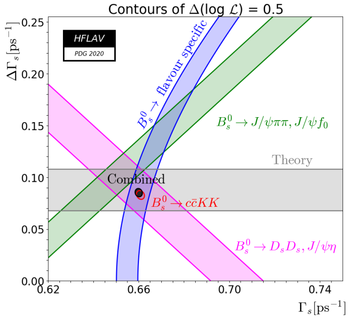
- Our interest: $\Delta\Gamma_s$ from $B_s^0 - \bar{B}_s^0$
- Experimental value (HFLAV 2020 average)

$$\Delta\Gamma^{\text{exp}} = (0.085 \pm 0.004)\ \text{ps}^{-1}$$

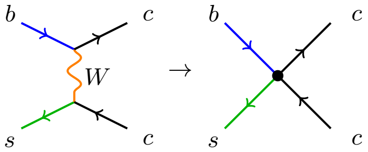
- Theory prediction (NLO + n_f-piece of NNLO QCD corrections) [Beneke et al., 1999; Ciuchini et al., 2002, 2003; Lenz & Nierste, 2007; Asatrian et al., 2020, 2017]

$$\Delta\Gamma_{\text{OS}} = (0.077 \pm 0.015_{\text{pert.}} \pm 0.002_{B,\tilde{B}_S} \pm 0.017_{\Lambda_{\text{QCD}}/m_b})\ \text{ps}^{-1}$$
$$\Delta\Gamma_{\overline{\text{MS}}} = (0.088 \pm 0.011_{\text{pert.}} \pm 0.002_{B,\tilde{B}_S} \pm 0.014_{\Lambda_{\text{QCD}}/m_b})\ \text{ps}^{-1}$$

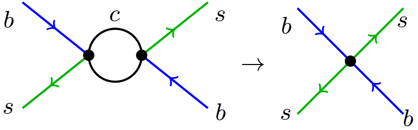
- Large perturbative uncertainty from the uncalculated NNLO corrections (pert.)
- Theory under pressure, full NNLO corrections highly desirable



$|\Delta B| = 1$ effective theory ($m_b \ll m_W, m_t$)

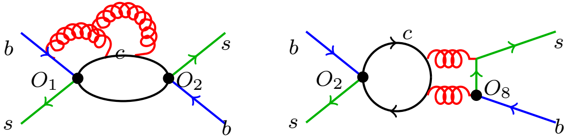


$|\Delta B| = 2$ effective theory (via HQE)

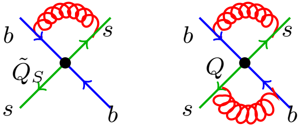


$$\Gamma_{12} \sim \frac{1}{m_b^3} \sum_i \left(\frac{\alpha_s}{4\pi}\right)^j \Gamma_3^{(i)} + \frac{1}{m_b^4} \sum_i \left(\frac{\alpha_s}{4\pi}\right)^j \Gamma_4^{(i)} + \ldots$$

- Calculation done using $\mathcal{H}_{\text{eff}}^{|\Delta B|=1}$ in the CMM operator basis for $b \to s c \bar{c}$ [Chetyrkin et al., 1998]
- Representative diagrams in the $|\Delta B| = 1$ EFT needed for the NNLO accuracy



matched to the $|\Delta B| = 2$ EFT



- 2-loop $O_{1-2} \times O_{3-6}$ available: $\Delta\Gamma_s^{p,12\times 36,\alpha_s} / \Delta\Gamma_s = 1.4\% (\overline{\text{MS}})$ [Gerlach, Nierste, VS, Steinhauser, 2021]
- All relevant 2-loop and 3-loop correlators already computed, two more publications in preparation

$|\Delta B| = 1$ effective Hamiltonian in the CMM basis for $b \to sc\bar{c}$ decays [Chetyrkin et al., 1998]

$$\mathcal{H}_{\text{eff}}^{|\Delta B|=1} = \frac{4G_F}{\sqrt{2}} \Bigg[-V_{ts}^* V_{tb}^\dagger \left(\sum_{i=1}^{6} C_i Q_i + C_8 Q_8 \right) - V_{us}^* V_{ub}^\dagger \sum_{i=1}^{2} C_i (Q_i - Q_i^u) + V_{us}^* V_{cb} \sum_{i=1}^{2} C_i Q_i^{cu} + V_{cs}^* V_{ub} \sum_{i=1}^{2} C_i Q_i^{uc} \Bigg] + \text{h.c.}\,,$$

- Current operators

$$Q_1 = \bar{s}_L \gamma_\mu T^a c_L \; \bar{c}_L \gamma^\mu T^a b_L,$$
$$Q_2 = \bar{s}_L \gamma_\mu c_L \; \bar{c}_L \gamma^\mu b_L,$$
$$Q_1^u = \bar{s}_L \gamma_\mu T^a u_L \; \bar{u}_L \gamma^\mu T^a b_L,$$
$$Q_2^u = \bar{s}_L \gamma_\mu u_L \; \bar{u}_L \gamma^\mu b_L,$$
$$Q_1^{cu} = \bar{s}_L \gamma_\mu T^a u_L \; \bar{c}_L \gamma^\mu T^a b_L,$$
$$Q_2^{cu} = \bar{s}_L \gamma_\mu u_L \; \bar{c}_L \gamma^\mu b_L,$$
$$Q_1^{uc} = \bar{s}_L \gamma_\mu T^a c_L \; \bar{u}_L \gamma^\mu T^a b_L,$$
$$Q_2^{uc} = \bar{s}_L \gamma_\mu c_L \; \bar{u}_L \gamma^\mu b_L,$$

- Penguin operators

$$Q_3 = \bar{s}_L \gamma_\mu b_L \sum\nolimits_q \bar{q} \gamma^\mu q\,,$$
$$Q_4 = \bar{s}_L \gamma_\mu T^a b_L \sum\nolimits_q \bar{q} \gamma^\mu T^a q\,,$$
$$Q_5 = \bar{s}_L \gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} b_L \sum\nolimits_q \bar{q} \gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} q\,,$$
$$Q_6 = \bar{s}_L \gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} T^a b_L \sum\nolimits_q \bar{q} \gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} T^a q\,,$$
$$Q_8 = \frac{g_s}{16\pi^2} m_b \, \bar{s}_L \sigma^{\mu\nu} T^a b_R \, G_{\mu\nu}^a\,,$$

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- 4-fermion vertices generate Dirac structures with multiple insertions of γ matrices

$$(P_L)_{ij} \times (P_L)_{kl}, \quad (\gamma^\mu P_L)_{ij} \times (\gamma_\mu P_L)_{kl}, \quad (\gamma^\mu \gamma^\nu P_L)_{ij} \times (\gamma_\mu \gamma_\nu P_L)_{kl},$$
$$(\gamma^\mu \gamma^\nu \gamma^\rho P_L)_{ij} \times (\gamma_\mu \gamma_\nu \gamma_\rho P_L)_{kl}, \quad (\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma P_L)_{ij} \times (\gamma_\mu \gamma_\nu \gamma_\rho \gamma_\sigma P_L)_{kl},$$
$$(\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \gamma^\tau P_L)_{ij} \times (\gamma_\mu \gamma_\nu \gamma_\rho \gamma_\sigma \gamma_\tau P_L)_{kl}, \ldots$$

- 4-dimensions: Products of γ matrices reducible via the Chisholm identity

$$\gamma^\mu \gamma^\nu \gamma^\rho = g^{\mu\nu} \gamma^\rho + g^{\nu\rho} \gamma^\mu - g^{\mu\rho} \gamma^\nu + i \varepsilon^{\mu\nu\rho\sigma} \gamma_\sigma \gamma^5$$

- In d-dimensions such a reduction is not possible (unambiguously).
- Proper treatment using evanescent operators [Dugan & Grinstein, 1991; Herrlich & Nierste, 1995]

$|\Delta B| = 1$ effective Hamiltonian in the CMM basis for $b \to sc\bar{c}$ decays [Chetyrkin et al., 1998]

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$$+ V_{us}^* V_{cb} \sum_{i=1}^{2} C_i Q_i^{cu} + V_{cs}^* V_{ub} \sum_{i=1}^{2} C_i Q_i^{uc} \Big] + \text{h.c.}\,,$$

- $|\Delta B| = 1$ LO evanescent operators

$$E_1^{(1)} = \bar{s}_L \gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} T^a c_L \; \bar{c}_L \gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} T^a b_L - 16 Q_1\,,$$

$$E_2^{(1)} = \bar{s}_L \gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} c_L \; \bar{c}_L \gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} b_L - 16 Q_2\,,$$

$$E_3^{(1)} = \bar{s}_L \gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} \gamma^{\mu_4} \gamma^{\mu_5} b_L \textstyle\sum_q \bar{q} \gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} \gamma_{\mu_4} \gamma_{\mu_5} q \; - \; 20 Q_5 \; + \; 64 Q_3\,,$$

$$E_4^{(1)} = \bar{s}_L \gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} \gamma^{\mu_4} \gamma^{\mu_5} T^a b_L \textstyle\sum_q \bar{q} \gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} \gamma_{\mu_4} \gamma_{\mu_5} T^a q \; - \; 20 Q_6 \; + \; 64 Q_4$$

$|\Delta B| = 1$ effective Hamiltonian in the CMM basis for $b \to s c\bar{c}$ decays [Chetyrkin et al., 1998]

$$\mathcal{H}_{\text{eff}}^{|\Delta B|=1} = \frac{4G_F}{\sqrt{2}} \Big[-V_{ts}^* V_{tb}^\dagger \Big(\sum_{i=1}^{6} C_i Q_i + C_8 Q_8 \Big) - V_{us}^* V_{ub}^\dagger \sum_{i=1}^{2} C_i (Q_i - Q_i^u)$$

$$+ V_{us}^* V_{cb} \sum_{i=1}^{2} C_i Q_i^{cu} + V_{cs}^* V_{ub} \sum_{i=1}^{2} C_i Q_i^{uc} \Big] + \text{h.c.} \, ,$$

- $|\Delta B| = 1$ NLO evanescent operators

$$E_1^{(2)} = \bar{s}_L \gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3}\gamma^{\mu_4}\gamma^{\mu_5} T^a c_L \; \bar{c} \gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3}\gamma_{\mu_4}\gamma_{\mu_5} T^a b_L - 20 E_1^{(1)} - 256 Q_1 \, ,$$

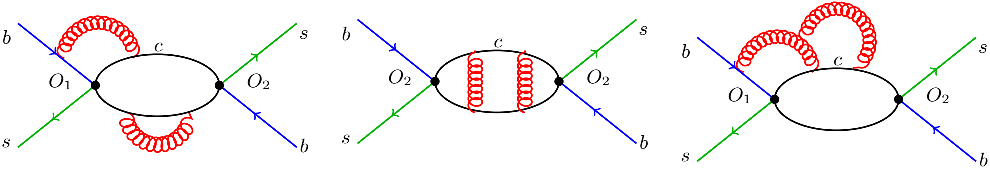
$$E_2^{(2)} = \bar{s}_L \gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3}\gamma^{\mu_4}\gamma^{\mu_5} c_L \; \bar{c}_L \gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3}\gamma_{\mu_4}\gamma_{\mu_5} b_L - 20 E_2^{(1)} - 256 Q_2 \, ,$$

$$E_3^{(2)} = \bar{s}_L \gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3}\gamma^{\mu_4}\gamma^{\mu_5}\gamma^{\mu_6}\gamma^{\mu_7} b_L \sum_q \bar{q} \gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3}\gamma_{\mu_4}\gamma_{\mu_5}\gamma_{\mu_6}\gamma_{\mu_7} q \; - \; 336 Q_5 \; + \; 1280 Q_3 \, ,$$

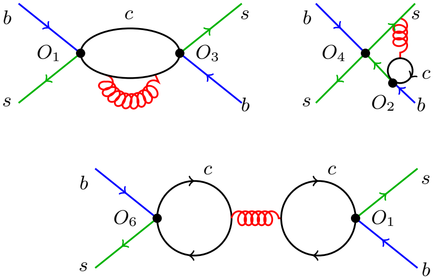
$$E_4^{(2)} = \bar{s}_L \gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3}\gamma^{\mu_4}\gamma^{\mu_5} T^a b_L \sum_q \bar{q} \gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3}\gamma_{\mu_4}\gamma_{\mu_5} T^a q \; - \; 336 Q_6 \; + \; 1280 Q_4$$

Representative diagrams in the $|\Delta B| = 1$ theory (1-loop, 2-loop, 3-loop)

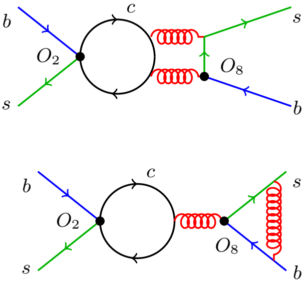
3-loop $O_{1,2} \times O_{1,2}$ correlators



2-loop $O_{1,2} \times O_{3-6}$ correlators



2-loop $O_{1,2} \times O_8$ correlators



- $\Delta\Gamma_s$ described by local $|\Delta B| = 2$ operators [Beneke et al., 1999; Lenz & Nierste, 2007; Asatrian et al., 2017]
- Using Heavy Quark Expansion [Khoze & Shifman, 1983; Shifman & Voloshin, 1985; Khoze et al., 1987; Chay et al., 1990; Bigi & Uraltsev, 1992; Bigi et al., 1992, 1993; Blok et al., 1994; Manohar & Wise, 1994] (expansion in $\Lambda_{\rm QCD}/m_b$) one arrives at

$$\Gamma_{12} = -(\lambda_c^q)^2 \Gamma_{12}^{cc} - 2\lambda_c^q \lambda_u^q \Gamma_{12}^{uc} - (\lambda_u^q)^2 \Gamma_{12}^{uu}, \quad \lambda_{q'}^q \equiv V_{q'q}^* V_{q'b}$$

$$\Gamma_{12}^{ab} = \frac{G_F^2 m_b^2}{24\pi M_{B_s}} \left[H^{ab}(z) \langle B_s | Q | \bar{B}_s \rangle + \widetilde{H}_S^{ab}(z) \langle B_s | \widetilde{Q}_S | \bar{B}_s \rangle \right] + \mathcal{O}\left(\Lambda_{\rm QCD}/m_b\right)$$

- Physical $|\Delta B| = 2$ operators

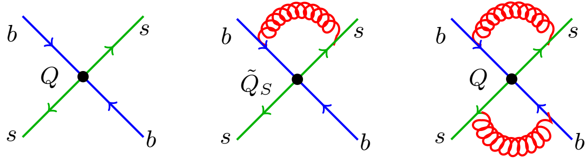
$$Q = \bar{s}_i \gamma^\mu \left(1 - \gamma^5\right) b_i \; \bar{s}_j \gamma_\mu \left(1 - \gamma^5\right) b_j \quad \widetilde{Q}_S = \bar{s}_i \left(1 - \gamma^5\right) b_j \; \bar{s}_j \left(1 - \gamma^5\right) b_i$$

- Additional operators needed at intermediate stages (e. g. basis changes, def. of evanescent operators)

$$\widetilde{Q} = \bar{s}_i \gamma^\mu \left(1 - \gamma^5\right) b_j \; \bar{s}_j \gamma_\mu \left(1 - \gamma^5\right) b_i \,, \quad Q_S = \bar{s}_i \left(1 - \gamma^5\right) b_i \; \bar{s}_j \left(1 - \gamma^5\right) b_j \,,$$

- Not shown here: evanescent $|\Delta B| = 2$ operators and the $1/m_b$ suppressed operator R_0
- $H(z)$ and $\widetilde{H}_S(z)$: Wilson coefficients from the perturbative matching of $|\Delta B| = 1$ to $|\Delta B| = 2$.
- Nonperturbative ME $\langle B_s | Q | \bar{B}_s \rangle$ and $\langle B_s | \widetilde{Q}_S | \bar{B}_s \rangle$ (also for B_d mesons) from QCD/HQET sum rules [Ovchinnikov & Pivovarov, 1988; Reinders & Yazaki, 1988; Korner et al., 2003; Mannel et al., 2011; Grozin et al., 2016; Kirk et al., 2017; King et al., 2019], lattice QCD [Bazavov et al., 2016; Dowdall et al., 2019] or combined [Di Luzio et al., 2019]

Representative diagrams in the $|\Delta B| = 2$ theory (tree-level, 1-loop, 2-loop)



Wilson coefficients of the $|\Delta B| = 2$ theory determined in the matching to $|\Delta B| = 1$

$|\Delta B| = 1$ contributions needed for NNLO

$$C_i O_i \sim \begin{cases} 1 & \text{for } i = 1, 2 \\ \alpha_s & \text{for } i = 3,4,5,6 \quad (C_{3-6} \text{ numerically small}) \\ \alpha_s & \text{for } i = 8 \quad (\text{explicit strong coupling in the definition of } O_8 \end{cases}$$

Important scale: $z \equiv m_c^2/m_b^2$

- LO contributions to $\Delta\Gamma_s$
  - 1-loop $O_{1-2} \times O_{1-2}$ correlators (z-exact) [Hagelin, 1981; Franco et al., 1982; Chau, 1983; Buras et al., 1984; Khoze et al., 1987; Datta et al., 1987, 1988]
- NLO contributions to $\Delta\Gamma_s$ (z-exact)
  - 2-loop $O_{1-2} \times O_{1-2}$ correlators (z-exact) [Beneke et al., 1999]
  - 1-loop $O_{1-2} \times O_{3-6}$ correlators (z-exact) [Beneke et al., 1999]
  - 1-loop $O_{1-2} \times O_8$ correlators (z-exact) [Beneke et al., 1999]

$|\Delta B| = 1$ contributions needed for NNLO

$$C_i O_i \sim \begin{cases} 1 & \text{for } i = 1,2 \\ \alpha_s & \text{for } i = 3,4,5,6 \quad (C_{3-6} \text{ numerically small}) \\ \alpha_s & \text{for } i = 8 \quad (\text{explicit strong coupling in the definition of } O_8 \end{cases}$$

Important scale: $z \equiv m_c^2/m_b^2$

- NNLO contributions to $\Delta\Gamma_s$
  - 3-loop $O_{1-2} \times O_{1-2}$ correlators [Asatrian et al., 2017, 2020] (n_f piece only, $\mathcal{O}(z^3)$)
  - 2-loop $O_{1-2} \times O_{3-6}$ correlators [Asatrian et al., 2017, 2020] (n_f piece only, z-exact)
  - 2-loop $O_{1-2} \times O_8$ correlators [Asatrian et al., 2017, 2020] (n_f piece only, z-exact)
  - 1-loop $O_{3-6} \times O_{3-6}$ correlators (z-exact) [Beneke et al., 1996]
  - 1-loop $O_{3-6} \times O_8$ correlators [Asatrian et al., 2017, 2020] (n_f piece only, z-exact)
  - 1-loop $O_8 \times O_8$ correlators [Asatrian et al., 2017, 2020] (n_f piece only, z-exact)
- This work
  - Full (n_f + non-n_f) results for all 2-loop correlators at $\mathcal{O}(z)$ (including $O_8 \times O_8 \Rightarrow$ N^{3}LO)
  - Full (n_f + non-n_f) results for the 3-loop $O_{1-2} \times O_{1-2}$ at $\mathcal{O}(z^0)$
  - WIP: Final checks for the 3-loop result, higher order expansions in z, possibly z-exact results for selected correlators

- Calculational strategy
  - Matching done on-shell: $p_b^2 = m_b^2$
  - The s-quark mass is neglected $\Rightarrow p_s = 0$
  - Asymptotic expansion in $z \equiv m_c^2/m_b^2$ (at first up to $\mathcal{O}(z)$ for 2-loop and $\mathcal{O}(z^0)$ for 3-loop)
  - Only the imaginary part of the $|\Delta B| = 1$ diagrams enters the matching
- Regularization
  - Dimensional regularization used both for UV- and IR-divergences
  - Cross-check: massive gluons in IR-divergent diagrams at 2-loops
  - $\varepsilon_{\rm UV} + m_g$: renormalized amplitudes manifestly finite $\Rightarrow$ the limit $d \to 4$ is safe
  - $\varepsilon = \varepsilon_{\rm UV} = \varepsilon_{\rm IR}$: products of $1/\varepsilon_{\rm IR}$ and evanescent ME are of $\mathcal{O}(\varepsilon^0)$

NLO matching with $\varepsilon = \varepsilon_{\mathrm{IR}} = \varepsilon_{\mathrm{UV}}$ (no gluon mass) [Ciuchini et al., 2002]

- Normally, only the matching coefficients of physical $|\Delta B| = 2$ operators are relevant
- Here matching coefficients of evanescent operators are also needed (at intermediate stages)
- $|\Delta B| = 2$ matching coefficients obtain $\mathcal{O}(\varepsilon)$ pieces

$$C = f_0^{(0)} + \varepsilon\, f_1^{(0)} + \frac{\alpha_s}{4\pi} f_0^{(1)}, \qquad C_E = f_{E,0}^{(0)} + \varepsilon\, f_{E,1}^{(0)} + \frac{\alpha_s}{4\pi} f_{E,0}^{(1)}$$

- LO matching must be carried out up to $\mathcal{O}(\varepsilon)$: fixes $f_0^{(0)}, f_1^{(0)}, f_{E,0}^{(0)}, f_{E,1}^{(0)}$
- At NLO we only need $\mathcal{O}(\varepsilon^0)$
- Upon inserting $f_0^{(0)}, f_1^{(0)}, f_{E,0}^{(0)}, f_{E,1}^{(0)}$ at NLO all $1/\varepsilon_{\mathrm{IR}}$ poles must cancel.
- Finally, the difference

$$A_{\mathrm{ren}}^{|\Delta B|=1} - A_{\mathrm{ren}}^{|\Delta B|=2}$$

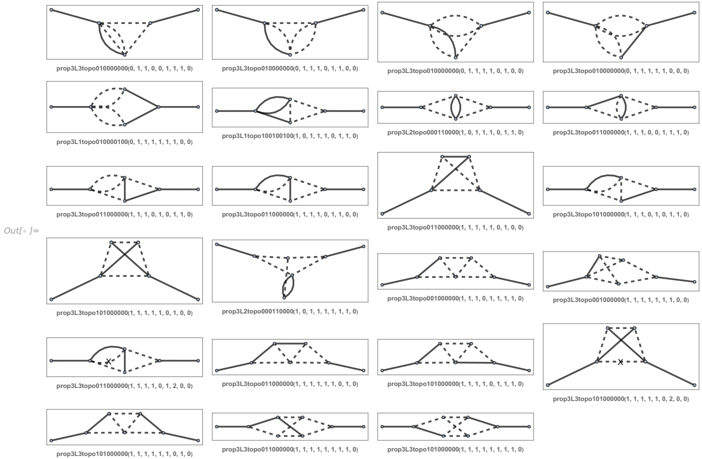
  is manifestly finite $\Rightarrow$ determine $f_0^{(1)}$
- Only $f_0^{(0)}$ and $f_0^{(1)}$ enter the physical matching coefficient
- What about $f_{E,1}^{(0)}$? Not needed at NLO, must be determined for the NNLO calculation!
- At NNLO, the LO matching must be done up $\mathcal{O}(\varepsilon^2)$, the NLO matching up to $\mathcal{O}(\varepsilon)$
- The explicit cancellation of IR poles (and of ξ) is a highly nontrivial cross-check of the whole calculation

- All computations done using our well-tested automatic setup
  - Diagram generation with QGRAF [Nogueira, 1993]
  - Insertion of Feynman rules and topology identification using Q2E/EXP [Seidensticker, 1999; Harlander et al., 1998] or TAPIR [Gerlach, Herren, 2021]
  - Feynman amplitude evaluation: in-house CALC setup written in FORM [Ruijl et al., 2017]
  - IBP-reduction: FIRE 6 [Smirnov & Chuharev, 2020]
  - Analytic computation of master integrals: HyperInt [Panzer, 2015], HyperlogProceedings [Schnetz], PolyLogTools [Duhr & Dulat, 2019]
  - All master integrals checked numerically using FIESTA [Smirnov, 2016] and pySecDec [Borowka et al., 2018]
- Cross-checks of selected intermediate results using FeynArts [Hahn, 2001], FeynRules [Christensen & Duhr, 2009; Alloul et al., 2014] and FeynCalc [VS et al., 2020]

- 23 on-shell 3-loop integrals with massive (solid) lines
- Only imaginary parts are relevant and turn out to be very simple
- Appearing constants

$$\pi, \ln(2), \zeta_2, \zeta_3, \zeta_4, \mathrm{Cl}_2(\pi/3), \sqrt{3},$$
$$\mathrm{Li}_4(1/2), \ln\left((1+\sqrt{5})/2\right)$$

- Real parts (obtained as a byproduct) more complicated but irrelevant for $\Delta\Gamma_s$



- New contributions to Γ_{12}^s computed in the course of this project ($z = m_c^2/m_b^2$)

Correlator	Perturbative order	z-dependence
$O_{1,2} \times O_{3-6}$	2 loops	$\mathcal{O}(z)$
$O_{1,2} \times O_8$	2 loops	$\mathcal{O}(z)$
$O_{3-6} \times O_{3-6}$	2 loops	$\mathcal{O}(z)$
$O_{3-6} \times O_8$	2 loops	$\mathcal{O}(z)$
$O_8 \times O_8$	1 loop	exact
$O_8 \times O_8$	2 loops	$\mathcal{O}(z)$
$O_{1,2} \times O_{1,2}$	3 loops	$\mathcal{O}(z^0)$

- All 2-loop contributions to the NNLO correction already computed and cross-checked
- New theory predictions for the width difference $\Delta\Gamma_s$ and the CP asymmetry a_{fs}^s under way

$$\frac{\Delta\Gamma_s}{\Delta M_s} = -\text{Re}\left(\frac{\Gamma_{12}^s}{M_{12}^s}\right), \quad a_{\text{fs}}^s = \text{Im}\left(\frac{\Gamma_{12}^s}{M_{12}^s}\right)$$

- Ingredients

$$\Gamma_{12}^s = -(\lambda_t^s)^2 \left[\Gamma_{12}^{s,cc} + 2\frac{\lambda_u^s}{\lambda_t^s}\left(\Gamma_{12}^{s,cc} - \Gamma_{12}^{s,uc}\right) + \left(\frac{\lambda_u^s}{\lambda_t^s}\right)^2 \left(\Gamma_{12}^{s,uu} + \Gamma_{12}^{s,cc} - 2\Gamma_{12}^{s,uc}\right)\right]$$

$$\Gamma_{s,12}^{ab} = \frac{G_F^2 m_b^2}{24\pi M_{B_s}}\left[H^{ab}(z)\underbrace{\langle B_s|Q|\bar{B}_s\rangle}_{\frac{8}{3}M_{B_s}^2 f_{B_s}^2 B_{B_s}} + \widetilde{H}_S^{ab}(z)\underbrace{\langle B_s|\widetilde{Q}_S|\bar{B}_s\rangle}_{\frac{1}{3}M_{B_s}^2 f_{B_s}^2 \widetilde{B}'_{S,B_s}}\right] + \mathcal{O}\left(\Lambda_{\text{QCD}}/m_b\right)$$

$$M_{12} = (\lambda_t^s)^2 \frac{G_F^2 M_{B_s}}{12\pi^2} M_W^2 \hat{\eta}_B S_0\left(\frac{m_t^2}{M_W^2}\right) f_{B_s}^2 B_{B_s}$$

- Cancellation of $(\lambda_t^s)^2 = (V_{ts}^* V_{tb}^\dagger)^2$, f_{B_s}, M_{B_s} and to large extent bag parameters in the ratio Γ_{12}^s/M_{12}^s
- Following [Asatrian et al., 2020] we can calculate

$$\Delta\Gamma_s = \left(\frac{\Delta\Gamma_s}{\Delta M_s}\right)\Delta M_s^{\text{exp}}$$

- $|V_{cb}|$ controversy irrelevant!

- Theoretical predictions for the $\overline{\text{MS}}$ and pole schemes
- m_b^2 in the prefactor of Γ_{12} treated as $\left(m_b^{\text{OS}}\right)^2$ in the pole scheme and $\left(m_b^{\text{MS}}(m_b)\right)^2$ in the $\overline{\text{MS}}$ scheme
- In both schemes we use $\bar{z} = (m_c^{\text{MS}}(m_b)/m_b^{\text{MS}}(m_b))^2$
- NLO result for M_{12} from [Buras et al., 1990]
- $1/m_b$ LO corrections to Γ_{12} [Beneke et al., 1996; Lenz & Nierste, 2007] are included
- Experimental value (HFLAV 2020 average): $\Delta\Gamma_s^{\text{exp}} = (0.085 \pm 0.004)\ \text{ps}^{-1}$

- Numerical input [Tanabashi et al., 2018; Dowdall et al., 2019; Bazavov et al., 2018; Amhis et al., 2021]

$$M_{B_s} = 5366.88\ \text{MeV} \quad f_{B_s} = (0.2307 \pm 0.0013)\ \text{GeV},$$
$$B_{B_s} = 0.813 \pm 0.034, \quad \widetilde{B}'_{S,B_s} = 1.31 \pm 0.09,$$
$$\frac{\lambda_u^s}{\lambda_t^s} = -(0.00865 \pm 0.00042) + (0.01832 \pm 0.00039)i$$
$$\Delta M_s^{\text{exp}} = (17.749 \pm 0.020)\ \text{ps}^{-1}$$

- Theoretical predictions for the $\overline{\text{MS}}$ and pole schemes
- m_b^2 in the prefactor of Γ_{12} treated as $\left(m_b^{\text{OS}}\right)^2$ in the pole scheme and $\left(m_b^{\text{MS}}(m_b)\right)^2$ in the $\overline{\text{MS}}$ scheme
- In both schemes we use $\bar{z} = (m_c^{\text{MS}}(m_b)/m_b^{\text{MS}}(m_b))^2$
- NLO result for M_{12} from [Buras et al., 1990]
- $1/m_b$ LO corrections to Γ_{12} [Beneke et al., 1996; Lenz & Nierste, 2007] are included
- Experimental value (HFLAV 2020 average): $\Delta\Gamma_s^{\text{exp}} = (0.085 \pm 0.004)$ ps^{-1}

- **Preliminary** results (no scale variation, 3-loop corrections not included)

Included correlators	Re (Γ_{12}^s/M_{12}^s)	$\Delta\Gamma_s$
$O_{1,2} \times O_{1,2}$ (2 loops), $O_{1,2} \times O_{3-6}$ (1 loop), $O_{1,2} \times O_8$ (2 loops), $O_{3-6} \times O_{3-6}$ (1 loop)	$(5.31 \pm 0.67) \times 10^{-3}$ ($\overline{\text{MS}}$) $(4.73 \pm 0.08) \times 10^{-3}$ (pole)	$(0.094 \pm 0.012)\text{ps}^{-1}$ ($\overline{\text{MS}}$) $(0.084 \pm 0.014)\text{ps}^{-1}$ (pole)
as above + $O_{1,2} \times O_{3-6}$ (2 loops)	$(5.21 \pm 0.67) \times 10^{-3}$ ($\overline{\text{MS}}$) $(4.71 \pm 0.08) \times 10^{-3}$ (pole)	$(0.093 \pm 0.012)\text{ps}^{-1}$ ($\overline{\text{MS}}$) $(0.084 \pm 0.014)\text{ps}^{-1}$ (pole)
as above + $O_{3-6} \times O_{3-6}$ (2 loops)	$(5.24 \pm 0.67) \times 10^{-3}$ ($\overline{\text{MS}}$) $(4.74 \pm 0.08) \times 10^{-3}$ (pole)	$(0.094 \pm 0.012)\text{ps}^{-1}$ ($\overline{\text{MS}}$) $(0.084 \pm 0.014)\text{ps}^{-1}$ (pole)
as above + $O_{3-6} \times O_8$ (2 loops) and $O_8 \times O_8$ (1 loop and 2 loops)	$(5.24 \pm 0.67) \times 10^{-3}$ ($\overline{\text{MS}}$) $(4.73 \pm 0.08) \times 10^{-3}$ (pole)	$(0.094 \pm 0.012)\text{ps}^{-1}$ ($\overline{\text{MS}}$) $(0.084 \pm 0.014)\text{ps}^{-1}$ (pole)

- The error is dominated by the uncertainty in the hadronic ME entering $1/m_b$ LO corrections
- The main contribution comes from the current-penguin $O_{12} \times O_{3-6}$ 2-loop corrections

- Theoretical predictions for the $\overline{\text{MS}}$ and pole schemes
- m_b^2 in the prefactor of Γ_{12} treated as $\left(m_b^{\text{OS}}\right)^2$ in the pole scheme and $\left(m_b^{\text{MS}}(m_b)\right)^2$ in the $\overline{\text{MS}}$ scheme
- In both schemes we use $\bar{z} = (m_c^{\text{MS}}(m_b)/m_b^{\text{MS}}(m_b))^2$
- NLO result for M_{12} from [Buras et al., 1990]
- $1/m_b$ LO corrections to Γ_{12} [Beneke et al., 1996; Lenz & Nierste, 2007] are included
- Experimental value (HFLAV 2020 average): $\Delta\Gamma_s^{\text{exp}} = (0.085 \pm 0.004)$ ps^{-1}

- **Preliminary** results (no scale variation, 3-loop corrections not included)

$$a_{\text{fs}}^{s,2-\text{loop}} = (2.02 \pm 0.08) \times 10^{-5} (\overline{\text{MS}}),$$
$$a_{\text{fs}}^{s,2-\text{loop}} = (2.05 \pm 0.08) \times 10^{-5} (\text{pole})$$

Summary

- Experimental precision of $\Delta\Gamma_s$ calls for the NNLO calculation!
- We calculated all building blocks needed to obtain the NNLO correction to $B_s^0 - \bar{B}_s^0$ mixing
- All the occurring 3-loop MI from the current-current contribution calculated analytically (for $m_c = 0$)
- The result for the 2-loop current-penguin contribution already published [Gerlach, Nierste, VS, Steinhauser, 2021]

Outlook

- Results for all the remaining 2-loop contributions and the 3-loop current-current piece to appear soon
- New theory predictions for $\Delta\Gamma_s$ and the CP asymmetry a_{fs}^s
- Higher order expansions in $z \equiv m_c^2/m_b^2$, ideally z-exact results at least for the 2-loop contributions

Anomalies in the flavor physics (incomplete list)

- Hints for Lepton Flavor Universality (LFU) violation in semileptonic decays
  - Loop-level (FCNC): $b \to s\ell\ell$ (e. g. $B_d \to K^* \mu^+ \mu^-$) [Aaij et al., 2017, 2019; Abdesselam et al., 2021; Choudhury et al., 2021; Aaij et al., 2021]
  - Tree-level: $b \to c\tau\nu$ (e. g. $\bar{B}^0 \to D^{*+} \tau^- \bar{\nu}_\tau$) [Lees et al., 2019; Hirose et al., 2017; Aaij et al., 2015, 2018]
- Dimuon charge asymmetry (related to CP-asymmetry in B-mixing) [Abazov et al., 2006, 2010a, 2010b, 2011, 2014]
- $g-2$ of the muon [Abi et al., 2021]
- CP-violation in the neutral kaon system (ε/ε') [Alavi-Harati et al., 1999; Fanti et al., 1999]
- a_{fb} in $Zb\bar{b}$ decays [Abbaneo et al., 1996]
- LFU violation in leptonic τ decays [Aubert et al., 2010; Anastassov et al., 1997; Albrecht et al., 1992]
- $B \to \pi K$ puzzle [Buras et al., 2003, 2004b, 2004a]
- Inclusive vs exclusive determinations of V_{cb} [Waheed et al., 2019; Lees et al., 2019]