INCLUSIVE HADROPRODUCTION OF P-WAVE HEAVY QUARKONIA IN POTENTIAL NRQCD

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A Virtual Tribute to Quark Confinement and the Hadron Spectrum 2021

NRQCD FACTORIZATION

- NRQCD provides a factorization formalism for inclusive production cross sections.

NRQCD matrix elements

$$\sigma = \sum_n \sigma_{Q\bar{Q}(n)} \langle \Omega | \mathcal{O}_n | \Omega \rangle$$

Perturbatively calculable
***$Q\bar{Q}$* cross sections**

Bodwin, Braaten, Lepage, PRD51, 1125 (1995)

- In general it is not known how to compute matrix elements from first principles, so they are usually determined from cross section measurements. So far this approach has not lead to a comprehensive description of measurements.

- We aim to compute the matrix elements in potential NRQCD, which is obtained by integrating out scales above mv^2.

QUARKONIUM IN PNRQCD

- We work in the strong coupling regime where $mv^2 \ll \Lambda_{\rm QCD}$, which is valid for non-Coulombic quarkonia, such as P-wave quarkonia. The degree of freedom is the singlet field $S(x_1,x_2)$, which describe $Q\bar{Q}$ in a color-singlet state.

$$\mathcal{L}_{\rm pNRQCD} = {\rm Tr}\{S^\dagger(i\partial_0 - h)S\}$$

Pineda, Soto, NPB Proc. Suppl. 64, 428 (1998)
Brambilla, Pineda, Soto, Vairo, NPB566, 275 (2000)
Brambilla, Pineda, Soto, Vairo, Rev. Mod. Phys. 77, 1423 (2005)

- Matching to NRQCD is done nonperturbatively.
- pNRQCD provides expressions for decay matrix elements in terms of wavefunctions and universal gluonic correlators.
- We extend the formalism for production matrix elements.

MATCHING IN PNRQCD

- Matching to NRQCD in strongly coupled pNRQCD is done as an expansion in powers of $1/m$:

Brambilla, Eiras, Pineda, Soto, Vairo, PRL88, 012003 (2002)
Brambilla, Eiras, Pineda, Soto, Vairo, PRD67, 034018 (2003)
Brambilla, HSC, Müller, Vairo, JHEP04 (2020) 095

NRQCD Hamiltonian $$H_{\rm NRQCD} = H^{(0)}_{\rm NRQCD} + H^{(1)}_{\rm NRQCD}/m + \ldots$$

Eigenstates $$|\underline{\rm n}; \boldsymbol{x}_1, \boldsymbol{x}_2\rangle = |\underline{\rm n}; \boldsymbol{x}_1, \boldsymbol{x}_2\rangle^{(0)} + \frac{1}{m}|\underline{\rm n}; \boldsymbol{x}_1, \boldsymbol{x}_2\rangle^{(1)} + \ldots$$

- $|\underline{0}; \boldsymbol{x}_1, \boldsymbol{x}_2\rangle$ is the ground state, x_1, x_2 are positions of Q, $\bar{Q}$.

- A quarkonium state in vacuum is described by $$\int d^3x_1 d^3x_2 \phi(\boldsymbol{x}_1, \boldsymbol{x}_2)|\underline{0}; \boldsymbol{x}_1, \boldsymbol{x}_2\rangle$$

$\phi(\boldsymbol{x}_1, \boldsymbol{x}_2)$ is the quarkonium wavefunction.

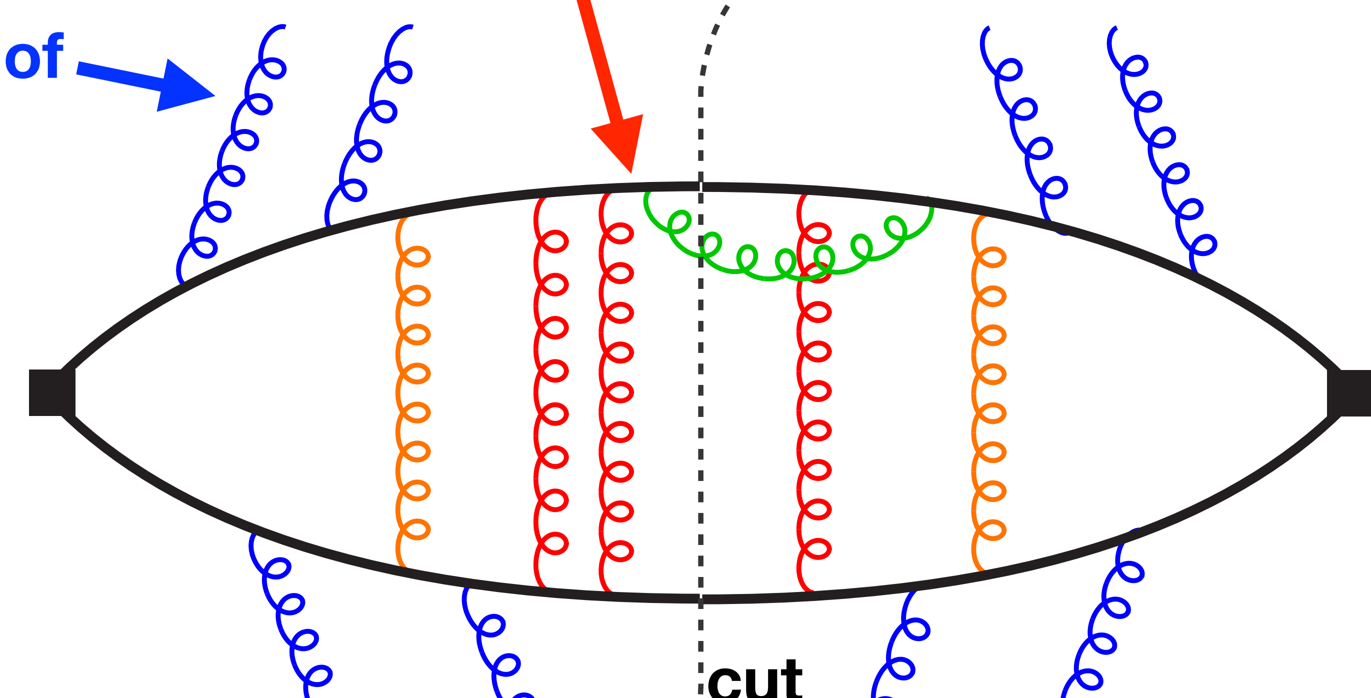
PRODUCTION MATRIX ELEMENTS

- Production matrix elements for production of quarkonium $\mathcal{Q}$

$$\langle \Omega | \chi^\dagger \mathcal{K} \psi \mathcal{P}_{\mathcal{Q}} \psi^\dagger \mathcal{K}' \chi | \Omega \rangle$$

color/spin matrices and covariant derivatives, gauge-completion Wilson lines (color octet)

- $\mathcal{P}_{\mathcal{Q}} = \sum_X |\mathcal{Q} + X\rangle\langle \mathcal{Q} + X| = a^\dagger_{\mathcal{Q}} a_{\mathcal{Q}}$
  : projection onto quarkonium + anything
- A schematic view of production matrix element



PRODUCTION MATRIX ELEMENTS

- We want to compute

$$\langle \Omega | \chi^\dagger \mathcal{K} \psi \mathcal{P}_{\mathcal{Q}} \psi^\dagger \mathcal{K}' \chi | \Omega \rangle$$

color/spin matrices and covariant derivatives, gauge-completion Wilson lines (color octet)

- To compute production matrix elements we need to

1. Express $\mathcal{P}_{\mathcal{Q}} = a^\dagger_{\mathcal{Q}} a_{\mathcal{Q}}$ in terms of $|\underline{\mathrm{n}}; \boldsymbol{x}_1, \boldsymbol{x}_2\rangle$ states.
2. Describe a quarkonium in background of light particles in terms of wavefunctions. While quarkonium is always color singlet, background can be color octet.

- We need to do this nonperturbatively, but we still expand in powers of $1/m$.

QUARKONIUM PROJECTION OPERATOR

- $\mathcal{P}_\mathcal{Q} = a^\dagger_\mathcal{Q} a_\mathcal{Q}$ is essentially a number operator : $\mathcal{P}_\mathcal{Q}$ and $H_{\rm NRQCD}$ are simultaneously diagonalizable. Simultaneous eigenstates are given by

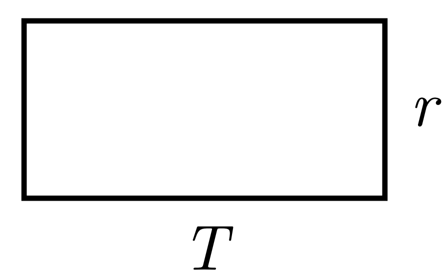
$$|\mathcal{Q}(n)\rangle = \int d^3x_1 d^3x_2\, \phi_{\mathcal{Q}(n)}(\boldsymbol{x}_1, \boldsymbol{x}_2)|\underline{n}; \boldsymbol{x}_1, \boldsymbol{x}_2\rangle$$

- We obtain the expression $\mathcal{P}_\mathcal{Q} = \sum_n |\mathcal{Q}(n)\rangle\langle\mathcal{Q}(n)|$
- For $n{=}0$, $|\mathcal{Q}(0)\rangle$ is just the quarkonium in vacuum and ϕ is the usual quarkonium wavefunction.
- For $n{>}0$, $|\mathcal{Q}(n)\rangle$ describe quarkonium + light particles. The "wavefunctions" ϕ are in general unknown for $n{>}0$.

QUARKONIUM WAVEFUNCTIONS

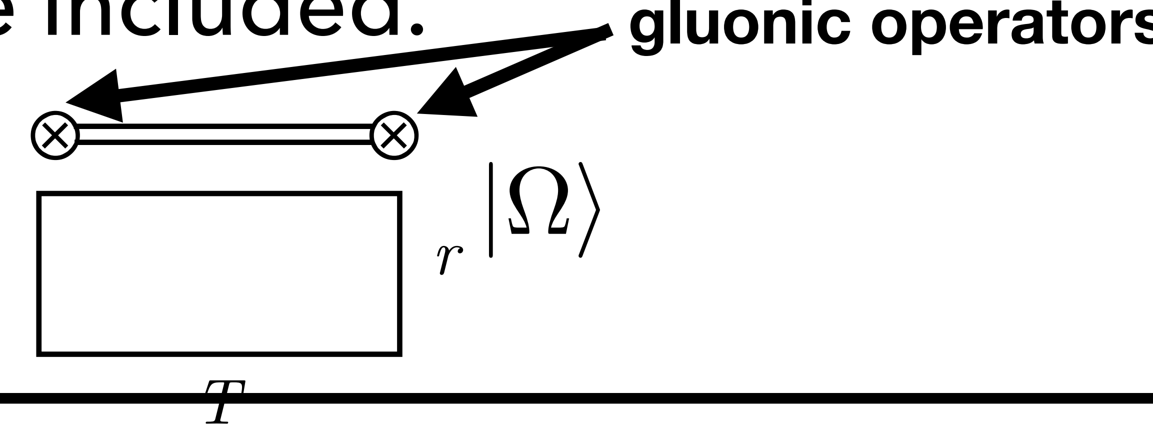
- We need to identify "wavefunctions" of quarkonium in the background of gluons and light particles.

- Potential for quarkonium in vacuum is given in terms of the vacuum expectation value (VEV) of a Wilson loop:

$$V(r) = \lim_{T\to\infty} \frac{i}{T} \log \langle \Omega |$$


$$| \Omega \rangle$$

- For the potential for the $n>0$ states, the light excitations in the $|\underline{\mathrm{n}}; \boldsymbol{x}_1, \boldsymbol{x}_2\rangle$ states should be included.

$$V(r) = \lim_{T\to\infty} \frac{i}{T} \log \langle \Omega |$$


$$| \Omega \rangle$$

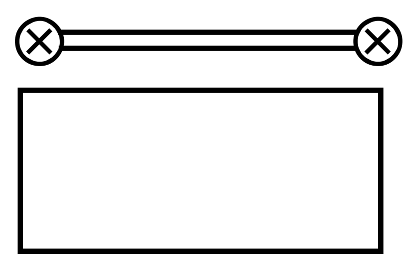
QUARKONIUM WAVEFUNCTIONS

- In general, VEVs of products of color-singlet operators factorize into products of VEVs of individual operators.

$$\langle\Omega|AB|\Omega\rangle = \langle\Omega|A|\Omega\rangle\langle\Omega|B|\Omega\rangle[1+O(1/N_c^2)]$$

Makeenko, Migdal, PLB88, 135 (1979)
Witten, NATO Sci. Ser. B 59, 403 (1980)

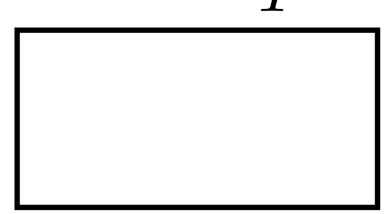
- So the $n>0$ potentials reduce to

$V(r) = \lim_{T\to\infty} \frac{i}{T} \log\langle\Omega|$  $_r\,|\Omega\rangle$

T

$= \lim_{T\to\infty} \frac{i}{T} \log\Big(\langle\Omega|$  $_r|\Omega\rangle \times \langle\Omega|$  $|\Omega\rangle\Big)$

T

$= \lim_{T\to\infty} \frac{i}{T} \log\langle\Omega|$  $_r\,|\Omega\rangle + \text{constant}$

T

QUARKONIUM WAVEFUNCTIONS

- Hence, the $n>0$ potentials are just the $n=0$ potential, plus constants that have no effect to the wavefunctions.
- Therefore, the wavefunctions ϕ are independent of n, and the projection operator is just

$$\mathcal{P}_{\mathcal{Q}} = \sum_n |\mathcal{Q}(n)\rangle\langle\mathcal{Q}(n)|$$

$$|\mathcal{Q}(n)\rangle = \int d^3x_1 d^3x_2\, \phi_{\mathcal{Q}}(\boldsymbol{x}_1, \boldsymbol{x}_2)|\underline{n}; \boldsymbol{x}_1, \boldsymbol{x}_2\rangle$$

- These are valid up to corrections of relative order $1/N_c^2$.

PRODUCTION MATRIX ELEMENTS IN PNRQCD

- Now we can compute the production matrix elements

$$\langle\Omega|\chi^\dagger\mathcal{K}\psi\mathcal{P}_{\mathcal{Q}}\psi^\dagger\mathcal{K}'\chi|\Omega\rangle = \int d^3x_1 d^3x_2 \int d^3x'_1 d^3x'_2 \phi(\boldsymbol{x}_1,\boldsymbol{x}_2)\phi(\boldsymbol{x}'_1,\boldsymbol{x}'_2) \sum_n \langle\Omega|\chi^\dagger\mathcal{K}\psi|\underline{\mathrm{n}};\boldsymbol{x}_1,\boldsymbol{x}_2\rangle\langle\underline{\mathrm{n}};\boldsymbol{x}'_1,\boldsymbol{x}'_2|\psi^\dagger\mathcal{K}'\chi|\Omega\rangle$$

Contact term,
computed order by order in $1/m$

- Contact terms can be computed in terms of universal gluonic correlators and differential operators that act on wavefunctions.

- This allows calculation of production matrix elements in strongly coupled pNRQCD.

PRODUCTION OF P-WAVE QUARKONIA

- We apply this formalism for $\chi_{QJ}\,(Q{=}c$ or $b,\ J{=}1,\,2)$
- At leading order in v, the cross section is given by

$$\sigma_{\chi_{QJ}+X} = (2J+1)\sigma_{Q\bar{Q}(^3P_J^{[1]})}\langle\mathcal{O}^{\chi_{Q0}}(^3P_0^{[1]})\rangle + (2J+1)\sigma_{Q\bar{Q}(^3S_1^{[8]})}\langle\mathcal{O}^{\chi_{Q0}}(^3S_1^{[8]})\rangle$$

Bodwin, Braaten, Yuan, Lepage, PRD46, R3703 (1992)
Bodwin, Braaten, Lepage, PRD51, 1125 (1995)

color singlet : $$\mathcal{O}^{\chi_{Q0}}(^3P_0^{[1]}) = \frac{1}{3}\chi^\dagger\left(-\frac{i}{2}\overleftrightarrow{\boldsymbol{D}}\cdot\boldsymbol{\sigma}\right)\psi\mathcal{P}_{\chi_{Q0}}\psi^\dagger\left(-\frac{i}{2}\overleftrightarrow{\boldsymbol{D}}\cdot\boldsymbol{\sigma}\right)\chi$$

color octet : $$\mathcal{O}^{\chi_{Q0}}(^3S_1^{[8]}) = \chi^\dagger\sigma^i T^a\psi\Phi_\ell^{\dagger ab}\mathcal{P}_{\chi_{Q0}}\Phi_\ell^{bc}\psi^\dagger\sigma^i T^c\chi$$

Nayak, Qiu, Sterman, PLB613, 45 (2005)

- We compute both color singlet and color octet matrix elements in strongly coupled pNRQCD.

P-WAVE PRODUCTION MATRIX ELEMENTS

- Color-singlet matrix element: $\langle \mathcal{O}^{\chi_{Q0}}({}^3P_0^{[1]})\rangle = \frac{3N_c}{2\pi}|R^{(0)\prime}_{\chi_{Q0}}(0)|^2$ we reproduce the known result in vacuum-saturation approximation. $R(r)$: radial wavefunction

- Color-octet matrix element: result is given in terms of a universal gluonic correlator.
$$\langle \mathcal{O}^{\chi_{Q0}}({}^3S_1^{[8]})\rangle = \frac{3N_c}{2\pi}|R^{(0)\prime}_{\chi_{Q0}}(0)|^2 \, \frac{\mathcal{E}}{9N_c m^2}$$

- $\mathcal{E}$ is a universal quantity that does not depend on quark flavor or radial excitation. Determination of $\mathcal{E}$ directly leads to determination of all χ_{cJ} and $\chi_{bJ}(nP)$ cross sections, as well as h_c and h_b production rates.

P-WAVE PRODUCTION MATRIX ELEMENTS

- The correlator $\mathcal{E}$ is defined in terms of chromoelectric fields gE at time t and t', with Wilson lines extending to infinity in the ℓ direction.

$$\mathcal{E} = \frac{3}{N_c} \int_0^\infty t\, dt \int_0^\infty t'\, dt' \langle \Omega | \Phi_\ell^{\dagger ab} \Phi_0^{\dagger da}(0,t) gE^{d,i}(t) gE^{e,i}(t') \Phi_0^{ec}(t',0) \Phi_\ell^{bc} | \Omega \rangle.$$

- $\mathcal{E}$ has a one-loop scale dependence that is consistent with the evolution equation for NRQCD matrix elements

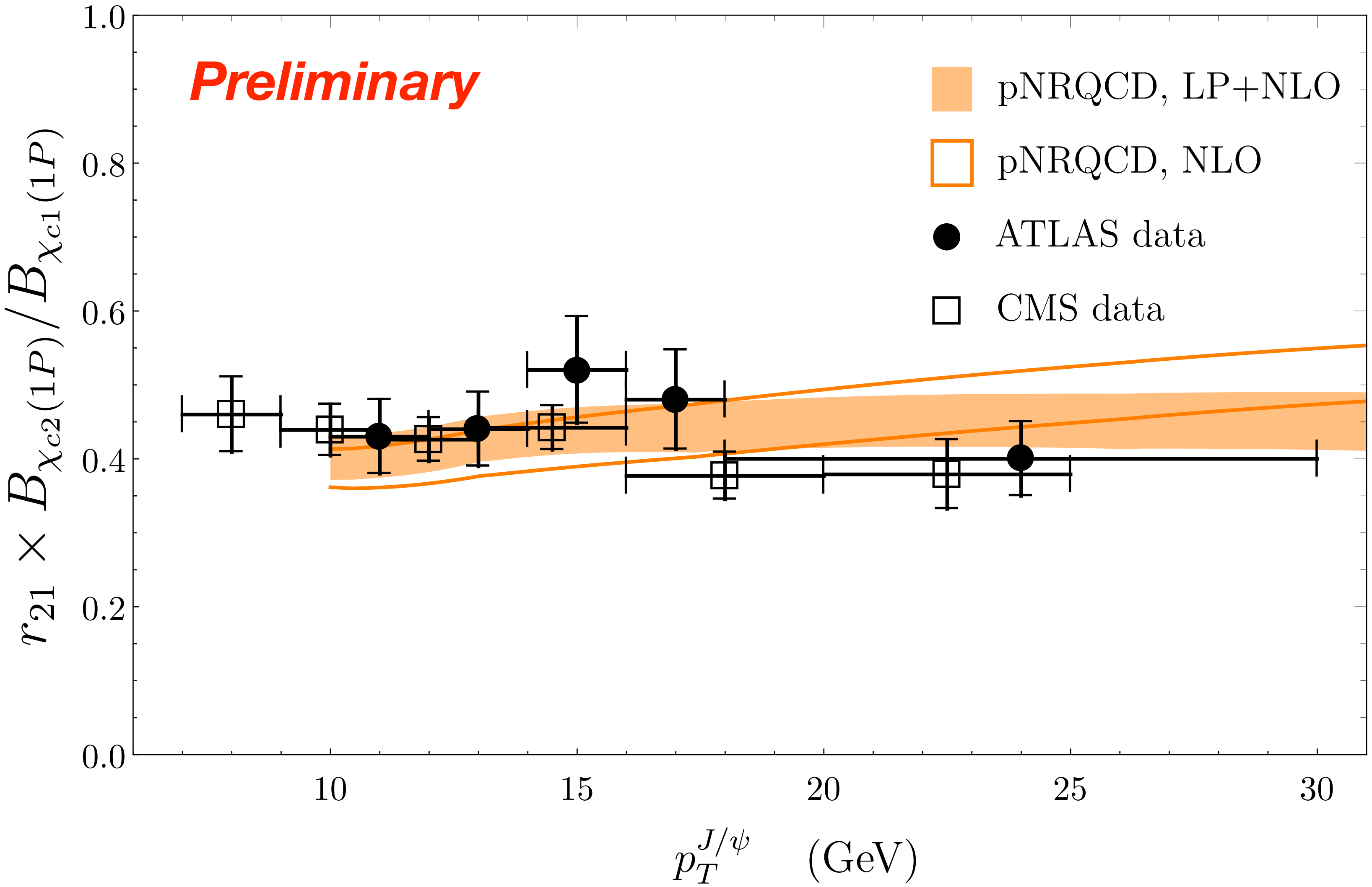
$$\frac{d}{d\log\Lambda}\mathcal{E}(\Lambda) = 12 C_F \frac{\alpha_s}{\pi} \qquad \frac{d}{d\log\Lambda}\langle \mathcal{O}^{\chi_{QJ}}({}^3S_1^{[8]})\rangle = \frac{4C_F\alpha_s}{3N_c\pi m^2}\langle \mathcal{O}^{\chi_{QJ}}({}^3P_J^{[1]})\rangle$$

- In principle, $\mathcal{E}$ can be determined from lattice QCD. Since lattice calculation is unavailable, we determine $\mathcal{E}$ from measured χ_{cJ} cross section ratios to obtain $\mathcal{E}(\Lambda = 1.5\ \mathrm{GeV}) = 2.8 \pm 1.7$ *Preliminary*

P-WAVE CHARMONIUM PRODUCTION

- Cross section ratio $\sigma(\chi_{c2})/\sigma(\chi_{c1})$ at the LHC compared to ATLAS and CMS data.

CMS, EPJC72, 2251 (2012)
ATLAS, JHEP07, 154 (2014)



←including resummed logarithms
←without resummed logarithms

Perturbative $Q\bar{Q}$ cross sections computed at NLO in α_s + resummed logarithms from Bodwin, Chao, HSC, Kim, Lee, Ma, PRD93, 034041 (2016)

P-WAVE CHARMONIUM PRODUCTION

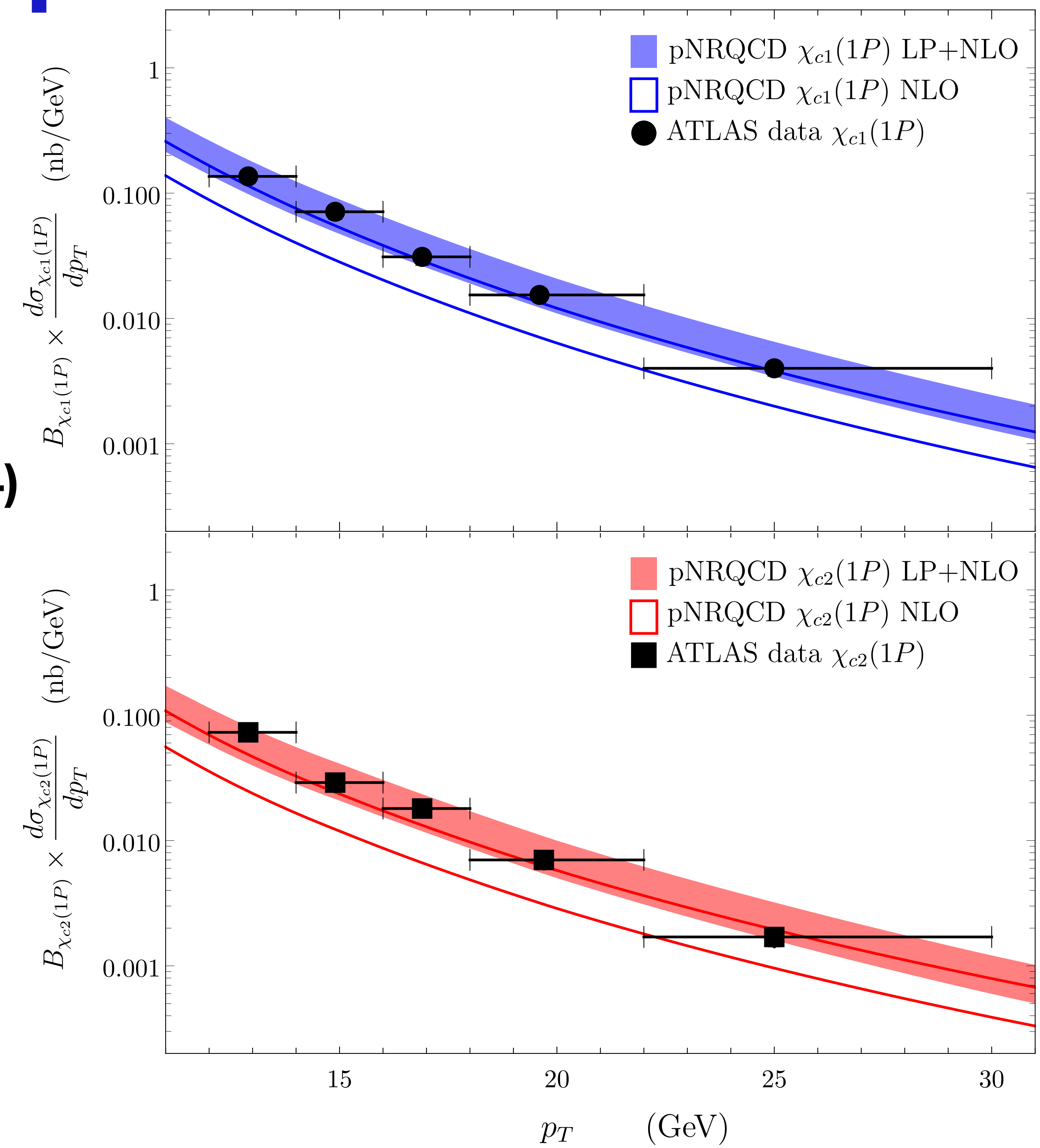
- χ_{c2} and χ_{c1} cross sections at the LHC, compared to ATLAS data.

ATLAS, JHEP07, 154 (2014)

- Wavefunctions at the origin obtained from two-photon decay rates of χ_{c2} and χ_{c0}.

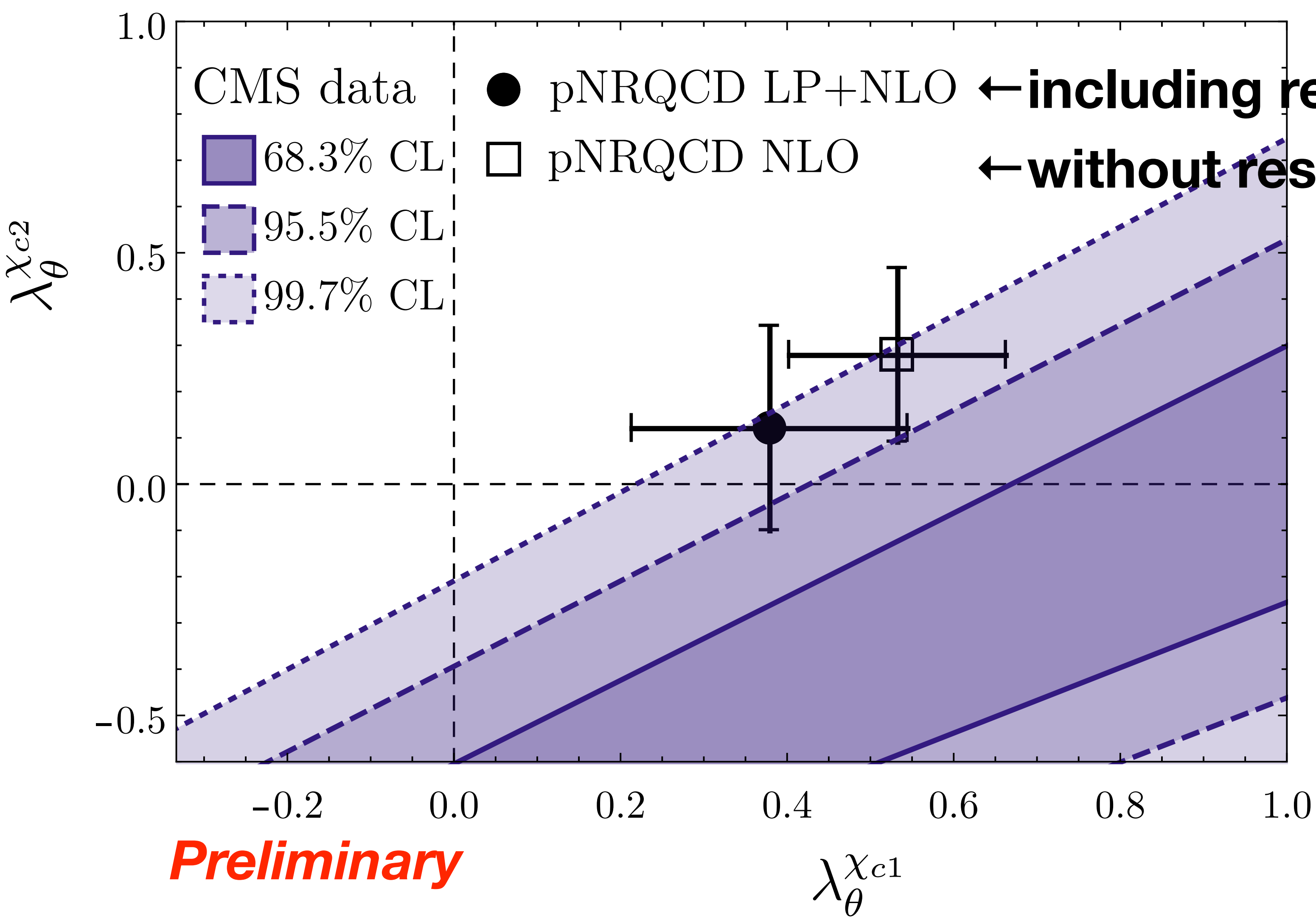
Perturbative $Q\bar{Q}$ cross sections computed at NLO in α_s + resummed logarithms from Bodwin, Chao, HSC, Kim, Lee, Ma, PRD93, 034041 (2016)

Preliminary



P-WAVE CHARMONIUM POLARIZATION

- χ_{c2} and χ_{c1} polarization at the LHC compared to experimental constraints from CMS. **CMS, PRL124, 162002 (2020)**



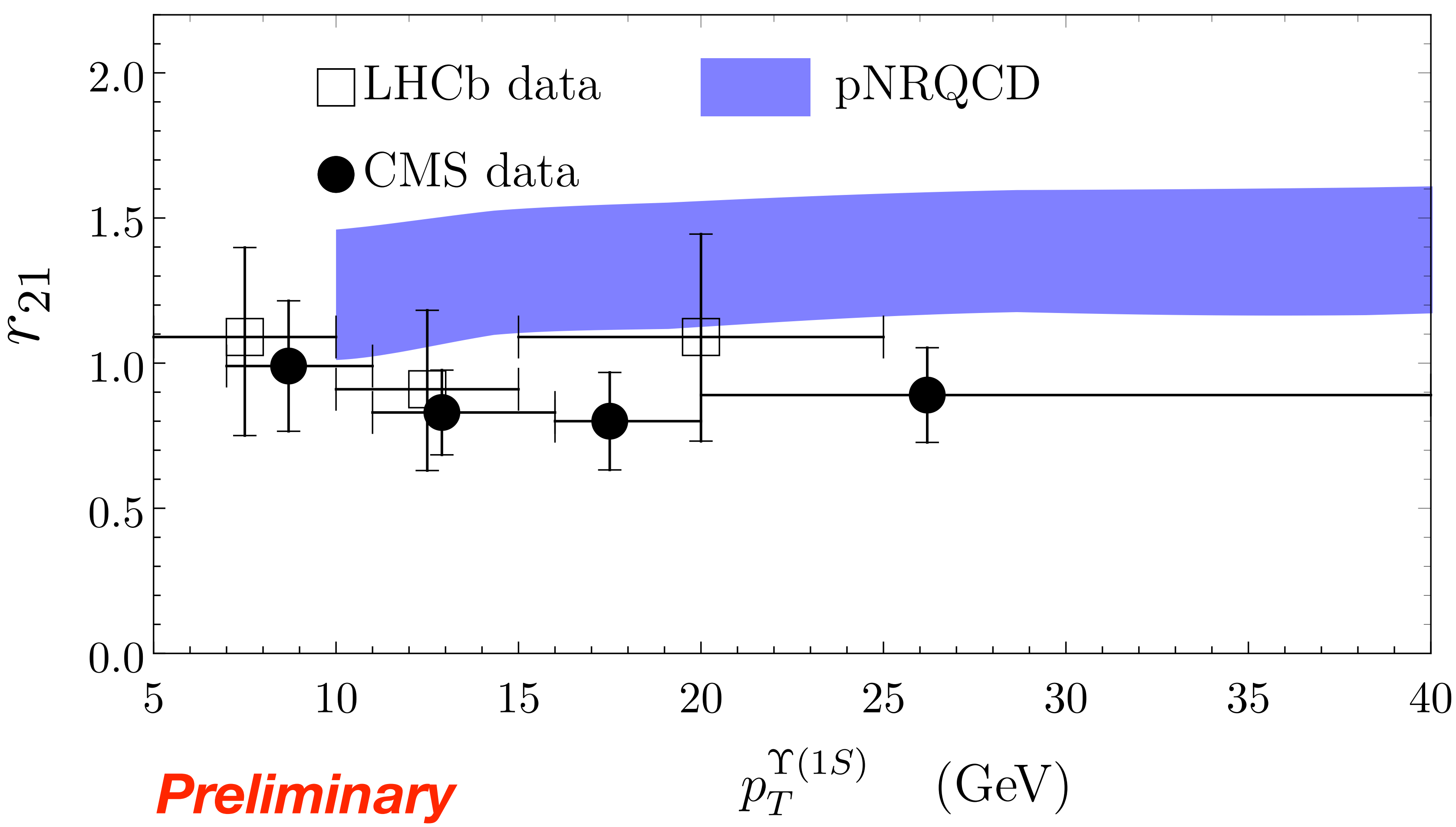
←including resummed logarithms

←without resummed logarithms

Perturbative $Q\bar{Q}$ cross sections computed at NLO in α_s + resummed logarithms from Bodwin, Chao, HSC, Kim, Lee, Ma, PRD93, 034041 (2016)

P-WAVE BOTTOMONIUM PRODUCTION

- Cross section ratio $\sigma(\chi_{b2})/\sigma(\chi_{b1})$ for $1P$ states at the LHC compared to LHCb and CMS measurements.



Preliminary

LHCb, JHEP10, 088 (2014)
CMS, PLB743, 383 (2015)

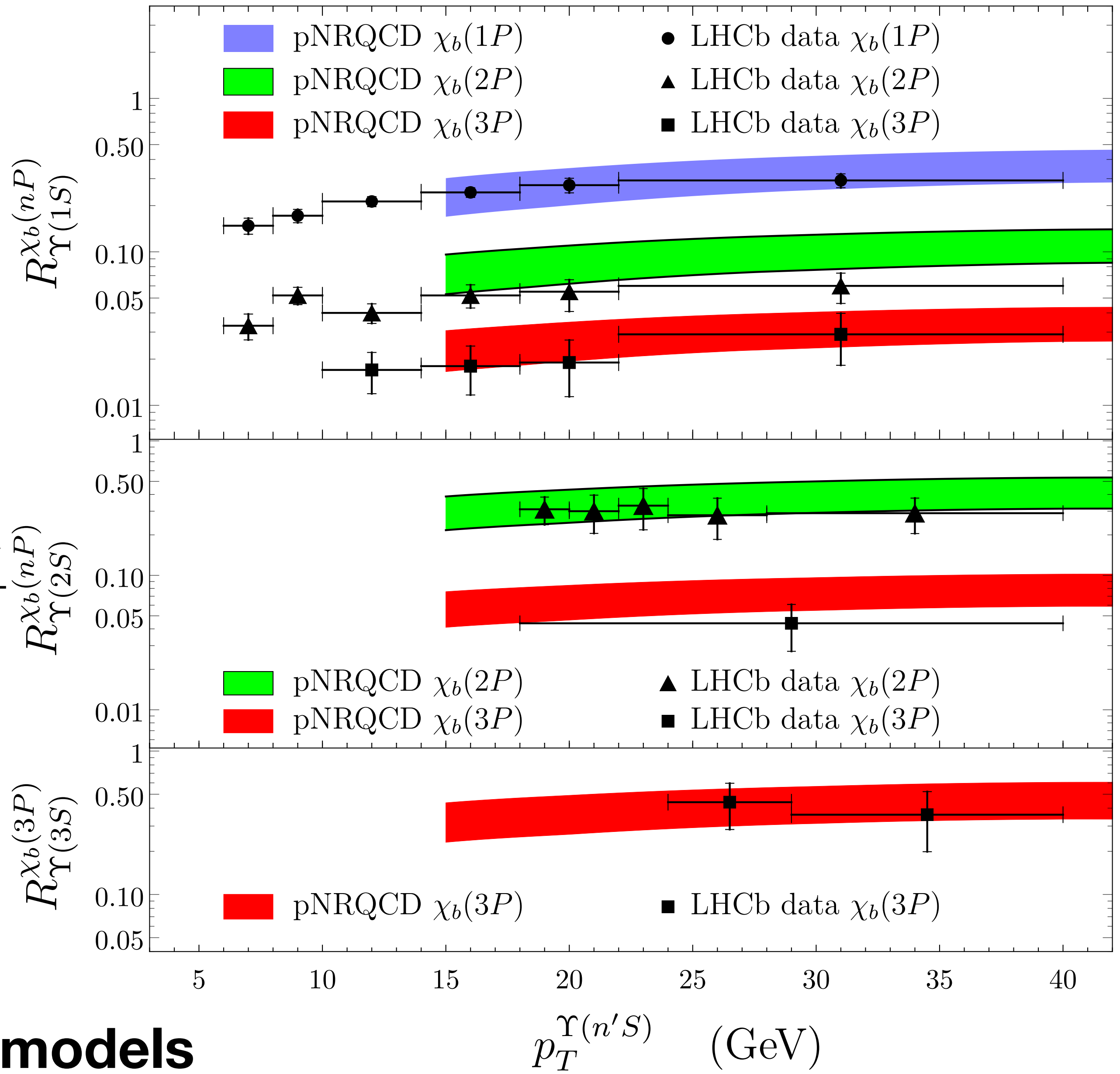
Perturbative $Q\bar{Q}$ cross sections computed at NLO in α_s using FDCHQHP Package from Wan and Wang, Comput. Phys. Commun. 185, 2939 (2014)

P-WAVE BOTTOMONIUM PRODUCTION

- $\chi_{bJ}(nP)$ production rates relative to $\Upsilon(n'S)$ cross sections at the LHC compared to LHCb measurement of feeddown fractions.

LHCb, EPJC74, 3092 (2014)

Preliminary



$$R^{\chi_b(nP)}_{\Upsilon(n'S)} = \sum_{J=1,2} \frac{\sigma_{\chi_{bJ}(nP)} \times \mathrm{Br}_{\chi_{bJ}\to\Upsilon(nS)+\gamma}}{\sigma_{\Upsilon(n'S)}}$$

Perturbative $Q\bar{Q}$ cross sections computed at NLO in α_s using FDCHQHP Package from Wan and Wang, Comput. Phys. Commun. 185, 2939 (2014)

χ_{bJ} **wavefunctions computed from potential models**

$\Upsilon(nS)$ **matrix elements taken from fits to data in Han, Ma, Meng, Shao, Zhang, Chao, PRD94, 014028 (2016)**

SUMMARY

- We developed a formalism for inclusive production of heavy quarkonium in strongly coupled potential NRQCD.
- For the first time, this allows first-principles determination of color-octet production matrix elements.
  A single gluonic correlator leads to determination of all *P*-wave charmonium and bottomonium cross sections.
- We computed production rates of χ_{cJ} and χ_{bJ} at the LHC, which are in agreements with measurements.
- Lattice determination of the gluonic correlator is desirable.
- We are working on *S*-wave quarkonia $(J/\psi,\ \eta_c,\ \Upsilon,\ \eta_b)$