INCLUSIVE HADROPRODUCTION OF P-WAVE HEAVY QUARKONIA IN POTENTIAL NRQCD

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NRQCD FACTORIZATION

- NRQCD provides a factorization formalism for inclusive production cross sections.

\[ \sigma = \sum_n \sigma_{Q\bar{Q}(n)} \langle \Omega | \mathcal{O}_n | \Omega \rangle \]

- In general it is not known how to compute matrix elements from first principles, so they are usually determined from cross section measurements. So far this approach has not lead to a comprehensive description of measurements.

- We aim to compute the matrix elements in potential NRQCD, which is obtained by integrating out scales above \( m v^2 \).
We work in the strong coupling regime where $m v^2 \ll \Lambda_{\text{QCD}}$, which is valid for non-Coulombic quarkonia, such as $P$-wave quarkonia. The degree of freedom is the singlet field $S(x_1, x_2)$, which describe $Q\bar{Q}$ in a color-singlet state.

$$\mathcal{L}_{\text{pNRQCD}} = \text{Tr}\{S^\dagger (i\partial_0 - h) S\}$$

Matching to NRQCD is done nonperturbatively.

pNRQCD provides expressions for decay matrix elements in terms of wavefunctions and universal gluonic correlators.

We extend the formalism for production matrix elements.
Matching to NRQCD in strongly coupled pNRQCD is done as an expansion in powers of $1/m$:

$$H_{\text{NRQCD}} = H_{\text{NRQCD}}^{(0)} + \frac{H_{\text{NRQCD}}^{(1)}}{m} + \ldots$$

$|n; x_1, x_2\rangle = |n; x_1, x_2\rangle^{(0)} + \frac{1}{m} |n; x_1, x_2\rangle^{(1)} + \ldots$

$|0; x_1, x_2\rangle$ is the ground state, $x_1, x_2$ are positions of $Q, \bar{Q}$.

A quarkonium state in vacuum is described by

$$\int d^3 x_1 d^3 x_2 \phi(x_1, x_2) |0; x_1, x_2\rangle$$

$\phi(x_1, x_2)$ is the quarkonium wavefunction.
Production matrix elements for production of quarkonium $Q$

$$\langle \Omega | \chi^\dagger K\psi \mathcal{P}_Q \psi^\dagger K' \chi | \Omega \rangle$$

$$\mathcal{P}_Q = \sum_X |Q + X \rangle \langle Q + X| = a_Q^\dagger a_Q$$

- Projection onto quarkonium + anything
- A schematic view of production matrix element

$Q\bar{Q}$ evolve into quarkonium state

Low-energy radiation of light particles

$Q\bar{Q}$ is produced locally in either color singlet or color octet

Color/spin matrices and covariant derivatives, gauge-completion Wilson lines (color octet)
PRODUCTION MATRIX ELEMENTS

- We want to compute

\[ \langle \Omega | \chi^\dagger K \psi \mathcal{P}_Q \psi^\dagger K' \chi | \Omega \rangle \]

- To compute production matrix elements we need to

1. Express \( \mathcal{P}_Q = a_Q^\dagger a_Q \) in terms of \( |n; x_1, x_2 \rangle \) states.

2. Describe a quarkonium in background of light particles in terms of wavefunctions. While quarkonium is always color singlet, background can be color octet.

- We need to do this nonperturbatively, but we still expand in powers of \( 1/m \).
QUARKONIUM PROJECTION OPERATOR

- $\mathcal{P}_Q = a_Q^\dagger a_Q$ is essentially a number operator: $\mathcal{P}_Q$ and $H_{\text{NRQCD}}$ are simultaneously diagonalizable. Simultaneous eigenstates are given by

$$|Q(n)\rangle = \int d^3x_1 d^3x_2 \phi_Q(n)(x_1, x_2)|n; x_1, x_2\rangle$$

- We obtain the expression $\mathcal{P}_Q = \sum |Q(n)\rangle\langle Q(n)|$

- For $n=0$, $|Q(0)\rangle$ is just the quarkonium in vacuum and $\phi$ is the usual quarkonium wavefunction.

- For $n>0$, $|Q(n)\rangle$ describe quarkonium + light particles. The “wavefunctions” $\phi$ are in general unknown for $n>0$. 
QUARKONIUM WAVEFUNCTIONS

- We need to identify “wavefunctions” of quarkonium in the background of gluons and light particles.

- Potential for quarkonium in vacuum is given in terms of the vacuum expectation value (VEV) of a Wilson loop:

\[
V(r) = \lim_{T \to \infty} \frac{i}{T} \log \langle \Omega | \begin{array}{c} r \\ T \end{array} | \Omega \rangle
\]

- For the potential for the \( n > 0 \) states, the light excitations in the \( |\underline{n}; x_1, x_2\rangle \) states should be included.

\[
V(r) = \lim_{T \to \infty} \frac{i}{T} \log \langle \Omega | \begin{array}{c} r \\ T \end{array} | \Omega \rangle
\]
In general, VEVs of products of color-singlet operators factorize into products of VEVs of individual operators.

\[
\langle \Omega | AB | \Omega \rangle = \langle \Omega | A | \Omega \rangle \langle \Omega | B | \Omega \rangle [1 + O(1/N_c^2)]
\]

So the \( n > 0 \) potentials reduce to

\[
V(r) = \lim_{T \to \infty} \frac{i}{T} \log \langle \Omega | \begin{array}{c} \bigotimes \bigotimes \\ T \end{array} r | \Omega \rangle
\]

\[
= \lim_{T \to \infty} \frac{i}{T} \log \left( \langle \Omega | \begin{array}{c} \bigotimes \\ T \end{array} r | \Omega \rangle \times \langle \Omega | \begin{array}{c} \bigotimes \\ T \end{array} | \Omega \rangle \right)
\]

\[
= \lim_{T \to \infty} \frac{i}{T} \log \langle \Omega | \begin{array}{c} \bigotimes \\ T \end{array} r | \Omega \rangle + \text{constant}
\]
QUARKONIUM WAVEFUNCTIONS

- Hence, the $n>0$ potentials are just the $n=0$ potential, plus constants that have no effect to the wavefunctions.

- Therefore, the wavefunctions $\phi$ are independent of $n$, and the projection operator is just

\[
P_Q = \sum_n |Q(n)\rangle\langle Q(n)|
\]

\[
|Q(n)\rangle = \int d^3x_1 d^3x_2 \phi_Q(x_1, x_2)|\overline{n}; x_1, x_2\rangle
\]

- These are valid up to corrections of relative order $1/N_c^2$. 
Now we can compute the production matrix elements

\[ \langle \Omega | \chi^\dagger K \psi \mathcal{P}_Q \psi^\dagger K' \chi | \Omega \rangle = \int d^3 x_1 d^3 x_2 \int d^3 x'_1 d^3 x'_2 \phi(x_1, x_2) \phi(x'_1, x'_2) \]

\[ \sum_n \langle \Omega | \chi^\dagger K \psi | n ; x_1, x_2 \rangle \langle n ; x'_1, x'_2 | \psi^\dagger K' \chi | \Omega \rangle \]

Contact term, computed order by order in \( 1/m \)

Contact terms can be computed in terms of universal gluonic correlators and differential operators that act on wavefunctions.

This allows calculation of production matrix elements in strongly coupled pNRQCD.
PRODUCTION OF P-WAVE QUARKONIA

- We apply this formalism for $\chi_{QJ} (Q = c \text{ or } b, \ J = 1, \ 2)$

- At leading order in $\nu$, the cross section is given by

$$\sigma_{\chi_{QJ} + X} = (2J + 1) \sigma_{Q\bar{Q}(3 P^1_J)} \langle \mathcal{O}^{\chi_Q 0} (3 P^1_0) \rangle$$

$$+ (2J + 1) \sigma_{Q\bar{Q}(3 S^8_1)} \langle \mathcal{O}^{\chi_Q 0} (3 S^8_1) \rangle$$


Bodwin, Braaten, Yuan, Lepage, PRD46, R3703 (1992)

**color singlet:**

$$\mathcal{O}^{\chi_Q 0} (3 P^1_0) = \frac{1}{3} \chi^\dagger \left( -\frac{i}{2} \vec{D} \cdot \sigma \right) \psi \mathcal{P}_{\chi_Q 0} \psi^\dagger \left( -\frac{i}{2} \vec{D} \cdot \sigma \right) \chi$$

**color octet:**

$$\mathcal{O}^{\chi_Q 0} (3 S^8_1) = \chi^\dagger \sigma^a T^a \psi \Phi^ab \mathcal{P}_{\chi_Q 0} \Phi^b_{c} \psi^\dagger \sigma^c T^c \chi$$


Nayak, Qiu, Sterman, PLB613, 45 (2005)

- We compute both color singlet and color octet matrix elements in strongly coupled pNRQCD.
P-WAVE PRODUCTION MATRIX ELEMENTS

- Color-singlet matrix element: \( \langle O^{XQ_0}(3P_0^{[1]}) \rangle = \frac{3N_c}{2\pi} |R^{(0)'}_{XQ_0}(0)|^2 \) we reproduce the known result in vacuum-saturation approximation. \( R(r) \): radial wavefunction

- Color-octet matrix element: result is given in terms of a universal gluonic correlator.
  \[
  \langle O^{XQ_0}(3S_1^{[8]}) \rangle = \frac{3N_c}{2\pi} |R^{(0)'}_{XQ_0}(0)|^2 \frac{E}{9N_cm^2}
  \]

- \( E \) is a universal quantity that does not depend on quark flavor or radial excitation. Determination of \( E \) directly leads to determination of all \( \chi_{cJ} \) and \( \chi_{bJ}(nP) \) cross sections, as well as \( h_c \) and \( h_b \) production rates.
P-WAVE PRODUCTION MATRIX ELEMENTS

- The correlator $\mathcal{E}$ is defined in terms of chromoelectric fields $gE$ at time $t$ and $t'$, with Wilson lines extending to infinity in the $\ell$ direction.

$$\mathcal{E} = \frac{3}{N_c} \int_0^\infty t \, dt \int_0^\infty t' \, dt' \langle \Omega | \Phi_{\ell}^{\dagger ab} \Phi_0^{\dagger da}(0, t) gE^{d', i}(t) gE^{e', i}(t') \Phi_0^{ec}(t', 0) \Phi_\ell^{bc} | \Omega \rangle.$$ 

- $\mathcal{E}$ has a one-loop scale dependence that is consistent with the evolution equation for NRQCD matrix elements

$$\frac{d}{d \log \Lambda} \mathcal{E}(\Lambda) = 12C_F \frac{\alpha_s}{\pi} \frac{d}{d \log \Lambda} \langle O^{\chi q J} (3 S_{1}^{[8]} ) \rangle = \frac{4C_F \alpha_s}{3N_c \pi m^2} \langle O^{\chi q J} (3 P_{1}^{[1]} ) \rangle$$ 

- In principle, $\mathcal{E}$ can be determined from lattice QCD. Since lattice calculation is unavailable, we determine $\mathcal{E}$ from measured $\chi_{cJ}$ cross section ratios to obtain

$$\mathcal{E}(\Lambda = 1.5 \text{ GeV}) = 2.8 \pm 1.7 \quad \text{Preliminary}$$
P–WAVE CHARMONIUM PRODUCTION

- Cross section ratio \( \sigma(\chi_{c2})/\sigma(\chi_{c1}) \) at the LHC compared to ATLAS and CMS data.

CMS, EPJC72, 2251 (2012)
ATLAS, JHEP07, 154 (2014)

Preliminary

Perturbative \( \overline{Q}Q \) cross sections computed at NLO in \( \alpha_s + \) resummed logarithms from Bodwin, Chao, HSC, Kim, Lee, Ma, PRD93, 034041 (2016)
HEAVY QUARKONIUM PRODUCTION IN POTENTIAL NRQCD

P–WAVE CHARMONIUM PRODUCTION

- $\chi_{c2}$ and $\chi_{c1}$ cross sections at the LHC, compared to ATLAS data.
  
  ATLAS, JHEP07, 154 (2014)

- Wavefunctions at the origin obtained from two-photon decay rates of $\chi_{c2}$ and $\chi_{c0}$.

Perturbative $Q\bar{Q}$ cross sections computed at NLO in $\alpha_s +$ resummed logarithms from Bodwin, Chao, HSC, Kim, Lee, Ma, PRD93, 034041 (2016)
P–WAVE CHARMONIUM POLARIZATION

- $\chi^c_2$ and $\chi^c_1$ polarization at the LHC compared to experimental constraints from CMS. CMS, PRL124, 162002 (2020)

Perturbative $Q\bar{Q}$ cross sections computed at NLO in $\alpha_s$ + resummed logarithms from Bodwin, Chao, HSC, Kim, Lee, Ma, PRD93, 034041 (2016)

Preliminary
Cross section ratio $\sigma(\chi_{b2})/\sigma(\chi_{b1})$ for $1P$ states at the LHC compared to LHCb and CMS measurements.

Preliminary $\gamma_{21}$ cross section ratio compared to LHCb and CMS measurements.

LHCb, JHEP10, 088 (2014)
CMS, PLB743, 383 (2015)

Perturbative $Q\bar{Q}$ cross sections computed at NLO in $\alpha_s$ using FDCHQHP Package from Wan and Wang, Comput. Phys. Commun. 185, 2939 (2014)
P–WAVE BOTTOMONIUM PRODUCTION

\( \chi_{bJ}(nP) \) production rates relative to \( \Upsilon(n'S) \) cross sections at the LHC compared to LHCb measurement of feeddown fractions.

LHCb, EPJC74, 3092 (2014)

\[
R_{\Upsilon(n'S)}^{\chi_{bJ}(nP)} = \sum_{J=1,2} \frac{\sigma_{\chi_{bJ}(nP)} \times Br_{\chi_{bJ} \rightarrow \Upsilon(n) + \gamma}}{\sigma_{\Upsilon(n'S)}}
\]

Perturbative \( \bar{Q}Q \) cross sections computed at NLO in \( \alpha_s \) using FDCHQHP Package from Wan and Wang, Comput. Phys. Commun. 185, 2939 (2014)

\( \chi_{bJ} \) wavefunctions computed from potential models

\( \Upsilon(nS) \) matrix elements taken from fits to data in Han, Ma, Meng, Shao, Zhang, Chao, PRD94, 014028 (2016)
We developed a formalism for inclusive production of heavy quarkonium in strongly coupled potential NRQCD. For the first time, this allows first-principles determination of color-octet production matrix elements. A single gluonic correlator leads to determination of all $P$-wave charmonium and bottomonium cross sections. We computed production rates of $\chi_{cJ}$ and $\chi_{bJ}$ at the LHC, which are in agreement with measurements. Lattice determination of the gluonic correlator is desirable. We are working on $S$-wave quarkonia ($J/\psi$, $\eta_c$, $\Upsilon$, $\eta_b$).