Static force with and without gradient flow

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Based on:

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Static energy $E_0(r)$

- Perturbatively known to $\mathrm{N^3LL}$ [1]:

$$E_0(r) = \Lambda_\mathrm{s} - \frac{C_\mathrm{F}\alpha_\mathrm{s}}{r}\left(1 + \#\alpha_\mathrm{s} + \#\alpha_\mathrm{s}^2 + \#\alpha_\mathrm{s}^3 + \#\alpha_\mathrm{s}^3 \ln\alpha_\mathrm{s} + \dots\right)$$

- From lattice can be measured from Wilson loop

$$E(r) = -\lim_{T\to\infty} \frac{\ln\langle \mathrm{Tr}(W_{r\times T})\rangle}{T}, \qquad W_{r\times T} = P\left\{\exp\left(i\oint_{r\times T} dz_\mu\, gA_\mu\right)\right\}$$

- Useful: Scale setting, strong coupling extraction
- Arbitrary shift needed to get rid of constant contributions:
  - Continuum : renormalon
  - Lattice : linear UV divergence
- Interesting physics encoded in the shape $\rightarrow$ Static force

$$F(r) = \partial_r E_0(r)$$

[1] For review of perturbative results, see: X. Tormo Mod. Phys. Lett. A28 (2013)

Static force $F(r)$

- On lattice requires noisy numerical derivative by default
- Alternatively define directly[1]:

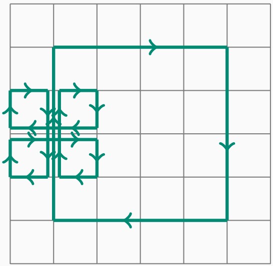
$$F(r) = -\lim_{T\to\infty} \frac{i}{\langle \mathrm{Tr}(W_{r\times T})\rangle} \left\langle \mathrm{Tr}\left(P\left\{ \exp\left(i \oint_{r\times T} \mathrm{d}z_\mu \, gA_\mu \right) \hat{\mathrm{r}} \cdot g\mathrm{E}(\mathrm{r}, t^*) \right) \right\} \right\rangle$$

  as chromoelectric field E inserted to Wilson (or Polyakov) loop

- On lattice E has finite size and Different discretizations
- The self energy contributions of E converge slowly to continuum[2]
  $\rightarrow$ need renormalization Z_E

[1] A. Vairo Mod. Phys. Lett. A 31 (2016) & EPJ Web Conf. 126 (2016), Brambilla *et.al.*PRD63 (2001)
[2] See e.g. Lepage *et.al.*PRD48 (1993), G. Bali Phys. Rept. 343 (2001), and many others ...

Measured operators

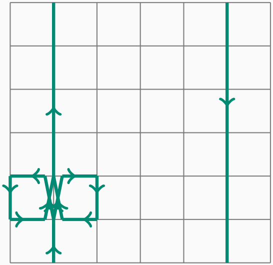


Wilson loop with Clover

$$E_i = \frac{1}{2iga^2}\left(\Pi_{i0} - \Pi_{i0}^\dagger\right)$$

$$\Pi_{\mu\nu} = \frac{1}{4}\left(P_{\mu,\nu} + P_{\nu,-\mu} + P_{-\mu,-\nu} + P_{-\nu,\mu}\right)$$

Used with multilevel and flow



Polyakov loop with Butterfly

$$E_i = \frac{1}{2}\left(F_{0i} + F_{-i0}\right)$$

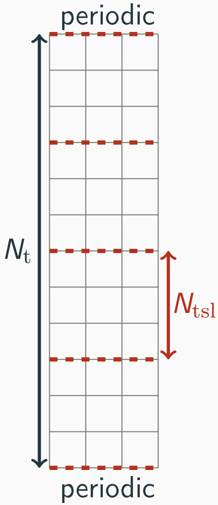
$$F_{\mu\nu} = \frac{1}{2iga^2}\left(P_{\mu,\nu} - P_{\mu,\nu}^\dagger\right)$$

$$P_{\mu,\nu}(x) = U_\mu(x)U_\nu(x+\hat{\mu})U_\mu^\dagger(x+\hat{\nu})U_\nu^\dagger(x)$$

Only used with multilevel

For discretizations, see e.g. Bali *et.al.*PRD56 (1997) or Koma & Koma Nucl. Phys. B769 (2007)

Algorithm 1: Multilevel algorithm

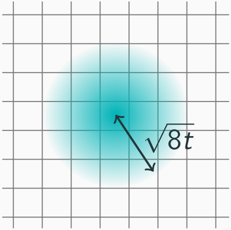


- Algorithm for quenched simulations
  - Cannot be generalized to un-quenched
- Improves signal of noisy observables
- Idea: Divide the lattice to temporal slices of size N_{tsl}
- Update each sub-lattice independently keeping boundaries fixed
- Average over different boundary configurations
  + Allows reaching better statistics with less configurations
- Spatial Wilson lines located at the boundaries

Lüscher and Weisz JHEP09 (2001)

Algorithm 2: Gradient flow

$$\partial_t B_{t,\mu} = -\frac{\delta S_{YM}}{\delta B} = D_{t,\mu} G_{t,\mu\nu}\,,$$
$$G_{t,\mu\nu} = \partial_\mu B_{t,\nu} - \partial_\nu B_{t,\mu} + [B_{t,\mu}, B_{t,\nu}]\,.$$
$$B_{0,\mu} = A_\mu \;\leftarrow \text{ the original gauge field}$$



- Evolve gauge along fictitious time t
- Drives B_μ towards minima of S_{YM}
- Diffuses the initial gauge field with radius $\sqrt{8t}$
- We use Lüscher-Weisz action for S_{YM}
- \+ Automatically renormalizes gauge invariant observables
- \+ Can be used un-quenched (This work: quenched)
- Generally needs zero flowtime limit

Lüscher JHEP 1008, 071 (2010)

Simulation details

- Use Wilson gauge action, pure gauge
- Heath bath with overrelaxation
- 3 Ensembles A: a=0.06fm, B: a=0.05fm, C: a=0.04fm
- Scale setting with[1]:

  $$\ln(a/r_0) = -1.6804 - 1.7331\,(\beta-6) + 0.7849\,(\beta-6)^2 - 0.4428\,(\beta-6)^3$$

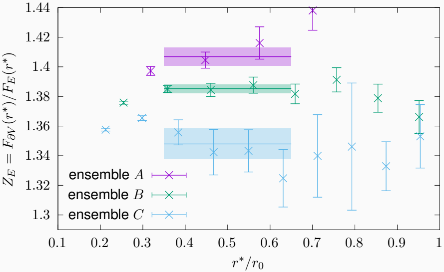
- Tree-level improve the force[1]:

  $$r_{\mathrm{I}} = \sqrt{\frac{2a}{4\pi\,[G(r+a) - G(r-a)]}} \qquad G(r) = \frac{1}{a}\int_{-\pi}^{\pi} \frac{\mathrm{d}k^3}{(2\pi)^3}\,\frac{\cos(rk_3/a)}{4\sum_j \sin(k_j/2)}$$

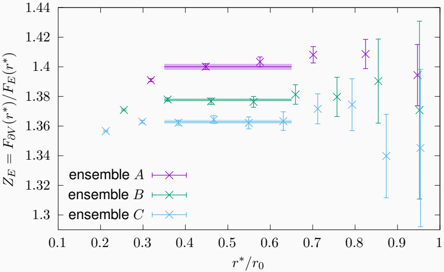
- Multilevel & Wilson loops: APE-smearing for spatial links $\alpha_{\mathrm{APE}} = 0.5, N_{\mathrm{APE}} = 50$

[1] S. Necco & R. Sommer. Nucl. Phys. B622 (2002)

Renormalization constant $Z_{\mathrm{E}} = \partial_r E(r)/F_E(r)$



Wilson loop

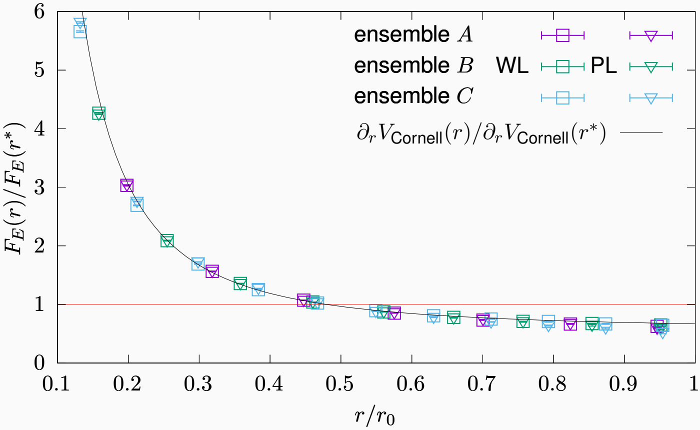


Polyakov loop

Ensemble	a in fm	Z_E from Wilson loops	Z_E from Polyakov loops
A	0.060	1.4068(63)	1.4001(20)
B	0.048	1.3853(30)	1.3776(10)
C	0.040	1.348(11)	1.3628(13)

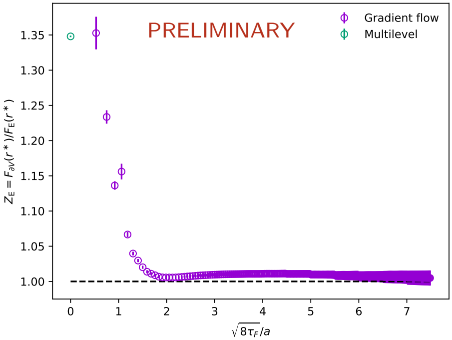
- Force from numerical derivative of E_0 differs from force from F_E
- Nonperturbative Z_E. Very little r-dependence

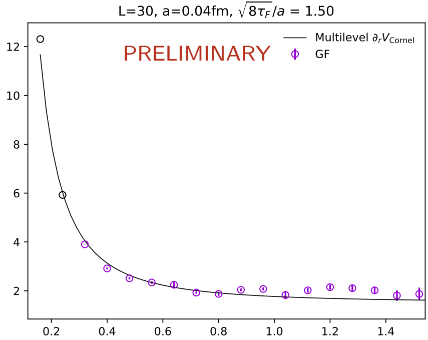
Multilevel result



- Remove Z_E by dividing with measurement at $r^* = 0.48 r_0$
- Proof of concept:
  - Both derivative of potential and direct force agree
  - Both Wilson loop and Polyakov loops agree

Gradient flow results





- Gradient flow automatically renormalizes the force at finite flowtime
  $\rightarrow$ No need for Z_{E}
- Divide with the leading flow time dependence for potential
- Early GF results indicate a good agreement to multilevel results
- The continuum and zero flowtime limits still need to be done

Conclusions

- Proof of concept: Static force can be measured directly from lattice by inserting chromoelectric field to a Wilson loop.
- Issue with self energy of chromoelectric field can be solved by:
    - Dividing the force with force at fixed separation r^*
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- This work can be expanded in future to many operators appearing in NREFTs

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Thank you!