The dipole picture and the non-relativistic expansion

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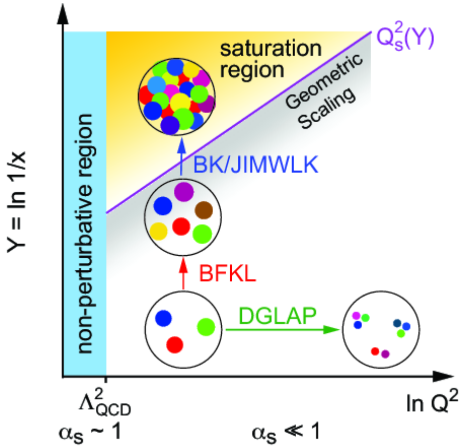
Plan

1. Introduction
2. Quarkonium light cone wave function in the non-relativistic limit
3. Cross-checks
4. Exclusive quarkonium production in the $Q \gg m$ limit
5. Conclusions

Why is exclusive quarkonium production interesting?



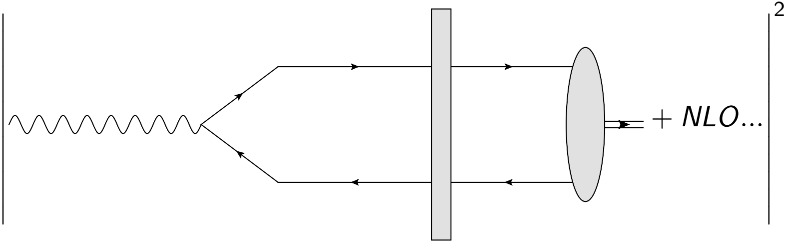
Picture taken from Marquet (2013)

Exclusive quarkonium production is an ideal way to study the gluon distribution at low x in DIS and UPCs

- Exclusive processes depend on the gluon density quadratically.
- Non-perturbative contributions are suppressed with respect to other exclusive processes.

The dipole picture

Application of light cone perturbation theory to exclusive processes



At LO

$$\frac{d\sigma_{T,L}^{\gamma^*+N\to HQ+N}}{dt} = \frac{1}{16\pi}\left|\int d^2r_\perp \int_0^1 \frac{dz}{4\pi}\left(\Psi^*_{HQ}\Psi_{\gamma^*}\right)_{T,L}\sigma_{q\bar{q}}\right|^2$$

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- Ψ_{γ^*} is the virtual photon wave function. Its transverse extent is of the order of the inverse of the virtuality Q. Dominated by perturbative physics.

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- $\sigma_{q\bar{q}}$ is the dipole cross-section. Contains the information about the gluon content of the nucleus and the saturation scale Q_s.

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- Ψ_{HQ} is the quarkonium wave function.
- At higher orders the wave functions have to take into account the presence of gluons inside the photon and quarkonium. We also need to take into account $\sigma_{q\bar{q}g}$, $\sigma_{q\bar{q}gg}$ and so on.

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- At higher orders the wave functions have to take into account the presence of gluons inside the photon and quarkonium. We also need to take into account $\sigma_{q\bar{q}g}$, $\sigma_{q\bar{q}gg}$ and so on.
- Our aim is to determine the properties of the nucleus. But in order to do this we need an accurate description of quarkonium wave function.

Quarkonium's light cone wave function

- Phenomenological approaches, boosted gaussian, gaussLC... See Kowalski, Motyka and Watt (2006) for a review.
- Obtained by solving the Schrödinger equation using a phenomenological potential. Cepila, Nemchik, Krelina and Pasechnik (2019).
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Non-relativistic assumption

- Widely used in the literature in other contexts, generally combined with EFT approaches (NRQCD, pNRQCD). Inclusive production, spectroscopy, decays.
- Well-defined limit of QCD. Theoretically interesting.
- It has already been used in the dipole model in its simplest form. Ryskin (1993) and Brodsky, Frankfurt, Gunion, Mueller and Strikman (1994).

Energy scales in a non-relativistic quarkonium

- The mass of the heavy quark m is by definition bigger than Λ_{QCD}. Perturbative physics.
- The typical velocity of the heavy quarks around the center of mass v is small. Hence $p \sim \frac{1}{r} \sim mv$. It is estimated that for charmonium $v^2 \sim 0.3$ while for bottomonium $v^2 \sim 0.1$.
- The binding energy is, due to the Virial theorem, of the same order of magnitude as the kinetic energy, mv^2.

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The non-relativistic limit simplifies the treatment because...

- It allows to separate the computation of scale m effects, which are pertubative.
- From the point of view of the scales smaller than m the production of heavy quarks is a local process.

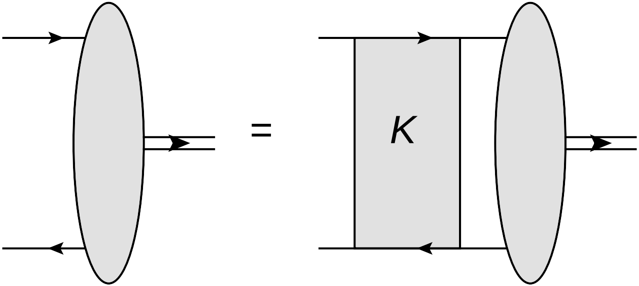
Plan

The non-relativistic limit

Basic assumptions

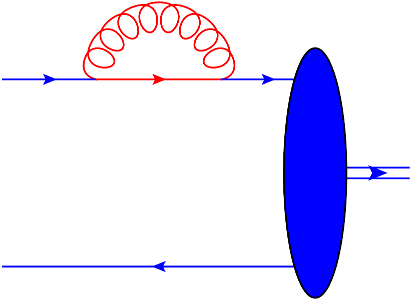
- The leading order light cone wave function can be computed taking into account only non-relativistic quarks and their interaction.
- Relativistic degrees of freedom can appear, but they are a perturbation $\rightarrow$ can appear during small times.
- Non-relativistic quarks (in light-cone perturbation theory) are defined by having a $p_\perp$ much smaller than m and a momentum fraction very close to $\frac{1}{2}$.

The leading order wave function



- Contains only non-relativistic components.
- Fulfils a Bethe-Salpeter equation which can be expressed as a Schrödinger equation.
- Relation between this and the wave function in potential models/NRQCD studied in Bodwin, Kang and Lee (2006).

Relativistic corrections to the non-relativistic wave function

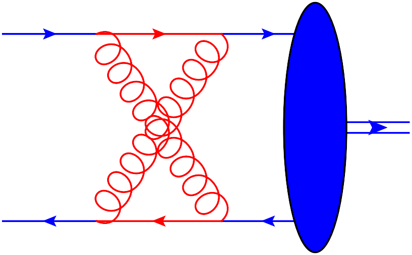


Type 1. Contribution of the relativistic degrees of freedom to the wave function renormalization of the non-relativistic quark.
Computed in Mustaki, Pinsky, Shigemitsu and Wilson (1991).

Color code

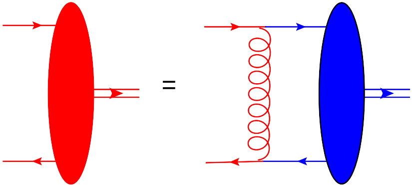
- Relativistic particles and gluons with virtuality of order m^2.
- Non-relativistic particles and softer gluons.

Relativistic corrections to the non-relativistic wave function



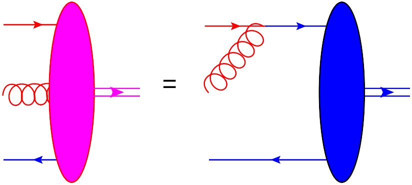
Type 2. Contributions that can be encoded as a redefinition of the potential between non-relativistic quarks.
In our case, it will not appear explicitly in our equations. However, it modifies the value of the non-relativistic wave-function at the origin.

Relativistic quark-antiquark pair component



- Can be computed in perturbation theory. $\rightarrow$ Proportional to $\alpha_s(m)$.
- Point like interaction from the point of view of non-relativistic quarks. $\rightarrow$ Proportional to the leading order non-relativistic wave function at the origin.

Relativistic quark, hard gluon and non-relativistic antiquark



- Proportional to $g(m)$.
- Cross-check. One can recover the wave-function renormalization by computing the square of this contribution.

Mathematical structure

$$\int dzf(z)\Psi^n_{HQ}(z,\mathrm{x}_\perp)=\sum_{m,k}\int dzf(z){\color{red}C^k_{n\leftarrow m}(z,\mathrm{x}_\perp)}{\color{blue}\left(\frac{\nabla}{m}\right)^k\int\frac{d\lambda}{4\pi}\phi^m(\lambda,0)}$$

- In this formula it is assumed that $x_\perp \sim \frac{1}{m}$ or smaller. The momentum fraction of a non-relativistic quark is $\lambda+\frac{1}{2}$ where $\lambda \ll 1$.
- ϕ represents the non-relativistic part. n and m label de components of the Fock space. For example, $C^k_{q\bar{q}\leftarrow q\bar{q}}(z,\mathrm{x}_\perp)$ means how the $q\bar{q}$ component of the full wave function depends on the same component of the non-relativistic wave function.
- The terms in the rhs scale as v^k. Note that if $mv^2 \gg \Lambda_{QCD}$ then $v \sim \alpha_s(mv)$.
- $C^k_{n\leftarrow m}(z,\mathrm{x}_\perp)$ can be computed as and expansion in $\alpha_s(m)$.

The cross-section in the non-relativistic expansion

$$16\pi\frac{d\sigma_{T,L}^{\gamma^*+N\to HQ+N}}{dt}=$$

$$\left|\sum_{n,m,k}\left(\left(\frac{\nabla}{M}\right)^k\int\frac{d\lambda}{4\pi}\phi^m(\lambda,0)\right)\int d^2r_\perp\int_0^1\frac{dz}{4\pi}\left(\left(C^k_{n\leftarrow m}(z,r_\perp)\right)^*\Psi_{\gamma^*}\right)^n_{T,L}\sigma_n\right|^2$$

Power counting

- The first correction from terms with $k\neq 0$ will enter at NNLO in α_s. Studied by Lappi, Mäntysaari and Penttala (Phys.Rev.D 102 (2020) 5, 054020).
- At NLO we only need to take into account $C^0_{q\bar{q}\leftarrow q\bar{q}}$ and $C^0_{q\bar{q}g\leftarrow q\bar{q}}$.
- In this power counting we did not consider the difference between $\alpha_s(m)$ and $\alpha_s(mv)$.

Results: $q\bar{q}$ longitudinal polarization

$$\int d\theta_r C^0_{q\bar{q}\leftarrow q\bar{q}}(z, \mathrm{r}_\perp; \lambda_1, \lambda_2; \lambda_1', \lambda_2')_{long} = 8\pi^2\delta(z-\frac{1}{2})(1+\delta Z)\delta_{\lambda_1\lambda_1'}\delta_{\lambda_2\lambda_2'}$$
$$+\frac{4g^2C_F z(1-z)}{\left(z-\frac{1}{2}\right)^2}\delta_{\lambda_1,\lambda_1'}\delta_{\lambda_2,\lambda_2'}\Bigg\{K_0(\tau)$$
$$+\left(\theta(z-\frac{1}{2})(1-z)+\theta(\frac{1}{2}-z)z\right)\left[\frac{\left(z-\frac{1}{2}\right)^2-\frac{1}{2}}{z(1-z)}\Big(K_0(\tau)\right.$$
$$\left.-\frac{\tau}{2}K_1(\tau)\Big)-\frac{\left(z-\frac{1}{2}\right)^2}{2z(1-z)}\tau K_1(\tau)\right]\Bigg\}$$

Longitudinal polarization. We do the transverse angle integration to get a more compact expression. $\tau = 2m\left(z-\frac{1}{2}\right)r_\perp$.

Results: $q\bar{q}$ transverse polarization

$$\int d\theta_r C^0_{q\bar{q}\leftarrow q\bar{q}}(z, \mathrm{r}_\perp; \lambda_1, \lambda_2; \lambda_1', \lambda_2')_{trans} = 8\pi^2\delta(z-\frac{1}{2})(1+\delta Z)\delta_{\lambda_1\lambda_1'}\delta_{\lambda_2\lambda_2'}$$
$$+\frac{4g^2C_F z(1-z)\delta_{\lambda_1,\lambda_1'}\delta_{\lambda_2,\lambda_2'}}{\left(z-\frac{1}{2}\right)^2}\left[K_0(\tau)\right.$$
$$\left.-\frac{(\theta(z-\frac{1}{2})(1-z)+\theta(\frac{1}{2}-z)z)}{2z(1-z)}\left(K_0(\tau)-\frac{\tau}{2}K_1(\tau)\right)\right]$$

Transverse polarization. We do the transverse angle integration to get a more compact expression. $\tau = 2m\left(z-\frac{1}{2}\right)r_\perp$.

Results: $q\bar{q}g$

$$\int \frac{d\lambda}{4\pi}\left\{\Psi^{q\bar{q}g}_{HQ}\right\}^{i}_{\lambda_{RQ},\lambda_G,\lambda_{\bar{Q}}}(x,\mathrm{l}_\perp;\lambda,\mathrm{r}_\perp=0)=$$

$$-\frac{ig\left(1-\frac{x}{2}\right)xl^i_\perp\epsilon^{*\,A,j}_\perp(\lambda_G)}{2\pi\sqrt{2x(1-x)}l_\perp}\left(\frac{mx}{2\pi l_\perp\mu^2}\right)^{\frac{D-4}{2}}K_{\frac{D-2}{2}}(mxl_\perp)\sum_{\lambda_Q}\bar{u}(\hat{p}_{RQ},\lambda_{RQ})T^A$$

$$\times\left[\delta^{ij}-\frac{x}{4\left(1-\frac{x}{2}\right)}[\gamma^i_\perp,\gamma^j_\perp]\right]\not{n}u(mv,\lambda_Q)\int\frac{d\lambda}{4\pi}\phi^i_{q\bar{q}}(\lambda,0)$$

$$+\frac{g\sqrt{x(1-x)}x}{8\sqrt{2}\pi(1-x)}\left(\frac{mx}{2\pi l_\perp\mu^2}\right)^{\frac{D-4}{2}}K_{\frac{D-4}{2}}(mxl_\perp)$$

$$\times\sum_{\lambda_Q}\bar{u}(\hat{p}_{RQ},\lambda_{RQ})\not{\epsilon}^*_\perp(\lambda_G)\not{n}u(mv,\lambda_Q)\int\frac{d\lambda}{4\pi}\phi^i_{q\bar{q}}(\lambda,0)$$

$l_\perp$ is the distance between the gluon and the relativistic quark. x is the momentum fraction of the gluon with respect to the non-relativistic quark.

Plan

Light-cone distribution amplitude

- The light-cone distribution amplitude is a function that is defined in collinear factorization.
- In light-cone gauge, it is equal to the light-cone wave function at $r_\perp = 0$.
- It must fulfil the Efremov-Radyushkin-Brodsky-Lepage (ERBL)[1] equation.

[1] Lepage and Brodsky (1980), Chernyak and Zhitnitsky (1984)

Light-cone distribution amplitude

The limit $r_\perp \to 0$ introduces ultraviolet divergences, they can be regulated in dimensional regularization.

$$\lim_{r_\perp \to 0} f(z, \mathrm{r}_\perp) = \int \frac{d^{D-2}p_\perp}{(2\pi)^{D-2}} f(z, \mathrm{p}_\perp)$$

For the longitudinal polarization

$$D^3(z) = 4\pi(1+\delta Z)\delta\left(z-\frac{1}{2}\right)\int \frac{d\lambda}{4\pi}\phi^3_{q\bar{q}}(\lambda,0)$$
$$-\frac{2g^2 C_F z(1-z)}{\pi\left(z-\frac{1}{2}\right)^2}\left[\left(\frac{1}{D-4}+\frac{1}{2}\log\left(\frac{m^2\left(z-\frac{1}{2}\right)^2}{\pi\mu^2}\right)+\frac{\gamma_E}{2}\right)\right.$$
$$\times\left(1+(\theta(z-\frac{1}{2})(1-z)+\theta(\frac{1}{2}-z)z)\frac{(D-2)\left(z-\frac{1}{2}\right)^2-1}{2z(1-z)}\right)$$
$$\left.+\frac{(\theta(z-\frac{1}{2})(1-z)+\theta(\frac{1}{2}-z)z)}{4z(1-z)}\left(2(D-2)\left(z-\frac{1}{2}\right)^2-1\right)\right]\int \frac{d\lambda}{4\pi}\phi^3_{q\bar{q}}(\lambda,0)$$

Light-cone distribution amplitude

For the transverse polarization

$$D^i(z) = 4\pi(1+\delta Z)\delta\left(z-\frac{1}{2}\right)\int \frac{d\lambda}{4\pi}\phi^3_{q\bar{q}}(\lambda,0)$$
$$-\frac{2g^2 C_F z(1-z)}{\pi\left(z-\frac{1}{2}\right)^2}\Bigg[\left(\frac{1}{D-4}+\frac{1}{2}\log\left(\frac{m^2\left(z-\frac{1}{2}\right)^2}{\pi\mu^2}\right)+\frac{\gamma_E}{2}\right)$$
$$\left(1-\frac{(\theta(z-\frac{1}{2})(1-z)+\theta(\frac{1}{2}-z)z)}{2z(1-z)}\right)$$
$$-\frac{(\theta(z-\frac{1}{2})(1-z)+\theta(\frac{1}{2}-z)z)}{4z(1-z)}\Bigg]\int \frac{d\lambda}{4\pi}\phi^i_{q\bar{q}}(\lambda,0)$$

The ERBL equation

We have checked that it is fulfilled that

$$\frac{\partial D^i(z)}{\partial \log \mu^2} = \frac{\alpha_s C_F}{2\pi} \int_0^1 dz' K_{L,T}(z,z') D^i(z')$$

where

$$K_L(z,z') = \theta(z-z')\frac{1-z}{1-z'}\left(1+\left[\frac{1}{z-z'}\right]_+\right) + \theta(z'-z)\frac{z}{z'}\left(1+\left[\frac{1}{z'-z}\right]_+\right) + \frac{3}{2}\delta(z-z')$$

and

$$K_T(z,z') = \theta(z-z')\frac{1-z}{1-z'}\left[\frac{1}{z-z'}\right]_+ + \theta(z'-z)\frac{z}{z'}\left[\frac{1}{z'-z}\right]_+ + \frac{3}{2}\delta(z-z')$$

Quarkonium decay into leptons

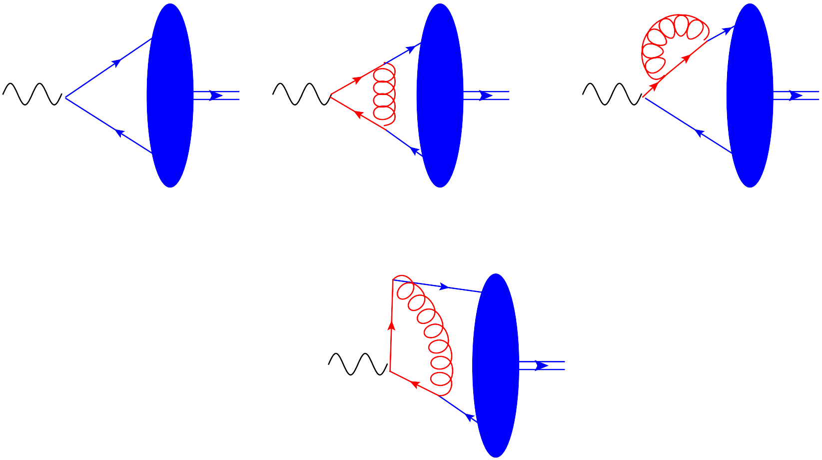
This is a quantity that can be computed knowing the light-cone wave function. It has also been computed at one loop using NRQCD and related approaches[2] (in dimensional regularization). Therefore, we know

$$\int_0^1 dz \sum_n \Psi^n_{HQ}(z, 0_\perp) = \left(1 - \frac{2\alpha_s C_F}{\pi}\right) \int d\lambda \phi(\lambda, 0)$$

However, we are not using dimensional regularization.

[2] Barbieri, Gatto, Kogerler and Kunszt (1975)

Diagrams



Remark: The last diagram vanishes in the longitudinal polarization case.

Result

$$\int_0^1 dz \Psi_{HQ}^{q\bar{q}}(z, 0_\perp) = \left(1 + \frac{\alpha_s C_F}{\pi}\left(\frac{1}{x_0} - 2\right)\right)\int d\lambda \phi(\lambda, 0)$$

Result

$$\int_0^1 dz \Psi_{HQ}^{q\bar{q}}(z, 0_\perp) = \left(1 + \frac{\alpha_s C_F}{\pi}\left(\frac{1}{x_0} - 2\right)\right) \int d\lambda \phi(\lambda, 0)$$

- We are using a cut-off x_0 to regulate the integration in z and DR to regulate the transverse component.
- Power like divergences do not appear in dimensional regularization (DM) but they can appear in our case.
- The divergence comes from the region in which $p_\perp \sim m x_0 \ll m$ and $\left|z - \frac{1}{2}\right| \sim \frac{x_0}{2} \ll 1$. Coulomb singularity.

Result

$$\int_0^1 dz \Psi^{q\bar{q}}_{HQ}(z, 0_\perp) = \left(1 + \frac{\alpha_s C_F}{\pi}\left(\frac{1}{x_0} - 2\right)\right) \int d\lambda \phi(\lambda, 0)$$

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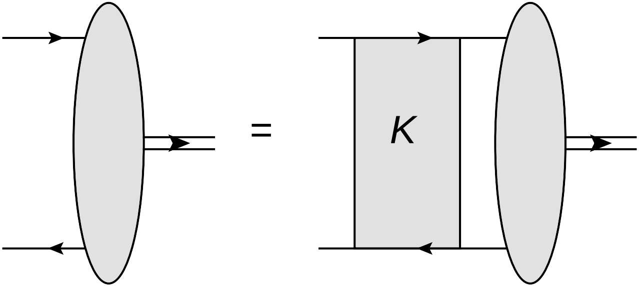
We need that...

$$\frac{d}{dx_0} \int_0^1 dz \Psi^{q\bar{q}}_{HQ}(z, 0_\perp) = -\frac{\alpha_s C_F}{\pi x_0^2} \int d\lambda \phi(\lambda, 0) + \frac{d}{dx_0} \int d\lambda \phi(\lambda, 0) = 0$$

Coulomb singularity

We want to check if the condition is fulfilled.



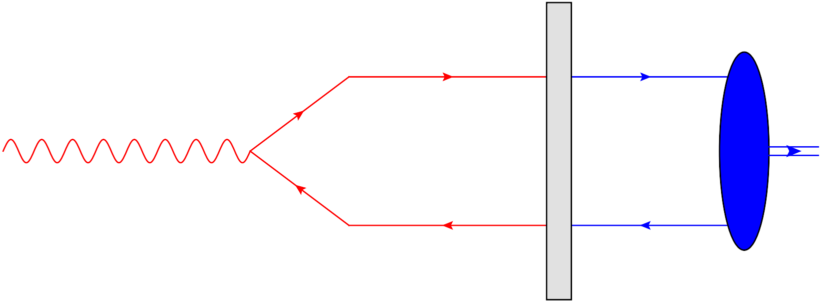
- We look at the ultraviolet behaviour, we can substitute the kernel by a Coulomb exchange.
- We focus on the case in which $\left|z-\frac{1}{2}\right| \sim \frac{x_0}{2}$ and $m \gg p_\perp \sim m x_0 \gg m v$.
- Indeed, the dependence of $\int d\lambda\phi$ on x_0 makes decay width finite.

Plan

Exclusive quarkonium production in the $Q \gg m$ limit at NLO

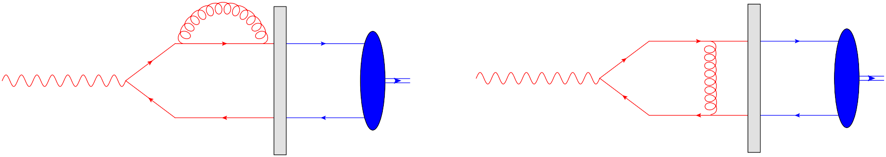
- The general case $Q \sim m$ has been very recently studied using this quarkonium wave function. ArXiv:2104.02349, Mäntysaari and Penttala.
- For this we need the NLO photon wave function with massive quarks. This result was recently published, ArXiv:2103.14549, Beuf, Lappi and Paatelainen.
- Here, we check that all divergences cancel in the $Q \gg m$ limit. We focus on the simpler, longitudinal polarization case.
- We get consistent results compatible with B-JIMWLK evolution.

Tree level



Dependence with x_0 hidden in two terms. $\sigma_{q\bar{q}}$, which fulfils B-JIMWLK evolution, and the non-relativistic wave function, which depends on x_0 in the way we described when discussing the decay into leptons.

One loop corrections to photon wave function

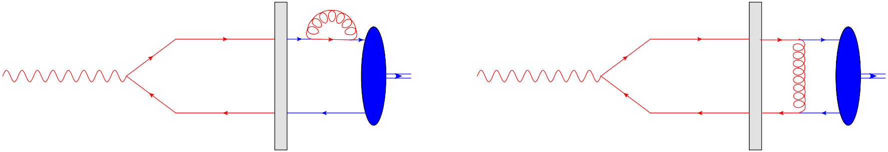


$$\Psi_\gamma(z, r_\perp)|_{NLO} = \Psi_\gamma(z, r_\perp)|_{LO}\,(1 + \delta Z_\gamma(z, r_\perp))$$

Recently computed in Beuf (2017). In our case we need the value at $z = \frac{1}{2}$.

$$\delta Z_\gamma(\frac{1}{2}, r_\perp) = -\frac{2C_F\alpha_s}{\pi}\left(\left(\log x_0 + \frac{3}{4}\right)\left(\frac{1}{D-4} - \frac{\gamma_E}{2} - \frac{1}{2}\log(\pi\mu^2 r_\perp^2)\right)\right.$$
$$\left. + \frac{\pi^2}{24} - \frac{3}{4}\right)$$

One loop corrections to quarkonium wave function



Dependence with μ

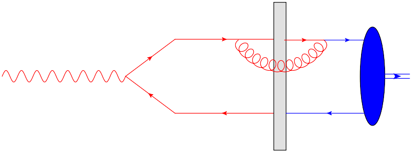
Comes only from the wave function renormalization and fulfils that $\frac{d\delta Z}{d\mu}=\frac{d\delta Z_\gamma\left(\frac{1}{2},r_\perp\right)}{d\mu}$

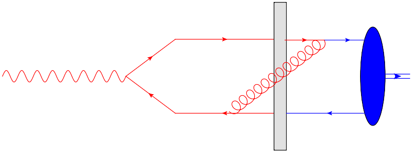
Dependence with x_0

Can be divided into two pieces:

- One which cancels the x_0 dependence of the non-relativistic wave function. Coulomb singularity.
- One whose derivative is proportional to $\frac{d\delta Z_\gamma\left(\frac{1}{2},r_\perp\right)}{dx_0}$.

Contribution of the $q\bar{q}g$ Fock state

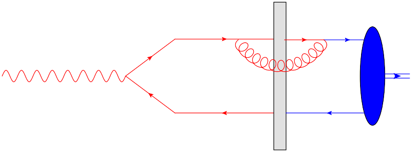


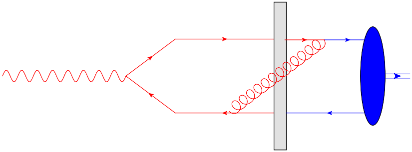


Dependence with μ

Note that in the ultraviolet $\sigma_{q\bar{q}g} \to \sigma_{q\bar{q}}$. It has a divergence that cancels that of the wave functions of the photon and quarkonium.

Contribution of the $q\bar{q}g$ Fock state





Dependence with x_0

Can be divided in two terms:

- One proportional to $(\sigma_{q\bar{q}g} - \sigma_{q\bar{q}})$ which cancels the B-JIMWLK evolution of the target.
- One proportional to $\sigma_{q\bar{q}}$ which cancels the divergences of the wave functions of the photon and quarkonium, except for the piece related with the Coulomb singularity

Plan

Conclusions

- We have computed the NLO corrections to the quarkonium wave function in the non-relativistic limit.
- We have checked that the light-cone distribution amplitude obtained in this framework fulfils ERBL equation.
- We recovered known results for the decay of quarkonium into leptons. To our knowledge, first computation in light-cone gauge.
- We have checked that when the wave function is applied to compute exclusive quarkonium production all divergences will cancel.
- Ours results were recently used by Mäntysaari and Penttala to compute exclusive quarkonium production.