

Long and Short Distance Behavior of the Imaginary Part of the Heavy-Quark Potential

ConfXIV, August 4, 2021

A Virtual Tribute to Quark Confinement and the Hadron Spectrum 2021

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Arxiv ePrint: [2102.12587](https://arxiv.org/abs/2102.12587)

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Outline of the talk

- In-medium Heavy quarkonia → **imaginary part of the potential**
- Considering the classical regime (effectively) of Yang-Mills theory/
static limit of the potential
- Perturbative and nonperturbative methods
- Larger lattices/ smaller spacing/ various temperatures (or β)
- Compact functional forms/ **Relation to κ**

In-medium Heavy quarks, $\text{Im}[V]$

$$V_{\text{KMS}}(r, T) = \left(\frac{1 - e^{-m_D r}}{m_D r} \right) \sigma r - \frac{\alpha}{r} e^{-m_D r}$$

Matsui and Satz: *Phys. Lett. B* 178 (1986)
Karsch, Mehr, and Satz : *Z. Phys. C*37, 617 (1988)

$\text{Im}[V]$

Thermal gluonic medium → Emergence of $\text{Im}[V]$:
[Laine et al JHEP 03 (2007) 054]

Melting and in-medium break-up of quarkonium.

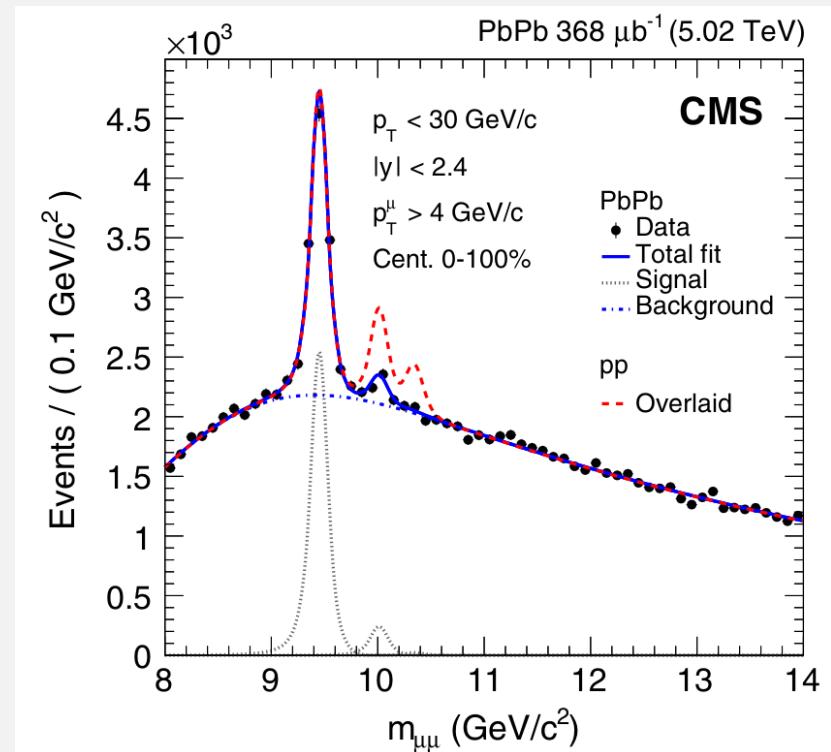
Similar studies, phenomenological applications:

Burnier, Laine, and Vepsäläinen (2009): *Phys. Lett. B* 678:86-8

Dumitru, Guo, and Strickland (2009): *Phys. Rev. D* 79, 114003

Islam and Strickland (2020): *Phys. Lett. B.*, 811, 135949

Lehmann and Rothkopf (2021): *JHEP* 07, 067



Plot from: **Phys. Rev. Lett. 120, 142301 (2018)**

$\text{Im}[V]$ from stochastic Hamiltonian

Stochastic Hamiltonian

$$H(\mathbf{r}, t) = -\frac{\nabla_{\mathbf{r}}^2}{M} + V(\mathbf{r}) + \tilde{\nu}\left(\mathbf{R} + \frac{\mathbf{r}}{2}, t\right) - \tilde{\nu}\left(\mathbf{R} - \frac{\mathbf{r}}{2}, t\right)$$

$$\langle \tilde{\nu}(\mathbf{r}, t) \rangle = 0, \quad \langle \tilde{\nu}(\mathbf{r}, t) \tilde{\nu}(\mathbf{r}', t') \rangle = D(\mathbf{r} - \mathbf{r}') \delta(t - t')$$

$$i \frac{d}{dt} \langle \psi_{Q\bar{Q}}(\mathbf{r}, t) \rangle = \left(-\frac{\nabla_{\mathbf{r}}^2}{M} + V(\mathbf{r}) - i\{D(\mathbf{0}) - D((\mathbf{r}))\} \right) \langle \psi_{Q\bar{Q}}(\mathbf{r}, t) \rangle$$

Kajimoto et al (2018): Phys. Rev. D97, 014003,

Rothkopf (2020) : Phys. Rept. 858, 1-117

Related talk in ConfXIV

Akamatsu (Tuesday)

Im[V]: 2nd order pt (HTL, HCL)

HTL result: [2nd order pt with dimensional regularization]

$$\text{Im}[V^{(2)}(r)] = -\frac{C_F g^2 T}{4\pi} \phi(m_{D,\text{HTL}} r)$$

Laine et al: JHEP 03
(2007) 054

$$\phi(x) \equiv 2 \int_0^\infty dz \frac{z}{(1+z^2)^2} \left[1 - \frac{\sin(zx)}{zx} \right]$$

$$m_{D,\text{HTL}}^2 = 4g^2 N_c \int \frac{d^3 p}{(2\pi)^3} \frac{f_{\text{BE}}(p)}{p} = \frac{N_c g^2 T^2}{3}$$

HCL result: [2nd order pt with cubic lattice regularization]

$(N \rightarrow \infty, t \rightarrow \infty)$

$$\text{Im}[V_{\text{cl}}^{(2)}(r)]/g^2 T = -\frac{\pi C_F N_c^2}{\beta} \int_0^1 d^3 \mathbf{x} \frac{1 - \cos(\pi x_3 r/a)}{(\tilde{\mathbf{x}}^2 + N_c^2 \Sigma/\pi \beta)^2} \int_{-1}^1 d^3 \mathbf{y} \frac{\delta(\tilde{\mathbf{x}} \cdot \mathring{\mathbf{y}})}{(\tilde{\mathbf{y}}^2)^{1/2}}$$

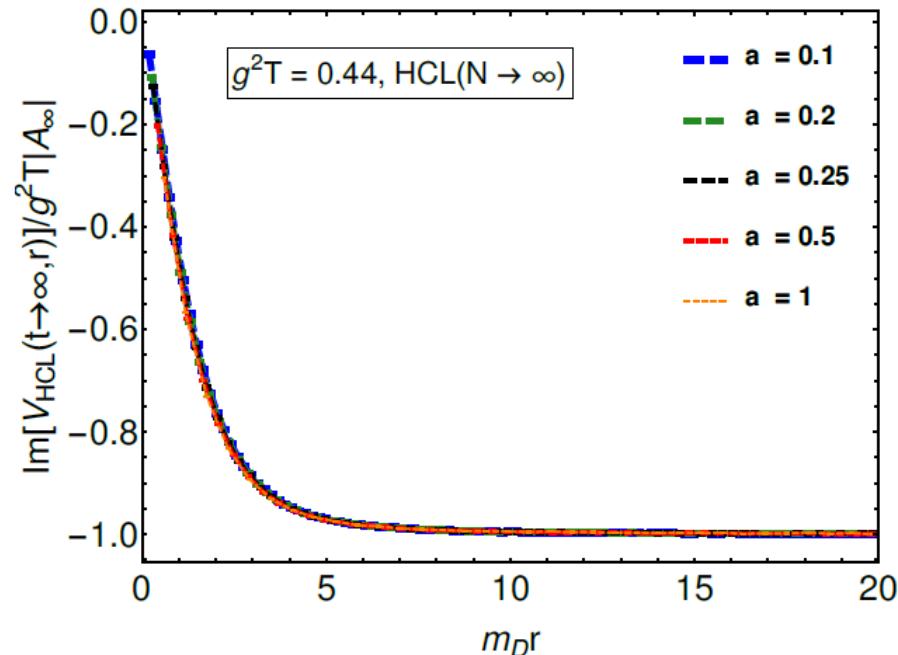
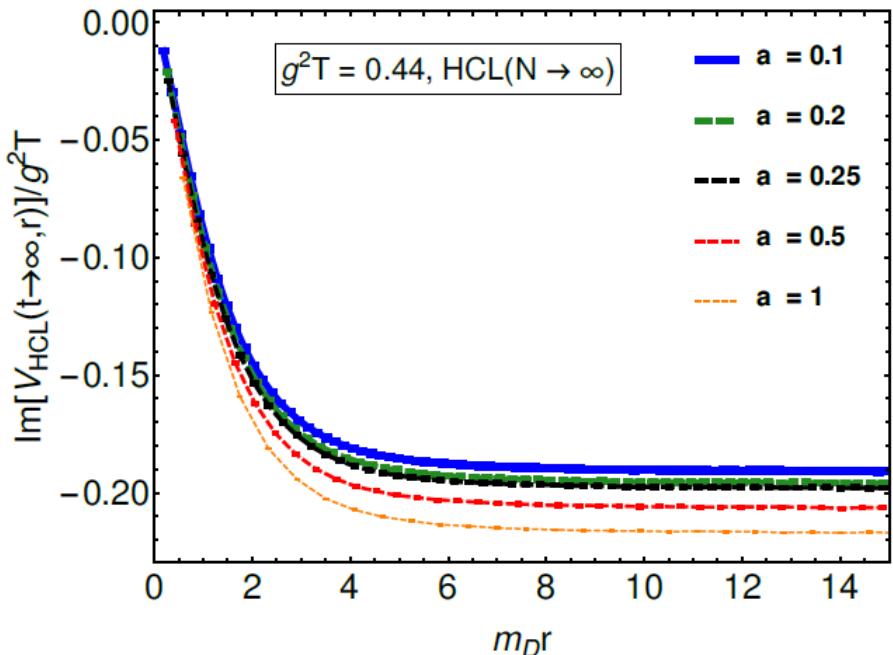
$$\tilde{x}_i \equiv 2 \sin\left(\frac{\pi x_i}{2}\right), \quad \dot{x}_i \equiv \sin(\pi x_i), \quad x_i \in (-1, 1)$$

Laine, Philipsen,
Tassler: JHEP
0709:066, (2007)

$$\beta = \frac{2N_c}{g^2 T a}$$

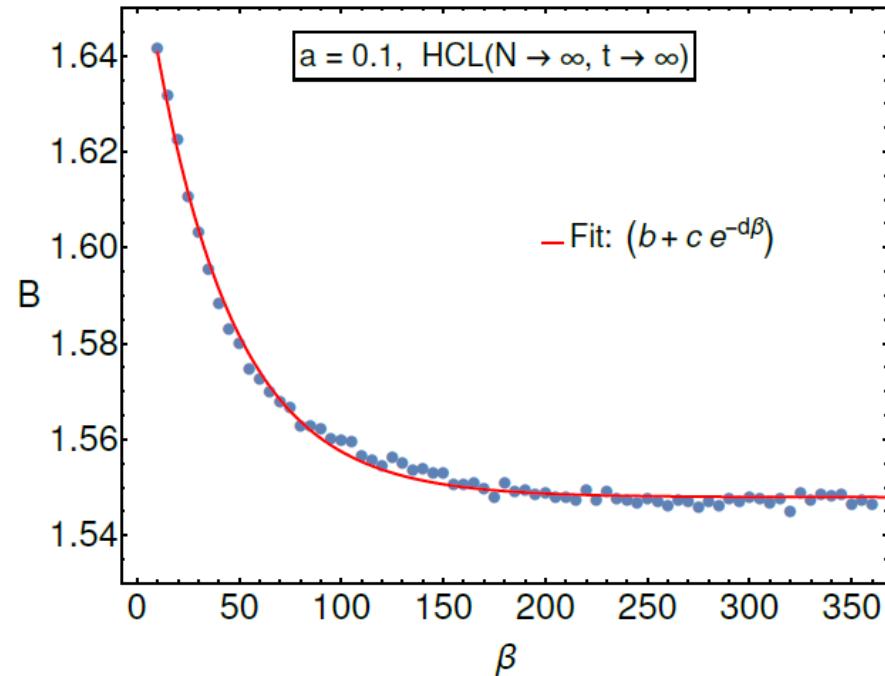
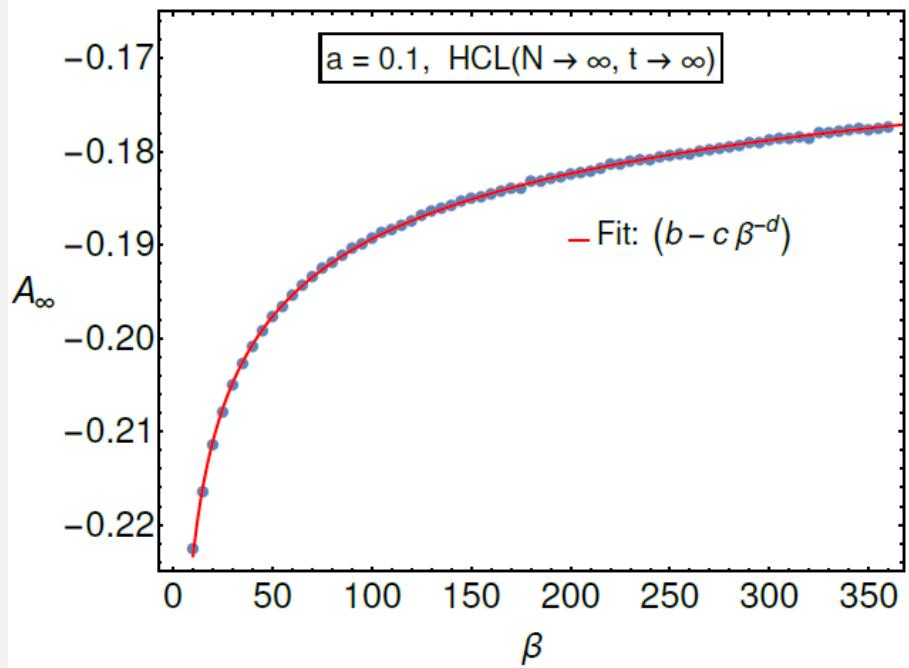
$$m_{D,\text{HCL}}^2 = \frac{g^2 T N_c}{2a} \left(\frac{3\Sigma}{2\pi} - 1 \right)$$

HCL plots, $A\phi$ ansatz



$$\text{Im}[V_{\text{cl}}^{(2)}(r)] = g^2 T A_\infty \phi(B m_D r)$$

HCL large- β



$$A_{\infty}^{\text{HCL}}(\beta) \approx -0.144 - 0.139 \beta^{-0.24}$$
$$B^{\text{HCL}}(\beta) \approx 1.548 + 0.12 e^{-0.025\beta},$$

Classical-Statistical Lattice Yang-Mills

SU(3) gauge theory, YM action:

$$S[A] = -\frac{1}{2} \int d^4x \operatorname{Tr} (F^{\mu\nu} F_{\mu\nu})$$

In terms of link variables

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]$$

Classical equations of motion:

$$U_j(t, \mathbf{x}) \approx \exp(igaA_j(t, \mathbf{x}))$$

$$U_j(t + a_t/2, \mathbf{x}) = \exp(ia_t a g E^j(t, \mathbf{x})) U_j(t - a_t/2, \mathbf{x}),$$

$$gE^j(t + a_t, \mathbf{x}) = gE^j(t, \mathbf{x}) - \frac{a_t}{a^3} \sum_{j \neq i} [U_{ij}(t - a_t/2, \mathbf{x}) + U_{i(-j)}(t - a_t/2, \mathbf{x})]_{\text{ah}}$$

Plaquette:

$$U_{ij}(\mathbf{x}) = U_i(\mathbf{x}) U_j(\mathbf{x} + \mathbf{a}_i) U_i^\dagger(\mathbf{x} + \mathbf{a}_j) U_i^\dagger(\mathbf{x})$$

Grigoriev and Rubakov (1988): Nucl. Phys. B 299 / Ambjorn and Krasnitz (1995): Phys. Lett. B 362 / Bödeker, McLellan, and Smilga (1995): Phys. Rev. D 52, 4675 / Arnold (1997): Phys. Rev. D 55, 7781 / Iancu (1998): hep-ph/9809535 / Moore and Rummukainen (2000) Phys. Rev. D 61, 105008 / Kurkela and Moore (2012): Phys. Rev. D 86, 056008

Setup

Solve the EOM with small time-steps: $a_t/a = 0.01$

Observables:

$$\langle O \rangle(t) \equiv \frac{1}{n} \sum_{j=1}^n O_j(t)$$

Berges et al 2014: Phys. Rev. D 89 (2014)
114007 [1311.3005]

Boguslavski et al 2018: Phys. Rev. D 98 (2018)
014006 [1804.01966]

Initialize such that:

$$g^2 \langle EE \rangle_T = g^2 T_0 , \quad g^2 \langle EE \rangle_L = 0$$

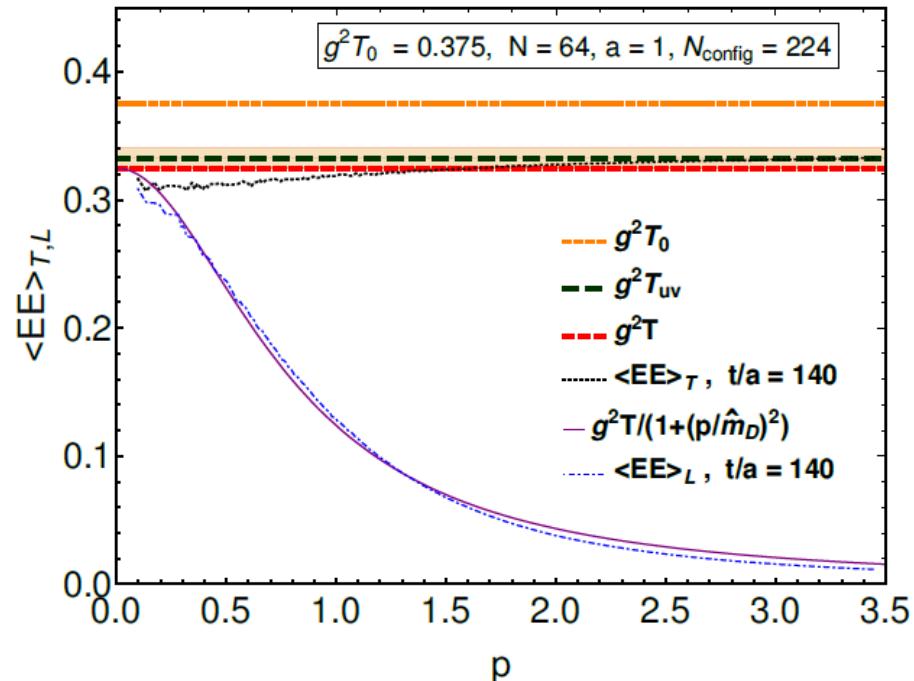
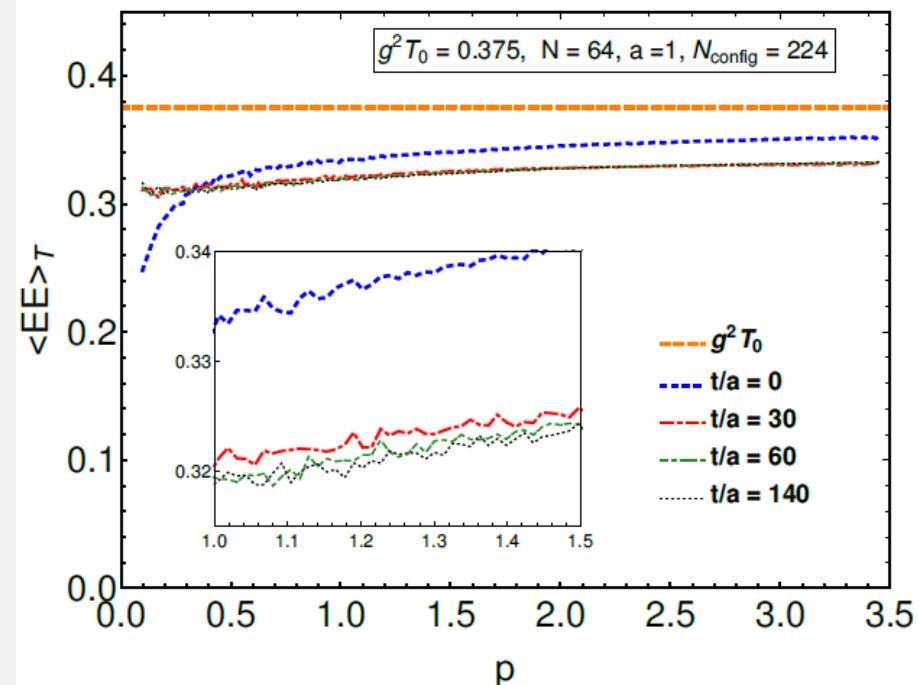
$$g^2 \langle AA \rangle_T = \frac{g^2 T_0}{p^2} , \quad g^2 \langle AA \rangle_L = 0$$

Related talks in ConfXIV
Boguslavski (Monday)
Peuron (Thursday)
Schuh (Thursday)

Temperature and Debye mass from chromo-electric field correlations:

$$g^2 \langle EE \rangle_T \approx g^2 T , \quad g^2 \langle EE \rangle_L \approx g^2 T \frac{\hat{m}_D^2}{p^2 + \hat{m}_D^2}$$

Thermalization, extraction of T and m_D



$$g^2 \langle EE \rangle_T \approx g^2 T , \quad g^2 \langle EE \rangle_L \approx g^2 T \frac{\hat{m}_D^2}{p^2 + \hat{m}_D^2}$$

$\text{Im}[V]$: nonperturbative

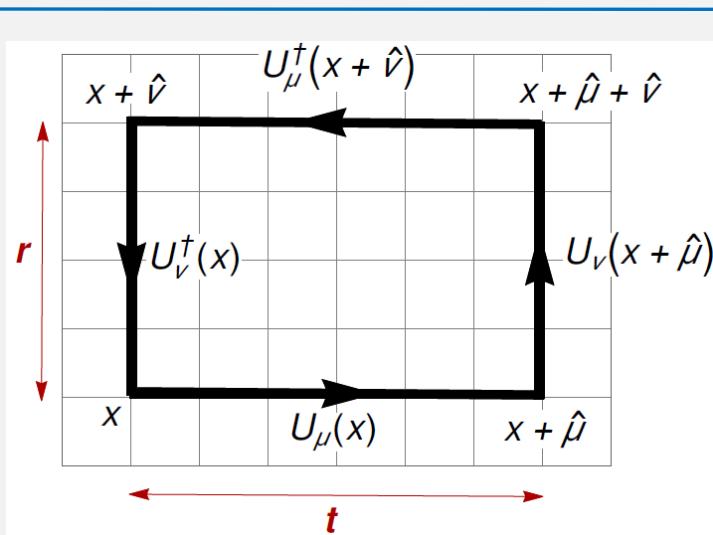
Following Laine et al 2007 [JHEP 03 (2007) 054 (hep-ph/0611300)]:

$$i\partial_t C_{\text{cl}}(t, r) = V_{\text{cl}}(t, r)C_{\text{cl}}(t, r)$$

Static limit:

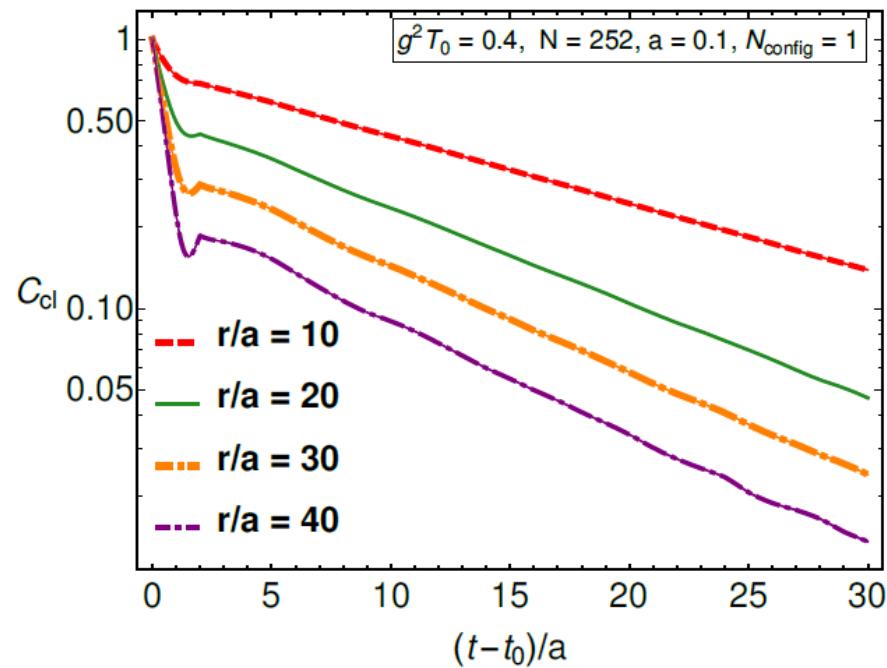
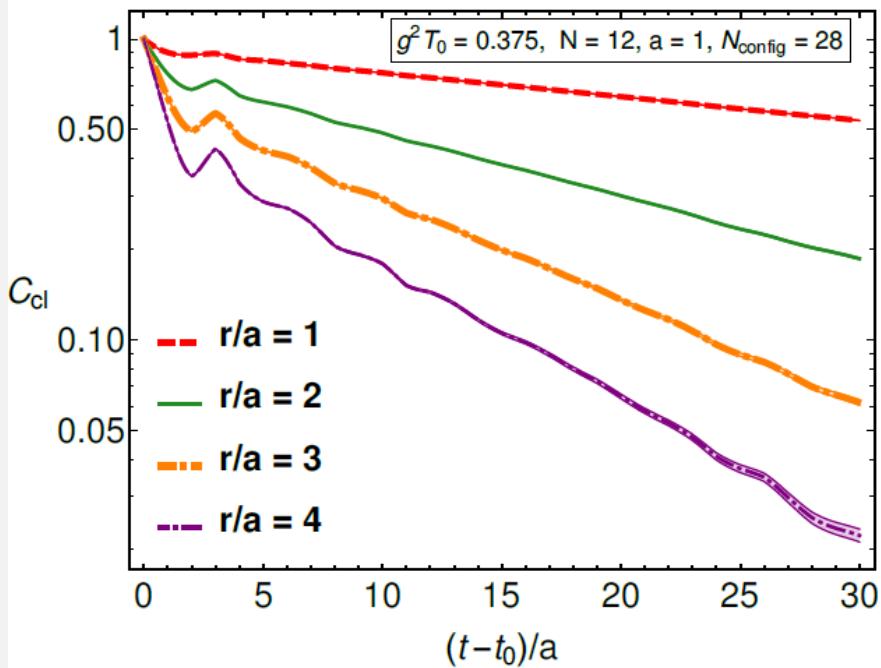
$$\text{Im}[V_{\text{cl}}(r)] \equiv \lim_{t \rightarrow \infty} \text{Im}[V_{\text{cl}}(t, r)]$$

With classical Wilson loop:

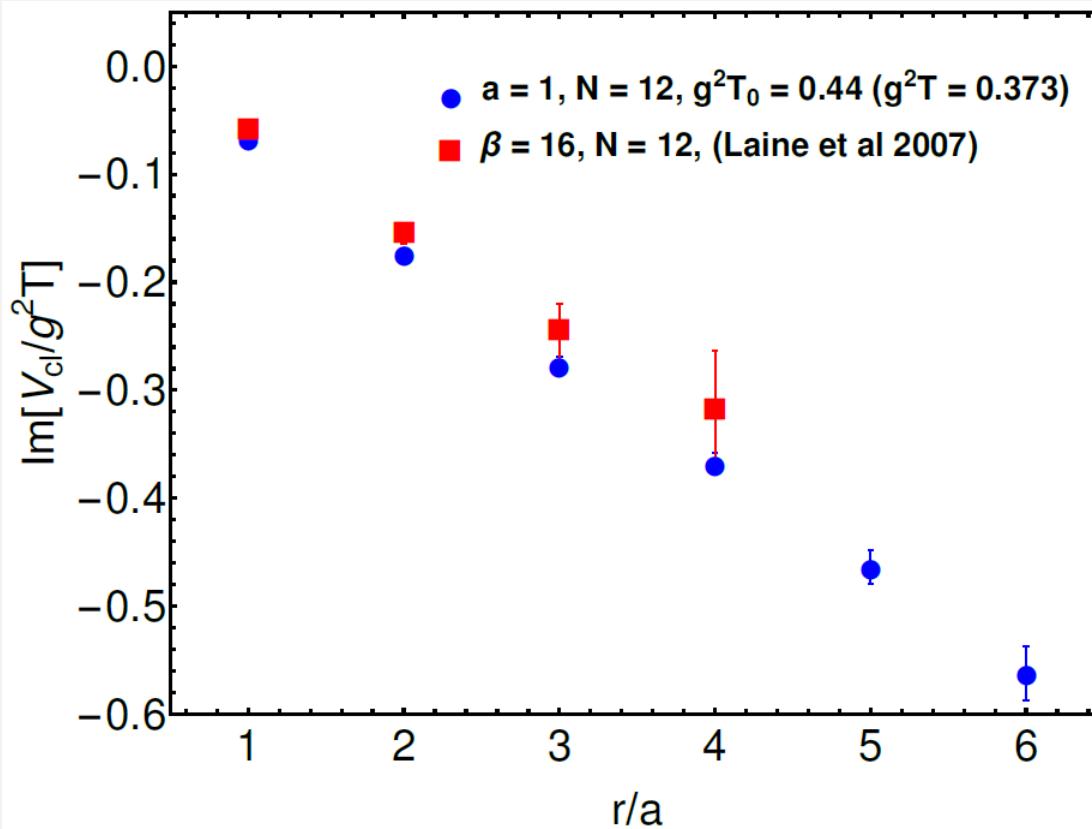


$$C_{\text{cl}}(t, r) \equiv \frac{1}{N_c} \text{Tr} \langle W[(t_0, \mathbf{x}); (t, \mathbf{x})] W[(t, \mathbf{x}); (t, \mathbf{0})] W[(t, \mathbf{0}); (t_0, \mathbf{0})] W[(t_0, \mathbf{0}); (t_0, \mathbf{x})] \rangle$$

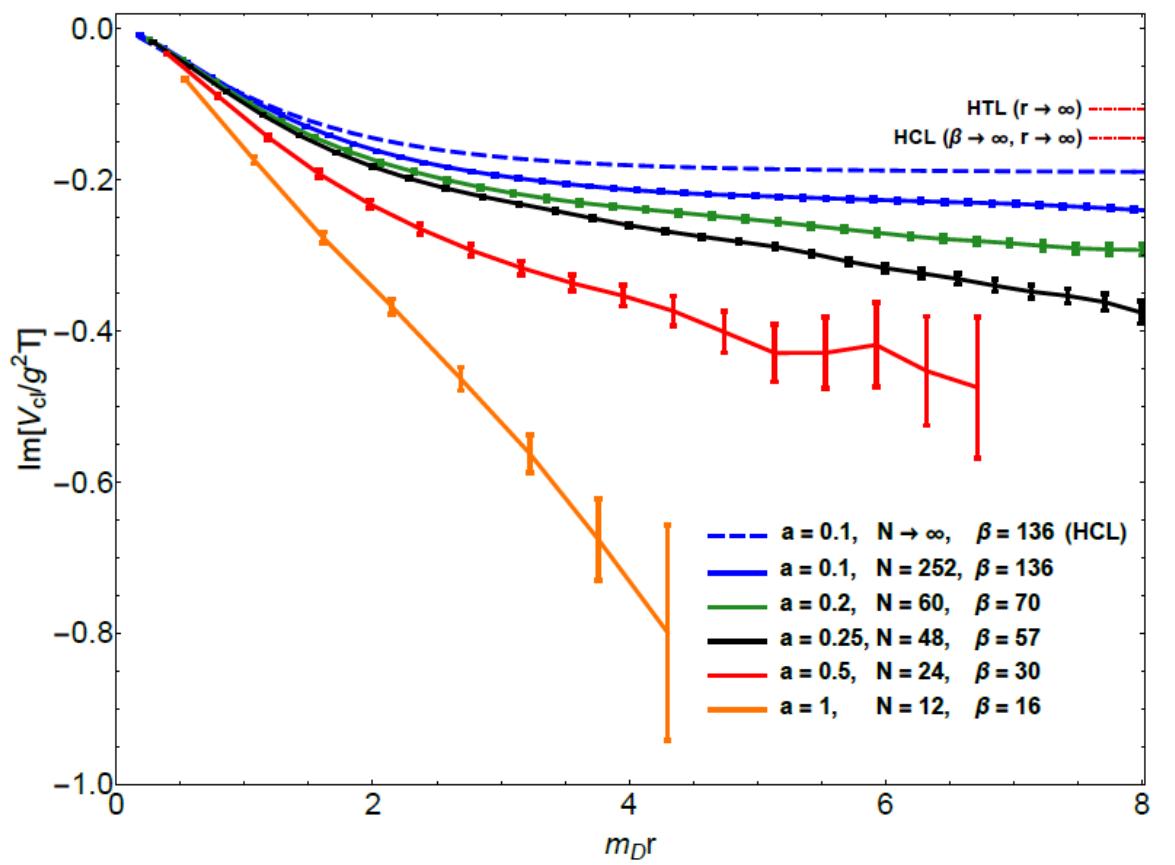
Evolution of Classical Wilson loop



$\text{Im}[V]/g^2T$, comparison with previous results



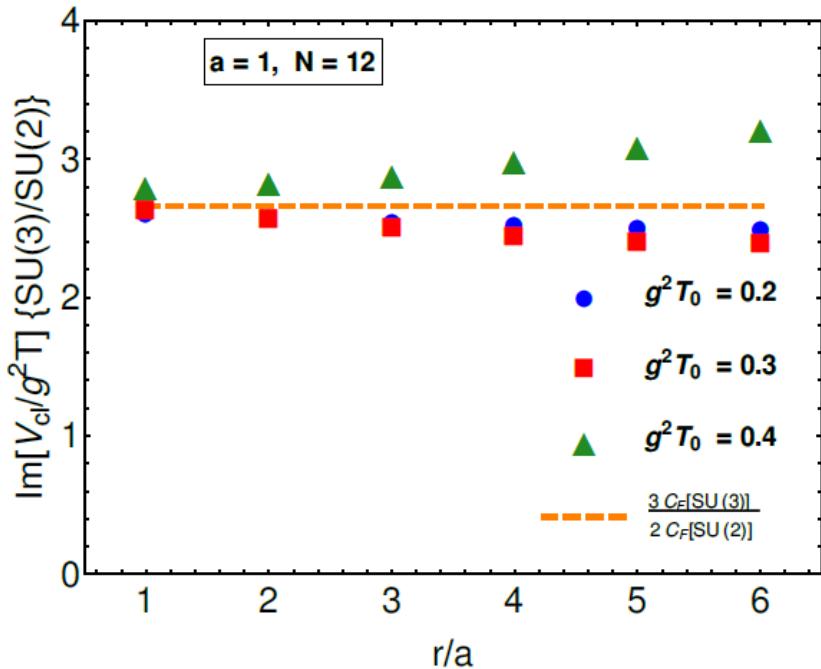
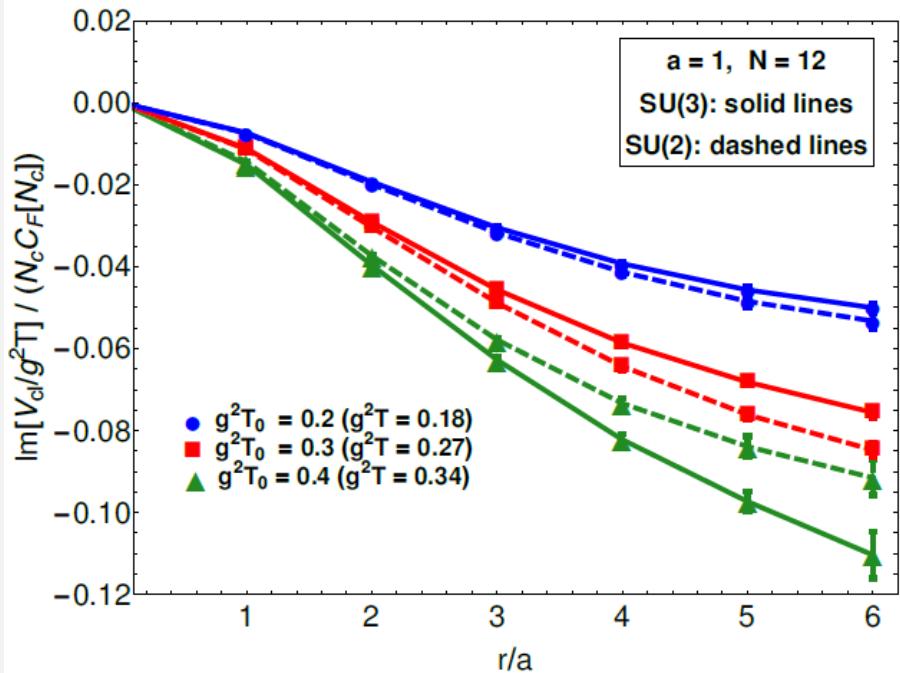
$\text{Im}[V]$: main results



Classical divergence in $\text{Im}[V]$ is moved into the Debye mass $m_D \sim 1/\sqrt{a}$

$$r_{\min} \ll 1/m_D \ll r_{\max}$$

SU(2) case



Momentum diffusion

Heavy-quark diffusion coefficient

$$\frac{d}{dt} \langle (\Delta p)^2 \rangle = \kappa$$

Langevin equation for momentum of heavy-quarks of mass M in a medium of temperature T ($M \gg T$)

$$\frac{dp_i}{dt} = -\eta_D p_i + \xi_i(t), \quad \langle \xi_i(t) \xi_j(t') \rangle = \kappa \delta_{ij} \delta(t - t')$$

$$\kappa = \frac{1}{3} \int_{-\infty}^{\infty} dt \sum_i \langle \xi_i(t) \xi_i(0) \rangle$$

See for example:

Moore and Teaney (2005): Phys. Rev. C 71, 064904

Francis et al (2015): Phys. Rev. D 92, 116003

Brambilla et al (2018): Phys. Rev. D 97, 074009

Related talks in ConfXIV

Altenkort (Wednesday)

Peuron (Thursday)

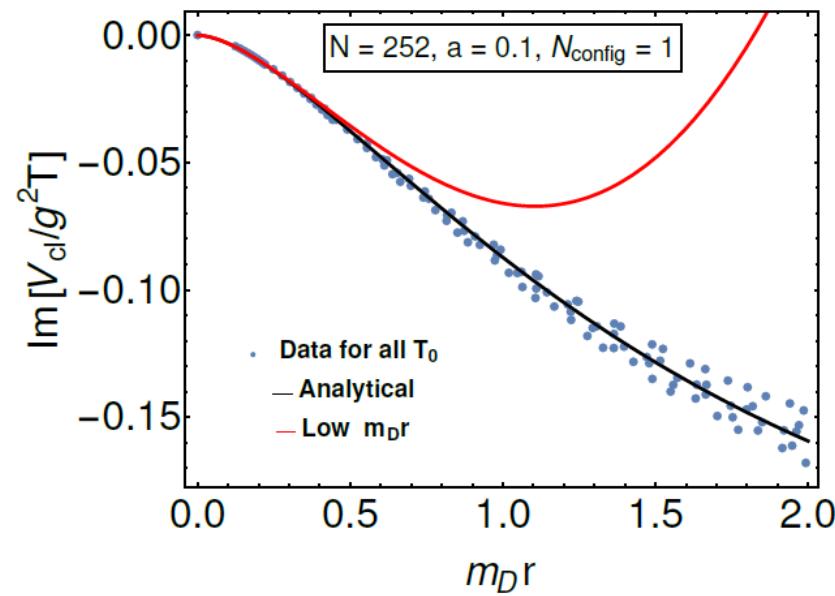
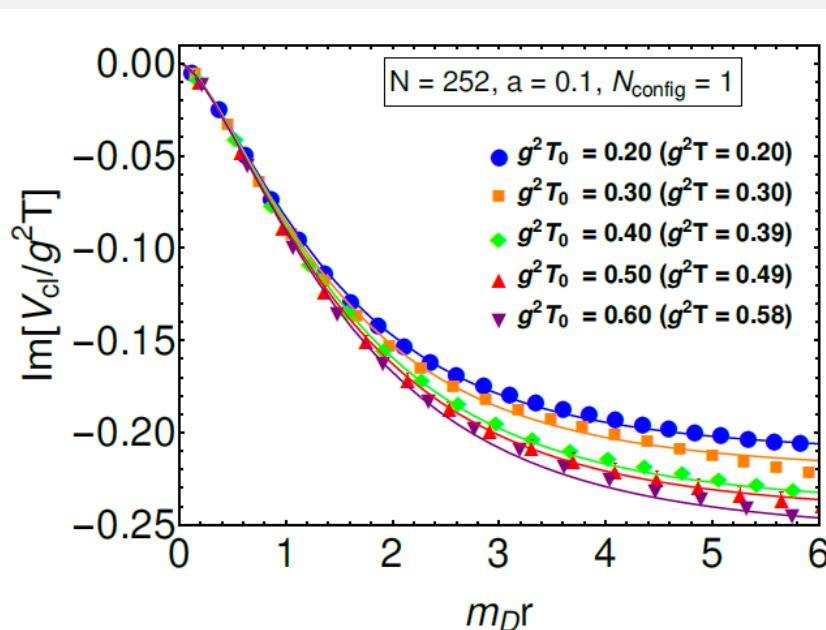
Shu (Friday)

Momentum diffusion from $\text{Im}[V]$

Low distance:

$$\text{Im}[V_{\text{cl}}^{(2)}(r)] \simeq -\frac{1}{2} \kappa r^2$$

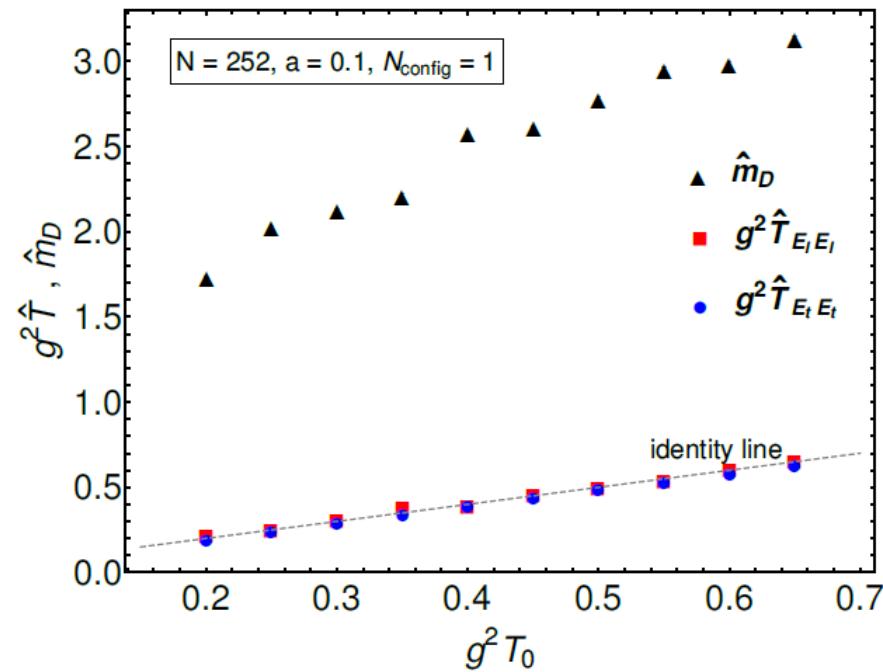
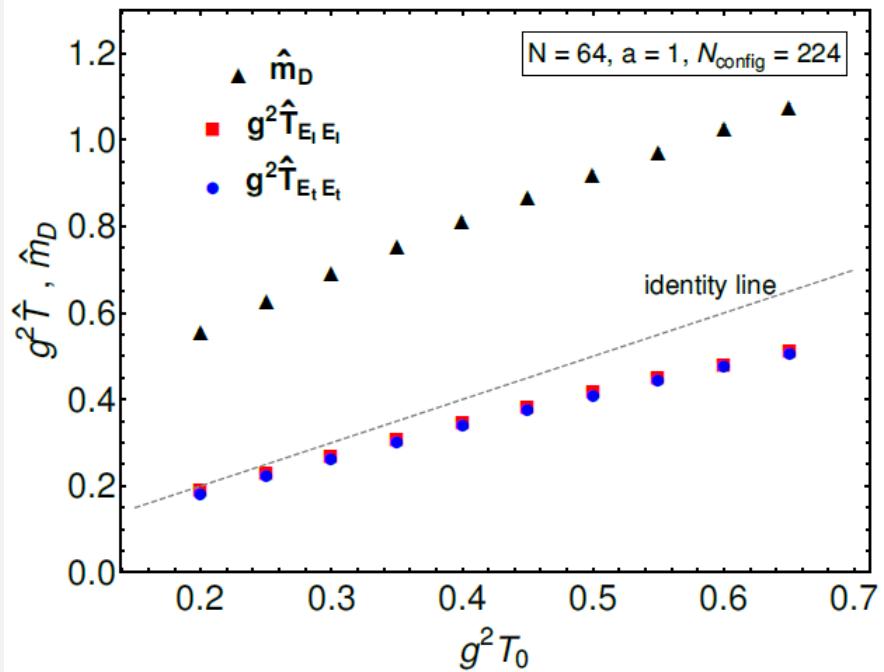
$$\kappa_V^{\text{latt}} \approx C_F g^2 T \hat{m}_D^2 \left(0.086 \log \left(\frac{T}{\hat{m}_D} \right) + 0.018 \right)$$



Summary and outlook

- Nonperturbative $\text{Im}[V]$, larger N , smaller a
- $A \phi$ ansatz
- κ extraction
- **Outlook: expanding and anisotropic cases**

Backup: $T(T_0)$



Backup: Low distance $\text{Im}[V]$

$$\text{Im}[V_{\text{cl}}(r)] \simeq -r^2 \frac{1}{9} |A_\infty| B^2 g^2 T m_D^2 (4 - 3\gamma - 3 \log(Bm_D r))$$

$$\simeq -\frac{1}{2} r^2 \left[\frac{2}{9} |A_\infty| B^2 g^2 T m_D^2 \left(3 \log\left(\frac{T}{m_D}\right) + 4 - 3\gamma - 3 \log\left(\frac{2B}{D}\right) - 3 \log\left(\frac{DrT}{2}\right) \right) \right]$$

$$D \sim \mathcal{O}(1)$$

$$r \approx 2/T$$



$$\text{Im}[V_{\text{cl}}^{(2)}(r)] \simeq -\frac{1}{2} \kappa_V r^2$$