Heavy quark diffusion in an overoccupied gluon plasma

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Heavy quarks & Transport far-from equilibrium

- Heavy quarks probe the entire history of the event, $M \gg$ other scales in the medium.

Recent interest in transport out-of-equilibrium:

- Fokker-Planck evolution of heavy quarks in a non-equilibrium gluon plasma:
  - Carrington, Czajka, Mrowczynski NPA 1001 (2020) 121914
  - Mrowczynski, EPJA 54 (2018) 3, 43
- Pre-equilibrium heavy quark transport coefficients:
  - R_{AA} and v_2: Das, Ruggieri, Scardina, Plumari, Greco J. Phys. G44(2017) no. 9 095102
  - Charm transport coefficients: Song, Moreau, Aichelin, Bratkovskaya PRC 101 (2020) 4, 044901
- Anisotropic momentum broadening in the 2+1D Glasma
  - Ipp, Müller, Schuh PLB 810 (2020) 135810

This talk: Heavy quark diffusion in pre-equilibrium QGP

Pre-equilibrium dynamics & classical field simulations

- Gluon $f \sim 1/g^2 \rightarrow$ classical $\rightarrow$ real time lattice simulations.
- Assumptions: M large, measure force felt by a "test particle".
- Non-thermal fixed points and self-similar scaling
- Dynamically generated hierarchy of scales reminiscent of PT:
  - Coupling $g \iff$ time t
  - Temperature $T \iff$ hard scale $\Lambda \sim t^{1/7}$
  - Soft scale $gT \iff$ mass scale $\omega_{\text{pl}} \sim t^{-1/7}$
  - Ultrasoft scale $g^2T \iff$ magnetic scale $m_{\text{mag}} \sim t^{-1/3}$

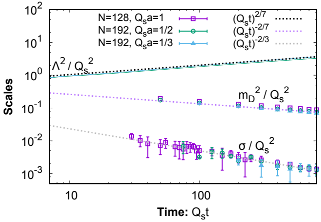


Figure: Schlichting et. al. PRD 93 (2016) 7, 074036.

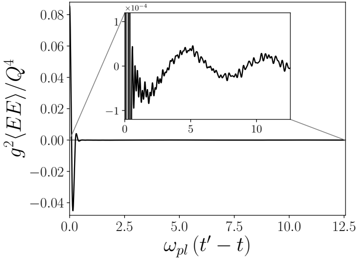
Heavy quark in a color field

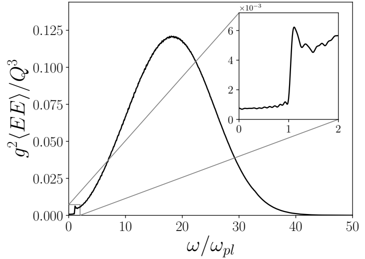
Classical equation of motion

$$\dot{p}_i(t) = \mathscr{F}_i(t)\,. \tag{1}$$

The chromoelectric force has $\langle \dot{p} \rangle = 0$ but nonzero variance

$$\langle \dot{p}_i(t)\dot{p}_i(t')\rangle = \frac{g^2}{N_c}\mathrm{Tr}\langle E_i(t)U_0(t,t')E_i(t')U_0(t',t)\rangle = \frac{g^2}{2N_c}\langle EE\rangle(t,t'). \tag{2}$$

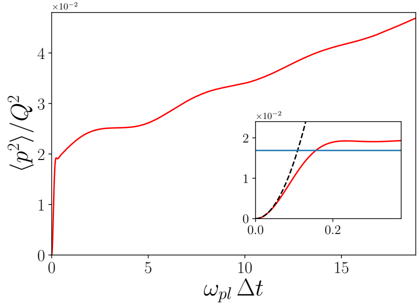




- RHS fig: sharp rise at $\omega = \omega_{pl}$ QP contribution. Intercept with y-axis $\approx \kappa$. Landau damping of longit. gluon fields.

Momentum broadening

$$\langle p^2(t,\Delta t)\rangle = \frac{g^2}{2N_c}\int_t^{t+\Delta t} \mathrm{d}t' \int_t^{t+\Delta t} \mathrm{d}t'' \langle EE\rangle(t',t'')\,. \qquad (3)$$



3 distinct features

- Rapid growth $\Delta t \approx 2\pi/\Lambda$, broad peak dominates the integration
- Damped oscillations $\Delta t \approx 2\pi/\omega_{pl}$, new feature! Transverse QP excitations.
- Linear growth in Δt, $1/\Lambda \ll \Delta t \ll t$, consistent with Langevin description.

Heavy quark diffusion vs. momentum broadening

Define $\kappa(t, \Delta t)$ as

$$3\kappa(t,\Delta t) = \frac{\mathrm{d}}{\mathrm{d}\Delta t}\langle p^2(t,\Delta t)\rangle \tag{4}$$

As in thermal equilibrium, diffusion coefficient $\kappa(t)$ defined at $\Delta t \to \infty$ limit

$$\frac{g^2}{2N_c}\langle EE\rangle(t,\omega=0) \;=\; 3\,\kappa_\infty(t)\,. \tag{5}$$

On the lattice $\kappa(t, \Delta t)$ given by

$$\kappa(t,\Delta t) \approx \frac{g^2}{3N_c}\int_t^{t+\Delta t}\mathrm{d}t'\int\frac{\mathrm{d}^3x}{V}\langle E_i^a(t,\mathbf{x})E_i^a(t',\mathbf{x})\rangle, \tag{6}$$

To identify $\kappa(t,\Delta t) = \kappa(t)$ we want to have $t \gg \Delta t \gg 1/\gamma_{pl}$, where γ_{pl} is the largest inverse lifetime of QP excitations.

Understanding the Δt dependence in $\kappa(t, \Delta t)$

$\kappa(t, \Delta t)$ oscillates in Δt with the frequency ω_{pl}

$$\omega_{pl}^2 = \frac{4}{3} N_c \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \frac{g^2 f(t,p)}{\omega(p)} \sim t^{-2/7} \tag{7}$$

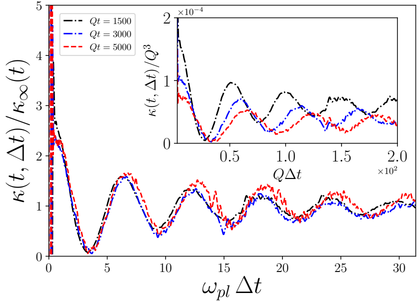


Figure: Lattice extraction of $\kappa(t, \Delta t)$

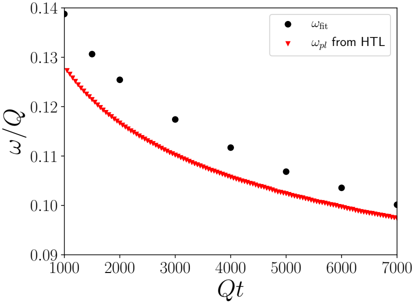


Figure: Frequency extracted from $\kappa(t, \Delta t)$ and using (7)

Spectral Reconstruction (SR) method

$$3\kappa(t,\Delta t) = \frac{g^2}{N_c}\int_{-\infty}^{\infty}\frac{\mathrm{d}\omega}{2\pi}\,\frac{\sin(\omega\,\Delta t)}{\omega}\int\frac{\mathrm{d}^3p}{(2\pi)^3}\langle EE\rangle(t,\omega,p) \qquad (8)$$

Generalized FDR (PRD 98 (2018) 1, 014006):

$$\langle EE\rangle_{T,L}(t,\omega,p) \approx \dot{\rho}(t,\omega,p)\langle EE\rangle_{T,L}(t,t,p) \qquad (9)$$

$$\dot{\rho}(\omega,p) = 2\mathfrak{Im}G_R^{\mathrm{HTL}}(\omega,p) \qquad (10)$$

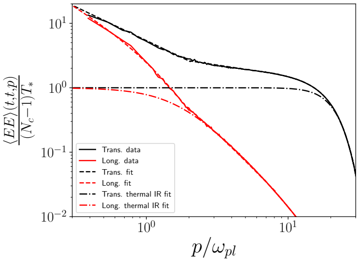
$\langle EE\rangle$: HTL $\neq$ far-from-equilibrium

- HTL: IR is thermal $\langle EE\rangle_T \approx T_* \rightarrow f(p) \approx \frac{T_*}{\sqrt{m^2+p^2}}$
- Far-from-equilibrium: IR enhanced compared to thermal $\langle EE\rangle_T \gg T_*$

$\dot{\rho}$ from HTL, use our data for QP properties:

- Dispersion $\omega(p)$
- Damping rate $\gamma(p)$.

Spectral Reconstruction (SR) method cont.



- Gauge choice: $A^0 = 0\ \forall t$. At readout times $\nabla \cdot \boldsymbol{A}(t) = 0$.

At the $\Delta t \to \infty$ limit only longitudinal Landau damping contributes. Using the thermal IR assumption

$$\kappa^{\mathrm{SR}}_{\infty,LL}(t) \approx \frac{N_c^2 - 1}{12\pi N_c} m_D^2(t) g^2 T_*(t) \log\left(\frac{\Lambda(t)}{m_D(t)}\right) \sim (Qt)^{-5/7} \log(Qt) \tag{11}$$

This will be our expectation for the time dependence of $\kappa(t)$.

Kinetic theory (generalizing PRC 71 (2005) 064904)

In the kinetic theory framework κ is given by ($gq \to gq$, t-channel gluon exchange, Compton amplitude suppressed)

$$\kappa(\omega) = \frac{\langle \Delta k^2 \rangle}{\Delta t} = \frac{1}{6M} \int \frac{\mathrm{d}^3\boldsymbol{k}\,\mathrm{d}^3\boldsymbol{q}}{(2\pi)^6\, 8|\boldsymbol{k}||\boldsymbol{k}+\boldsymbol{q}|M} 2\pi\delta(|\boldsymbol{k}+\boldsymbol{q}| - |\boldsymbol{k}| - \omega) \times \boldsymbol{q}^2 |\mathscr{M}|^2_{\text{gluon}}(\omega) f(k) f(|\boldsymbol{k}+\boldsymbol{q}|) \tag{12}$$

k and k' gluon momenta, $q = k - k'$, p and p' incoming and outgoing heavy quark momenta.

$$|\mathscr{M}(\omega)|^2_{\text{gluon}} = \left[N_c C_H g^4\right] \frac{4M^2(k_0 + k_0')^2\left(1 + \cos^2\theta_{\boldsymbol{k}\boldsymbol{k}'}\right)}{(q^2 - \omega^2 + m_D^2)^2}, \tag{13}$$

with $k_0' = k_0 + \omega$. The limit $\Delta t \to \infty$ obtained at $\omega \to 0$ limit.

Understanding the oscillations in $\kappa(t, \Delta t)$

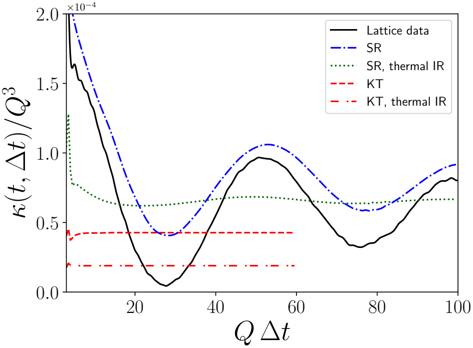
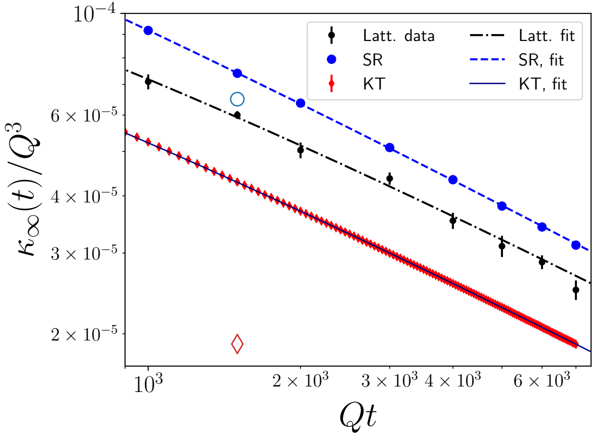


Figure: $\kappa(Qt = 1500, \Delta t)$.

- IR enhancement $\rightarrow$ similar oscillations as on the lattice. Frequency: Trans. QP modes, Offset: Long. Landau damping.
- KT: no oscillations. Trivialized frequency structure.
- Lattice vs. IR enhanced SR model $\rightarrow$ IR enhancement exists ($\kappa(t, \Delta t)$ is gauge invariant).

$\kappa(t)$ - time-evolution

- Expectation as (11) $\kappa(t) \approx A t^{-5/7}(\log(t) + B)$, use as a fit.
- Closed symbols: IR enhancement, open symbols: Thermal IR.



Conclusions

We have

- Extracted heavy quark momentum diffusion coefficient κ, t-evol consistent with $\kappa \sim t^{-5/7} \log(t)$, predicted by HTL and self-similarity.
- Observed IR enhancement (relative to thermal case) in equal time correlation functions $\rightarrow$ interesting oscillations in $\langle p^2 \rangle(\Delta t)$ and $\kappa(\Delta t)$ with frequency ω_{pl}.
- Heavy quarks, quarkonia, jets may reveal IR dynamics of non-equilibrium QGP.

Future plans

- Extract κ during the bottom-up thermalization scenario using EKT.

Breaking down the different HTL contributions

- Transverse QP contributions → oscillations at ω_{pl}.
- Longitudinal Landau cut → κ_∞

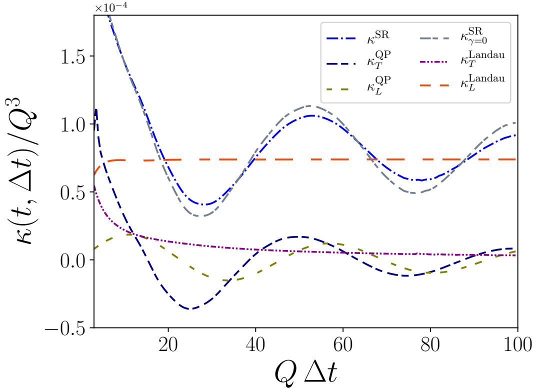


Figure: Different HTL contributions

Comparison with thermal case

Compare far-from-equilibrium to equilibrium by defining an effective coupling via scale separation:

$$\epsilon \sim T^4, \quad m_D^2 \sim g^2 T^2 \quad \rightarrow \frac{m_D^2}{\sqrt{\epsilon}} \sim g_\epsilon^2 \tag{14}$$

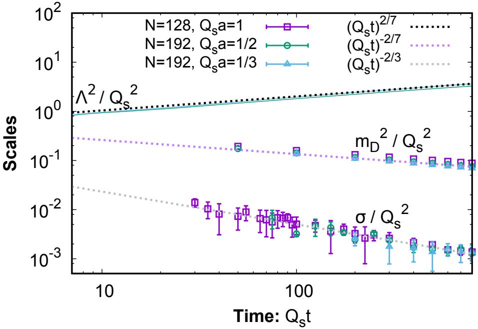


Figure: Schlichting et. al. Phys.Rev.D 93 (2016) 7, 074036. Time evolution in self-sim. regime $\longleftrightarrow$ decreasing g in pert. theory

Comparison with thermal case

Expressing everything in tems of g:

$$\kappa^{\text{therm}} \approx 0.016\left(\ln\frac{1}{g} + 0.25\right) g^4 \varepsilon^{3/4} \qquad (15)$$

$$\kappa_\infty(t) \approx 0.0050\left(\ln\frac{g}{\tilde{g}_\varepsilon^2} + 0.148\right) \tilde{g}_\varepsilon^5\, g^{-1} \varepsilon^{3/4} \qquad (16)$$

$$\kappa_\infty(t) \approx 0.0047\left(\ln\frac{1}{\tilde{g}_\Lambda} + 0.177\right) \tilde{g}_\Lambda^{5/2}\, g^{3/2} \varepsilon^{3/4}. \qquad (17)$$

- $g_\epsilon, g_\Lambda \gg g$ for overoccupied system. Extrapolate to $f \sim 1$ to compare to thermal.
- Coefficient of κ^{therm} larger than far-from equilibrium: IR enhancement increases m_D more than it increases κ.

SR method and generalized Fluctuation Dissipation Relation

Generalized FDR: connecting spectrum and statistical properties (Boguslavski, Kurkela, Lappi, JP: Phys.Rev.D 98 (2018) 1, 014006):

$\langle EE\rangle_{T,L}(t,\omega,p) \approx \overbrace{\langle[\hat{E},\hat{A}]\rangle(t,\omega,p)}^{\dot{\rho}(t,\omega,p)}\langle EE\rangle(t,t,p)$. The spectral function $\dot{\rho}$ is defined as

$$\dot{\rho}(\omega,p) = 2\Im\mathfrak{m}G_R^{HTL}(\omega,p), \tag{18}$$

with $G_R^T = \dfrac{-1}{\omega^2 - p^2 - \Pi_T(\omega,p)}$ and $G_R^L = \dfrac{p^2}{\omega^2}\dfrac{-1}{p^2 - \Pi_L(\omega,p)}$

$$\Pi_T(x) = m^2 x\left(x + (1-x^2)Q_0(x)\right)$$
$$\Pi_L(x) = -2m^2\left(1 - x\,Q_0(x)\right), \tag{19}$$

and $(x = \omega/p)$

$$Q_0(x) = \frac{1}{2}\ln\frac{x+1}{x-1} = \frac{1}{2}\ln\left|\frac{x+1}{x-1}\right| - \frac{i\pi}{2}\theta(1-x^2). \tag{20}$$

Spectral reconstruction method, quasiparticles and Landau damping

2 types of contributions:

- Particle-like excitations, at LO proportional to $\delta(\omega - \omega_{T,L})$.
- Landau damping contributions, nonzero only for $\theta(1-\omega^2/p^2)$, damping of space charge waves in plasma.

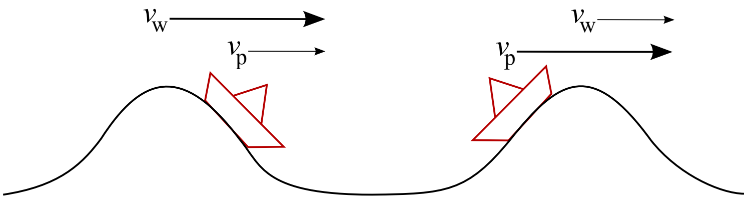


Figure: (From Wikipedia) Illustration of the Landau damping

$\Delta t \to \infty$ limit in the SR framework

At the $\Delta t \to \infty$ limit only longitudinal Landau damping contributes. Using the thermal IR assumption

$$\langle EE\rangle_L^{LL}(t,t,p) = (N_c^2-1)\, T_*\, \frac{m_D^2}{p^2+m_D^2}\, \theta(\Lambda - p)\,. \tag{21}$$

We get

$$\kappa^{\mathrm{SR}}_{\infty,LL}(t) \approx \frac{N_c^2-1}{12\pi N_c}\, m_D^2(t) g^2 T_*(t) \log\left(\frac{\Lambda(t)}{m_D(t)}\right) \sim (Qt)^{-5/7} \log(Qt) \tag{22}$$

This will be our expectation for the time dependence of $\kappa(t)$.