Quarkonium production in soft gluon factorization

Yan-Qing Ma

Peking University

References:
1703.08402: YQM, Chao
1911.05886: Li, Feng, YQM
2005.08786: Chen, YQM
2103.15121: Chen, Jin, YQM, Meng

A Virtual Tribute to Quark Confinement and the Hadron Spectrum 2021
2021/08/02-07

Outline

I. NRQCD factorization

II. Soft gluon factorization

III. Application to FF: $g \to Q\bar{Q}({}^3S_1^{[8]}) + X$

IV. Summary and outlook

NRQCD: factorization

➢ Factorization formula

Bodwin, Braaten, Lepage, 9407339

$$(2\pi)^3 2P_H^0 \frac{d\sigma_H}{d^3P_H} = \sum_n d\hat{\sigma}_n(P_H)\langle \mathcal{O}_n^H \rangle$$

Production of a heavy quark pair

Hadronization (LDMEs)

- n: quantum numbers of the pair: color, spin, orbital angular momentum, total angular momentum, spectroscopic notation ${}^{2S+1}L_J^{[c]}$

➢ A glory history

- Solved IR divergences in P-wave quarkonium decay
- Explained ψ' surplus
- Explained χ_{c2}/χ_{c1} production ratio
- ....

Thanks to color-octet mechanism

NRQCD: pheno difficulties

➢ Polarization puzzle (th. trans. VS exp. unpol.)

- J/ψ: transverse polarization largely canceled between ${}^3S_1^{[8]}$ and ${}^3P_J^{[8]}$
- ψ': cancelation is weak, still hard to understand

Chao,YQM,Shao,Wang, Zhang,1201.2675
Bodwin, Chung, Kim, Lee, 1403.3612

See also Pietro Faccioli's talk

➢ Hierarchy problem

- Best fit of J/ψ hadroproduction data at high p_T:

$$M_0 = \langle O\left({}^1S_0^{[8]}\right)\rangle + 3.9\,\langle O\left({}^3P_0^{[8]}\right)\rangle/m_c^2 \approx 0.074\,\mathrm{GeV}^3$$

YQM, Wang, Chao, 1009.3655

$$M_1 = \langle O\left({}^3S_1^{[8]}\right)\rangle - 0.56\,\langle O\left({}^3P_0^{[8]}\right)\rangle/m_c^2 \approx 0.0005\,\mathrm{GeV}^3$$

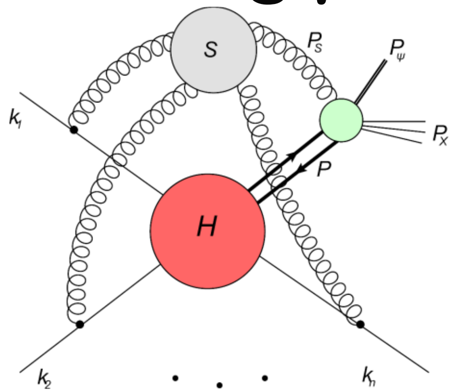
- Velocity scaling rule: $M_0 \sim M_1$

➢ Universality problem

- Upper bound of M_0 from e^+e^- collision: $M_0 < 0.02\,\mathrm{GeV}^3$ Zhang, YQM, Wang, Chao, 0911.2166
- Global fit of LDMEs: $\chi^2_{\mathrm{d.o.f.}} = 725/194 = 3.74$ Butenschoen, Kniehl, 1105.0820

Theoretical challenges: power corr.

- **Soft gluon emission in color-bleaching process**
  - P_ψ **is different from** P, $P = P_\psi[1 + O(\lambda)]$
  - **NRQCD expand** P **around** P_ψ



YQM, Vogt, 1609.06042

- **Bad convergence of NRQCD expansion**
  - **Cross section approximately** $\propto P^{-4} = P_\psi^{-4}[1 + O(\lambda)]^{-4}$

$$\int_{-1}^{1} \frac{d\cos\theta}{2(1+\lambda+\lambda\cos\theta)^4} = 0.42$$

$$= 1 - 4\lambda + 40/3\lambda^2 - 40\lambda^3 + \cdots$$

$$= 1 - 1.2 + 1.2 - 1.08 + 0.91 - 0.73 + \cdots$$

With $\lambda \approx v^2 \approx 0.3$

Mangano, Petrelli, 9610364

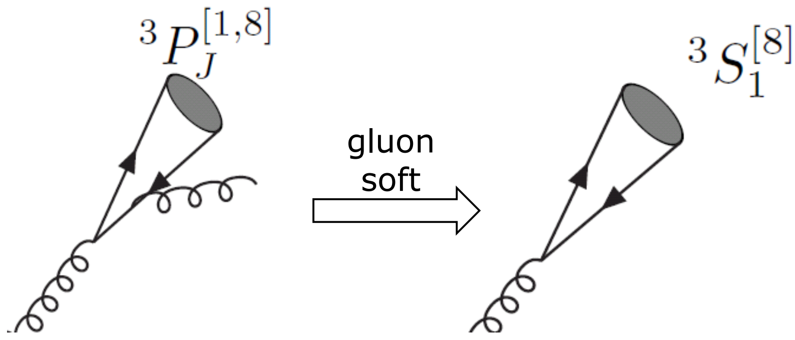
- **Solution: soft gluon momentum should be kept, but not expanded. Resummation of power corrections (or relativistic corrections).**

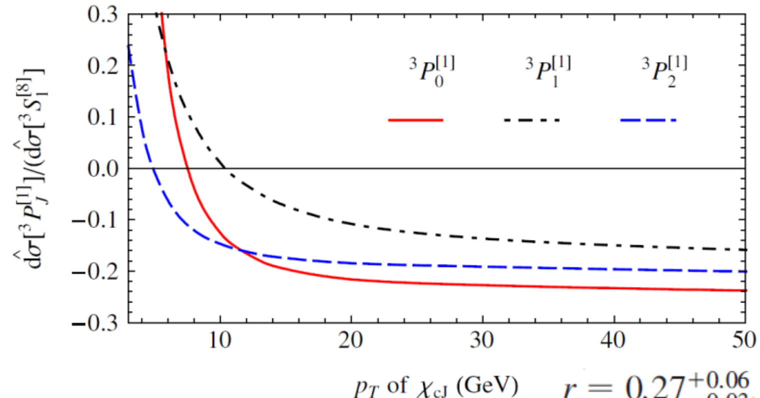
Theoretical challenges: over subtraction

Braaten, Chen, 9610401
YQM, Wang, Chao, 1002.3987

- **Eg. χ_{cJ} production:**

$$d\sigma_{\chi_{cJ}}/(2J+1) \approx d\hat{\sigma}_{^3P_J^{[1]}} \langle O\left(^3P_0^{[1]}\right)\rangle + d\hat{\sigma}_{^3S_1^{[8]}} \langle O\left(^3S_1^{[8]}\right)\rangle$$





$r = 0.27^{+0.06}_{-0.03}$

- **Soft gluon in P-wave: factorized to S-wave matrix element**
- **Subtraction scheme: at *zero momentum*, which contributes the largest production rate. Over subtracted! P-wave negative!**
- **Big cancellation between S-wave and P-wave! Perturbation unstable**
- **Solution: soft gluon momentum should be kept during subtraction process**

Theoretical challenges: threshold region

➢ At threshold region

- Large logarithms appear: can be resummed by introducing shape functions

  Beneke, Rothstein, Wise, 9705286
  Fleming, Leibovich, Mehen, 0306139
  Leibovich, Liu, 0705.3230

  See also Sean Fleming's talk

- Soft gluon momentum: has leading contribution for quarkonium momentum distribution, cannot be ignored

➢ Combination of logs resummation and powers resummation is needed

  - Keep soft gluon momentum unexpanded is the first step.

Summary of NRQCD factorization

- **Rigorousness**
  - Based on EFT of QCD: NRQCD
  - Factorization has been tested to NNLO

Nayak, Qiu, Sterman, 0509021
Bodwin, Chung, Ee, Kim, Lee, 1910.05497
Zhang, Meng, YQM, Chao, 2011.04905

- **Color-octet mechanism: great success in solving theoretical issues and explaining data**
- **Color-octet mechanism: final-state radiation of soft gluons results in large power and log corr.**
  - Should be responsible for phenomenological failures
- **Soft gluon factorization: resum a dominant series of power corrections (kinematic effects) and log corrections**

Outline

I. NRQCD factorization

II. Soft gluon factorization

III. Application to FF: $g \to Q\bar{Q}({}^3S_1^{[8]}) + X$

IV. Summary and outlook

Power corrections in NRQCD fac.

➢ Dominant power (relativistic) corrections

- Equations of motion of NRQCD EFT: $\left(iD_0 - \frac{\boldsymbol{D}^2}{2m} + \cdots\right)\psi = 0$

  For kinematic effects, ignore gluon field and replace D by ∇

- NRQCD factorization: use EOM to remove ∇_0, leaving operators like:

$$\chi^\dagger\psi \qquad \chi^\dagger \overleftrightarrow{\nabla}^2\psi \qquad \nabla^2(\chi^\dagger\psi)$$

Leading term

Relative (spatial) momentum

Total (spatial) momentum

Power (relativistic) corrections

Warning: needs proper gluon fields to make them gauge invariant

➢ Other power corrections in NRQCD fac.

- Usually less important, can also be calculated (ignored in this talk)

$$\chi^\dagger(g\boldsymbol{E}\cdot\boldsymbol{\sigma})\psi \;✘$$

SGF: exclusive processes

➢ Different way to use EOM

Li, Feng, YQM, 1911.05886
Chen, YQM, 2005.08786

- Use EOM to remove relative derivatives, leaving only total derivatives:

$$\langle 0|\nabla_0^{n_1}\nabla^{2n_2}\left(\chi^\dagger\psi\right)|H\rangle$$

the same degrees of freedom as NRQCD factorization

➢ Using integration by parts

- Remove operators unless $n_1 = n_2 = 0$

➢ Factorization

$$\mathcal{A}^Q = \sum_n \hat{\mathcal{A}}^n \overline{R}_Q^{n*} \qquad \overline{R}_Q^{n*} = \langle 0|[\overline{\Psi}\mathcal{K}_n\Psi](0)|Q\rangle_S$$

- Matching coefficients are functions of quarkonium mass

"S": field operators are in small momentum regions

SGF: inclusive processes

- Use EOM to remove relative derivatives

$$\langle H+X|\nabla_0^{n_1}\nabla^{2n_2}(\chi^\dagger\psi)|0\rangle$$

- Using integration by parts

YQM, Chao, 1703.08402
Chen, YQM, 2005.08786

  - Remove operators unless $n_1 = n_2 = 0$
  - Matching coefficients are functions of: $P_H^2, P_H\cdot P_X, P_X^2$

- Factorization

$$(2\pi)^3 2P_H^0 \frac{d\sigma_H}{d^3P_H} \approx \sum_n \int \frac{d^4P}{(2\pi)^4} \mathcal{H}_n(P) F_{n\to H}(P, P_H)$$

  - $n = {}^{2S+1}L_J^{[c]}$
  - P: momentum of $Q\bar{Q}$
  - $\mathcal{H}_n$: perturbatively calculable hard parts
  - $F_{n\to H}$: nonperturbative soft gluon distributions (SGDs)
  - UV renormalization scale is suppressed

Soft gluon distributions (SGDs)

➢ Operator definition

- **Expectation values of bilocal operators in QCD vacuum**

$$F_{n\to H}(P,P_H)=\int d^4b e^{-iP\cdot b}\langle 0|[\overline{\Psi}\mathcal{K}_n\Psi]^\dagger(0)\left(a_H^\dagger a_H\right)[\overline{\Psi}\mathcal{K}_n\Psi](b)|0\rangle_S$$

with

$$a_H^\dagger a_H=\sum_X\sum_{J_z^H}|H+X\rangle\langle H+X|$$

$$\mathcal{K}_n(rb)=\frac{\sqrt{M_H}}{M_H+2m}\frac{M_H+\not{P}_H}{2M_H}\Gamma_n\frac{M_H-\not{P}_H}{2M_H}\mathcal{C}^{[c]}$$

Spin project operators: $\Gamma_n=\sum_{L_z,S_z}\langle L,L_z;S,S_z|J,J_z\rangle\Gamma^o_{LL_z}\Gamma^s_{SS_z}$

Color project operators:

$$\mathcal{C}^{[1]}=\frac{\mathbf{1}_c}{\sqrt{N_c}}\qquad\qquad \mathcal{C}^{[8]}=\sqrt{2}t^{\bar{a}}\,\Phi^{(A)}_{a\bar{a}}(rb)$$

Soft gluon distributions (SGDs)

➢ Gauge link

$$\Phi^{(A)}(rb) = \mathcal{P}\exp\left\{-ig_s\int_0^\infty d\lambda\, b_\ell\cdot A^{(A)}(r\,b+\lambda\, b_\ell)\right\}$$

$$b_\ell^\mu = b^\mu + \varepsilon\ell^\mu \qquad\qquad 0 < \varepsilon \ll 1$$

- **When b is finite, gauge link along b direction (avoid gauge-link-collinear divergence)**
- **When $b\to 0$, gauge link unambiguously along l direction (agree with gauge-completed NRQCD matrix elements)**

Nayak, Qiu, Sterman, 0509021

➢ Evaluated in ***small*** region

- **Subscript "S": evaluate the matrix element in the region where off-shellness of all particles is much smaller than heavy quark mass**

➢ RGEs

Chen, Jin, YQM, Meng, 2103.15121

$$\frac{d}{d\ln\mu_f}F_{[L'\tilde{L}',\lambda']\to H}(z,M_H,m_Q,\mu_f)=\sum_{L,\tilde{L},\lambda}\int_z^1\frac{dx}{x}\boldsymbol{K}^{[L\tilde{L},\lambda]}_{[L'\tilde{L}',\lambda']}(\hat{z},M_H/x,m_Q,\mu_f)$$
$$\times F_{[L\tilde{L},\lambda]\to H}(x,M_H,m_Q,\mu_f),$$

- **Evolution kernels**

$$\boldsymbol{K}^{[L\tilde{L},\lambda],LO}_{[L'\tilde{L}',\lambda']}(\hat{z},M_H/x,m_Q,\mu_f)=\frac{d}{d\ln\mu_f}F^{NLO}_{[L'\tilde{L}',\lambda']\to Q\bar{Q}[L\tilde{L},\lambda]}(\hat{z},M_H/x,m_Q,\mu_f).$$

$$\boldsymbol{K}^{[SS],LO}_{[SS]}(z,M_H,m_Q,\mu_f)=\frac{\alpha_s}{\pi}\Bigg\{N_c\Bigg[\frac{2z}{(1-z)_+}-\ln\frac{\mu^2e^{-1}}{M_H^2}\delta(1-z)$$
$$-2\delta(1-z)\left(\frac{1}{2\Delta}\ln\frac{1+\Delta}{1-\Delta}-1\right)\Bigg]+\frac{1}{N_c}\left(\frac{1+\Delta^2}{2\Delta}\ln\frac{1+\Delta}{1-\Delta}-1\right)\delta(1-z)\Bigg\}.$$

Outline

I. NRQCD factorization

II. Soft gluon factorization

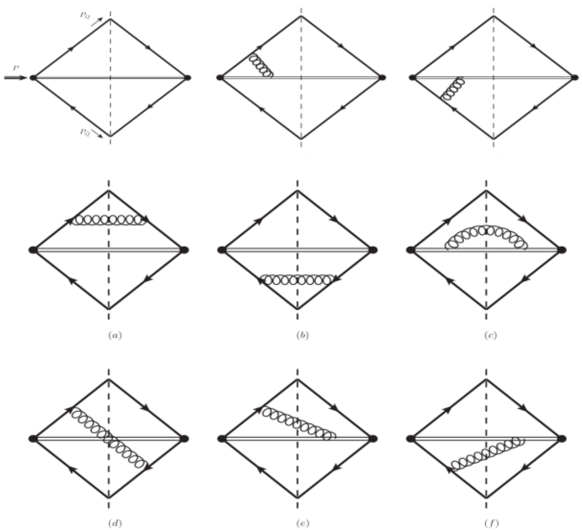
III. Application to FF: $g \to Q\bar{Q}({}^3S_1^{[8]}) + X$

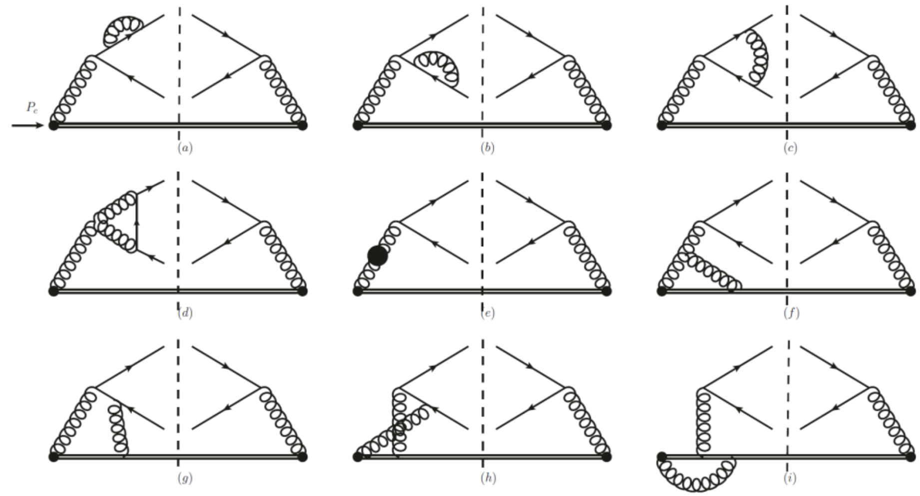
IV. Summary and outlook

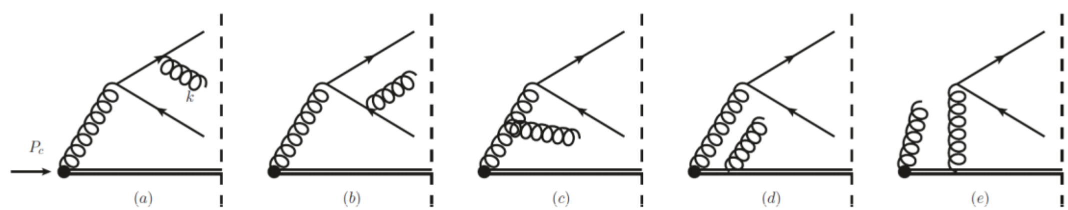
FF: $g \to Q\bar{Q}({}^3S_1^{[8]}) + X$ up to NLO

➢ Feynman diagrams

Chen, Jin, YQM, Meng, 2103.15121







Hard part for $g \to Q\bar{Q}({}^3S_1^{[8]}) + X$

➢ NRQCD

$$\hat{d}^{(2)}_{g\to{}^3S_1^{[8]}} = \frac{1}{12C_F}\Big[A(\mu_0)\delta(1-z) + \frac{1}{N_c}P_{gg}(z)\Big(\ln(\frac{\mu_0^2}{4m_Q^2}) - 1\Big) + \frac{2(1-z)}{z} - \frac{4(1-z+z^2)^2}{z}\Big(\frac{\ln(1-z)}{1-z}\Big)_+\Big],$$

Braaten, Lee, 0004228
YQM, Qiu, Zhang, 1311.7078

- **Double logs as $z \to 1$ (threshold logs)**

➢ SGF

Chen, Jin, YQM, Meng, 2103.15121

$$\hat{D}^{LO,(0)}_{[SS]}(\hat{z}, M_H/x, \mu, \mu_f) = \frac{\pi\alpha_s}{(N_c^2-1)}\frac{8x^3}{M_H^3}\delta(1-\hat{z}), \quad (5.28a)$$

$$\begin{aligned}
&\hat{D}^{NLO,(0)}_{[SS]}(\hat{z}, M_H/x, \mu, \mu_f) \\
&= \frac{4\alpha_s^2 N_c x^3}{(N_c^2-1)M_H^3}\Bigg[\frac{1}{2}\delta(1-\hat{z})\Bigg(2A(\mu, M_H/x) + \frac{2\beta_0}{N_c}\ln\Big(\frac{x^2\mu_f^2 e^{-1}}{M_H^2}\Big) + \ln^2\Big(\frac{x^2\mu_f^2 e^{-1}}{M_H^2}\Big) \\
&\quad + \frac{\pi^2}{6} - 1\Bigg) + \frac{1}{N_c}P_{gg}^{(0)}(\hat{z})\ln\Big(\frac{\mu^2}{\mu_f^2}\Big) + \Big(\frac{2(1-\hat{z})}{\hat{z}} + \hat{z}(4+2\hat{z}^2) + \frac{2\hat{z}^4}{9}(5+\hat{z})\Big) \\
&\quad \times\Big(\ln\Big(\frac{x^2\mu_f^2 e^{-1}}{M_H^2}\Big) - 2\ln(1-\hat{z})\Big) + \frac{2(1-\hat{z})}{\hat{z}} - \Big(\frac{4\hat{z}^4}{1-\hat{z}} - \frac{4\hat{z}^4}{9}(5+\hat{z})\Big)\ln\hat{z}\Bigg]. \quad (5.28b)
\end{aligned}$$

- **No threshold logs in hard part**
- **Logs are resummed by REGs of SGDs**

Numerical: $g \to Q\bar{Q}({}^3S_1^{[8]}) + X$

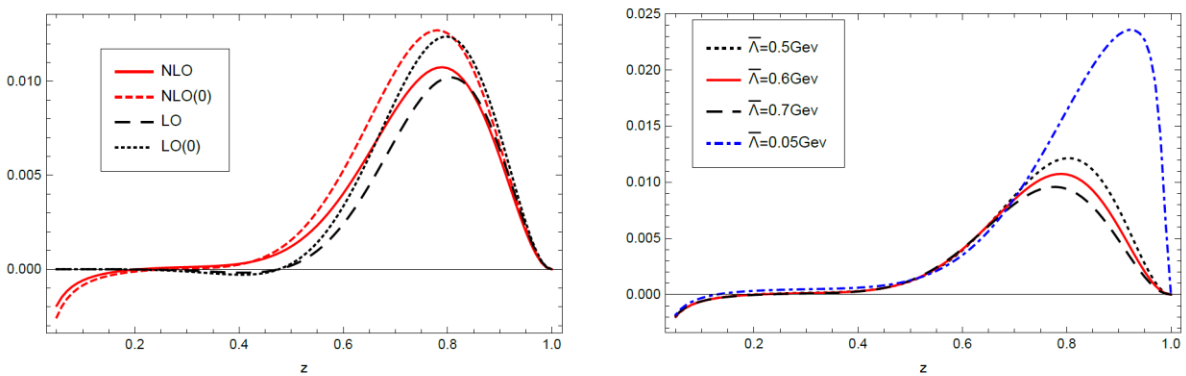
➢ Gluon FFs in ${}^3S_1^{[8]}$ channel

Chen, Jin, YQM, Meng, 2103.15121

$\bar{\Lambda}$: average momentum emitted

$$D_{g\to H}(z, M_H, m_Q, M_H) = \int_z^1 \frac{dx}{x} \hat{D}_{[SS]}(\hat{z}, M_H/x, m_Q, M_H, M_H) \times F_{[SS]\to H}(x, M_H, m_Q, M_H),$$

$$D^{(0)}_{g\to H}(z, M_H, m_Q, M_H) = \int_z^1 \frac{dx}{x} \hat{D}^{(0)}_{[SS]}(\hat{z}, M_H/x, M_H, M_H) \times F_{[SS]\to H}(x, M_H, m_Q, M_H).$$



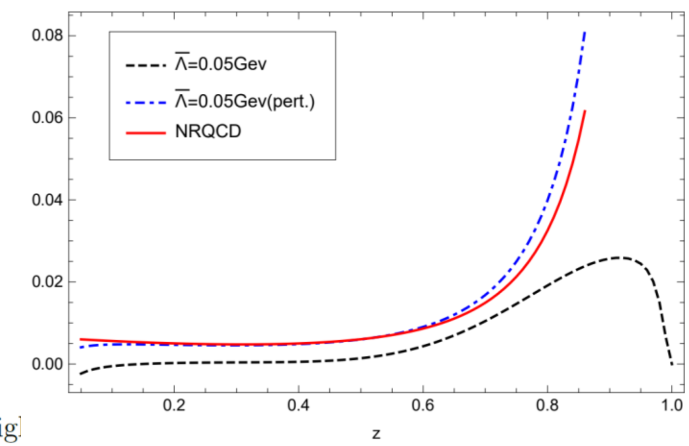


Figure 7. Left figure: Comparison of the gluon FF obtained in different approximations. Rig figure: $\bar{\Lambda}$ dependence of gluon FF at NLO.

$$R^X(n) \equiv \frac{\int_0^1 dz\, z^n D^X_{g\to H}(z, M_H, m_Q, \mu)}{\int_0^1 dz\, z^n D_{g\to H}(z, M_H, m_Q, \mu)},$$

$R^{NRQCD} \approx 6$

Summary

- **NRQCD factorization: polarization puzzle, hierarchy problem, universality problem**
  - Possible reason: convergence of v^2 expansion is bad because of soft gluon emission
- **Soft gluon factorization (SGF)**
  - Soft gluons effects considered; should have much better convergence in v^2 expansion
  - Rigorously defined; equivalent to NRQCD, but with power corrections originated from kinematic effects resummed, (partial) large logs resummed
  - Phenomenological difficulties encountered in NRQCD: to be studied in SGF

Thank you!

Sensitivity of nonperturbative model

The first class of models Fleming, Leibovich, Mehen, 0306139

$$F^{\mathrm{mod}}(\omega') = M_H N_H \frac{b^b}{\Gamma(b)} \frac{\omega'^{b-1}}{\bar{\Lambda}^b} e^{-b\omega'/\bar{\Lambda}}, \quad \omega' = M_H(1/x-1),$$

the zeroth, first and second moments are $M_H N_H$, $M_H N_H \bar{\Lambda}$ and $M_H N_H \bar{\Lambda}^2(\frac{1}{b}+1)$

Model-1: $F^{\mathrm{mod}}(\omega')|_{\bar{\Lambda}=0.6\mathrm{GeV},b=2}$, Model-2: $F^{\mathrm{mod}}(\omega')|_{\bar{\Lambda}=0.6\mathrm{GeV},b=1}$,

Model-3: $F^{\mathrm{mod}}(\omega')|_{\bar{\Lambda}=0.6\mathrm{GeV},b=3}$, Model-4: $F^{\mathrm{mod}}(\omega')|_{\bar{\Lambda}=0.5\mathrm{GeV},b=2}$,

Model-5: $F^{\mathrm{mod}}(\omega')|_{\bar{\Lambda}=0.7\mathrm{GeV},b=2}$,

- ☐ The other models

Model-6: $4M_H N_H[\theta(w' \geq \frac{19}{40}) - \theta(w' > \frac{29}{40})]$,

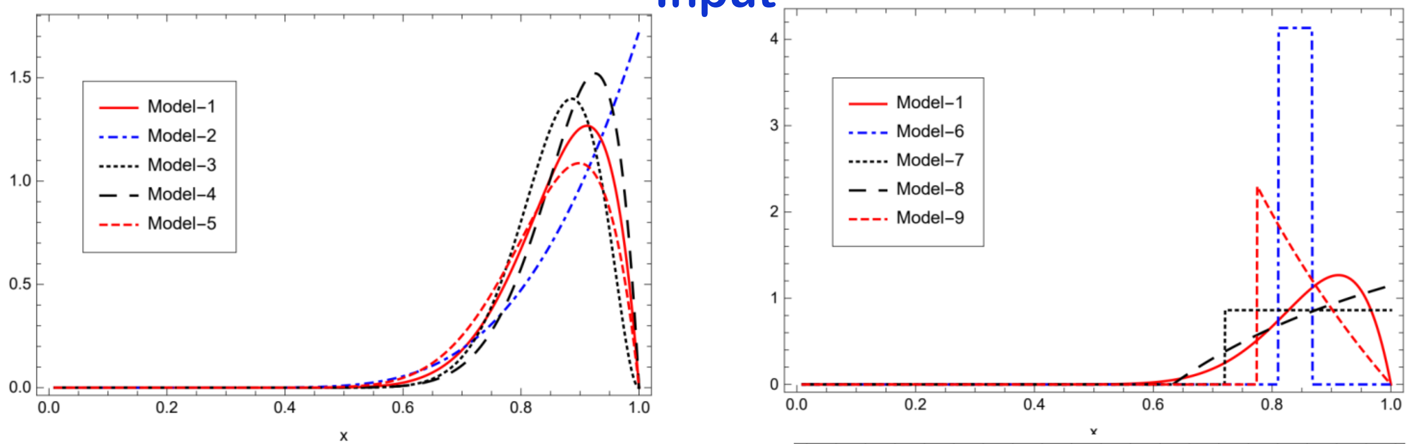
Model-7: $\frac{5}{6} M_H N_H[\theta(w' \geq 0) - \theta(w' > \frac{6}{5})]$,

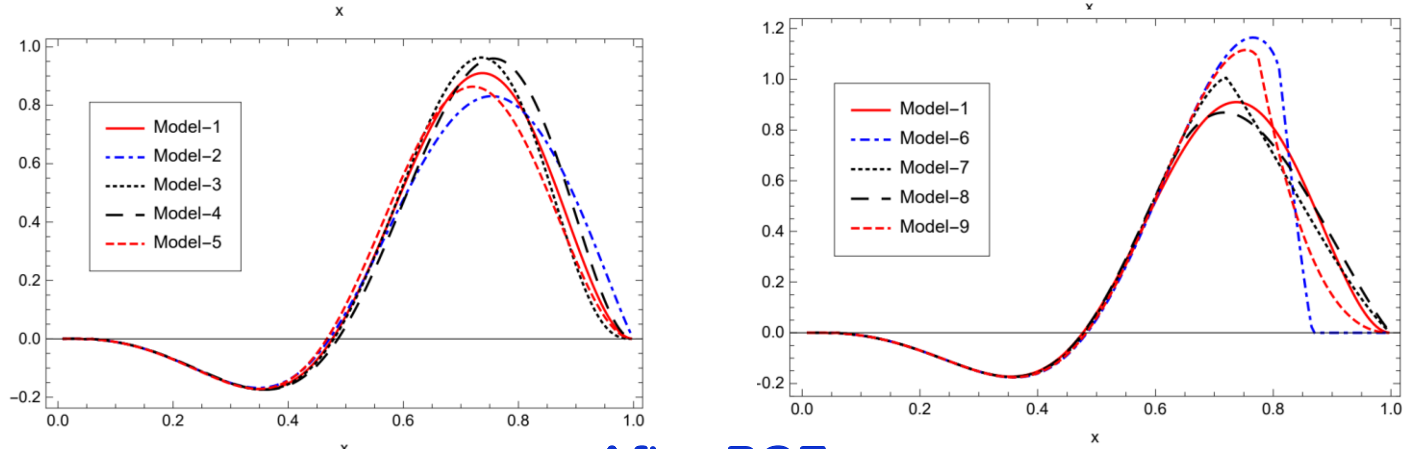
Model-8: $\begin{cases} M_H N_H(-\frac{50}{81}w' + \frac{10}{9}), & 0 \leq w' \leq \frac{9}{5}, \\ 0, & w' > \frac{9}{5}, \end{cases}$

Model-9: $\begin{cases} \frac{200}{81} M_H N_H w', & 0 \leq w' \leq \frac{9}{10}, \\ 0, & w' > \frac{9}{10}. \end{cases}$

RGEs effects

Input





After RGEs