Semi-inclusive decay of heavy quarkonium hybrids into quarkonium in the EFT framework

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Work in progress

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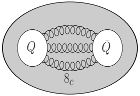
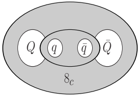
Confinement XIV

Exotic quarkonium

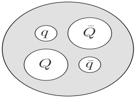
- Exotic quarkonia (XYZ particles) are quarkonium-like bound states that cannot be interpreted as a traditional quarkonium state.
- Interpretations for exotic quarkonia:



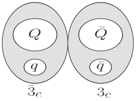
hybrid



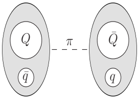
adjoint tetraquark



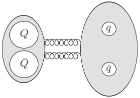
compact tetraquark



diquark-diquark



heavy-meson molecule



hadroquarkonium

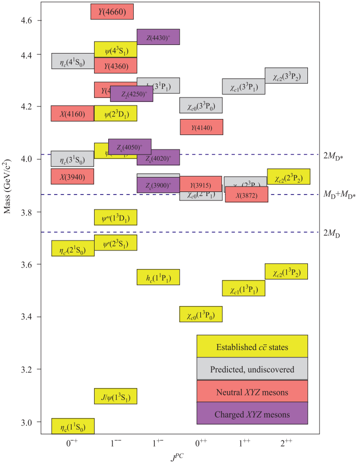


Figure from Front. Phys. 10 101401 (2015)

- A complete understanding of exotic quarkonia in terms of these pictures has not yet been obtained.
- EFT together with lattice QCD: a tool to understand the various pictures in a model-independent way.

Studies of quarkonium hybrids with EFT

- EFT framework and spectrum with leading operators:
  M. Berwein, N. Brambilla, J. Tarrús Castellà, A. Vairo, Phys. Rev. D92 (2015)
  N. Brambilla, G. Krein, J. Tarrús Castellà, A. Vairo, Phys. Rev. D97 (2018)

  R. Oncala, J. Soto, Phys. Rev. D96 (2017)
- Spin-dependent potential and spin splitting:
  N. Brambilla, **WKL**, J. Segovia, J. Tarrús Castellà, A. Vairo, Phys. Rev. D99 (2019)

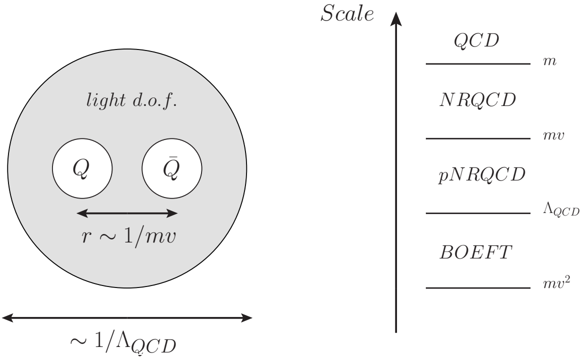
  N. Brambilla, **WKL**, J. Segovia, and J. Tarrús Castellà, Phys. Rev. D101 (2020)
- Decays to traditional quarkonia:
  R. Oncala, J. Soto, Phys. Rev. D96 (2017)

  J. Tarrús Castellà, E. Passemar (2021)
- This project:
  Comprehensive study of decays of quarkonium hybrids into traditional quarkonia

Quarkonium hybrids in the EFT picture

- Quarkonium hybrid: $Q\bar{Q}$ in color octet with gluon excitation (light d.o.f. with scale Λ_{QCD})
- Separation of scales for low-lying states:

  $m \gg mv \gg \Lambda_{QCD} \gg mv^2 \sim \sqrt{\frac{\Lambda_{QCD}^3}{m}} \Longrightarrow$ suitable for EFT description ($m \equiv m_Q$)
- Integrate out d.o.f.:
  QCD → NRQCD → pNRQCD → BOEFT (Born-Oppenheimer EFT)

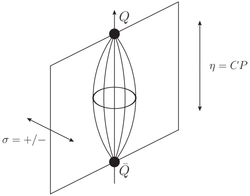


Static limit

- Dynamics of $Q\bar{Q}$ happens at time scale $\sim 1/mv^2 \gg 1/\Lambda_{QCD} \Rightarrow$ Born Oppenheimer approximation. E. Braaten, C. Langmack, D. H. Smith, Phys. Rev. D 90, (2014)
- Interquark potential given by energy eigenvalues with Q and $\bar{Q}$ at fixed positions.
- In the static limit of $Q\bar{Q}$, the system has symmetry group $D_{\infty h}$. Irreducible representation of $D_{\infty h}$: Λ_η^σ.

Irreducible representations of $D_{\infty h}$

- $\boldsymbol{K}$: angular momentum of light d.o.f.
  $\lambda = \hat{\boldsymbol{r}} \cdot \boldsymbol{K} = 0, \pm 1, \pm 2, \pm 3, \ldots$
  $\Lambda = |\lambda| = 0, 1, 2, 3, \ldots$ $(\Sigma, \Pi, \Delta, \Phi, \ldots)$
- Eigenvalue of CP: $\eta = +1\,(g), -1\,(u)$
- σ: eigenvalue of relfection about a plane containing $\hat{\boldsymbol{r}}$ (only for Σ states)



Static energies

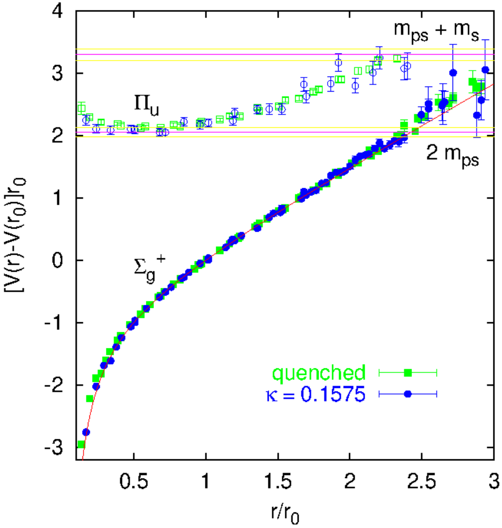
- In the static limit, when $r \to 0$ the symmetry group reduces to $O(3) \times C$, the quarkonium hybrid turns into a gluelump.
  State of light d.o.f. at $r = 0$ labeled by $\kappa = K^{PC}$.
- State of light d.o.f. labeled by $n = (\kappa, \Lambda_\eta^\sigma)$, with κ denoting the $r \to 0$ limit.
- Static energy $E_n^{(0)}(r)$: energy eigenvalue of $H^{(0)}$ (NRQCD Hamiltonian in the limit $m \to \infty$), with Q and $\bar{Q}$ at fixed positions $\boldsymbol{x}_1$, $\boldsymbol{x}_2$.
- In terms of Wilson loop:

$$E_n^{(0)}(r) = \lim_{T\to\infty} \frac{i}{T} \log\langle X_n(T/2)|X_n(-T/2)\rangle$$

$$|X_n(t)\rangle = \chi(\boldsymbol{x}_2, t)\phi(\boldsymbol{x}_2, \boldsymbol{R}, t)O_n(\boldsymbol{R}, t)\phi(\boldsymbol{R}, \boldsymbol{x}_1, t)\psi^\dagger(\boldsymbol{x}_1, t)|0\rangle$$

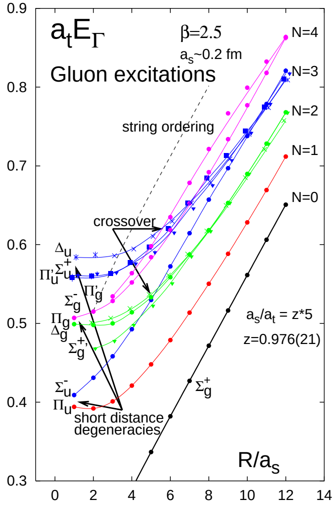
Λ_η^σ	K^{PC}	O_n
Σ_u^-	1^{+-}	$\hat{\boldsymbol{r}} \cdot \boldsymbol{B}$, $\hat{\boldsymbol{r}} \cdot (\boldsymbol{D} \times \boldsymbol{E})$
Π_u	1^{+-}	$\hat{\boldsymbol{r}} \times \boldsymbol{B}$, $\hat{\boldsymbol{r}} \times (\boldsymbol{D} \times \boldsymbol{E})$
$\Sigma_g^{+\prime}$	1^{--}	$\hat{\boldsymbol{r}} \cdot \boldsymbol{E}$, $\hat{\boldsymbol{r}} \cdot (\boldsymbol{D} \times \boldsymbol{B})$
Π_g	1^{--}	$\hat{\boldsymbol{r}} \times \boldsymbol{E}$, $\hat{\boldsymbol{r}} \times (\boldsymbol{D} \times \boldsymbol{B})$
Σ_g^-	2^{--}	$(\hat{\boldsymbol{r}} \cdot \boldsymbol{D})(\hat{\boldsymbol{r}} \cdot \boldsymbol{B})$
Π_g'	2^{--}	$\hat{\boldsymbol{r}} \times ((\hat{\boldsymbol{r}} \cdot \boldsymbol{D})\boldsymbol{B} + \boldsymbol{D}(\hat{\boldsymbol{r}} \cdot \boldsymbol{B}))$
Δ_g	2^{--}	$(\hat{\boldsymbol{r}} \times \boldsymbol{D})^i(\hat{\boldsymbol{r}} \times \boldsymbol{B})^j + (\hat{\boldsymbol{r}} \times \boldsymbol{D})^j(\hat{\boldsymbol{r}} \times \boldsymbol{B})^i$
Σ_u^+	2^{+-}	$(\hat{\boldsymbol{r}} \cdot \boldsymbol{D})(\hat{\boldsymbol{r}} \cdot \boldsymbol{E})$
Π_u'	2^{+-}	$\hat{\boldsymbol{r}} \times ((\hat{\boldsymbol{r}} \cdot \boldsymbol{D})\boldsymbol{E} + \boldsymbol{D}(\hat{\boldsymbol{r}} \cdot \boldsymbol{E}))$
Δ_u	2^{+-}	$(\hat{\boldsymbol{r}} \times \boldsymbol{D})^i(\hat{\boldsymbol{r}} \times \boldsymbol{E})^j + (\hat{\boldsymbol{r}} \times \boldsymbol{D})^j(\hat{\boldsymbol{r}} \times \boldsymbol{E})^i$

Lattice determination of static energies



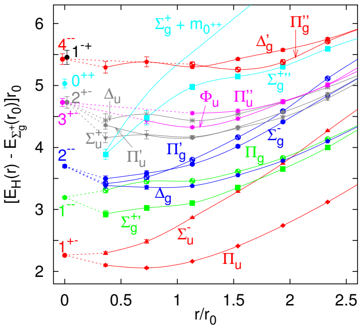
G. S. Bali *et al.*, Phys. Rev. D62, 054503 (2000)

Lattice determination of static energies



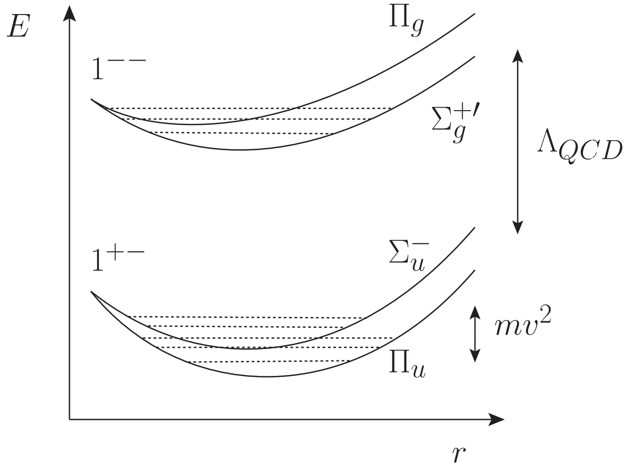
K. Juge, J. Kuti, C. Morningstar, Phys. Rev. Lett. 90 (2003)

- Ground state: Σ_g^+
- Gluelump energy Λ_κ:
  $E_n(r) \underset{r\to 0}{\to} V_o^{(0)}(r) + \Lambda_\kappa$
- Degeneracies at $r \to 0$



Gluelump energies (at $r = 0$):
M. Foster and C. Michael, Phys. Rev. D59 (1999)
Figure taken from
G. S. Bali and A. Pineda, Phys. Rev. D69 (2004)

BOEFT



BOEFT

Successive matchings to obtain BOEFT

- QCD: $\mathcal{L}_{\rm QCD} = \bar{Q}(i\not{D} - m)Q - \frac{1}{2}{\rm Tr}[G^{\mu\nu}G_{\mu\nu}]$
- QCD $\rightarrow$ NRQCD: Integrate out modes of scale m.

  $$\mathcal{L}_{\rm NRQCD} = \psi^\dagger\left(iD_0 + \frac{\boldsymbol{D}^2}{2m} + \frac{g\boldsymbol{B}\cdot\boldsymbol{\sigma}}{2m}\right)\psi - \chi^\dagger\left(iD_0 + \frac{\boldsymbol{D}^2}{2m} + \frac{g\boldsymbol{B}\cdot\boldsymbol{\sigma}}{2m}\right)\chi + \dots - \frac{1}{2}{\rm Tr}[G^{\mu\nu}G_{\mu\nu}]$$

  W. Caswell, G. Lepage, Phys. Lett. 167B (1986); G. Bodwin, E. Braaten, G. Lepage, Phys. Rev. D51 (1995)
- NRQCD $\rightarrow$ weakly-coupled pNRQCD: Integrate out modes of scale $mv \sim 1/r$ in the short distance regime ($r \ll 1/\Lambda_{QCD}$).

  $$L_{\rm pNRQCD} = \int d^3R\Big\{\int d^3r \left({\rm Tr}\left[{\rm S}^\dagger\,(i\partial_0 - h_s)\,{\rm S} + {\rm O}^\dagger\,(iD_0 - h_o)\,{\rm O}\right] + g{\rm Tr}\left[{\rm S}^\dagger\boldsymbol{r}\cdot\boldsymbol{E}\,{\rm O} + {\rm O}^\dagger\boldsymbol{r}\cdot\boldsymbol{E}\,{\rm S} + \frac{1}{2}{\rm O}^\dagger\boldsymbol{r}\cdot\{\boldsymbol{E},{\rm O}\}\right] + \cdots\right) - \frac{1}{2}{\rm Tr}[G^{\mu\nu}G_{\mu\nu}]\Big\}$$

  A. Pineda, J. Soto, Nucl. Phys. Proc. Suppl. 64 (1998); N. Brambilla, A. Pineda, J. Soto, A. Vairo, Nucl. Phys. B566 (2000)
- Weakly-coupled pNRQCD $\rightarrow$ BOEFT: Integrate out modes of scale Λ_{QCD}.

  $$L_{\rm BOEFT} = \int d^3R d^3r \sum_{\kappa\lambda\lambda'} \hat{\Psi}^\dagger_{\kappa\lambda}(\boldsymbol{r},\boldsymbol{R},t)\left\{i\partial_t - V_{\kappa\lambda\lambda'}(r) + P^{i\dagger}_{\kappa\lambda}\frac{\boldsymbol{\nabla}^2_r}{m}P^i_{\kappa\lambda'}\right\}\hat{\Psi}_{\kappa\lambda'}(\boldsymbol{r},\boldsymbol{R},t) + \dots$$

  M. Berwein, N. Brambilla, J. Tarrús Castellà, A. Vairo, Phys. Rev. D92 (2015); N. Brambilla, G. Krein, J. Tarrús Castellà, A. Vairo, Phys. Rev. D97 (2018); R. Oncala, J. Soto, Phys. Rev. D96 (2017);N. Brambilla, **WKL**, J. Segovia, and J. Tarrús Castellà, Phys. Rev. D101 (2020)

BOEFT

Lagrangian of BOEFT

$$L_{\rm BOEFT} = \int d^3R d^3r \sum_{\kappa} \sum_{\lambda\lambda'} \hat{\Psi}^{\dagger}_{\kappa\lambda}(\boldsymbol{r}, \boldsymbol{R}, t)\Big\{ i\partial_t - V_{\kappa\lambda\lambda'}(r) + P^{i\dagger}_{\kappa\lambda} \frac{\boldsymbol{\nabla}_r^2}{m} P^{i}_{\kappa\lambda'} \Big\} \hat{\Psi}_{\kappa\lambda'}(\boldsymbol{r}, \boldsymbol{R}, t) + \dots$$

- d.o.f.: color-singlet field $\hat{\Psi}_{\kappa\lambda}(\boldsymbol{r}, \boldsymbol{R}, t) \propto P^i_{\kappa\lambda} O^{a\dagger}(\boldsymbol{r}, \boldsymbol{R}, t) G^{ia}_{\kappa}(\boldsymbol{R}, t)$
  - $G^{ia}_{\kappa}(\boldsymbol{R}, t)$: gluelump operator
    $H^{(0)} O^{a\dagger}(\boldsymbol{r}, \boldsymbol{R}, \boldsymbol{t}) G^{ia}_{\kappa}(\boldsymbol{R}, t)|0\rangle \to (V_o^{(0)}(r) + \Lambda_\kappa) O^{a\dagger}(\boldsymbol{r}, \boldsymbol{R}, t) G^{ia}_{\kappa}(\boldsymbol{R}, t)|0\rangle$
    as $r \to 0$
  - $P^i_{\kappa\lambda}$ projects $G^{ia}_{\kappa}(\boldsymbol{R}, t)$ to a representation of $D_{\infty h}$.
- BOEFT: EFT for dynamics of $Q\bar{Q}$ at scale $mv^2 \Rightarrow$ Schrödinger equation

Schrödinger equation

$$\left[-P^{i\dagger}_{\kappa\lambda} \frac{\boldsymbol{\nabla}_r^2}{m} P^{i}_{\kappa\lambda'} + V_{\kappa\lambda\lambda'}(r) \right] \Psi^n_{\kappa\lambda'}(\boldsymbol{r}) = E_n \Psi^n_{\kappa\lambda}(\boldsymbol{r})$$

$\Psi^n_{\kappa\lambda}(\boldsymbol{r}) \equiv \langle 0|\hat{\Psi}(\boldsymbol{r}, \boldsymbol{R} = 0, t = 0)|n\rangle$
$\Psi^n_{\kappa\lambda}(\boldsymbol{r})$: wave function of $Q\bar{Q}$ in quarkonium hybrid $|n\rangle$

Semi-inclusive decay

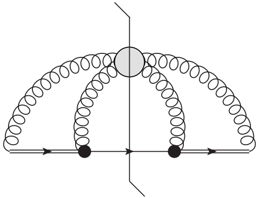
N. Brambilla, **WKL**, A. Mohapatra, A. Vairo (in progress)

Typical observed decay channels of XYZ into traditional quarkonia:

$$\mathrm{XYZ} \to \mathrm{quarkonium} + \mathrm{lightmesons} + \mathrm{photons}$$

Consider the decay $H \to S + X$, with $H =$ low-lying hybrid, $S =$ low-lying quarkonium.

- Define $\Delta \equiv m_H - m_S$ (~ 1 GeV). We observe that $\Delta \gg \Lambda_{\mathrm{QCD}} \gg mv^2$.
- Assume the hierarchy $mv \gg \Delta \gg \Lambda_{\mathrm{QCD}} \gg mv^2$.
  Integrate out modes of scale $\sim \Delta$ and $\sim \Lambda_{\mathrm{QCD}}$ to obtain BOEFT of hybrid at scale mv^2.
- At scale $mv \gg \mu \gg \Lambda_{\mathrm{QCD}}$, we have weakly-coupled pNRQCD.
  Leading relevant contribution to $\mathrm{Im}V$ of BOEFT:



Black dot: vertex of pNRQCD

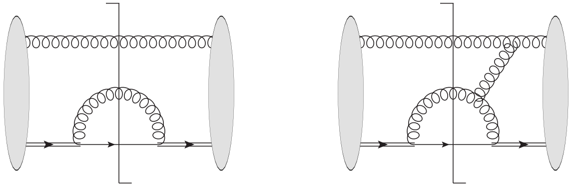
Weakly-coupled pNRQCD Lagrangian

$$L_{\rm pNRQCD} = \int d^3R \Big\{ \int d^3r \Big({\rm Tr}\left[{\rm S}^\dagger \left(i\partial_0 - h_s \right) {\rm S} + {\rm O}^\dagger \left(iD_0 - h_o \right) {\rm O} \right]$$
$$+ g{\rm Tr}\left[{\rm S}^\dagger \boldsymbol{r}\cdot\boldsymbol{E}\,{\rm O} + {\rm O}^\dagger \boldsymbol{r}\cdot\boldsymbol{E}\,{\rm S} + \frac{1}{2}{\rm O}^\dagger \boldsymbol{r}\cdot\{\boldsymbol{E},{\rm O}\}\right] + \frac{g}{4m}{\rm Tr}\left[{\rm O}^\dagger \boldsymbol{L}_{Q\bar{Q}}\cdot[\boldsymbol{B},{\rm O}]\right]$$
$$+ \frac{gc_F}{m}{\rm Tr}\left[{\rm S}^\dagger(\boldsymbol{S}_1-\boldsymbol{S}_2)\cdot\boldsymbol{B}\,{\rm O} + {\rm O}^\dagger(\boldsymbol{S}_1-\boldsymbol{S}_2)\cdot\boldsymbol{B}\,{\rm S} + {\rm O}^\dagger\boldsymbol{S}_1\cdot\boldsymbol{B}\,{\rm O} - {\rm O}^\dagger\boldsymbol{S}_2{\rm O}\cdot\boldsymbol{B}\right] - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu\,a}\Big\}$$

- Spin-preserving decay
- Spin-flipping decay

$$\Gamma(H\to S+X) = \Gamma_{\rm pert} + \mathcal{O}\left(\Lambda^2_{\rm QCD}/\Delta^2\right)$$

Perturbative diagrams:



Weakly-coupled pNRQCD Lagrangian

$$L_{\text{pNRQCD}} = \int d^3R \Big\{ \int d^3r \Big(\text{Tr}\Big[\text{S}^\dagger \left(i\partial_0 - h_s\right) \text{S} + \text{O}^\dagger \left(iD_0 - h_o\right) \text{O} \Big]$$
$$+ g\text{Tr}\Big[\text{S}^\dagger \boldsymbol{r}\cdot\boldsymbol{E}\,\text{O} + \text{O}^\dagger \boldsymbol{r}\cdot\boldsymbol{E}\,\text{S} + \frac{1}{2}\text{O}^\dagger \boldsymbol{r}\cdot\{\boldsymbol{E},\text{O}\} \Big] + \frac{g}{4m}\text{Tr}\Big[\text{O}^\dagger \boldsymbol{L}_{Q\bar{Q}}\cdot[\boldsymbol{B},\text{O}] \Big]$$
$$+ \frac{gc_F}{m}\text{Tr}\Big[\text{S}^\dagger(\boldsymbol{S}_1 - \boldsymbol{S}_2)\cdot\boldsymbol{B}\,\text{O} + \text{O}^\dagger(\boldsymbol{S}_1 - \boldsymbol{S}_2)\cdot\boldsymbol{B}\,\text{S} + \text{O}^\dagger\boldsymbol{S}_1\cdot\boldsymbol{B}\,\text{O} - \text{O}^\dagger\boldsymbol{S}_2\text{O}\cdot\boldsymbol{B} \Big] - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu\,a} \Big\}$$

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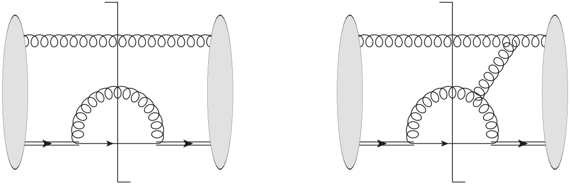
$$\Gamma(H \to S + X) = \Gamma_{\text{pert}} + \mathcal{O}\left(\Lambda^2_{\text{QCD}}/\Delta^2\right)$$

Perturbative diagrams:



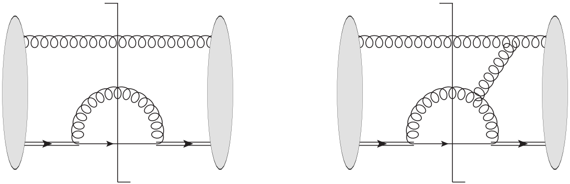
Weakly-coupled pNRQCD Lagrangian

$$L_{\rm pNRQCD} = \int d^3R \Big\{ \int d^3r \Big({\rm Tr}\left[{\rm S}^\dagger \left(i\partial_0 - h_s \right) {\rm S} + {\rm O}^\dagger \left(iD_0 - h_o \right) {\rm O} \right]$$
$$+ g{\rm Tr}\left[{\rm S}^\dagger \boldsymbol{r}\cdot\boldsymbol{E}\,{\rm O} + {\rm O}^\dagger \boldsymbol{r}\cdot\boldsymbol{E}\,{\rm S} + \frac{1}{2}{\rm O}^\dagger \boldsymbol{r}\cdot\{\boldsymbol{E},{\rm O}\} \right] + \frac{g}{4m}{\rm Tr}\left[{\rm O}^\dagger \boldsymbol{L}_{Q\bar{Q}}\cdot[\boldsymbol{B},{\rm O}] \right]$$
$$+ \frac{gc_F}{m}{\rm Tr}\left[{\rm S}^\dagger(\boldsymbol{S}_1 - \boldsymbol{S}_2)\cdot\boldsymbol{B}\,{\rm O} + {\rm O}^\dagger(\boldsymbol{S}_1 - \boldsymbol{S}_2)\cdot\boldsymbol{B}\,{\rm S} + {\rm O}^\dagger\boldsymbol{S}_1\cdot\boldsymbol{B}\,{\rm O} - {\rm O}^\dagger\boldsymbol{S}_2{\rm O}\cdot\boldsymbol{B} \right] - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu\,a} \Big\}$$

- Spin-preserving decay
- Spin-flipping decay

$$\Gamma(H \to S + X) = \Gamma_{\rm pert} + \mathcal{O}\left(\Lambda^2_{\rm QCD}/\Delta^2 \right)$$

Perturbative diagrams:



Spin-preserving decay

Spin-preserving inclusive decay rate for $H_m \to S_n + X$ (perturbative, LO in α_s)

$$\Gamma(H_m \to S_n + X) = \frac{4\alpha_s T_F}{3N_c} \sum_{q,q'} \int dE \int dE' \, f^i_{mq}(E) g^j_{qn'}(E) g^{j\dagger}_{q'n'}(E') f^{i\dagger}_{mq'}(E') \times (\Lambda + E/2 + E'/2 - E^s_n)^3 \sum_{n'} |h_{nn'}|^2$$

where

$$f^i_{mq}(E) = \int d^3r \, \Psi^{i\dagger}_m(\boldsymbol{r}) \Phi^o_{E,q}(\boldsymbol{r})$$

$$g^j_{qn'}(E) = \int d^3r \, \Phi^{o\dagger}_{E,q}(\boldsymbol{r}) r^j \Phi^s_{n'}(\boldsymbol{r})$$

$$h_{nn'} = \int d^3r \, \Phi^{Q\dagger}_n(\boldsymbol{r}) \Phi^s_{n'}(\boldsymbol{r})$$

Several points to note on the decay formula:

- It is a double integral over the energies E, E' of the octet wave function $\Phi^o_{E,q}$ and $\Phi^o_{E',q'}$.
- The phase space factor $(\Lambda + E/2 + E'/2 - E^s_n)^3 \sim \Delta^3$ parametrically.

As a preliminary exploration, let us apply the assumptions $h_{nn'} \approx 1$, $E_n^Q \approx E_n^s$, and $f_{mq}^i(E) \neq 0$ only when $E_m \approx E + \Lambda$.

Spin-preserving inclusive decay rate for $H_m \to S_n + X$ (simplified)

$$\Gamma^{\text{sim}}(H_m \to S_n + X) = \frac{4\alpha_s T_F}{3N_c}(E_m - E_n^Q)^3 T^{ij}(T^{ij})^*$$

where

$$T^{ij} \equiv \int d^3r \, \Psi_m^{i\dagger}(\boldsymbol{r}) r^j \Phi_n^Q(\boldsymbol{r})$$

- This simplified formula is the same as the formula in R. Oncala, J. Soto (2017) and J. Tarrús Castellà, E. Passemar (2021).
- In R. Oncala, J. Soto (2017), in numerical evaluations the tensor structure $T^{ij}(T^{ij})^*$ was replaced by $T^{ii}(T^{ii})^*$, which led to a selection rule that hybrids with $L = J$ (such as Np_1 (H_2)) do not decay. With the correct tensor structure $T^{ij}(T^{ij})^*$, such selection rule does not exist.

Preliminary result

Spin-preserving decay rates for charmonium hybrids:

$NL_J \to N'L'$	ΔE (GeV)	$\alpha_s\,(\Delta E)$	$\sqrt{T^{ij}(T^{ij})^*}$ (GeV^{-1})	Γ (MeV)
$1p_0 \to 1s$	1.418	0.307	1.187	274.48
$1p_0 \to 2s$	0.808	0.408	1.748	146.23
$2p_0 \to 1s$	1.852	0.276	0.312	38.0
$2p_0 \to 2s$	1.242	0.326	1.633	370.8
$3p_0 \to 1s$	2.231	0.259	0.144	13.23
$3p_0 \to 2s$	1.620	0.290	0.388	41.2
$1p_1 \to 1s$	1.078	0.349	1.886	346.0
$2p_1 \to 1s$	1.444	0.305	0.862	151.48
$2p_1 \to 2s$	0.834	0.401	1.908	187.90
$3p_1 \to 1s$	1.796	0.279	0.447	71.73
$3p_1 \to 2s$	1.186	0.334	0.942	109.62
$2(s/d)_1 \to 1p$	0.861	0.394	2.322	301.56
$4(s/d)_1 \to 1p$	1.224	0.329	3.667	1802.13

Preliminary result

Spin-preserving decay rates for bottomonium hybrids:

$NL_J \to N'L'$	ΔE (GeV)	$\alpha_s\,(\Delta E)$	$\sqrt{T^{ij}(T^{ij})^*}$ (GeV^{-1})	Γ (MeV)
$1p_0 \to 1s$	1.569	0.294	0.520	68.20
$1p_0 \to 2s$	1.002	0.363	1.270	130.86
$2p_0 \to 1s$	1.857	0.276	0.264	27.26
$2p_0 \to 2s$	1.290	0.321	0.583	51.96
$2p_0 \to 3s$	0.943	0.375	1.910	254.63
$3p_0 \to 1s$	2.109	0.264	0.162	14.37
$3p_0 \to 2s$	1.542	0.296	0.290	20.35
$4p_0 \to 1s$	2.338	0.254	0.111	8.88
$4p_0 \to 2s$	1.770	0.281	0.170	10.06
$4p_0 \to 3s$	1.423	0.307	0.329	21.31
$1p_1 \to 1s$	1.320	0.317	0.721	84.29
$2p_1 \to 1s$	1.528	0.297	0.546	70.35
$2p_1 \to 2s$	0.961	0.371	0.059	0.25
$3p_1 \to 1s$	1.732	0.283	0.395	51.00
$3p_1 \to 2s$	1.165	0.336	0.417	20.54
$2(s/d)_1 \to 1p$	0.978	0.367	0.890	60.51
$3(s/d)_1 \to 1p$	1.176	0.335	1.073	141.05
$3(s/d)_1 \to 2p$	0.819	0.405	0.131	0.85
$4(s/d)_1 \to 1p$	1.248	0.326	3.056	1312.72
$4(s/d)_1 \to 2p$	0.890	0.386	4.484	1218.65
$5(s/d)_1 \to 1p$	1.376	0.311	0.467	39.28
$5(s/d)_1 \to 2p$	1.019	0.360	1.011	86.36

Spin-flipping decay

Spin-flipping inclusive decay rate for $H_m \to S_n + X$ (perturbative, LO in α_s)

$$\Gamma(H_m \to S_n + X) = \frac{4\alpha_s T_F}{3N_c} \sum_{q,q'} \int dE \int dE' \, f^i_{mq}(E) w^j_{qn'}(E) w^{j\dagger}_{q'n'}(E') f^{i\dagger}_{mq'}(E') \times (\Lambda + E/2 + E'/2 - E^s_n)^3 \sum_{n'} |h_{nn'}|^2$$

$$w^j_{qn'}(E) = \int d^3r \, \Phi^{o\dagger}_{E,q}(\boldsymbol{r})(S^j_1 - S^j_2)\Phi^s_{n'}(\boldsymbol{r})$$

Spin-flipping inclusive decay rate for $H_m \to S_n + X$ (simplified)

$$\Gamma^{\rm sim}(H_m \to S_n + X) = \frac{4\alpha_s T_F c_F^2}{3N_c m_Q^2}(E_m - E^Q_n)^3 U^{ij}(U^{ij})^*$$

$$U^{ij} \equiv \int d^3r \, \Psi^{i\dagger}_m(\boldsymbol{r})(S^j_1 - S^j_2)\Phi^Q_n(\boldsymbol{r})$$

- Spin flip: $|S_H = 1\rangle \to |S_Q = 0\rangle$ or $|S_H = 0\rangle \to |S_Q = 1\rangle$
- Formula of $\Gamma^{\rm sim}$ agrees with J. Tarrús Castellà, E. Passemar (2021)

Preliminary result

Spin-flipping decay rates for charmonium hybrids:

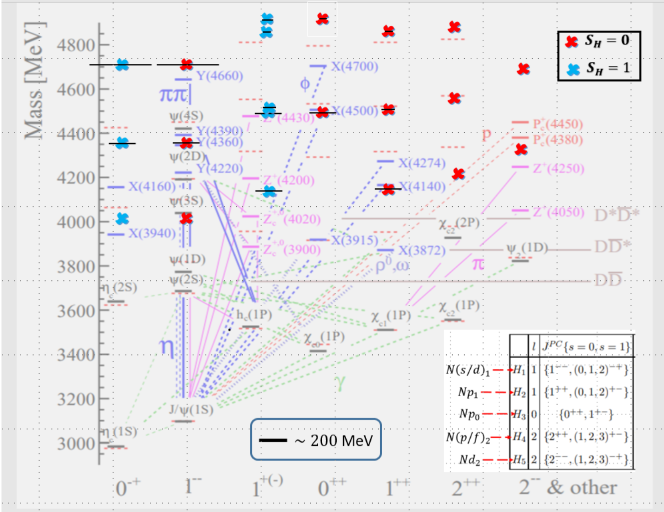
$NL_J \to N'L'$	ΔE (GeV)	$\alpha_S\,(\Delta E)$	$\Gamma(\mathrm{MeV})$ $(1 \to 0)$	$\Gamma(\mathrm{MeV})$ $(0 \to 1)$
$1p_0 \to 1p$	0.993	0.365	35.60	106.81
$2p_0 \to 1p$	1.426	0.307	1.44	4.31
$2p_0 \to 2p$	0.952	0.373	31.34	94.03
$3p_0 \to 1p$	1.804	0.279	0.41	1.22
$3p_0 \to 2p$	1.330	0.316	1.52	4.56
$1p_1 \to 1p$	0.653	0.467	46.57	139.72
$2p_1 \to 1p$	1.018	0.360	14.51	43.53
$2p_1 \to 2p$	0.544	0.531	26.13	78.39
$3p_1 \to 1p$	1.369	0.312	5.69	17.07
$3p_1 \to 2p$	0.895	0.385	10.97	32.90
$1(s/d)_1 \to 1s$	0.944	0.374	74.98	224.95
$2(s/d)_1 \to 1s$	1.288	0.321	32.51	97.53
$2(s/d)_1 \to 2s$	0.678	0.455	18.86	56.57
$3(s/d)_1 \to 1s$	1.624	0.290	7.73	23.18
$3(s/d)_1 \to 2s$	1.014	0.360	1.04	3.11
$4(s/d)_1 \to 1s$	1.650	0.288	7.35	22.06
$4(s/d)_1 \to 2s$	1.040	0.356	38.86	116.57

Preliminary result

Spin-flipping decay rates for bottomonium hybrids:

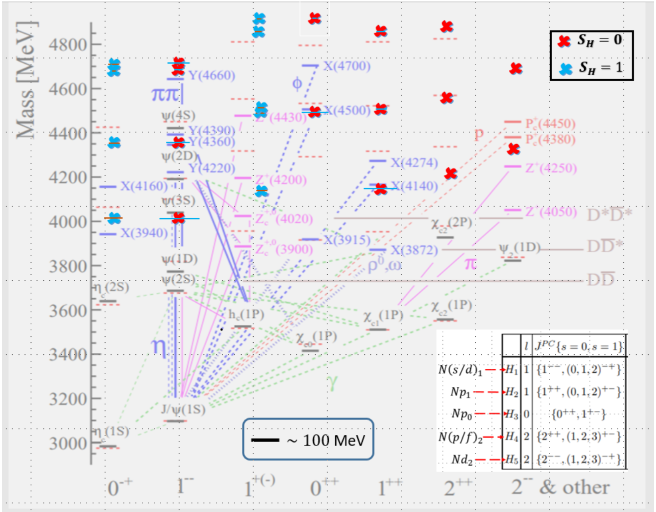
$NL_J \to N'L'$	ΔE (GeV)	$\alpha_S\,(\Delta E)$	Γ(MeV) $(1 \to 0)$	Γ(MeV) $(0 \to 1)$
$1p_0 \to 1p$	1.103	0.346	3.94	11.81
$2p_0 \to 1p$	1.391	0.310	0.54	1.61
$2p_0 \to 2p$	1.033	0.357	2.95	8.84
$3p_0 \to 1p$	1.643	0.289	0.19	0.56
$3p_0 \to 2p$	1.286	0.321	0.50	1.49
$4p_0 \to 1p$	1.872	0.275	0.09	0.28
$4p_0 \to 2p$	1.515	0.299	0.19	0.57
$4p_0 \to 3p$	1.226	0.328	0.45	1.35
$1p_1 \to 1p$	0.853	0.396	6.32	18.96
$2p_1 \to 1p$	1.062	0.352	3.22	9.67
$2p_1 \to 2p$	0.704	0.444	1.99	5.97
$3p_1 \to 1p$	1.266	0.323	1.62	4.85
$3p_1 \to 2p$	0.909	0.382	2.16	6.47
$1(s/d)_1 \to 1s$	1.247	0.326	8.6	25.86
$2(s/d)_1 \to 1s$	1.288	0.305	5.95	17.84
$2(s/d)_1 \to 2s$	0.876	0.390	0.56	1.68
$3(s/d)_1 \to 1s$	1.642	0.289	3.56	10.67
$3(s/d)_1 \to 2s$	1.075	0.350	0.95	2.84
$4(s/d)_1 \to 1s$	1.713	0.284	0.50	1.51
$4(s/d)_1 \to 2s$	1.146	0.339	2.56	7.68
$4(s/d)_1 \to 3s$	0.799	0.411	0.37	1.12

Preliminary result



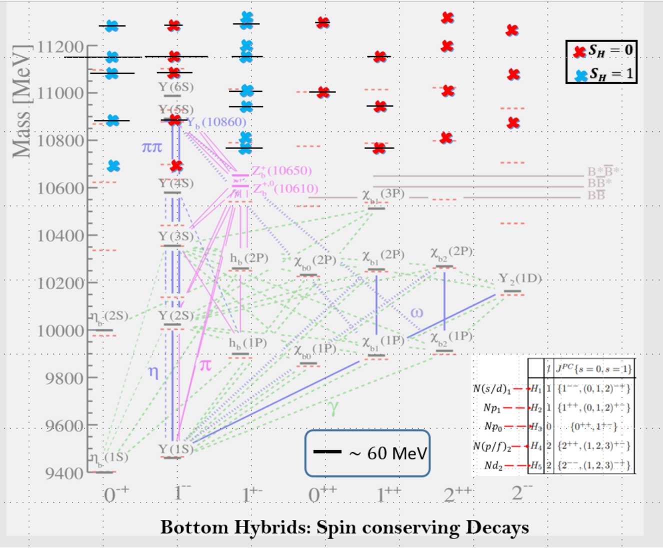
Charm Hybrids: Spin conserving Decays

Preliminary result

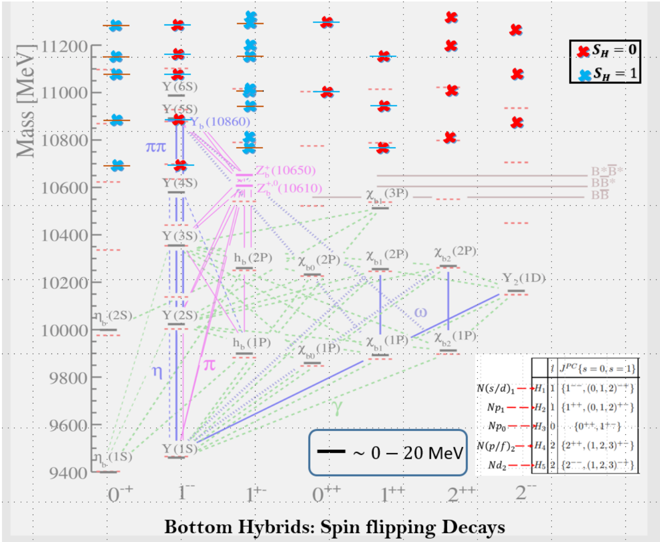


Charm Hybrids: Spin flipping Decays

Preliminary result



Preliminary result



		l	$J^{PC}\{s=0, s=1\}$
$N(s/d)_1$	H_1	1	$\{1^{--}, (0,1,2)^{-+}\}$
Np_1	H_2	1	$\{1^{++}, (0,1,2)^{+-}\}$
Np_0	H_3	0	$\{0^{++}, 1^{+-}\}$
$N(p/f)_2$	H_4	2	$\{2^{++}, (1,2,3)^{+-}\}$
Nd_2	H_5	2	$\{2^{--}, (1,2,3)^{-+}\}$

Summary and outlook

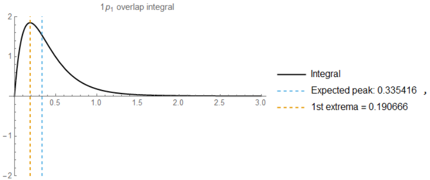
- BOEFT provides a model-independent way to study heavy quarkonium hybrids in a systematic way.
- We formulated a way to calculate inclusive decay rates of quarkonium hybrids into traditional quarkonia. The resulting decay formulae involve integrals of overlaps of wave functions. We evaluated the decay rates numerically by applying simplifying assumptions.
- We will
  - evaluate the decay formulae with integrals of wave function overlaps
  - quantify theoretical errors and compare with experimental data
  - apply the framework to exclusive decays
  - include mixing with excited quarkonia
  - extend the framework to the study of tetraquarks.

Thank you.

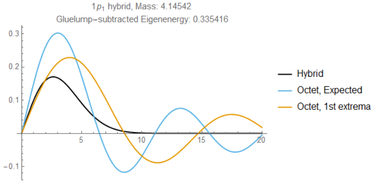
Back up slides

It is interesting to see how $f^i_{mq}(E) = \int d^3r\, \Psi^{i\dagger}_m(\boldsymbol{r})\Phi^o_{E,q}(\boldsymbol{r})$ looks like as a function of E:

$$H_2\text{-multiplet, } l = 1, J^{PC} = [1^{++}, (0,1,2)^{+-}]$$
$$H_2(4145):$$



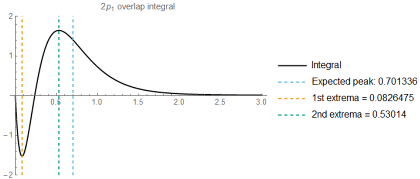
Radial integral of $f^i_{mq}(E)$ vs E (GeV)

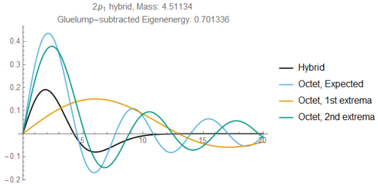


Radial hybrid wave function vs r (GeV^{-1})

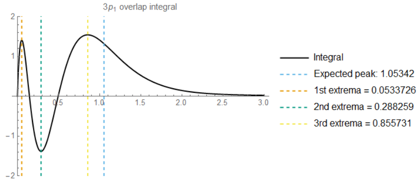
- The actual peak is slightly off (at a lower E) from the expected peak at $E = E_m - \Lambda$.
- The peak is broad, with width ~ 1 GeV. The assumption that $f^i_{mq}(E)$ is nonzero only when $E_m \approx E + \Lambda$ is not true.

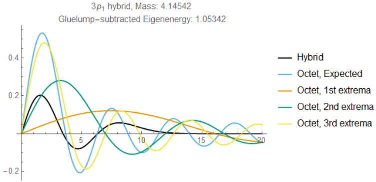
$H_2'(4511)$:





$H_2''(4863)$:





Radial integral of $f^i_{mq}(E)$ vs E (GeV) Radial hybrid wave function vs r (GeV^{-1})

- $f^i_{mq}(E)$ exhibits multiple peaks for excited hybrid states. All the peaks are at a lower E from the expected peak.

Preliminary result

Spin-preserving inclusive decay rates (with overlap integrals):

$NL_J \to N'L'$	Γ (MeV)
charmonium hybrid decay	
$1p_0 \to 1s$	240.0
$2(s/d)_1 \to 1p$	103.1
$4(s/d)_1 \to 1p$	450.4
bottomonium hybrid decay	
$1p_0 \to 1s$	61.8
$1p_0 \to 2s$	58.5
$2p_0 \to 2s$	14.3
$2p_0 \to 3s$	36.1
$4p_0 \to 1s$	10.9
$4p_0 \to 1s$	16.4
$4p_0 \to 2s$	11.3
$4p_0 \to 3s$	12.7
$2(s/d)_1 \to 1p$	43.0
$3(s/d)_1 \to 2p$	13,9
$4(s/d)_1 \to 2p$	181.5
$5(s/d)_1 \to 1p$	53.0
$5(s/d)_1 \to 2p$	47.0