Doubly heavy baryons in Born-Oppenheimer EFT

Jaume Tarrús Castellà

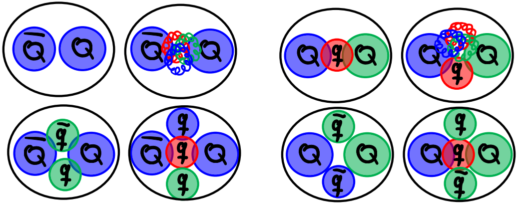
Institut de Física d'Altes Energies (Universitat Autònoma de Barcelona)

with: Joan Soto (Universitat de Barcelona)

based on: **Phys.Rev.D 102 (2020) 1, 014012.** arXiv:2005.00552
Phys.Rev.D 102 (2020) 1, 014013. arXiv:2005.00551
arXiv:2108.00496

A Virtual Tribute to Quark Confinement and the Hadron Spectrum, August 3rd, 2021.

- Doubly heavy hadrons: exotic quarkonium, doubly heavy baryons, pentaquarks and more...



- All doubly heavy hadrons have some common characteristics.
  - Heavy quarks are non relativistic $m_Q \gg \Lambda_{\rm QCD}$.
  - Adiabatic expansion $\Lambda_{\rm QCD} \gg m_Q v^2$.
- Born-Oppenheimer EFT built upon the expansions on these two ratios.
  - General formulation available for all doubly heavy hadrons Soto, JT Phys.Rev.D 102 (2020).
- Potentials are nonperturbative and lattice data is not always available!

BOEFT for doubly heavy baryons

Soto, JT Phys.Rev.D 102 (2020)

Why doubly heavy baryons?

- The simplest doubly heavy hadron with valence light quarks.
- Ground state, Ξ_{cc}, observed.

- The EFT is formulated in terms of the $\Psi_{\kappa^p} = \Psi^{\alpha\, i}_{\kappa^p}(t, \boldsymbol{r}, \boldsymbol{R})$ fields:
  - κ^p light-quark state quantum numbers.
  - α and i light-quark and heavy-quark spin indices.
  - $\boldsymbol{r}$ heavy quark distance, $\boldsymbol{R}$ heavy quark center of mass.
- The Lagrangian in the single baryon sector is

$$\mathcal{L}_{QQq} = \sum_{\kappa^p} \Psi^\dagger_{\kappa^p} \left[i\partial_t - h_{\kappa^p} \right] \Psi_{\kappa^p}$$

- The Hamiltonian density is organized as on expansion in $1/m_Q$

$$h_{\kappa^p} = \frac{\boldsymbol{p}^2}{m_Q} + \frac{\boldsymbol{P}^2}{4m_Q} + V^{(0)}_{\kappa^p}(\boldsymbol{r}) + \frac{1}{m_Q} V^{(1)}_{\kappa^p}(\boldsymbol{r},\, \boldsymbol{p}) + \mathcal{O}(1/m_Q^2)$$

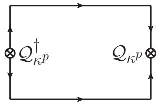
- Operators organized in representations of $D_{\infty h}$ (cylindrical symmetry).

- The leading term are the static potentials

$$V^{(0)}_{\kappa^p}(\boldsymbol{r}) = \sum_{\Lambda} V^{(0)}_{\kappa^p\Lambda}(r)\mathcal{P}_{\kappa\Lambda}$$

- Matching to NRQCD one finds the expressions of the potentials as a Wilson loop

$$V^{(0)}_{\kappa^p\Lambda}(\boldsymbol{r}) = \lim_{t\to\infty} \frac{i}{t} \log \mathrm{Tr}\left[\mathcal{P}_{\kappa\Lambda}\left\langle \quad \right\rangle\right]$$



$$\mathcal{Q}^{\alpha}_{(1/2)^+}(t,\boldsymbol{x}) = \underline{T}^{l}\left[P_+ q^{l}(t,\boldsymbol{x})\right]^{\alpha}$$

$$\mathcal{Q}^{\alpha}_{(1/2)^-}(t,\boldsymbol{x}) = \underline{T}^{l}\left[P_+ \gamma^5 q^{l}(t,\boldsymbol{x})\right]^{\alpha}$$

$$\mathcal{Q}^{\beta}_{(3/2)^-}(t,\boldsymbol{x}) = \mathcal{C}^{3/2\,\beta}_{1\,m\,1/2\,\alpha}\underline{T}^{l}\left[\left(\boldsymbol{e}^{\dagger}_{m}\cdot\boldsymbol{D}\right)\left(P_+ q(t,\boldsymbol{x})\right)^{\alpha}\right]^{l}$$

$$\mathcal{P}_{\frac{1}{2}\frac{1}{2}} = \mathbb{1}_2$$

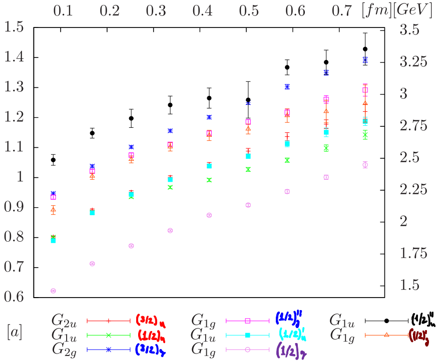
$$\mathcal{P}_{\frac{3}{2}\frac{1}{2}} = \frac{9}{8}\mathbb{1}_4 - \frac{1}{2}\left(\hat{\boldsymbol{r}}\cdot\boldsymbol{S}_{3/2}\right)^2$$

$$\mathcal{P}_{\frac{3}{2}\frac{3}{2}} = -\frac{1}{8}\mathbb{1}_4 + \frac{1}{2}\left(\hat{\boldsymbol{r}}\cdot\boldsymbol{S}_{3/2}\right)^2$$

($\underline{T}^{l}$ are the $\bar{3}$ tensor invariants)

Lattice static energies for doubly heavy baryons

Najjar, Bali PoS LAT2009 (2009)



$N_f = 2$, $a = 0.084$ fm, $L \simeq 1.3$ fm, $m_\pi \simeq 783$ MeV.

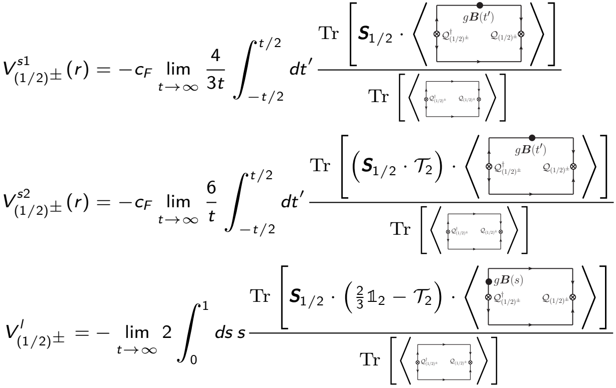
$O(3)$	$D_{\infty h}$
$(1/2)^+$	$(1/2)_g$
$(3/2)^-$	$(1/2)_u$, $(3/2)_u$
$(1/2)^-$	$(1/2)'_u$
$(3/2)^+$	$(1/2)'_g$, $(3/2)_g$

- Hyperfine operators appear at $1/m_Q$

$$V^{(1)}_{(1/2)\pm\,\mathrm{SD}}(\boldsymbol{r}) = V^{s1}_{(1/2)\pm}(r)\boldsymbol{S}_{QQ}\cdot\boldsymbol{S}_{1/2} + V^{s2}_{(1/2)\pm}(r)\boldsymbol{S}_{QQ}\cdot\left(\boldsymbol{\mathcal{T}}_2\cdot\boldsymbol{S}_{1/2}\right) + V^{l}_{(1/2)\pm}(r)\left(\boldsymbol{L}_{QQ}\cdot\boldsymbol{S}_{1/2}\right)$$

$(\mathcal{T}_2^{ij} = \hat{\boldsymbol{r}}^i\hat{\boldsymbol{r}}^j - \delta^{ij}/3$, for $\kappa = 3/2$ see Soto, JT Phys.Rev.D 102 (2020))

- Matching expressions of the potentials



Problem:

- No lattice data for the $1/m_Q$ suppressed potentials.

Problem:

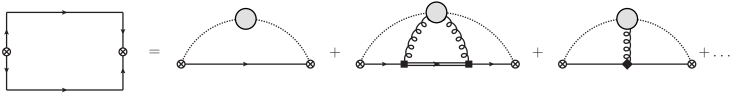
- No lattice data for the $1/m_Q$ suppressed potentials.

$\Rightarrow$ Parametrize the potentials from short-distance and long-distance descriptions.

Short distance: $r \ll 1/\Lambda_{\rm QCD}$

- Integrate out $m_Q v$ and $\Lambda_{\rm QCD}$ in two steps.
- The intermediate step is formally equivalent to weakly coupled pNRQCD. Pineda, Soto Nucl.Phys.Proc.Suppl.64 (1998); Brambilla, Pineda, Soto, Vairo Nucl.Phys.B566 (2000), Brambilla, Vairo, Rosch, Phys.Rev.D72 (2005)

- Compute the Wilson loop in weakly coupled pNRQCD



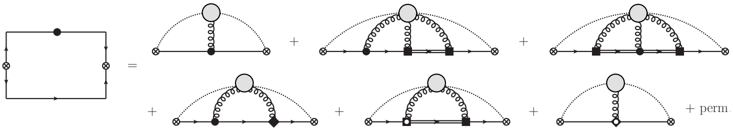
  - Triplet field (single line), sextet field (double line), light quark (dashed line).
  - ■ chromoelectric dipole, ◆ chromoelectric quadrupole.
- Static potentials:

$$V^{(0)}_{(1/2)\pm}(r) = -\frac{2}{3}\frac{\alpha_s}{r} + \overline{\Lambda}_{(1/2)\pm} + \overline{\Lambda}^{(1)}_{(1/2)\pm} r^2 + \dots$$

- Heavy quark-diquark duality Savage, Wise, Phys. Lett. B248,(1990):
  - $m_B = \overline{\Lambda}_{(1/2)+} + \mathcal{O}(1/m_b)$ (or D mesons).
  - $m_{D_0^*(2300)} - m_D = \overline{\Lambda}_{(1/2)-} - \overline{\Lambda}_{(1/2)+} + \mathcal{O}(1/m_c)$ (or from $m_{D_1(2420)} - m_{D^*(2007)}$).

Short-distance regime: Hyperfine operators potentials

- Wilson loops with operator insertion:



  - Black/white dots indicate heavy quark spin or angular momentum dependent operators.
- The hyperfine potentials can be expanded as

$$V^{s1}_{(1/2)\pm}(r) = c_F\left(\Delta^{(0)}_{(1/2)\pm} + \Delta^{(1,0)}_{(1/2)\pm} r^2 + \dots\right)$$

$$V^{s2}_{(1/2)\pm}(r) = c_F\Delta^{(1,2)}_{(1/2)\pm} r^2 + \dots$$

$$V^{l}_{(1/2)\pm} = \frac{1}{2}\left[\Delta^{(0)}_{(1/2)\pm} + \left(\Delta^{(1,0)}_{(1/2)\pm} - \frac{1}{3}\Delta^{(1,2)}_{(1/2)\pm}\right) r^2\right] + \dots$$

- From heavy quark-diquark duality Brambilla, Vairo, Rosch, Phys.Rev.D72 (2005):

$$m_{P^*_{\bar{Q}q}} - m_{P_{\bar{Q}q}} = \frac{2c_F(m_Q)}{m_Q}\Delta^{(0)}_{(1/2)\pm} + \mathcal{O}(1/m_Q^2)$$

Long-distance regime: Effective string theory

Hypotesis

- At long distances a flux tube emerges from each static sources joining at the position of the light quark.

- The flux tube is a bosonic string: $\xi^\mu(\tau,\lambda)$

$$S_{\rm g} = -\sigma \int d^2x \sqrt{|\det(g)|}$$

g induced metric on the string, σ string tension. Nambu, Phys.Lett.B80 (1979); Luscher, Symanzik, Weisz, Nucl.Phys.B173 (1980); Luscher, Weisz, JHEP07 (2002)

- Fermion constrained on the string

$$S_{\rm l.q} = \int d^2x \sqrt{g}\bar{\psi}(x)\left(i\rho^a \overleftrightarrow{\partial}_a - m_{\rm l.q.}\right)\psi(x)$$

$\bar{\psi}\rho^a\overleftrightarrow{\partial}_a\psi \equiv \left(\bar{\psi}(\rho^a\partial_a\psi) - (\partial_a\bar{\psi})\rho^a\psi\right)/2$, $\rho^a \equiv \gamma^\mu e^a_\mu$, $e^\alpha_a \equiv \partial\xi^\alpha/\partial x^a$.

- Expand for small fluctuations $\partial\xi \sim 1/(r\Lambda_{\rm QCD}) \ll 1$.
- Compute as in $1+1$ dimensional QFT.

Computation of the potentials in the Effective string theory

Mapping: Wilson loop to EST

- Match operators with same symmetry properties (parity, flavor, $D_{\infty h}$,...)
  Perez-Nadal, Soto, Phys.Rev.D79 (2009)
- $\mathcal{Q}_{(1/2)^{\pm}} \mapsto$ ground state string fermion with matching parity.
- Different components of $\boldsymbol{B}$ belong to different representations of $D_{\infty h}$.

$$\boldsymbol{B}^{l}(t,\, z) \mapsto \Lambda_f \bar{\psi}(t,\, z)\frac{\boldsymbol{\Sigma}^{l}}{2}\psi(t,\, z)\,, \quad l = 1, 2$$

$$\boldsymbol{B}^{3}(t,\, z) \mapsto \Lambda_f' \bar{\psi}(t,\, z)\frac{\boldsymbol{\Sigma}^{3}}{2}\psi(t,\, z)\,, \quad \boldsymbol{\Sigma} = \mathrm{diag}(\boldsymbol{\sigma},\, \boldsymbol{\sigma})$$

- The static potential

$$V^{(0)}_{(1/2)^{\pm}}(\boldsymbol{r}) = \sigma r + E_1\,, \quad E_1 = \sqrt{(\pi/r)^2 + m^2_{\rm l.q.}}$$

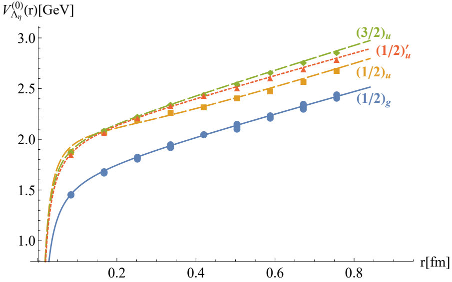
- Heavy-quark spin and angular momentum dependent potentials

$$V^{s1}_{(1/2)^{\pm}}(r) = \frac{c_F}{3r}\left(1 \mp \frac{m_{\rm l.q.}}{E_1}\right)\left(\Lambda_f' - 2\Lambda_f\right)$$

$$V^{s2}_{(1/2)^{\pm}}(r) = \frac{c_F}{r}\left(1 \mp \frac{m_{\rm l.q.}}{E_1}\right)\left(\Lambda_f' + \Lambda_f\right)$$

$$V^{l}_{(1/2)^{\pm}} = -\frac{1}{2r}\left(1 \mp \frac{4}{\pi^2}\frac{m_{\rm l.q.}}{E_1}\right)\Lambda_f$$

- We consider the 4 lowest laying static energies.
- The static potentials are obtained from constrained fits to the lattice data of Najjar, Bali PoS LAT2009 (2009).

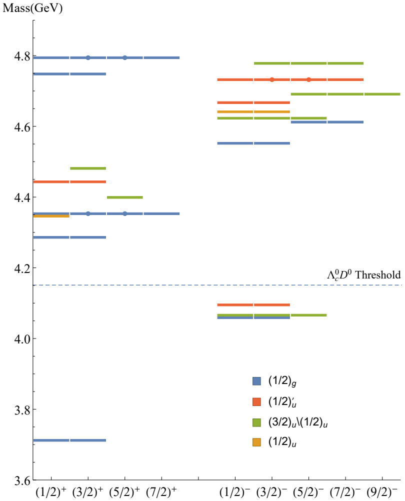


- Short distance:
  - Coulombic $-\frac{2}{3}\frac{\alpha_s}{r}+\overline{\Lambda}$
  - $\overline{\Lambda}_{(1/2)^+} = 0.555(31)$ GeV Bazavov et al. Phys. Rev. D98 (2018)
  - $\overline{\Lambda}$'s consistent with heavy-quark-diquark symmetry.
- Long distance:
  - Linear $\sigma r + c(m_{l.q.})$.
  - $\sigma = 0.21$ GeV^2

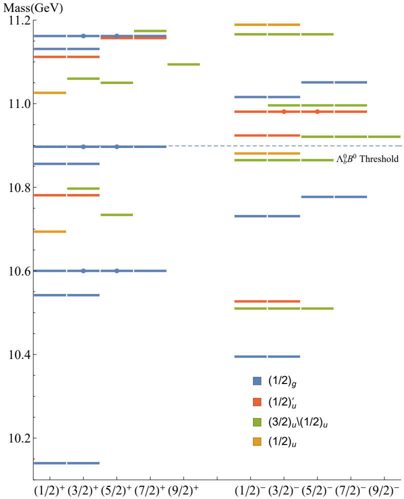
Spectra at leading order

Soto, JTC, Phys.Rev.D 102 (2020)

Ξ_{cc} spectrum

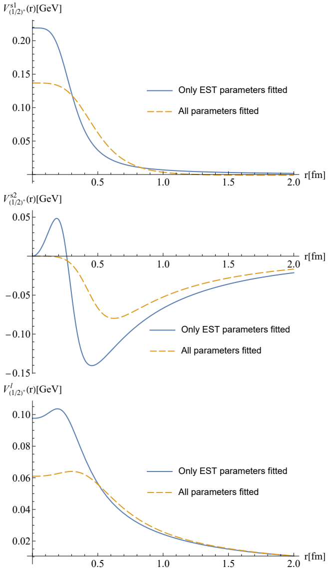


Ξ_{bb} spectrum



Spectra including hyperfine contributions for $\kappa = 1/2$

Soto, JTC, ??



- Minimal model for the hyperfine potentials:

$$V = V_{\rm short} \frac{r_0^n}{r^n + r_0^n} + V_{\rm long} \frac{r^n}{r^n + r_0^n}$$

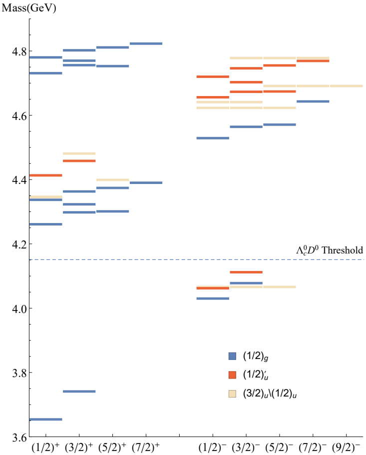
(n chosen to ensure correct limits)

- Unknown parameters fitted to lattice determinations of the hyperfine splittings.
- All inputs are QCD-based except interpolation choice.

Spectra including hyperfine contributions for $\kappa = 1/2$

Soto, JTC, ??

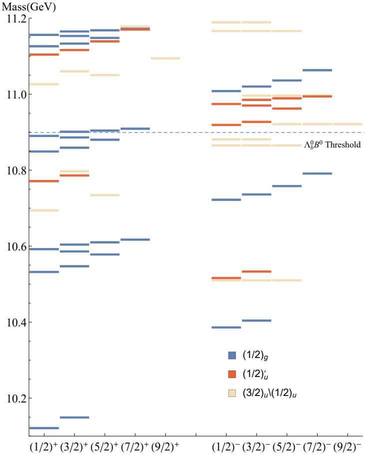
Ξ_{cc} spectrum



$m_{\Xi_{cc}}$ =3.654(75) GeV , $m_{\Xi^*_{cc}}$ = 3.741(75) GeV

δ_{hf} = 87(14) MeV

Ξ_{bb} spectrum



$m_{\Xi_{bb}}$ =10.121(52) GeV , $m_{\Xi^*_{bb}}$ = 10.149(52) GeV

δ_{hf} = 28(5) MeV

Summary

- We applied BOEFT to doubly heavy baryons.
- Lattice data is only available for (region of) the static potential.
- Developed parametrizations of the hyperfine potentials with minimal modeling.
  - Short-distance: multipole expansion (weakly coupled pNRQCD).
  - Long-distance: Effective String Theory with a fermion.
- Computed the spectrum for doubly heavy baryons including hyperfine contributions.

Thank you for your attention