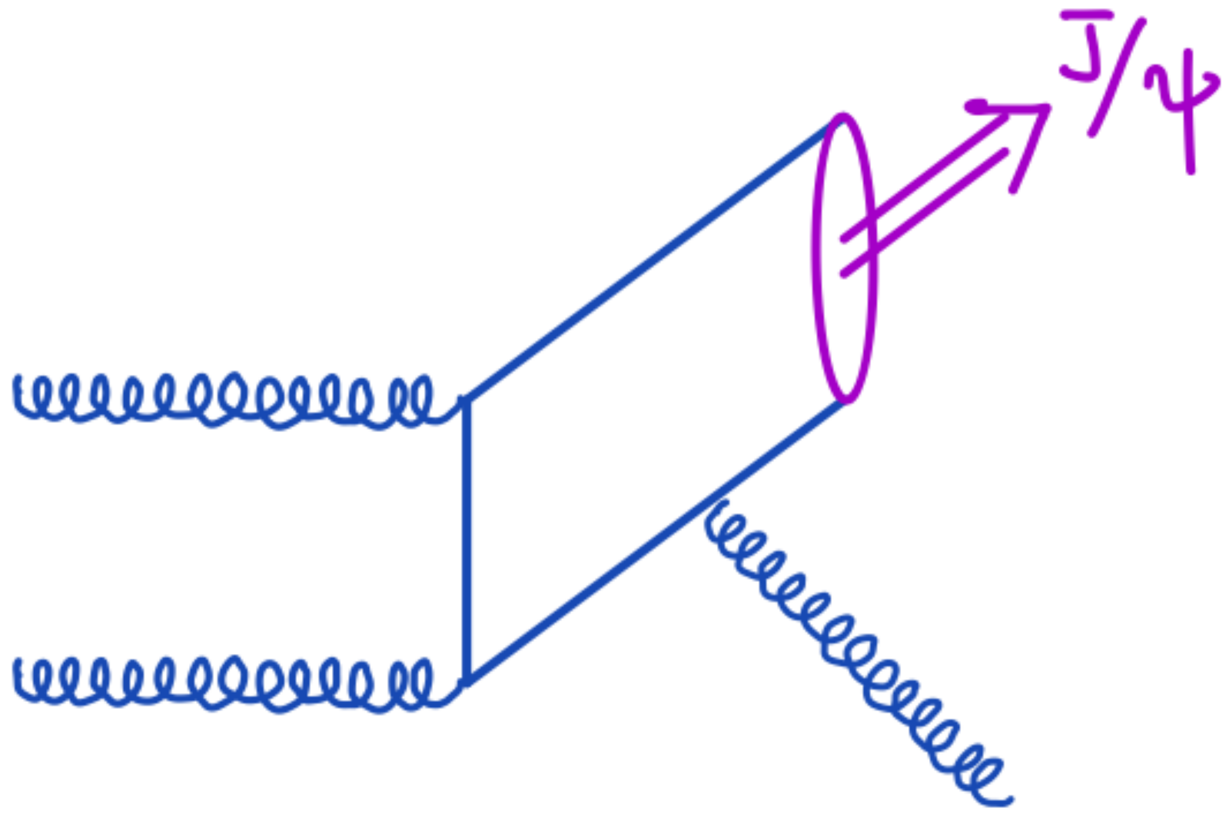
An effective field theory approach to quarkonium at small transverse momentum

Parallel presentation: A Virtual Tribute to Quark Confinement and the Hadron Spectrum 2021

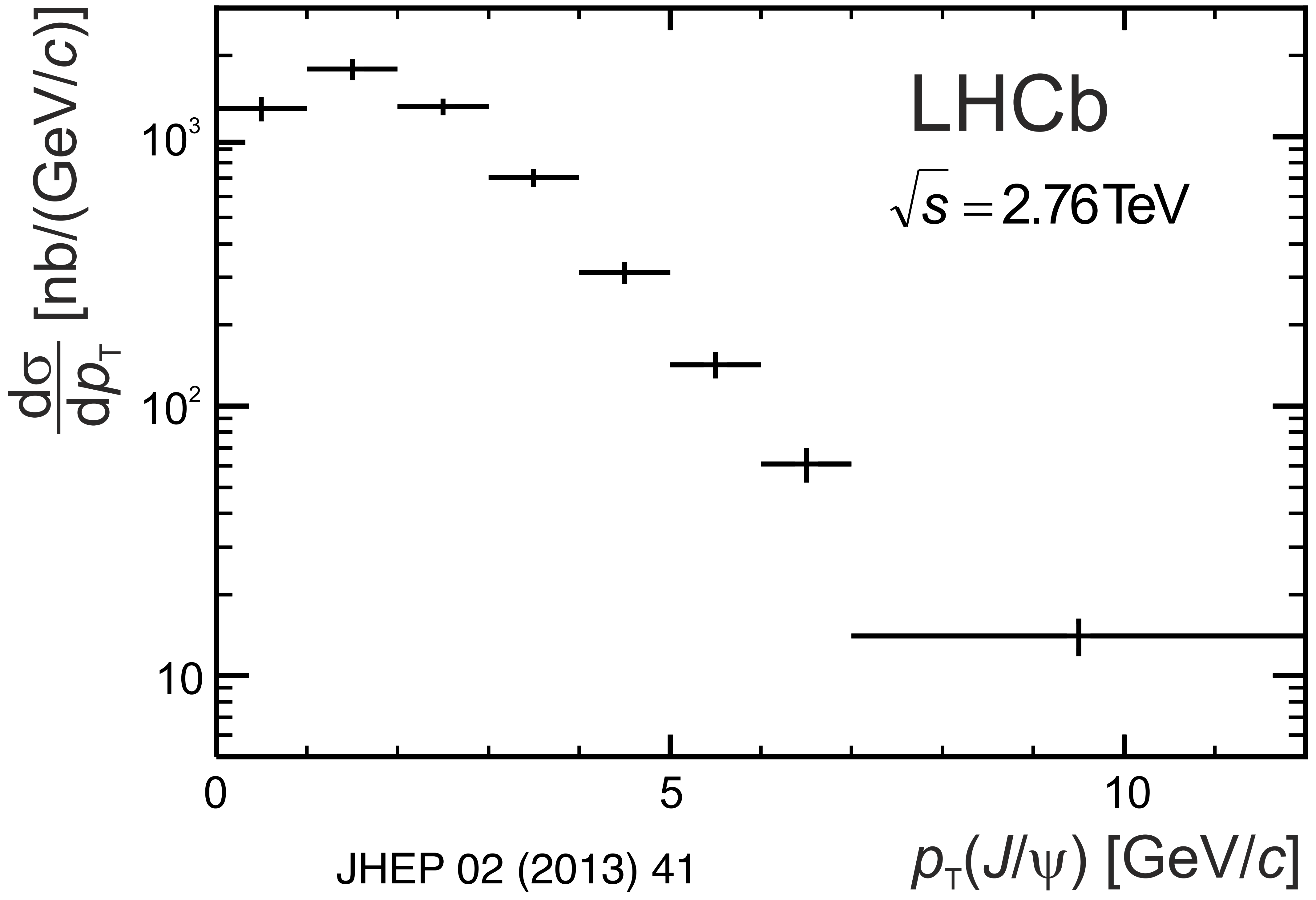
Sean Fleming, August 6 2021

Motivation

Quarkonium production in Hadronic collisions is dominated by initial state gluons



Good data is available even for small transverse momentum



Promising avenue for measuring the gluon transverse momentum dependent parton distribution function (TMDPDF)

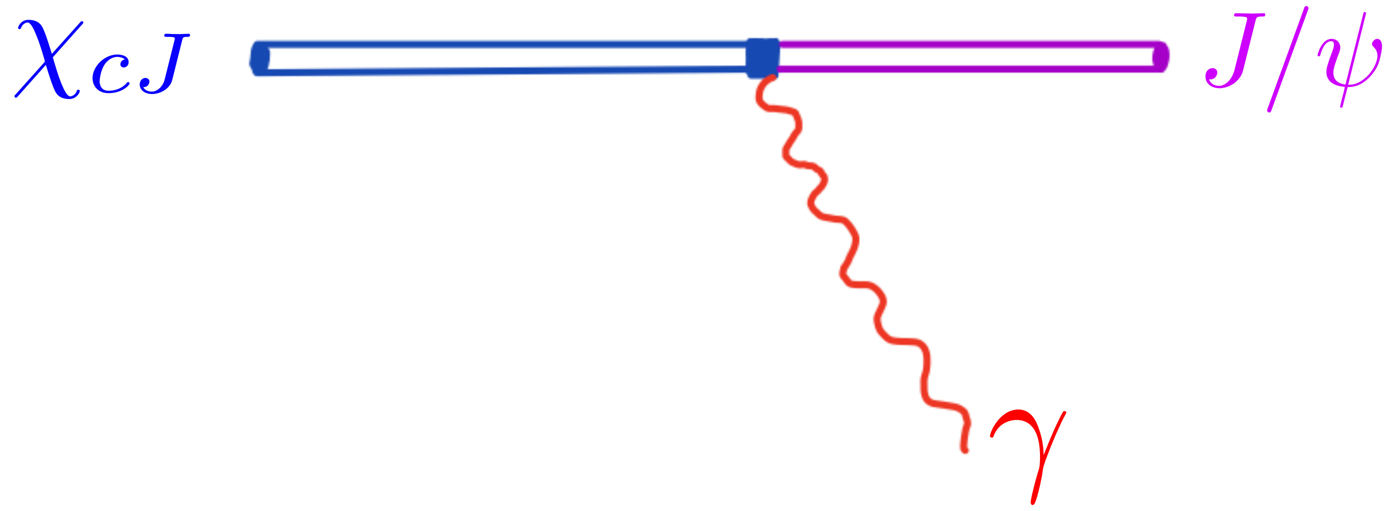
Motivation

Theoretical framework for attacking ${}^3S_1$ production while technically complicated is straightforward: my favorite approach is to combine non-relativistic QCD (NRQCD) and soft collinear effective theory (SCET)

So what's the problem?!?!?!?!

The P-wave

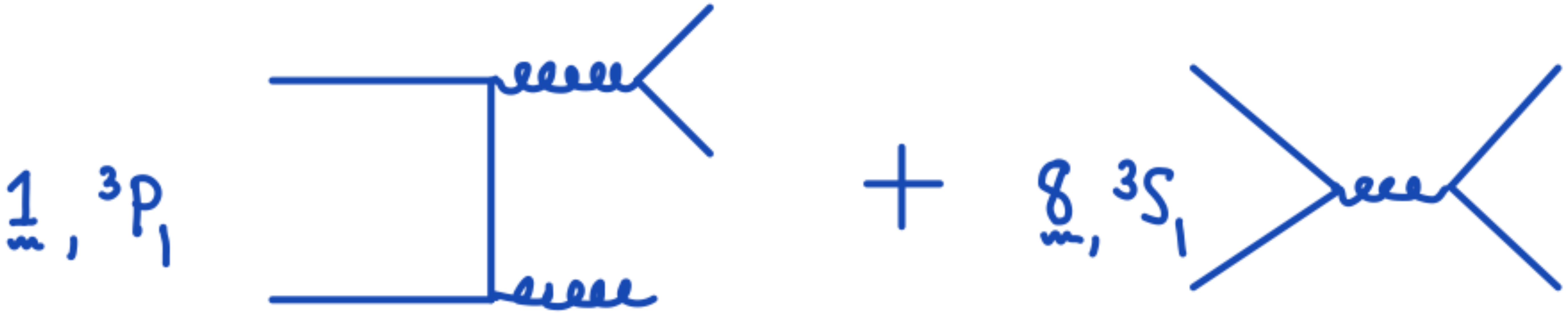
There is sizable feed down from the P-wave, which needs to be included



Plus it's interesting all on its own

Complication

Problem with P-wave was discovered in 1976 by Barbieri et al. and explained by Bodwin, Braaten and Lepage in 1992



An infrared divergence in the color-single P-wave contribution requires the presence of both a color-octet S-wave contribution

Complication

NRQCD solves this problem for inclusive decay, but not if the final state is restricted—Jets!

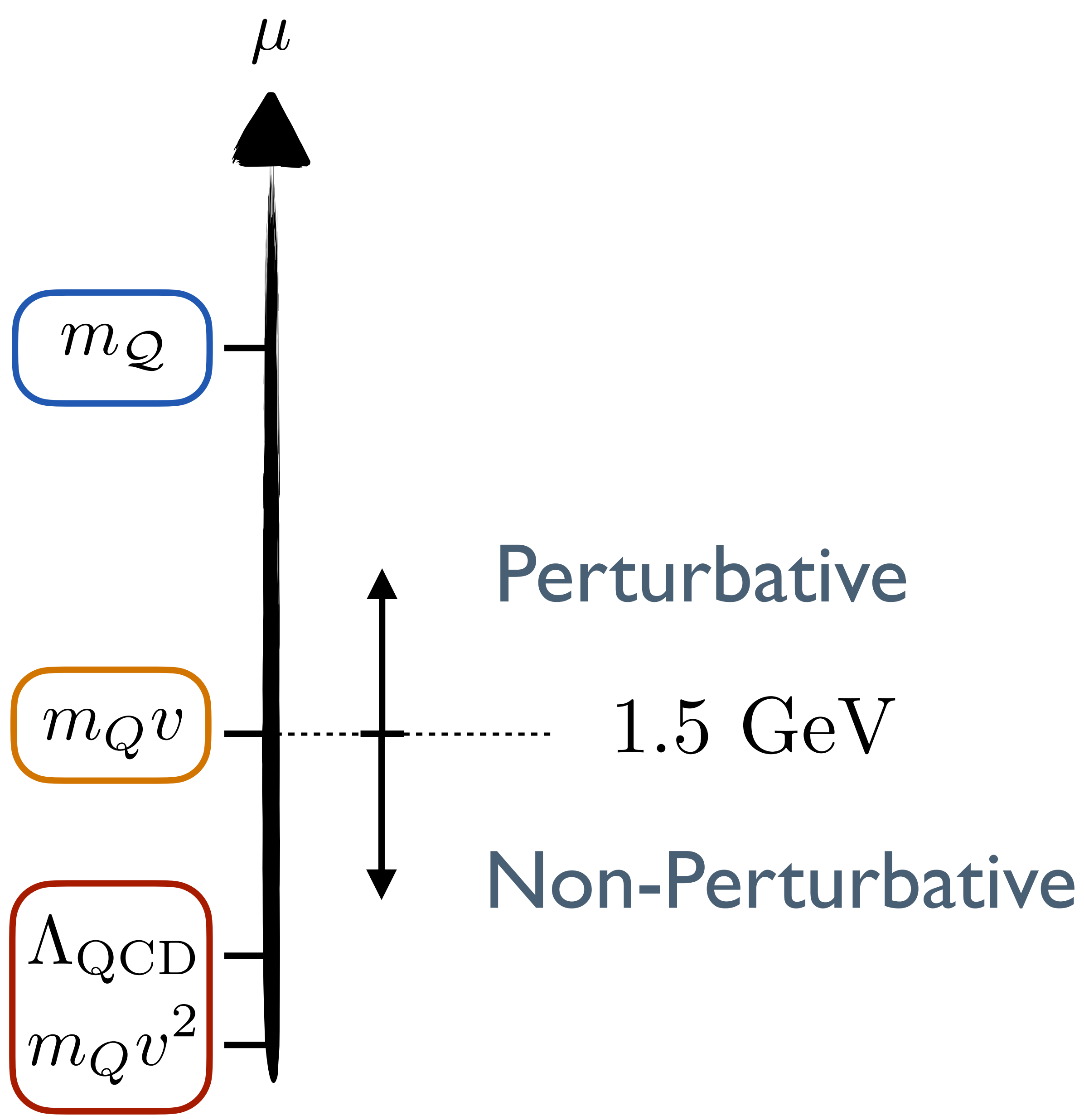
Need SCET to accurately describe the high energy light-like degrees of freedom

The formalism needs to include the cancelation of infrared divergences between singlet and octet contributions

So how do you do it?

Scales

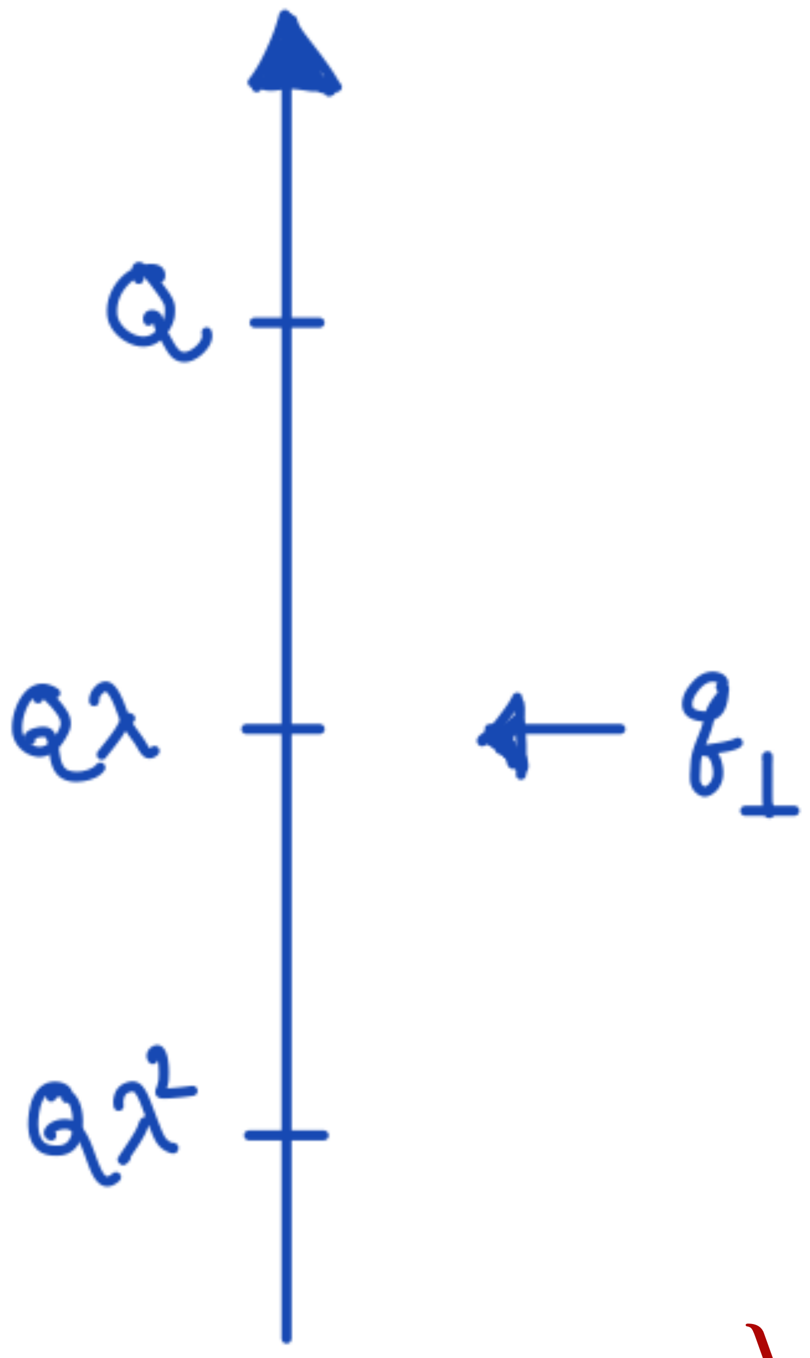
NRQCD Scales



$|\mathbf{p}_Q| \sim m_Q v$ (soft)

$K_Q \sim m_Q v^2$ (ultra-soft)

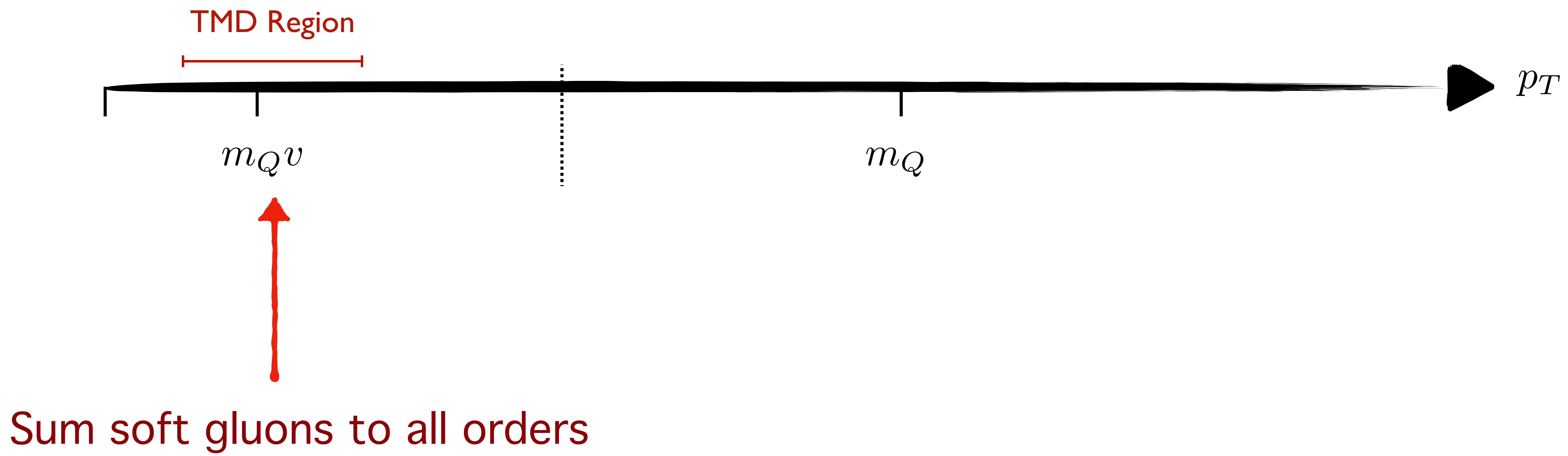
SCET Scales



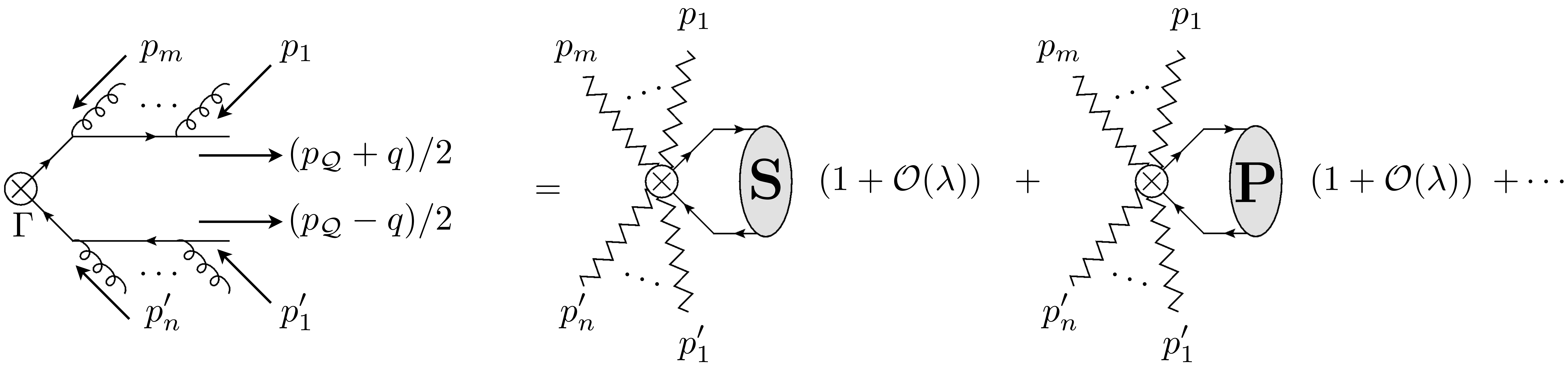
$\lambda = q_\perp / M$

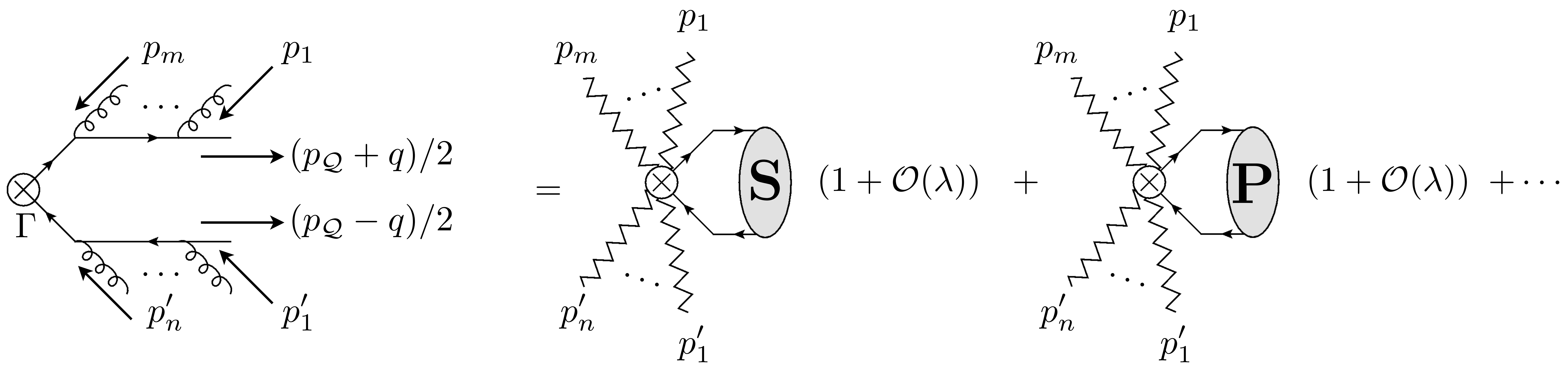
Gives recoil for quarkonium

Scales



Via diagrams





S-wave:
simple result

P-wave: Not so
simple result

$$d_\Gamma^{(0)} = \left(u^{(0)}\right)^\dagger S_v^\dagger\, \Gamma^{(0)}\; S_v v^{(0)}$$

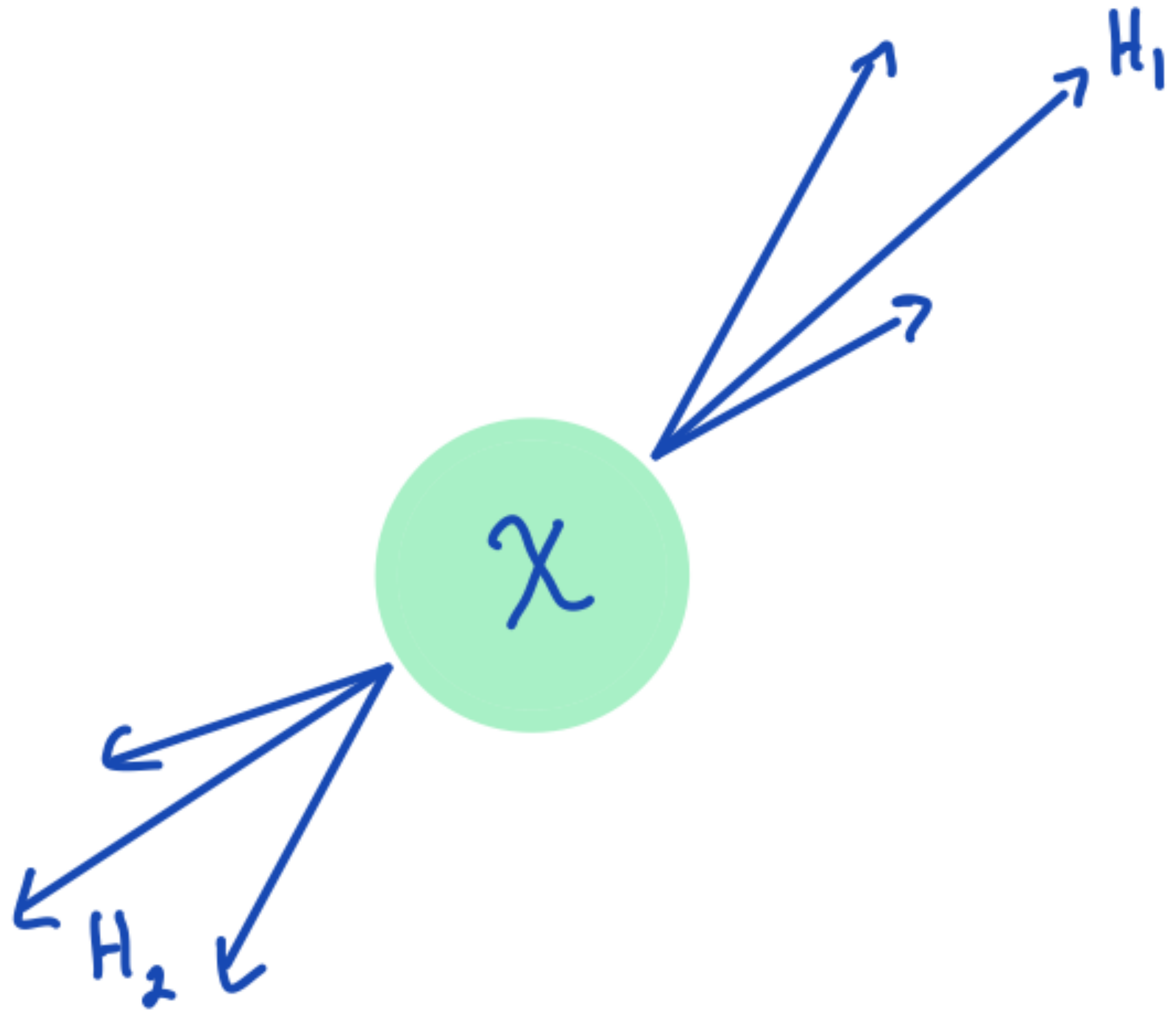
$$d_\Gamma^{(1)} = \frac{g}{2m}\left(u^{(0)}\right)^\dagger \left\{ S_v^\dagger \Gamma^{(0)} S_v, \left[\frac{1}{v\cdot\mathcal{P}}\mathbf{q}\cdot\mathbf{B}_s\right]\right\} v^{(0)} + \left(u^{(0)}\right)^\dagger S_v^\dagger\; \mathbf{q}\cdot\left(\mathbf{\Gamma}^{(1)} - \frac{1}{4m}\left\{\Gamma^{(0)},\boldsymbol{\gamma}\right\}\right) S_v v^{(0)}$$

$$B_s^\mu = -\frac{1}{g} S_v^\dagger\left[(\mathcal{P}^\mu - gA^\mu)S_v\right]$$

$$S_v(x,-\infty) = \mathrm{P}\left[\exp\left(-ig\int_{-\infty}^{0} d\tau\; v\cdot A_{soft}(x^\mu + v^\mu\tau)\right)\right]$$

Two operators are related by reperameterization invariance (RPI), as they are in NRQCD

“Easy” application: Quarkonium -> 2 Jets



$$\frac{d\Gamma}{dz_1 dz_2 d^2q_\perp} = \Gamma_0 \sum_{n={}^3S_1^{[8]},{}^3P_J^{[1]}} H_{[n]}(M_\chi,\mu) \int d^2k_{\bar{n}\perp} \int d^2k_{s\perp} \int d^2k_{n\perp} \delta^{(2)}(\mathbf{k}_{\bar{n}'\perp} + \mathbf{k}_{n\perp} + \mathbf{k}_{s\perp} - \mathbf{q}_\perp)\, S^\perp_{[n]}(\mathbf{k}_{s\perp}) D^\perp_{q/H_1}(z_1,\mathbf{k}_{n\perp}) D^\perp_{\bar{q}/H_2}(z_2,\mathbf{k}_{\bar{n}'\perp})$$

$$S^\perp_{\chi_J\to{}^3S_1^{[8]}}(\mathbf{k}_\perp) = \frac{d-2}{(d-1)t_F} \mathrm{tr}\Big\langle \chi_J \Big| \psi^\dagger \sigma^i T^a \chi \mathcal{S}_v^{ba} (S_{\bar{n}}^\dagger T^b S_n) \delta^{(2)}(\mathbf{k}_\perp - \boldsymbol{\mathcal{P}}_\perp) \times (S_n^\dagger T^c S_{\bar{n}}) \mathcal{S}_v^{dc} \chi^\dagger \sigma^i T^d \psi \Big| \chi_J \Big\rangle$$

$$S^\perp_{\chi_J\to{}^3P_J}(\mathbf{k}_\perp) = (2J+1)\frac{g^2}{N_c^2\, t_F}\, \mathcal{A}_J^{ij}\ \mathrm{tr}\Big\langle \chi_J \Big| \psi^\dagger \boldsymbol{\sigma}\cdot \overset{\leftrightarrow}{\boldsymbol{\mathcal{P}}}\, \chi \Big[\frac{B_s^{a,i}}{m\, v\cdot \mathcal{P}}\Big] \mathcal{S}_v^{ba}(S_{\bar{n}}^\dagger T^b S_n)\ \delta^{(2)}(\mathbf{k}_\perp - \boldsymbol{\mathcal{P}}_\perp)(S_n^\dagger T^c S_{\bar{n}}) \mathcal{S}_v^{dc} \Big[\frac{B_s^{d,j}}{m\, v\cdot \mathcal{P}}\Big] \chi^\dagger \boldsymbol{\sigma}\cdot \overset{\leftrightarrow}{\boldsymbol{\mathcal{P}}}\, \psi \Big| \chi_J \Big\rangle$$

Shape/Soft functions at NLO

The NLO S-wave shape function:

$$S^{\perp,\mathrm{NLO}}_{\chi\to{}^3S_1^{[8]}}(\mathbf{k}_\perp;\mu,\nu) = \frac{d-2}{d-1}\Bigg\{\Bigg[S^{\perp}_{\mathrm{DY}}(\mathbf{k}_\perp) + \frac{\alpha_s C_A}{2\pi}\left(\frac{1}{\epsilon}\delta^{(2)}(\mathbf{k}_\perp) - 2\mathcal{L}_0(\mathbf{k}_\perp^2,\mu^2)\right)\Bigg]\langle{}^3S_1^{[8]}\rangle_{\mathrm{LO}}$$

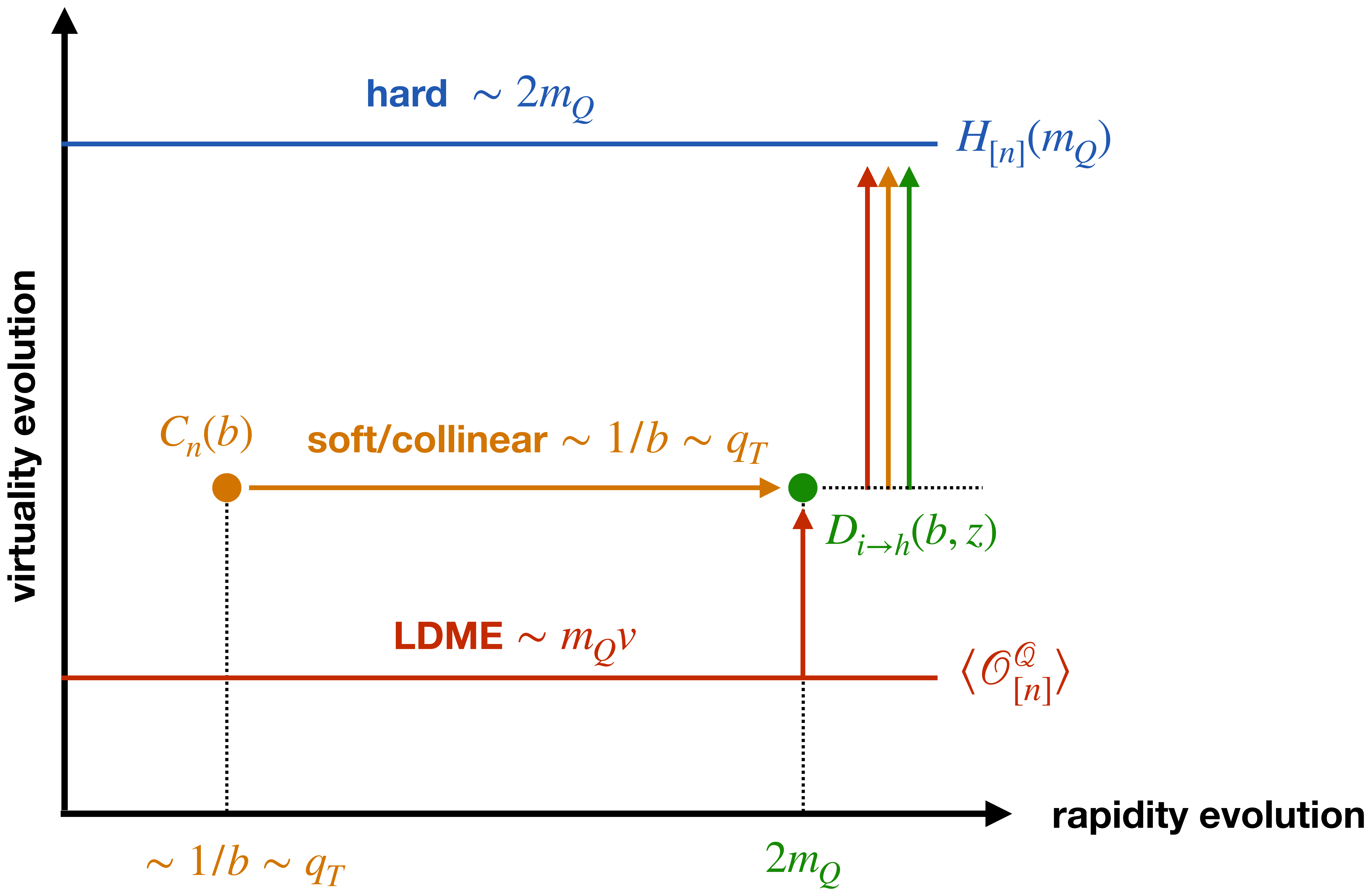
$$+\delta^{(2)}(\mathbf{k}_\perp)\Bigg[\frac{4\alpha_s}{3\pi m^2}\left(C_F\sum_J\langle{}^3P_J^{[1]}\rangle_{\mathrm{LO}} + B_F\sum_J\langle{}^3P_J^{[8]}\rangle_{\mathrm{LO}}\right)\left(\frac{1}{\epsilon_{\mathrm{UV}}} - \frac{1}{\epsilon_{\mathrm{IR}}}\right)\Bigg]\Bigg\}$$

Mixing terms in the renormalization.
The same cancelation as in NRQCD.

The NLO P-wave shape function:

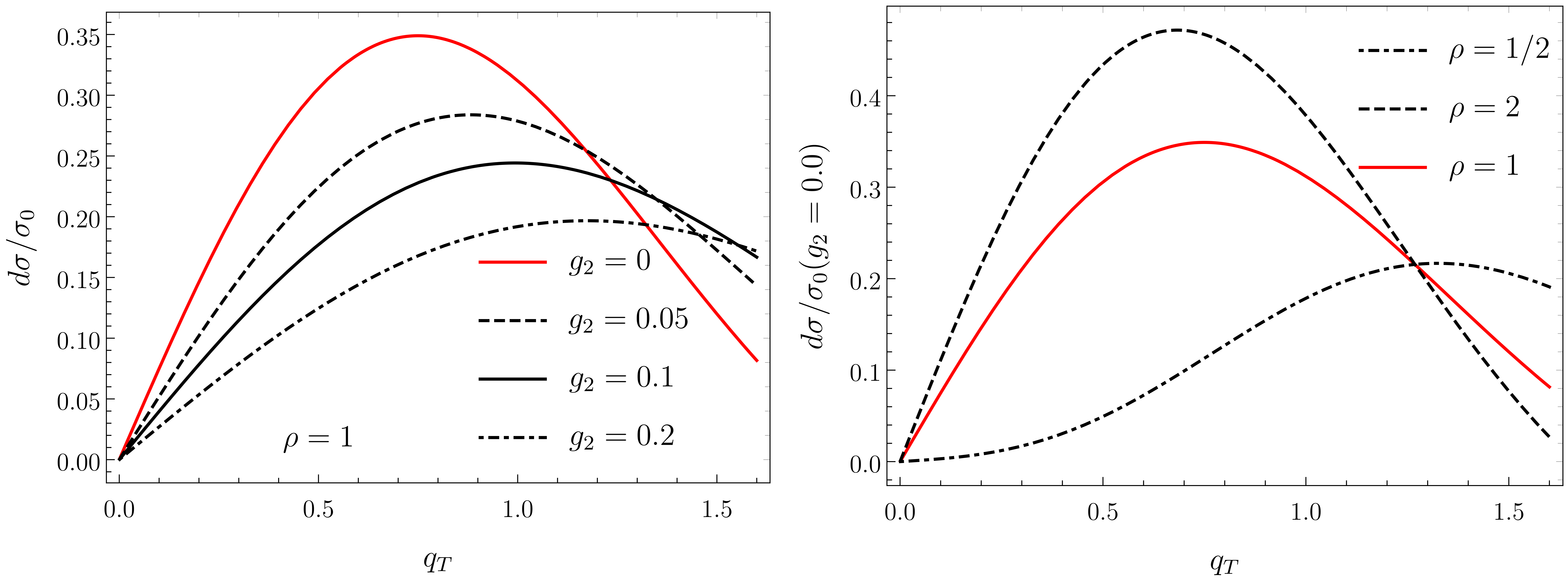
$$S^{\perp,\mathrm{NLO}}_{\chi\to{}^3P_J^{[1]}}(\mathbf{k}_\perp) = \frac{8\alpha_s C_F}{9\pi m^2}\sum_J\langle{}^3P_J^{[1]}\rangle_{\mathrm{LO}}\left(\frac{1}{\epsilon}\delta^{(2)}(\mathbf{k}_\perp) - 2\mathcal{L}_0(\mathbf{k}_\perp^2,\mu^2) + c_J\right)$$

Resummation



Results

$$\chi_{b0} \to \pi^+ + \pi^- + X$$



The future

- Apply this to hadronic production

Burning question: does factorization hold?