Heavy-quark mesons in the diabatic approach: string breaking and the quarkoniumlike spectrum

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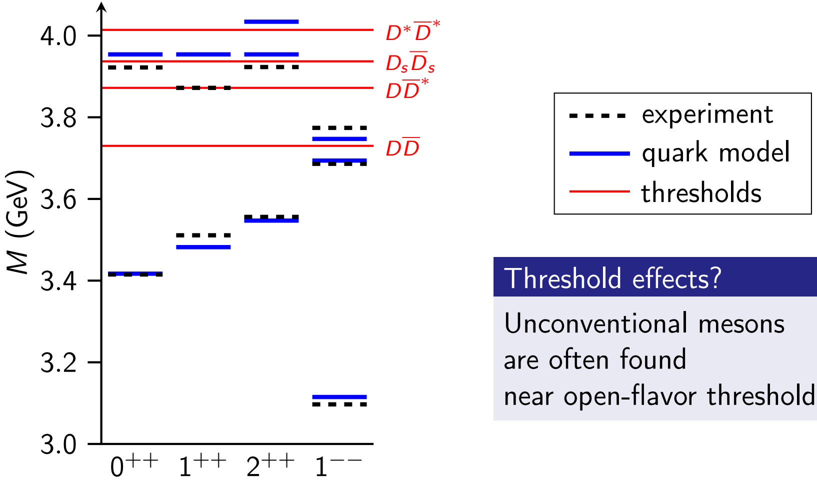
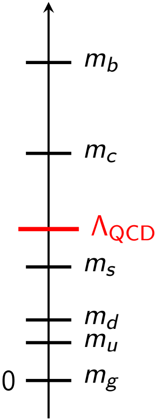
Unconventional Charmoniumlike Mesons



Threshold effects?

Unconventional mesons
are often found
near open-flavor thresholds.

Description in Heavy and Light Fields



Heavy ($m >> \Lambda_{QCD}$)

- heavy quarks c, b

Light ($m \lesssim \Lambda_{QCD}$)

- gluons g
- light quarks u, d, s

Integrating the Light Fields

1. Separate kinetic energy of the heavy quarks

$$H\,|\psi\rangle = E\,|\psi\rangle\,, \qquad H = K_{\text{heavy}} + H_{\text{light}}^{(\text{heavy})}.$$

2. Solve the light-field Hamiltonian for static heavy quarks

$$H_{\text{light}}^{(\text{heavy})} \to H_{\text{static}}(r), \qquad H_{\text{static}}(r)\,|\zeta_i(r)\rangle = V_i(r)\,|\zeta_i(r)\rangle\,.$$

Static energy levels

The static energies $V_i(r)$ can be calculated *ab initio* in Lattice QCD.

Adiabatic Wave Function

Adiabatic expansion

$$|\psi\rangle = \sum_i \int \mathrm{d}r\, \psi_i(r)\, |r\rangle\, |\zeta_i(r)\rangle$$

- Light field states calculated at the same position of the heavy quarks
- One wave function for each light-field energy

Adiabatic Schrödinger Equation

$$\sum_j \left[-\frac{\hbar^2}{2\mu}(\delta_{ij}\nabla + \tau_{ij}(r))^2 + \delta_{ij}(V_i(r) - E) \right] \psi_j(r) = 0$$

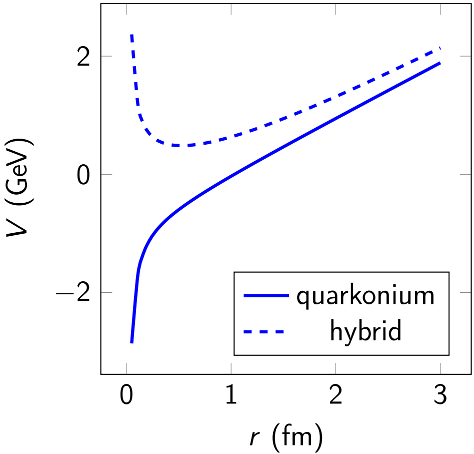
Non-adiabatic coupling terms

The kinetic energy term mixes different channels through the non-adiabatic couplings $\tau_{ij}(r) = \langle \zeta_i(r) | \nabla \zeta_j(r) \rangle$.

Adiabatic potentials

The potentials $V_i(r)$ in the Schrödinger equation are the energy levels calculated in Lattice QCD.

Adiabatic Potentials in Quenched Lattice QCD



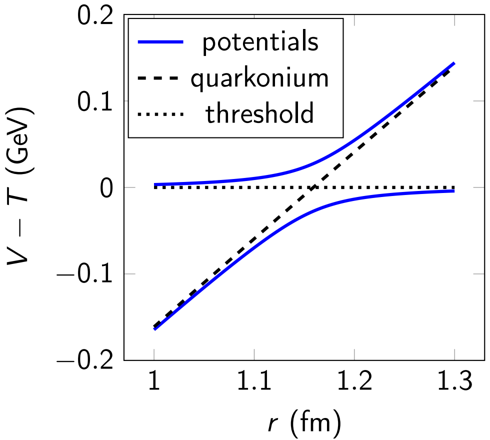
See for example [Bali, 2001].

Quenched Lattice QCD

Gluons only,
without light quarks

- Ground state: quarkonium potential
- Excited states: hybrid potentials

Adiabatic Potentials in Unquenched Lattice QCD



Unquenched Lattice QCD

Gluons and light quarks

String breaking

Channel mixing significant near the avoided crossing

See [Bali *et al.* (SESAM collaboration), 2005], [Bulava *et al.*, 2019].

From Adiabatic to Diabatic

Diabatic expansion

$$|\psi\rangle = \sum_i \int \mathrm{d}r \, \widetilde{\psi}_i(r, r_0) \, |r\rangle \, |\zeta_i(r_0)\rangle$$

Diabatic channels

$$\widetilde{\psi}_i(r, r_0) \to \psi_{Q\overline{Q}}(r), \, \psi_{M\overline{M}}(r)$$

- Light field states are calculated at a fixed position r_0.
- For r_0 far from the avoided crossing, they correspond to quark-antiquark and meson-meson.

The Diabatic Schrödinger Equation

$$\left[\begin{pmatrix} -\frac{\nabla^2}{2\mu_{Q\overline{Q}}} & 0 \\ 0 & -\frac{\nabla^2}{2\mu_{M\overline{M}}} \end{pmatrix} + \begin{pmatrix} V_{Q\overline{Q}}(r) & V_{\text{mix}}(r) \\ V_{\text{mix}}(r) & T_{M\overline{M}} \end{pmatrix} - E\right] \begin{pmatrix} \psi_{Q\overline{Q}}(r) \\ \psi_{M\overline{M}}(r) \end{pmatrix} = 0$$

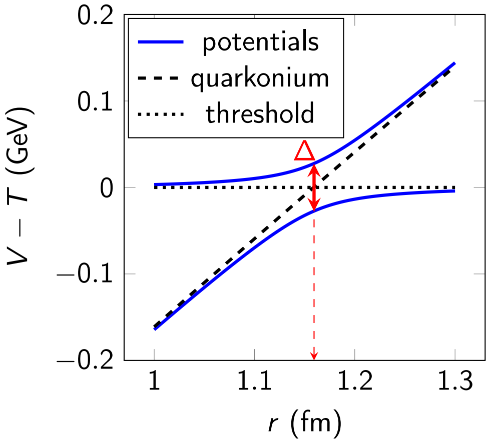
Diabatic potential matrix

The potential couples quark-antiquark and meson-meson.

Adiabatic-to-diabatic transformation

The eigenvalues of the diabatic potential matrix
are the adiabatic potentials calculated in Lattice QCD.

Adiabatic Potentials in Unquenched Lattice QCD



Unquenched Lattice QCD

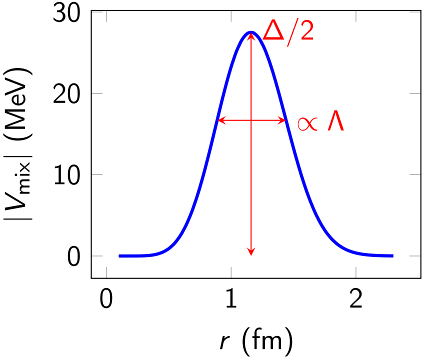
Gluons and light quarks

String breaking

Channel mixing significant near the avoided crossing

See [Bali *et al.* (SESAM collaboration), 2005], [Bulava *et al.*, 2019].

The Quark-Antiquark–Meson-Meson Mixing Potential



Gaussian parametrization

$$|V_{\text{mix}}(r)| = \frac{\Delta}{2} e^{-\frac{(V_{Q\overline{Q}}(r) - T_{M\overline{M}})^2}{2\Lambda^2}}$$

- Δ: mixing strength
- Λ: mixing width

Mixing Effects on the Quarkoniumlike Spectrum

Above threshold

- Meson states acquire decay width.
- Quarkonium masses are shifted.

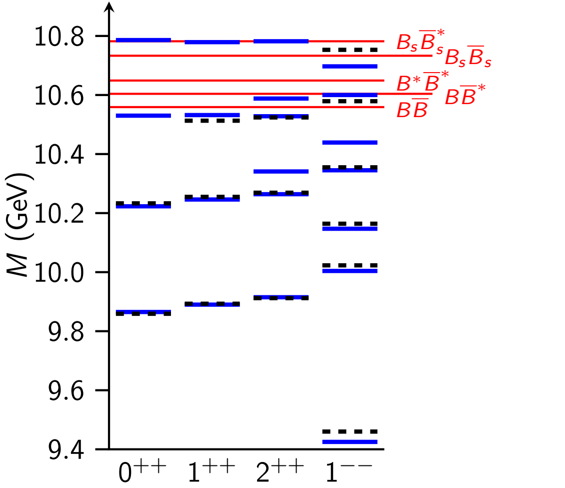
Below threshold

- Meson states acquire molecular components.
- Unconventional mesons may appear near threshold.

Unified description above and below threshold

Appearance of unconventional mesons and resonance decays are described by the same mixing potential.

Bottomoniumlike Spectrum



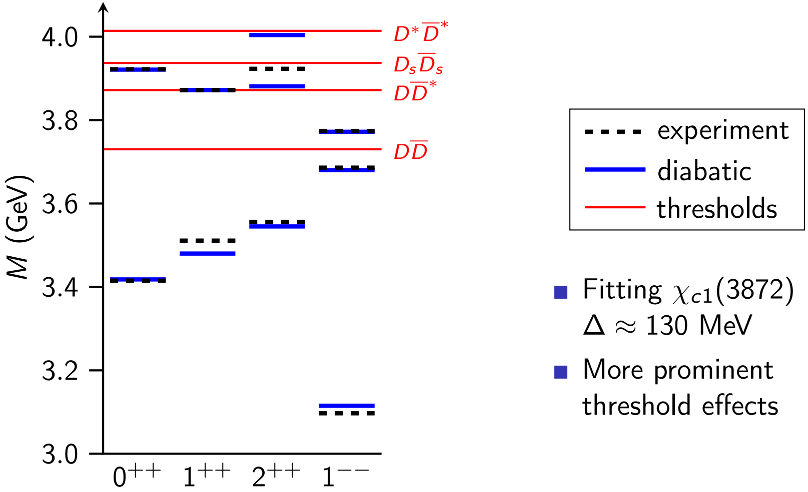
- From Lattice QCD $\Delta \approx 50$ MeV
- Relatively small threshold effects

Strong Decay Widths of Bottomoniumlike States

J^{PC}	M	$\Gamma_{B\overline{B}}$	$\Gamma_{B\overline{B}^*}$	$\Gamma_{B^*\overline{B}^*}$	$\Gamma_{B_s\overline{B}_s}$	$\Gamma^{\text{Theor}}_{\text{total}}$	$\Gamma^{\text{Expt}}_{\text{total}}$
0^{++}	10785.8	1.6		5.3	0.7	7.6	
1^{++}	10778.9		0.2	1.7		1.9	
2^{++}	10588.4	4.3				4.3	
2^{++}	10782.3	5.4	1.5	21.0	10.4	38.3	
1^{--}	10599.8	21.9				21.9	20.5 ± 2.5
1^{--}	10697.0	2.0	1.0	38.0		41.0	

Masses and widths in MeV units

Charmoniumlike Spectrum



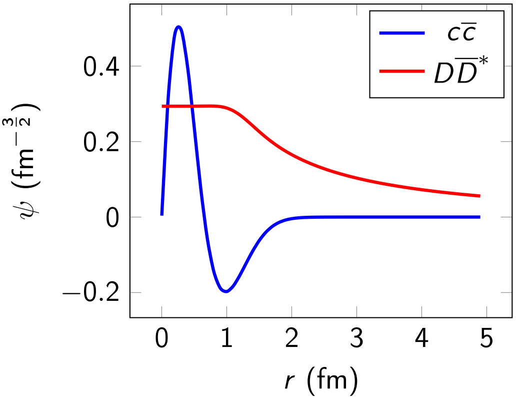
- Fitting $\chi_{c1}(3872)$
  $\Delta \approx 130$ MeV
- More prominent threshold effects

Strong Decay Widths of Charmoniumlike States

J^{PC}	M	$\Gamma_{D\overline{D}}$	$\Gamma_{D\overline{D}^*}$	$\Gamma_{D_s\overline{D}_s}$	$\Gamma^{\text{Theor}}_{\text{total}}$	$\Gamma^{\text{Expt}}_{\text{total}}$
0^{++}	3920.9	0.6			0.6	
2^{++}	3881.1	49.5	0.4		49.9	35.3 ± 2.8
2^{++}	4003.9	4.8	6.3	3.5	14.5	
1^{--}	3771.7	20.2			20.2	27.2 ± 1.0

Masses and widths in MeV units

Diabatic Wave Function of $\chi_{c1}(3872)$



- about 3% of $c\bar{c}$
- about 97% of $D\bar{D}^*$
- $\sqrt{\langle r^2 \rangle} \approx 11$ fm

Diabatic $\chi_{c1}(3872)$

It can be described as a $D\bar{D}^*$ molecule created by the mixing between $D\bar{D}^*$ and $c\bar{c}$.

Summary

- The Born-Oppenheimer approximation gives a description of quarkonium firmly based on Lattice QCD.
- The diabatic framework allows to include open-flavor mesons and string breaking into this description nonperturbatively.
- The diabatic potential matrix, calculable from Lattice QCD, can give account of the spectrum as well as strong decays of quarkoniumlike mesons.

For Further Reading

R. Bruschini and P. González.
Diabatic description of charmoniumlike mesons.
Phys. Rev. D **102**, 074002 (2020).

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Diabatic description of charmoniumlike mesons. II.
Mass corrections and strong decay widths.
Phys. Rev. D **103**, 074009 (2021).

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Diabatic description of bottomoniumlike mesons.
Phys. Rev. D **103**, 114016 (2021).