Highly excited pure gauge SU(3) flux tubes

talk at virtual Confinement 21

Pedro Bicudo

with Nuno Cardoso and Alireza Sharifian

CeFEMA, Departamento de Física, Instituto Superior Técnico (Universidade Técnica de Lisboa), Av. Rovisco Pais, 1049-001 Lisboa, Portugal

bicudo@tecnico.ulisboa.pt

August 02, 2021

TÉCNICO LISBOA

CeFEMA

Abstract

Spectra with full towers of levels are expected due to the quantization of the string vibrations.

Flux tube spectra are expected to have full towers of levels due to the quantization of the string vibrations. We study a spectrum of flux tubes with static quark and antiquark sources with pure gauge $SU(3)$ lattice QCD in 3+1 dimensions up to a significant number of excitations.

To go high in the spectrum, we specialize in the most symmetric case Σ_g^+, use a large set of operators, solve the generalized eigenvalue and compare different lattice QCD gauge actions and anisotropies.

Only a few references, mostly of our works, to help follow this talk, more colleagues will be cited in the proceedings contribution.

- C. Morningstar, Excitations of the static-quark potential, su(3) in 4 dimensions. URL https://www.andrew.cmu.edu/user/cmorning/static_potentials/SU3_4D/B30_AR3/plots/Rplots.html
- J. F. Arvis, The Exact $q\bar{q}$ Potential in Nambu String Theory, Phys. Lett. 127B (1983) 106–108. doi:10.1016/0370-2693(83)91640-4.
- O. Aharony, E. Karzbrun, On the effective action of confining strings, JHEP 06 (2009) 012. arXiv:0903.1927, doi:10.1088/1126-6708/2009/06/012.
- N. Cardoso, M. Cardoso, P. Bicudo, Inside the SU(3) quark-antiquark QCD flux tube: screening versus quantum widening, Phys. Rev. D88 (2013) 054504. arXiv:1302.3633, doi:10.1103/PhysRevD.88.054504.
- P. Bicudo, N. Cardoso, M. Cardoso, Color field densities of the quark-antiquark excited flux tubes in SU(3) lattice QCD, Phys. Rev. D 98 (11) (2018) 114507. arXiv:1808.08815, doi:10.1103/PhysRevD.98.114507.
- P. Bicudo, N. Cardoso, A. Sharifian, Spectrum of very excited Σ_g^+ flux tubes in SU(3) lattice QCD (5 2021). arXiv:2105.12159.
- K. G. Wilson, Confinement of Quarks, Phys. Rev. D 10 (1974) 2445–2459. doi:10.1103/PhysRevD.10.2445.
- C. Morningstar, Improved gluonic actions on anisotropic lattices, Nucl. Phys. B Proc. Suppl. 53 (1997) 914–916. arXiv:hep-lat/9608019, doi:10.1016/S0920-5632(96)00816-X.

Introduction - Motivation

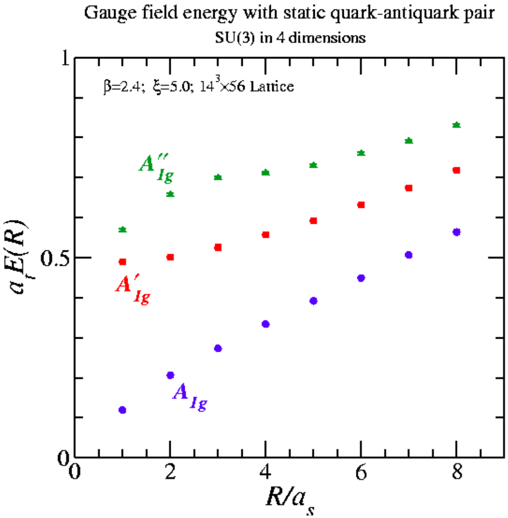

Figure: Most excited flux tube spectrum, with three quantum levels obtained By Colin Morningstar many years ago [1] : how high can we go now?

If we neglect the flux tube intrinsic width, it is equivalent to a quantum string. Flux tubes and strings have been studied for a long time.

- In 1911 Onnes discovered superconductivity. In 1933 Meissner and Ochsenfeld discovered the Meissner effect.
- In 1935, Rjabinin and Shubnikov experimentally discovered the Type-II superconductors. In 1950, Landau and Ginzburg, tnen continued by Abrikosov arrived at superconductor vortices, or flux tubes.
- When confinement was proposed for quarks inside hadrons in 1964 by Gell-Mann and Zweig, the analogy with flux tubes also led to a literature explosion in the quantum excitations of strings.
- The theoretical discovery of QCD in 1973 By Gross, Wilczek and Politzer shifted the interest back to particles.
- Nevertheless Lattice QCD by Wilson in 1974 was inspired in strings.
- The interest in strings exploded again when Maldacena and other proposed the AdS/CFT correspondence in 1997.
- The AdS/CFT and Holography has also been used as a model to compute spectra in hadronic physics, calling back string theory to QCD.

An approximation to the flux tube spectrum is given by Effective String Theories (EST), say Nambu-Goto transverse bosonic string. The action is the area of the string surface in time and its spatial direction,

$$S = -T \int d^2\sigma \sqrt{-\mathrm{det}\gamma} \, , \gamma_{\alpha\beta} = \frac{\partial X^\mu}{\partial \sigma^\alpha} \frac{\partial X^\nu}{\partial \sigma^\beta} \eta_{\mu\nu} \tag{1}$$

it is classically equivalent to the Polyakov action which introduces an auxiliary field, $g_{\alpha\beta}$ *einbein* to remove the square root,

$$S = -\frac{T}{2} \int d^2\sigma \sqrt{-\mathrm{det} g} \, g^{\alpha\beta} \frac{\partial X^\mu}{\partial \alpha} \frac{\partial X^\nu}{\partial \beta} \eta_{\mu\nu} \ . \tag{2}$$

Its spectrum for an open string with ends fixed at distance R with Dirichlet boundary conditions is given by the Arvis potential [2],

$$\begin{aligned} V_n(R) &= \sigma \sqrt{R^2 + \frac{2\pi}{\sigma} \left(\sum_{a_i=1}^{\infty} n_{a_i} - \frac{D-2}{24} \right)} \, , \qquad (3) \\ &= \sigma R + \frac{\pi}{R} \left(\sum_{a_i=1}^{\infty} n_{a_i} - \frac{D-2}{24} \right) + o\left(\frac{1}{R^3} \right) \, , \end{aligned}$$

where a_i are the principal transverse modes.
The Coulomb approximation, Lüscher term, can also be computed with a discretization of the String.

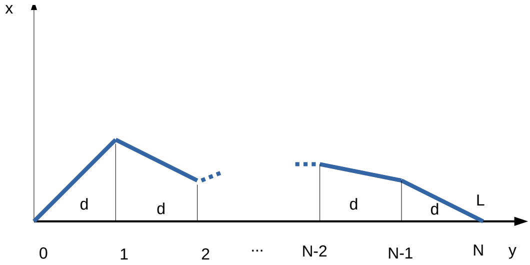

Figure: Discretization of the string transverse quantum fluctuations.

In any case, the continuum field theory computation of the zero mode energy is obtained using the Riemann Zeta regularization,

$$1 + 2 + 3 + \cdots = \sum_{n=1}^{\infty} n \qquad \to \qquad \zeta(-1) = -\frac{1}{12} \,, \tag{4}$$

which in lattice QCD is provided by the lattice regularization.

However the Nambu-Goto model is certainly not the EST of QCD flux tubes.

- There is a wider class of EST, the Nambu-Goto is just one of the possible EST [3].
- The zero mode of the QCD flux tubes has no tachyon *with negative square masses* at small distances R.
- There is lattice QCD evidence for an intrinsic width of the QCD flux tube [4].
- The QCD flux tube has a rich structure in chromoelectric and chromomagnetic field densities [5].

Moreover, a quarkonium with flux tube excitations corresponds to an exotic, hybrid excitation of a meson.

Thus we study the excited spectrum to learn more about the QCD flux tubes [6]. We restrict to Σ_g^+ flux tubes, the most symmetric ones, to go as high as possible in the spectrum.

To compute the very excited spectrum, we use a large basis of spacial operators composed of generalized Wilson lines.

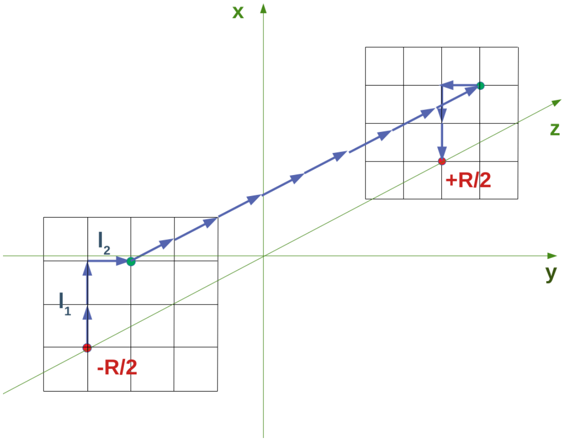

Figure: Example of a lattice QCD spacial operator, in this case $O(2, 1)$.

It turns out that using operators embedded only in axis planes, we decrease the degeneracy of states in our spectrum. This suppresses the Λ_g states, which due to the cubic symmetry of the lattice are as well generated by our operators.

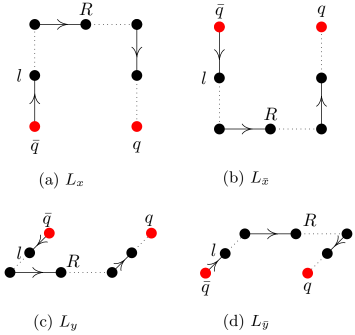

Figure: The four sub-operators, spatial Wilson line paths from the antiquark to the quark, used to construct the gauge field operators $O(l,0) = \frac{1}{\sqrt{4}}\left(L_x + L_{\bar{x}} + L_y + L_{\bar{y}}\right)$ at Euclidean time t_0. The inverse Wilson lines are used for the operators at time t.

The first step to compute the energy levels, is to diagonalise the Generalized Eigenvalue Problem for the correlation matrix of our operators $O, (l, 0)$

$$\mathcal{C}(t) v_n(t, t_0) = \lambda_n(t, t_0) \mathcal{C}(t_0) v_n(t_0) \ , \tag{5}$$

for each time extent t of the Wilson loop, and get a set of time dependent eigenvalues $\lambda_i(t)$. With the time dependence, we study the effective mass plot

$$E_i \simeq \log \frac{\lambda_i(t)}{\lambda_i(t+1)} \ , \tag{6}$$

and search for clear plateaux consistent with a constant energy E_i in intervals $t \in [t_{i\,\mathrm{ini}}, t_{i\,\mathrm{fin}}]$ between the initial and final time of the plateau.

We use the anisotropic Wilson action [7] computed with plaquettes,

$$S_{\text{Wilson}} = \beta \left(\frac{1}{\xi} \sum_{x,s>s'} W_{s,s'} + \xi \sum_{x,s} W_{s,t} \right) \tag{7}$$

where $W_c = \sum_c \frac{1}{3}\text{Re Tr}(1 - P_c)$, $P_{s,s'}$ denotes the spatial plaquette, $P_{s,t}$ the spatial-temporal plaquettes and ξ is the (unrenormalized) anisotropy. Moreover, to improve our signal we also resort to the improved anisotropic action S_{II} developed in Ref. [8],

$$\begin{aligned} S_{\text{II}} \;=\; & \beta \left(\frac{1}{\xi} \sum_{x,s>s'} \left[\frac{5 W_{s,s'}}{3 u_s^4} - \frac{W_{ss,s'} + W_{s's',s}}{12 u_s^6} \right] + \right. \\ & \left. + \xi \sum_{x,s} \left[\frac{4 W_{s,t}}{3 u_s^2 u_t^2} - \frac{W_{ss,t}}{12 u_s^4 u_t^2} \right] \right), \end{aligned} \tag{8}$$

with $u_s = \langle \frac{1}{3}\text{Re Tr}P_{ss} \rangle^{1/4},\ u_t = 1$. $W_{ss,s'}$ and $W_{ss,t}$, instead of plaquettes, include 2×1 rectangles. The results with more excited states shown in the literature [1], have been obtained with this action.

ens.	action	operators	β	Volume	u_s	u_t	ξ	ξ_R	$a_s\sqrt{\sigma}$	$a_t\sqrt{\sigma}$	# conf.
O_1	Wilson	11 $O(l_1, l_2)$	6.2	$24^3 \times 48$	-	-	1	1	0.1610	0.1610	1180
O_2	Wilson	11 $O(l_1, l_2)$	5.9	$24^3 \times 48$	-	-	2	2.1737(4)	0.3088(4)	0.1421(2)	2630
W_1	Wilson	13 $O(l, 0)$	6.2	$24^3 \times 48$	-	-	1	1	0.1610	0.1610	2500
W_2	Wilson	13 $O(l, 0)$	5.9	$24^3 \times 48$	-	-	2	2.1737(4)	0.3093(2)	0.1423(1)	2170
W_4	Wilson	13 $O(l, 0)$	5.6	$24^3 \times 96$	-	-	4	4.5459(9)	0.4986(4)	0.1097(1)	3475
S_4	S_{II}	13 $O(l, 0)$	4.0	$24^3 \times 96$	0.82006	1.0	4	3.6266(32)	0.3043(3)	0.0839(1)	3575

Table: Our ensembles, for the isotropic Wilson action and the improved anisotropic S_{II} action. ξ is the bare anisotropy in the Lagrangian and ξ_R is the renormalized anisotropy.

The anisotropy is used in order to have a smaller temporal lattice spacing a_t, for more precise excited results since we have more time slices for the same time intervals.
For the S_{II} ensemble and for the Wilson ensembles with anisotropy, we use MultiHit with 100 iterations in time followed by Stout smearing in space with $\alpha = 0.15$ and 20 iterations.
We use GPUS, and It turns out it is more economical to perform all our computations on the fly, rather than saving configurations.

Improved results for the Σ_g^+ spectrum

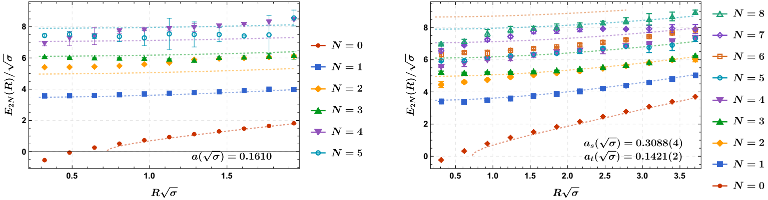

Figure: Results with the enlarged basis of 11 $O(l_1, l_2)$ operators with ensembles O_1 and O_2. The Nambu-Goto model spectrum is in dashed lines.

Using a set with off axis operators, we expect the degeneracies,
$n = 0$, 1 state : $n_r = 0$,
$n = 2$, 1 state : $n_r = 1$,
$n = 4$, 3 states: $n_r = 2$ or l=4,
...
Indeed we find in Fig. 5 nearly degenerate states $N = 2$ and $N = 3$, compatible with $n_r = 2$, $l = 0$ and $n_r = 0$, $l = 4$ states.

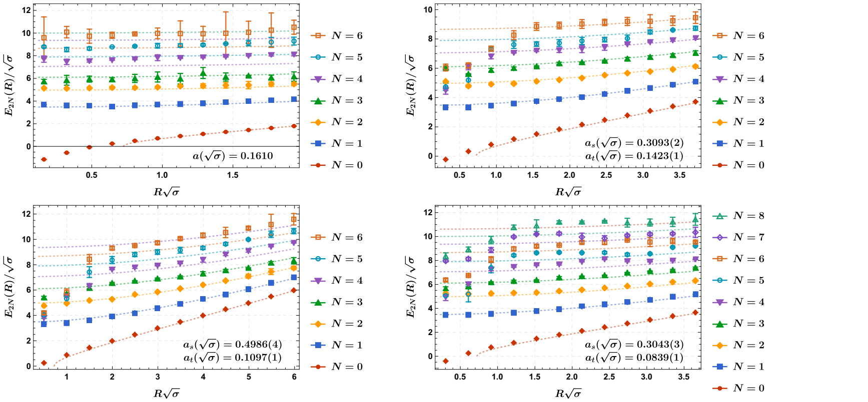

Figure: Results for the full spectrum, showing only the levels obtained with good Effective Mass Plots, from top to bottom respectively with ensembles W_1, W_2, W_4 and S_4. The Nambu-Goto model spectrum is in dashed lines.

Using only on axis operators the $l = 4$ degeneracy is suppressed at least for larger distances, we find up to $N = 8$ levels for the S_{II} ensemble. At smaller distances, there is still some degeneracy; possibly due to higher harmonics since $a_i = 1, 3, 5 \cdots$ are Σ_g^+ states.

Analysis of our Σ_g^+ spectrum

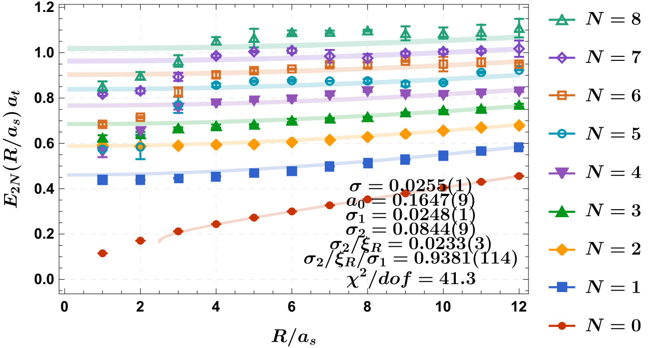

Figure: Very excited spectrum of the QCD flux tube fitted by a generalized Nambu-Goto.

We cannot fit accurately the very high spectrum with the Nambu-Goto spectrum. We generalize the Nambu-Goto model using two different string tensions, the σ_2 replaces the one inside the square root. With a global non-linear fit, we extract as well the renormalized anisotropy ξ and the string tensions.

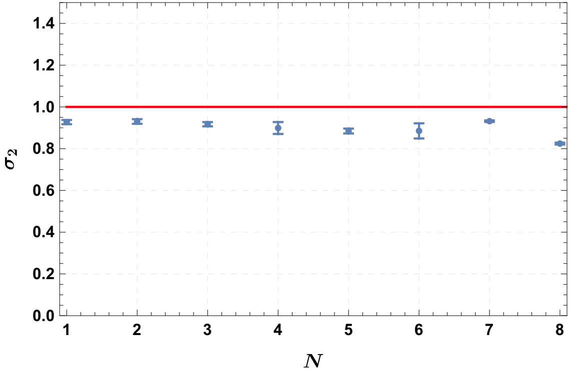

Figure: Small deviation from the Nambu-Goto spectrum, S_{II} action.

Parametrizing the deviation to the Nambu-Goto spectrum, using two different string tensions, the σ_2 inside the square root is apparently slightly smaller than σ. We find a deviation of up to 10% for the S_{II} action also present for the other ensembles. Only for the W_2 ensemble, the Nambu-Goto spectrum is rather close to our results.

Conclusion and discussion

We compute the potentials for several new excitations of the pure $SU(3)$ flux tubes produced by two static 3 and $\bar{3}$ sources, specializing in the radial excitations of the groundstate Σ_g^+.

Using a large basis of operators , employing the computational techniques with GPUs of Ref. [5] and utilizing different actions with smearing and anisotropy we go up to $N = 8$.

In general the excited states of the the Σ_g^+ flux tubes are comparable to the Nambu-Goto EST with transverse modes, only depending on the string tension σ and the radial quantum number n_r.
We find a deviation of up to 10% to the excited spectrum of the Nambu-Goto Model, leaving the confirmation of this deviation for futures studies.

A next feasible step is the computation of the widening in the different wavefunctions in the spectrum, starting from the zero mode.
An important outlook would be the study of hybrid quarkonium resonances.