Compositeness above the Electroweak scale and a proposed test at LHCb.

Gabriele Ferretti*, Cyberspace, Earth, August 2021

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The story so far...

The Higgs boson is nine years old but it’s acting like an old person, being very conservative and not throwing any tantrums...
Still, many of us hope that it will grow into a revolutionary fighter that will lead us to break the shackles of the Standard Model!
The reason for this hope is, of course, the fact that its mass is not “natural” and I still unapologetically embrace this argument.
There are basically two *symmetry-based* ways to attack the problem:

- **SUPERSYMMETRY** $\delta h \approx \epsilon \psi$
- **COMPOSITENESS** $\delta h \approx h + a$
Supersymmetry is a weakly coupled theory with natural elementary scalars and its problem is that it predicts too much (there are many unwanted $\text{dim} < 4$ operators.)

Compositeness is a strongly coupled theory without elementary scalars that predicts too little (we lack the $\text{dim} = 4$ Yukawa operators.)

I will of course discuss compositeness (lest I be kicked out of this conference!) and concentrate on the subcase in which the Higgs is realized as a (pseudo) Nambu-Goldstone boson. (Lattice also hints at other possibilities that might be even more interesting, but I don’t have anything to say about them.)
Contrary to other ideas in modern particle physics (Supersymmetry, Extra dimensions, Dilatons...) we already know that Nature does make use of the Nambu-Goldstone mechanism:

This is the picture of a pNGB scattering and creating another pNGB decaying into two more pNGBs, all swimming in a background of pNGBs...

*Life was easy then.*

Still...

*That’s a pretty dynamical mechanism you got there. It’d be a shame not to use it more often.*
So, what’s the idea?

The idea is to start with the Higgsless and massless Standard Model

\[ \mathcal{L}_{SM0} = -\frac{1}{4} \sum_{F=GWB} F_{\mu\nu}^2 + i \sum_{\psi=QudLe} \bar{\psi} \not{D}\psi \]

with gauge group \( G_{SM} = SU(3) \times SU(2) \times U(1) \) and couple it to a theory \( \mathcal{L}_{comp.} \) with hypercolor gauge group \( G_{HC} \) and global symmetry structure \( G_F \rightarrow H_F \) such that \( h \in G_F/H_F \) and

\[ \mathcal{L}_{comp.} + \mathcal{L}_{SM0} + \mathcal{L}_{int.} \rightarrow \mathcal{L}_{SM} + \cdots \]

( \( \mathcal{L}_{SM} + \cdots \) is the full SM plus possibly light extra matter from bound states of \( \mathcal{L}_{comp.} \).)
To make the point even more explicit, **at the cost of oversimplifying a bit**, dropping couplings, gauge and Lorentz indices, mixing...

<table>
<thead>
<tr>
<th></th>
<th>$G_{\text{SM}}$</th>
<th>$G_{\text{HC}}$</th>
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</thead>
<tbody>
<tr>
<td>$q$ (SM)</td>
<td>$R_1$</td>
<td>$x$</td>
</tr>
<tr>
<td>$\psi$ (BSM)</td>
<td>$R_2$</td>
<td>$R_3$</td>
</tr>
<tr>
<td>$\chi$ (BSM)</td>
<td>$R_4$</td>
<td>$R_5$</td>
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</table>

$L_{\text{int.}} \supset \frac{1}{\Lambda_{\text{UV}}^2} (q^2\psi\psi + q\psi\chi\psi \text{ (or } q\chi\psi\chi \text{) } + \cdots)$

\[ \rightarrow \left( \frac{\Lambda^2}{\Lambda_{\text{UV}}^2} \right)^{2+\gamma_\Phi} q^2\Phi + \left( \frac{\Lambda^2}{\Lambda_{\text{UV}}^2} \right)^{2+\gamma_\Psi} q\Psi + \cdots \]

Where $\Phi \approx \Lambda^{-2}\psi\psi$, $\Psi \approx \Lambda^{-2}\psi\chi\psi$ are the interpolating fields and $\gamma_\Phi$, $\gamma_\Psi$ their anomalous dimensions.
As far as the EW sector is concerned, the possible minimal custodial cosets of this type are generated by $\langle \tilde{\psi}^i \psi_j \rangle$ or $\langle \psi^i \psi^j \rangle$ for

<table>
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<tr>
<th>4 $(\psi, \tilde{\psi})$</th>
<th>Complex irrep</th>
<th>$SU(4) \times SU(4)'/SU(4)_D$</th>
</tr>
</thead>
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<td>4 $\psi$</td>
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<td>5 $\psi$</td>
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<td>$SU(5)/SO(5)$</td>
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E.g. $SU(4)/SO(4)$ is not acceptable since the pNGB are only in the symmetric irrep $(3, 3)$ of $SO(4) = SU(2)_L \times SU(2)_R$ and thus we do not get the Higgs irrep $(2, 2)$.

pNGB content under $SU(2)_L \times SU(2)_R$:
- $\text{Ad}$ of $SU(4)_D \rightarrow (3, 1) + (1, 3) + 2 \times (2, 2) + (1, 1)$
- $A_2$ of $Sp(4) \rightarrow (2, 2) + (1, 1)$
- $S_2$ of $SO(5) \rightarrow (3, 3) + (2, 2) + (1, 1)$
As far as fermion masses are concerned, we couple a SM fermion $q$ linearly to a $G_{HC}$-neutral fermionic bound state, $\Psi$. This requires additional hyper-fermions $\chi$ carrying color, schematically $\Psi \approx \psi\chi\psi$ or $\chi\psi\chi$.

We see that, to get the right top quark mass, we need $\gamma_{\Psi} \approx -2$ (since $\Lambda \ll \Lambda_{UV}$). This requires the theory to be strongly coupled in the conformal range.

Notice however that $\gamma_{\Psi} \approx -2$ is still strictly above the unitarity bound for fermions: $(\Delta[\Psi] \approx 9/2 - 2 = 5/2 > 3/2)$.

No new relevant operators are necessarily reintroduced in this case.
As an aside:

Since we have introduced a new set of hyper-fermions, we also need to embed the color group \( SU(3)_c \) into the unbroken global symmetry of \( \mathcal{L}_{\text{comp.}} \).

The choices of minimal field content allowing an anomaly-free embedding of unbroken \( SU(3)_c \) are

\[
\begin{array}{|c|c|}
\hline
3 \ (\chi, \tilde{\chi}) & \text{Complex irrep} \\
6 \ \chi & \text{Pseudoreal irrep} \\
6 \ \chi & \text{Real irrep} \\
\hline
\end{array}
\]

\[
\begin{align*}
SU(3) \times SU(3)' & \rightarrow SU(3)_D \equiv SU(3)_c \\
SU(6) & \rightarrow Sp(6) \supset SU(3)_c \\
SU(6) & \rightarrow SO(6) \supset SU(3)_c
\end{align*}
\]
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At $\Lambda$ some fermions decouple reducing $N_\psi, N_\chi$. The theory confines and breaks chiral symmetry. $\Psi$ creates a (light?) composite fermion $M_\Psi$. 
We narrowed it down to a list of twelve models likely to be outside the conformal window but with still enough matter to realize the mechanism of partial compositeness: [1604.06467,1610.06591]

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<th>$\psi$</th>
<th>$\chi$</th>
<th>$G_F/H_F$</th>
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<td>$SO(7)$</td>
<td>$5 \times F$</td>
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<td>$SU(5) \frac{SU(6)}{SO(5) SO(6)}$ $U(1)$</td>
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<td>$6 \times F$</td>
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<td>$SO(10)$</td>
<td>$5 \times F$</td>
<td>$3 \times (F, \bar{F})$</td>
<td>$SU(4) \frac{SU(6)}{SU(3) D}$ $U(1)$</td>
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<tr>
<td>$SO(11)$</td>
<td>$4 \times \text{Spin}$</td>
<td>$6 \times F$</td>
<td>$SU(4) \frac{SU(4) \times SU(4)'}{SU(4) D}$ $U(1)$</td>
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What’s in it for the lattice?

The first questions to be addressed concern the composite sector in isolation, before coupling to the SM. Then, the list of models reduces to

- $SU(4)$ with $N_F$ Fundamentals and $N_A$ Antisymmetric (possibly also $SU(5)$)
- $Sp(4)$ with $N_F$ Fundamentals and $N_A$ Antisymmetric
- $SO(N)$ with $N_F$ Fundamentals and $N_S$ Spin (with $N = 7, 9, 10, 11$)
In the first two cases, the hypercolor group is fixed and we scan over the two irreps:

- SU(4) case: \( \bullet = 1404.7137 \)
  - \( \bullet = \) “swapped”

- Sp(4) case: \( \bullet = 1311.6562 \)
  - \( \bullet = \) “swapped”
Some concrete questions that can be addressed are

- Where does the boundary of the conformal window start?
- For models **inside** the window, can we find an operator $\Psi \approx \psi \chi \psi$ (or $\chi \psi \chi$) of scaling dimension $\Delta \approx 5/2$?
- Does any of the four-fermi terms become relevant?
- Taking the models **outside** by removing some fermions, what is the mass of the composite fermionic resonances created by the remaining $\Psi$s?
- Can the mass be significantly lighter than the typical confinement scale $\Lambda$?
- Can we estimate the LEC in the pNGB potential?
- Can we estimate the **top Yukawa coupling**?
Some of these questions have already started to be answered.

For $G_{HC} = SU(4)$: [Ayyar, DeGrand, Golterman, Hackett, Jay, Neil, Shamir, Svetitskly, 1710.00806, 1801.05809, 1812.02727 ]

For $G_{HC} = Sp(4)$: [Bennett, Hong, Lee, Lin, Lucini, Piai, Vadacchino, 1710.07043, 1712.04220, 1811.00276, 1909.12662]

For $G_{HC} = SO(N)$: Nothing of direct relevance to PC yet. (Anybody who likes spinorial irreps?)

Just for fun, an iNSPIRE search on June 2021:

- $f \ t \ SU \ and \ t \ lattice \ 1321 \ hits$
- $f \ t \ Sp \ and \ t \ lattice \ 6 \ hits$
- $f \ t \ SO \ and \ t \ lattice \ 42 \ hits$
An additional light ALP:

There are two global $U(1)_\psi$ and $U(1)_\chi$ symmetries rotating all $\psi \rightarrow e^{i\alpha} \psi$ or all $\chi \rightarrow e^{i\beta} \chi$.

The linear combination $q_\psi \psi^\dagger \bar{\sigma}^\mu \psi + q_\chi \chi^\dagger \bar{\sigma}^\mu \chi$ free of anomalies:

$$q_\psi N_\psi T(\psi) + q_\chi N_\chi T(\chi) = 0$$

is associated to a light ALP $a$. 
The main point is that all the couplings $C_\psi, K_g, K_\gamma$ can be computed from the underlying theory and one is left with two continuous parameters $f_a$ and $m_a$ to describe the model.

$$L_{\text{eff}} = \frac{1}{2} (\partial_\mu a)(\partial^\mu a) - \frac{1}{2} m_a^2 a^2 - i \sum_{\psi=\text{QudLe}} C_\psi \frac{m_\psi}{f_a} a \bar{\psi} \gamma^5 \psi$$

$$+ \frac{a}{16\pi^2 f_a} \left( g_s^2 K_g G^a_{\mu\nu} \tilde{G}^{a\mu\nu} + e^2 K_\gamma F_{\mu\nu} \tilde{F}^{\mu\nu} \right)$$

In [Buarque Franzosi, Cacciapaglia, Cid Vidal, Ferretti, Flacke and Vázquez Sierra, 2106.12615] we studied the reach of LHCb for such object.
Existing bounds

The [LHCb 2007.03923] search for pseudoscalars di-muon resonances in the 2HDM can be re-cast to provide bounds between 10-60 GeV by noting that both cross sections and widths scale like $v/f$ in our models and $\sin \theta_H$ in the 2HDM.

$$\frac{v}{f} = \sqrt{\frac{\sigma(p p \rightarrow a \rightarrow \mu^+ \mu^-)_{\sin \theta_H=1}}{\sigma(p p \rightarrow a \rightarrow \mu^+ \mu^-)_{v=f}}} \sin \theta_H$$

![Graph showing the relationship between $v/f$ and $m_a$ (GeV) with different models indicated by lines of different colors and styles.](image-url)

- **Legend:**
  - M1
  - M2
  - M3
  - M4
  - M5
  - M6
  - M7
  - M8
  - M9
  - M10
  - M11
  - M12
Proposed search $pp \rightarrow a \rightarrow \tau^+ \tau^-$

This channel “wins” over the muons by an enhanced Branching Ratio $\propto (m_\tau/m_\mu)^2 = 283$ but “loses” by the presence of neutrinos in the final states and usual difficulties with taus.

Combining the leptonic and three-prong hadronic $\tau$ decay channels we estimate a sensitivity comparable to that of the di-muon channel (mostly from opposite flavor leptonic decays $\tau\tau \rightarrow e\mu$). At 15 fb$^{-1}$:
Proposed search $pp \rightarrow a \rightarrow D^+ D^-$

This only works in a small mass window above the $D^+ D^-$ threshold where $a \rightarrow D^+ D^-$ exclusively has a large fragmentation function. We use the fact that the decay $D^\pm \rightarrow K^\mp \pi^\pm \pi^\pm$ (9.38% BR)* is fully reconstructable at LHCb, as long as all the decay products are within the geometric acceptance, and the resolution on the invariant mass $m(D^+ D^-)$ can reach $\pm 10$ MeV. At 15 fb$^{-1}$:

* Unfortunately there is no such channel for $b \bar{b}$. 
CONCLUSIONS

▶ Realizing partial compositeness via ordinary 4D gauge theories provides a self contained concrete class of models to address the hierarchy problem.

▶ There are lots of open questions that go to the heart of strongly coupled theories, such as the range of the conformal window, anomalous dimensions and LEC.

▶ In the pNGB sector the models are fairly predictive, since many couplings can be computed from the underlying gauge theory.

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Thank you for your attention!