exploring hadron dynamics in the large N_c limit

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ift UAM

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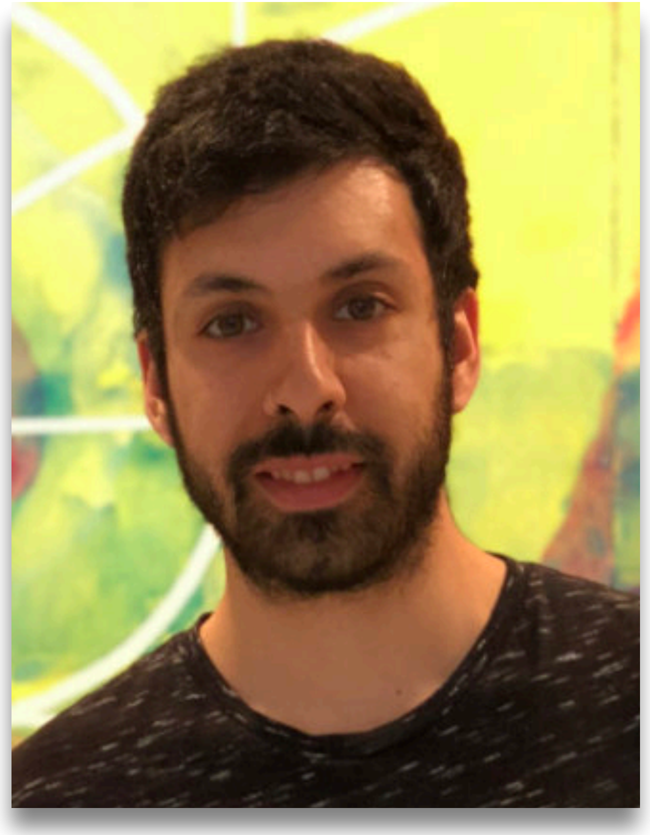
based on:

- **J.J. Baeza Ballesteros**, P. Hernández, **F. Romero-López**, in preparation [see JBB's talk @ Latt21]
- A. Donini, P. Hernández, CP, **F. Romero-López**, *Dissecting the* ΔI=1/2 *rule at large* N_c, EPJC 80 (2020) 638 [arXiv:2003.10293].
- **F. Romero-López,** A. Donini, P. Hernández, CP, *Mesons interactions at large* N_c *from lattice QCD*, PoS LATTICE2019 (2019) 005 [arXiv:1910.10418].
- P. Hernández, CP, **F. Romero-López** , *Large* N_c *scaling of meson masses and decay constants*, EPJC 79 (2019) 865 [arXiv:1907.11511].
- A. Donini, P. Hernández, CP, **F. Romero-López**, *Nonleptonic kaon decays at large* N_c, PRD 94 (2016) 114511 [arXiv:1607.03262].



FRL

JBB

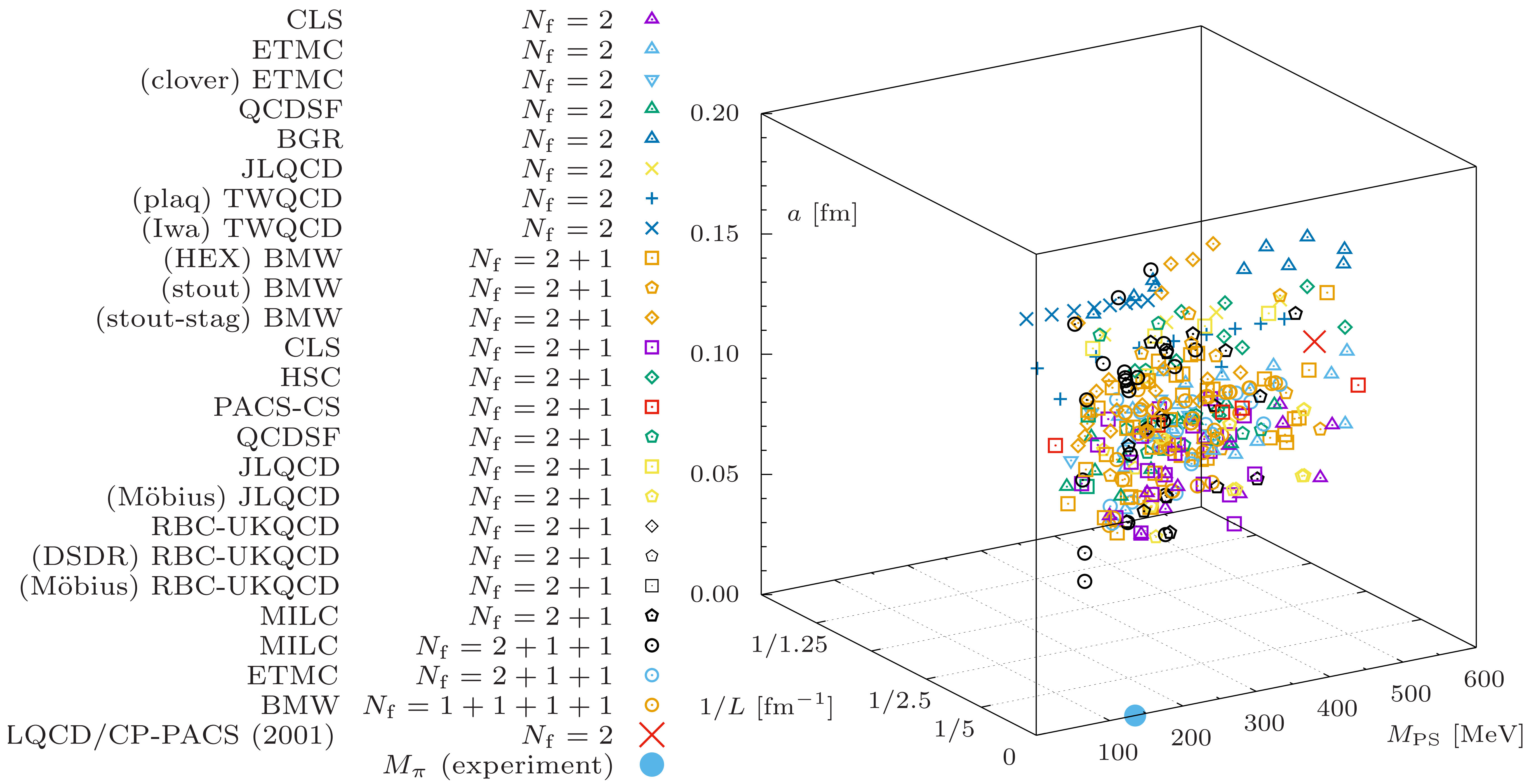
- E. Endress, CP, *Exploring the role of the charm quark in the* ΔI=1/2 *rule*, PRD 90 (2014) 094504.
- P. Hernández, M. Laine, CP, E. Torró, J. Wennekers, H. Wittig, *Determination of the* $\Delta S = 1$ *weak Hamiltonian in the SU(4) chiral limit through topological zero-mode wave functions*, JHEP 0805 (2008) 043.
- L. Giusti, P. Hernández, M. Laine, CP, J. Wennekers, H. Wittig, *On* $K \to \pi\pi$ *amplitudes with a light charm quark*, PRL 98 (2007) 082003.

motivation & plan

- lattice simulations can be used as a lab to explore quark mass, N_f, volume dependence of hadron dynamics

- ditto: **number of colours**

$\Rightarrow$ explore N_c dependence in problems where qualitative/quantitative insight can be expected



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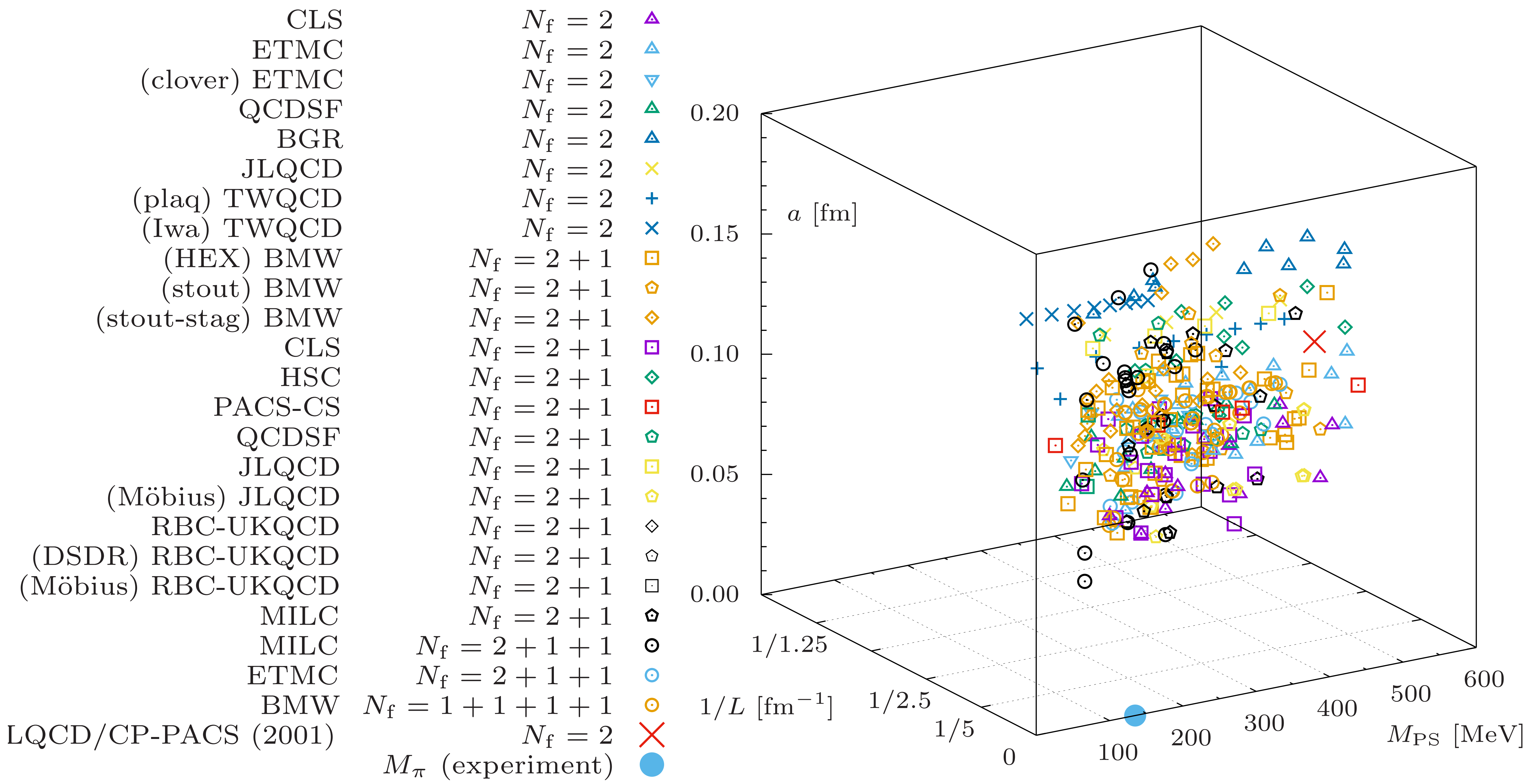

- original motivation: **non-leptonic kaon decay, especially $\Delta I = 1/2$ rule**

- spinoffs (current focus): **meson properties (mass, decay constants)** and interactions

QCD @ large N_c

varying-N_c lattice simulations have become a mature tool [cf. review by Hernández, Romero-López 2012.03331]

- Yang-Mills
  - string tension, glueball masses [Bali et al. 2013]
  - topological susceptibility [Cè, García Vera, Giusti, Schaefer 2016]
  - Wilson-flow coupling and factorisation [García Vera, Sommer 2019]
- quenched QCD
  - meson masses, decay constants [García Vera, Sommer 2019]
  - kaon weak decay matrix elements [this talk]
- dynamical
  - $N_f=2$: meson+baryon masses, topological susceptibility, finite T [DeGrand, Liu 2017; DeGrand 2021]
  - $N_f=4$: kaon weak decay matrix elements, ChiPT LECs, meson interactions [this talk]
- also: reduced models [cf. review by García Pérez, Lattice 2019]

QCD @ large N_c

't Hooft's large N_c limit of QCD: $N_c \to \infty|_{\lambda = g^2 N_c = \text{fixed}}$ ['t Hooft 1974]

à la 't Hooft proper: $\dfrac{N_f}{N_c} \to 0 \;\Rightarrow\; m^2_{\eta'} = m^2_\pi\,,\; U(N_f)$ chiral symmetry

à la Veneziano: $\dfrac{N_f}{N_c} = const \;\Rightarrow\; m^2_{\eta'} \gg m^2_\pi\,,\; SU(N_f)$ chiral symmetry [Veneziano 1979]
[also: Witten 1979]

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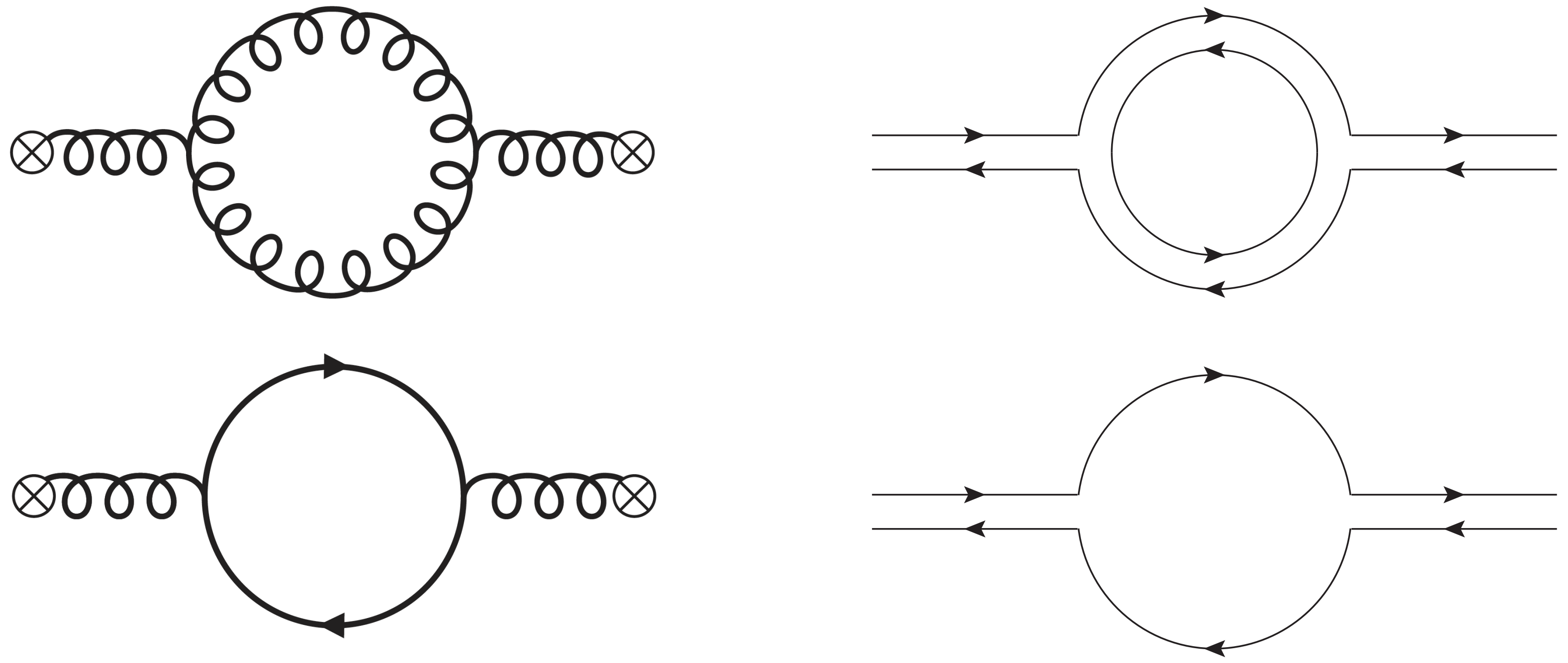
- preserves asymptotic freedom

$$\mu \frac{\mathrm{d}\lambda}{\mathrm{d}\mu} = -\left(\frac{11}{3} - \frac{2}{3}\frac{N_\mathrm{f}}{N_c}\right)\frac{\lambda^2}{8\pi^2} \;\to\; \Lambda_\infty$$

- captures most non-perturbative properties (confinement, chiral SSB, ...)
- simplifies the theory by suppressing dynamical quark effects



$\sim g^2 N_c^2 \sim N_c$

$\sim g^2 N_c \sim 1$

lattice setup

- simulate for N_c=3,...,6 (+7,8,17 quenched) at fixed lattice spacing (+1 at N_c=3), change quark mass along $\boldsymbol{m_u = m_d = m_s = m_c}$
  - **quenched:** use line of constant physics provided by quenched study of meson physics [Bursa et al. 2013]
  - **dynamical:** use gradient flow scale t_0 to set constant physics

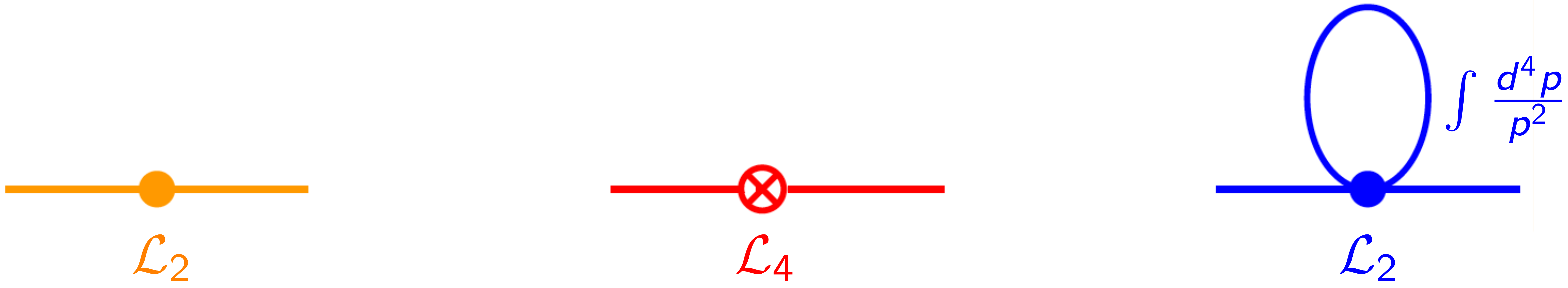
- use Wilson fermions for sea (**HiRep** code), **twisted-mass QCD** for valence
  - twisted valence à la Frezzotti-Rossi allows to avoid mixing with wrong-chirality operators [Frezzotti, Rossi 2004]
  - mixed-action approach requires matching of valence and sea, performed with meson mass
  - check for residual cutoff effects by changing value of c_{sw} + simulation on finer lattice
  - use perturbative renormalization and running (non-perturbative results unavailable)
    [Constantinou et al. 2011; Alexandrou et al. 2012]
    [Ciuchini et al. 1998; Buras et al. 2000]

- develop/check necessary bits of SU(4) χPT

N_c scaling of χPT LECs

Goldstone boson physics is well-parametrized by Chiral Perturbation Theory

$$F_\pi = F \left\{ 1 + \frac{M_\pi^2}{F_\pi^2} \left[4L_5(\mu) + 4N_\mathrm{f} L_4(\mu) \right] + \frac{N_\mathrm{f}}{2} \frac{M_\pi^2}{(4\pi F_\pi)^2} \log\left(\frac{M_\pi^2}{\mu} \right) \right\}$$



$$F_\pi^2 = \mathcal{O}(N_c)$$
$$L_5 = \mathcal{O}(N_c)$$
$$L_4 = \mathcal{O}(1)$$

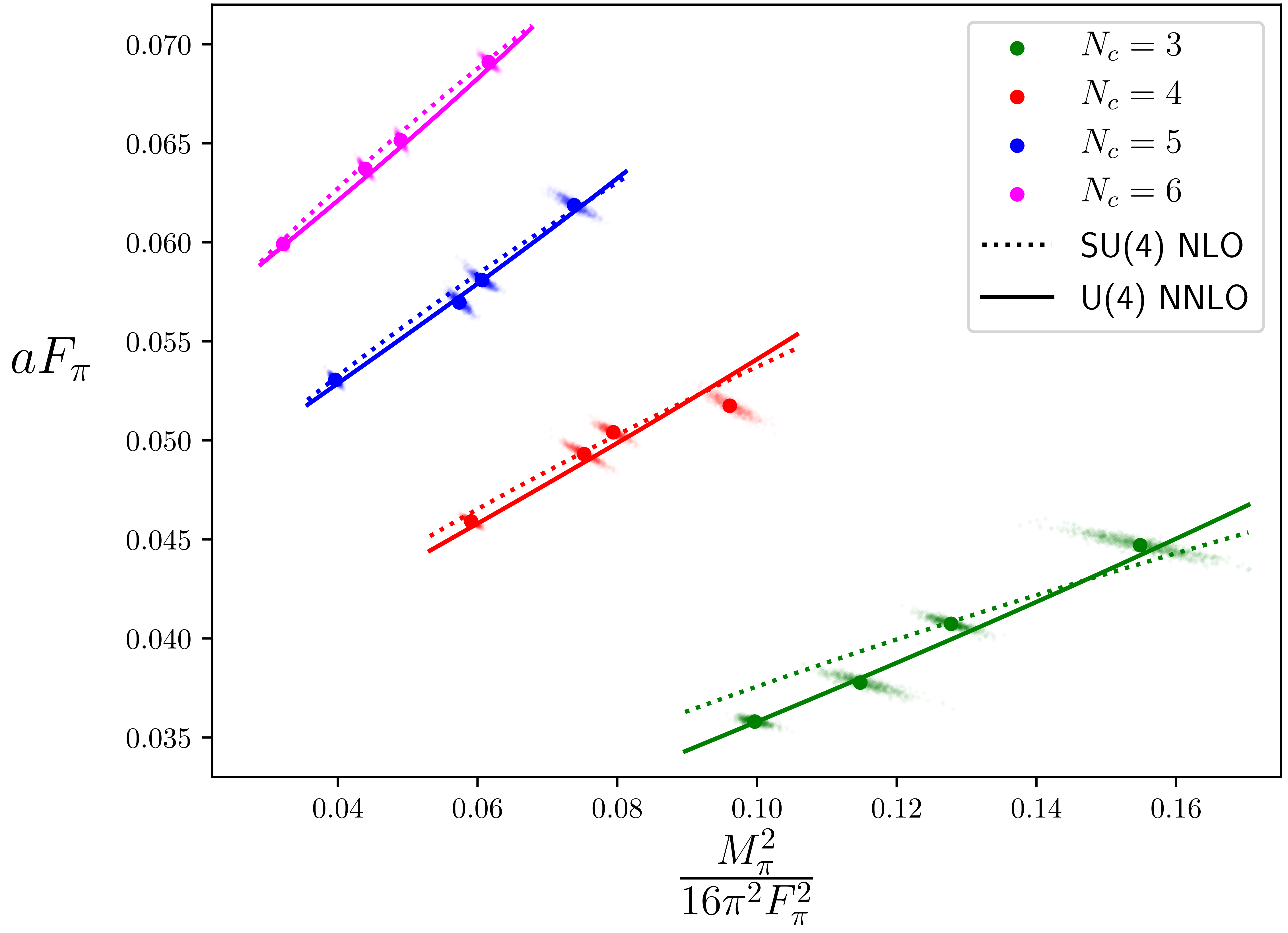
$$\longrightarrow \quad F_\pi \underset{N_c \to \infty}{\approx} F \left\{ 1 + 4 \frac{M_\pi^2}{F_\pi^2} L_5 + \text{logs} \right\}$$

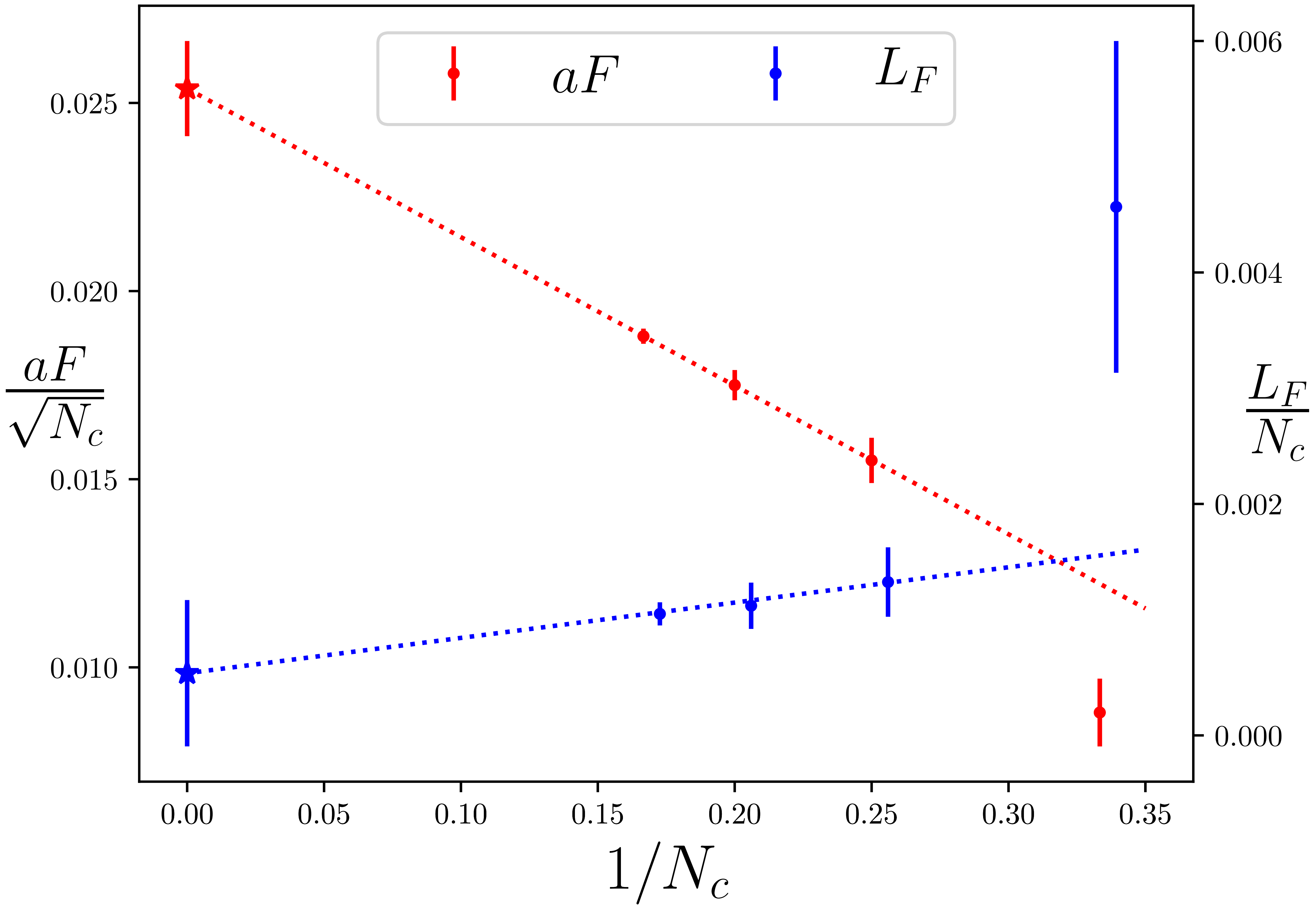
[Gasser, Leutwyler 1985]

N_c scaling of χPT LECs

$$F_\pi = F\left\{1 + \frac{M_\pi^2}{F_\pi^2}4L_F + \frac{N_\mathrm{f}}{2}\frac{M_\pi^2}{(4\pi F_\pi)^2}\log\left(\frac{M_\pi^2}{\mu^2}\right)\right]$$

$$F = \sqrt{N_c}\left(F^{(0)} + \frac{F^{(1)}}{N_c}\right) \qquad\qquad L_F = N_c L_F^{(0)} + L_F^{(1)}$$

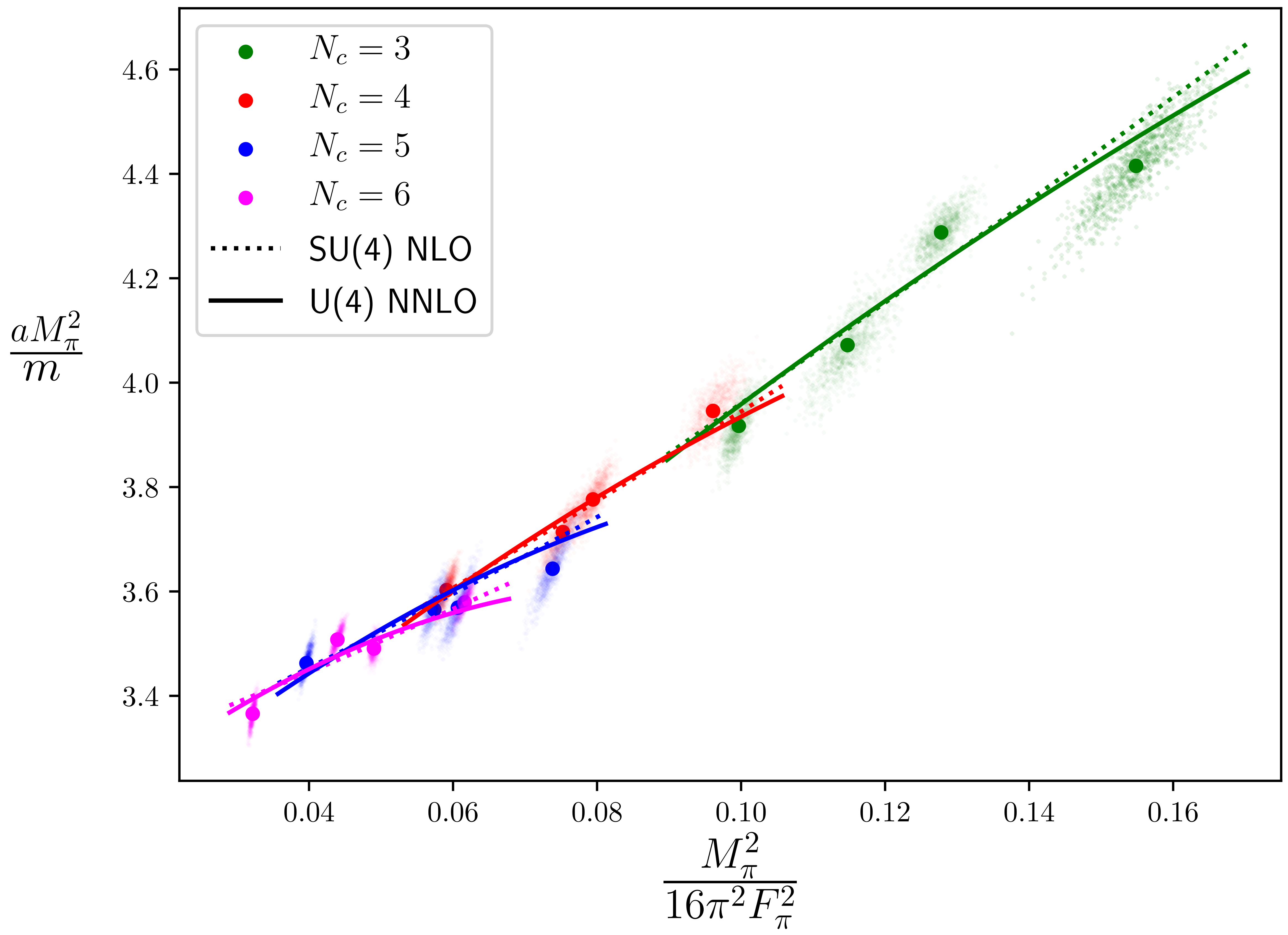


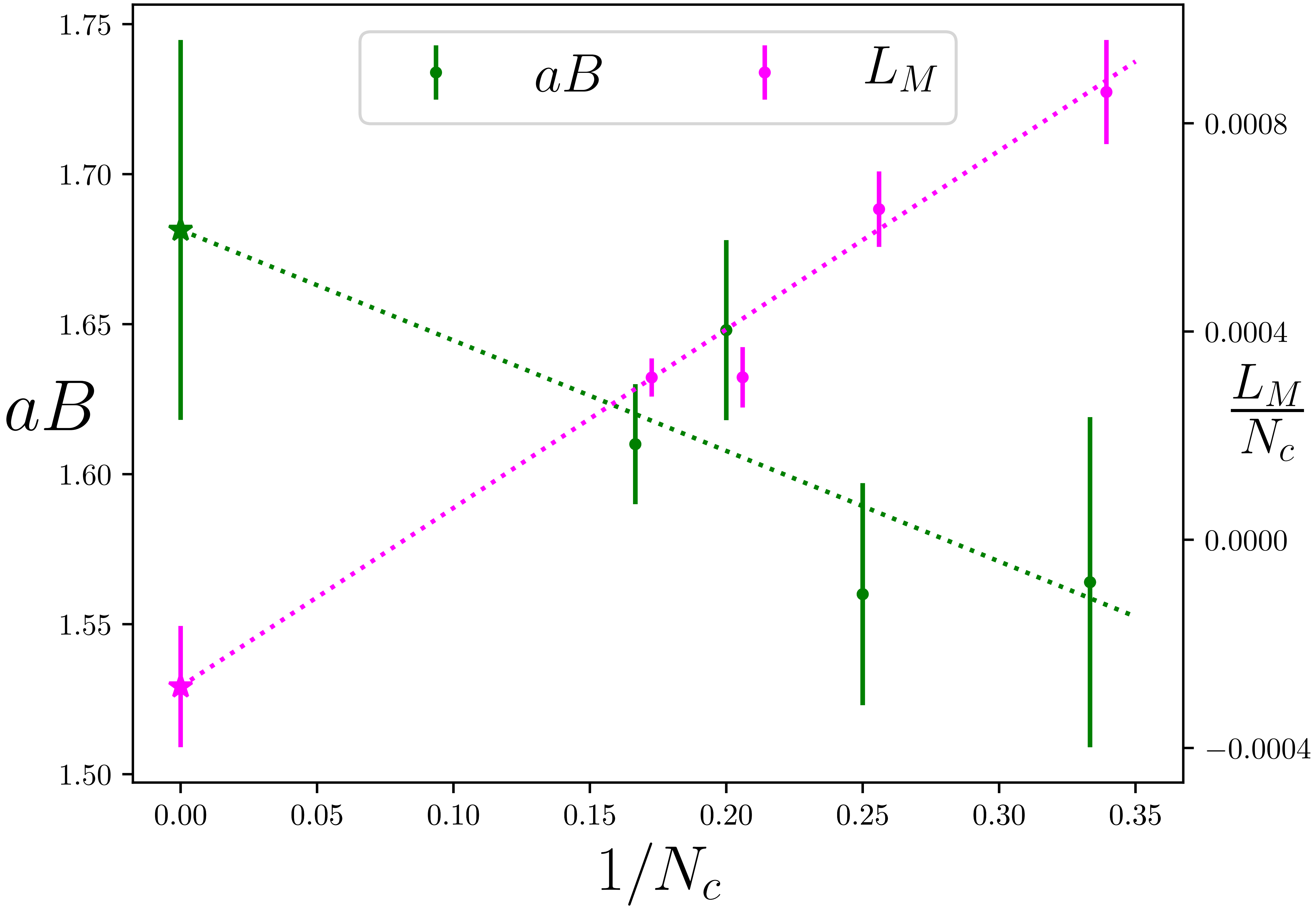


N_c scaling of χPT LECs

$$M_\pi^2 = 2Bm\left\{1 + \frac{M_\pi^2}{F_\pi^2}8L_M + \frac{1}{N_\mathrm{f}}\frac{M_\pi^2}{(4\pi F_\pi)^2}\log\left(\frac{M_\pi^2}{\mu^2}\right)\right\}$$

$$B = B^{(0)} + \frac{B^{(1)}}{N_c} \qquad L_M = N_c L_M^{(0)} + L_M^{(1)}$$





N_c scaling of χPT LECs

- LO LECs:

  - $$\frac{F}{\sqrt{N_c}} = \left[67(3) - 26(4)\frac{N_f}{N_c}\right] \text{ MeV} \quad \Rightarrow \quad F_{N_f=2} = 86(3) \text{ MeV} \qquad F_{N_f=3} = 71(3) \text{ MeV}$$

[FLAG 2019]

ETM 15A	[386]	86.3(2.8)
Engel 14	[50]	85.8(0.7)(2.0)
Brandt 13	[49]	84(8)(2)
QCDSF 13	[402]	86(1)
TWQCD 11	[394]	83.39(35)(38)
ETM 09C	[48]	$85.91(07)\left(^{+78}_{-07}\right)$
ETM 08	[53]	86.6(7)(7)
Hasenfratz 08	[397]	90(4)
JLQCD/TWQCD 08A	[376]	$79.0(2.5)(0.7)\left(^{+4.2}_{-0.0}\right)$
JLQCD/TWQCD 07	[398]	87.3(5.6)
Colangelo 03	[403]	86.2(5)

JLQCD/TWQCD 10A	[389]	71(3)(8)
MILC 10	[36]	80.3(2.5)(5.4)
MILC 09A	[17]	78.3(1.4)(2.9)
MILC 09	[129]	
PACS-CS 08	[162]	83.8(6.4)
RBC/UKQCD 08	[163]	66.1(5.2)

N_c scaling of χPT LECs

- LO LECs:
  - $$\frac{F}{\sqrt{N_c}} = \left[67(3) - 26(4)\frac{N_f}{N_c}\right] \text{ MeV} \quad \Rightarrow \quad F_{N_f=2} = 86(3) \text{ MeV} \qquad F_{N_f=3} = 71(3) \text{ MeV}$$
  - $\Sigma^{1/3}_{N_f=3} = 223(9)$ MeV vs $\Sigma^{1/3}_{N_f=3} = 214(6)(24)$ MeV [Fukaya et al. 2010]
  - $\frac{\Sigma_{N_f=3}}{\Sigma_{N_f=2}} = 1.49(10)$ vs $\frac{\Sigma_{N_f=3}}{\Sigma_{N_f=2}} = 1.51(11)$ [Bernard, Descotes-Genon, Toucas 2012]

- NLO LECs:
  - $\bar{\ell}_4 = 5.1(3)$ vs $\bar{\ell}_4 = 4.40(28)$ [FLAG 2019]
  - n.b. subleading corrections to LECs are sizable: $\frac{L_M^{N_f=4}}{N_c} \times 10^3 = -0.2(2) + \frac{2.9(6)}{N_c} + \mathcal{O}\left(\frac{1}{N_c^2}\right)$

weak decay and $\Delta I = 1/2$

K^0_S $\quad I(J^P) = \frac{1}{2}(0^-)$

Mean life $\tau = (0.8954 \pm 0.0004) \times 10^{-10}$ s (S = 1.1) Assuming *CPT*

Mean life $\tau = (0.89564 \pm 0.00033) \times 10^{-10}$ s Not assuming *CPT*

K^0_S DECAY MODES	Fraction (Γ_i/Γ)	Scale factor/ Confidence level	p (MeV/c)
Hadronic modes			
$\pi^0\pi^0$	(30.69±0.05) %		209
$\pi^+\pi^-$	(69.20±0.05) %		206

K^0_L $\quad I(J^P) = \frac{1}{2}(0^-)$

$m_{K_L} - m_{K_S}$

$= (0.5293 \pm 0.0009) \times 10^{10}\ \hbar\ \mathrm{s}^{-1}$ (S = 1.3) Assuming *CPT*

$= (3.484 \pm 0.006) \times 10^{-12}$ MeV Assuming *CPT*

$= (0.5289 \pm 0.0010) \times 10^{10}\ \hbar\ \mathrm{s}^{-1}$ Not assuming *CPT*

Mean life $\tau = (5.116 \pm 0.021) \times 10^{-8}$ s (S = 1.1)

K^0_L DECAY MODES			Fraction (Γ_i/Γ)	Scale factor/ Confidence level	p (MeV/c)
Semileptonic modes					
$\pi^\pm e^\mp \nu_e$		[o]	(40.55 ±0.11) %	S=1.7	229
$\pi^\pm \mu^\mp \nu_\mu$		[o]	(27.04 ±0.07) %	S=1.1	216
Hadronic modes, including Charge conjugation×Parity Violating (*CPV*) modes					
$3\pi^0$			(19.52 ±0.12) %	S=1.6	139
$\pi^+\pi^-\pi^0$			(12.54 ±0.05) %		133
$\pi^+\pi^-$	*CPV*	[q]	(1.967±0.010) × 10^{-3}	S=1.5	206
$\pi^0\pi^0$	*CPV*		(8.64 ±0.06) × 10^{-4}	S=1.8	209

[PDG 2019]

$$-iT[K^0 \to (\pi\pi)_I] = A_i e^{i\delta_I}$$

$$T[(\pi\pi)_I \to (\pi\pi)_I]_{l=0} = 2e^{i\delta_I} \sin\delta_I$$

$$\boxed{\left|\frac{A_0}{A_2}\right| = 22.45(6)}$$

(similar observations in baryon sector — e.g., $\Lambda/\Sigma \to N\pi$, heavy meson decay, ...)

[fully?] satisfactory understanding of result within SM lacking for almost 50 years

weak decay and $\Delta I = 1/2$

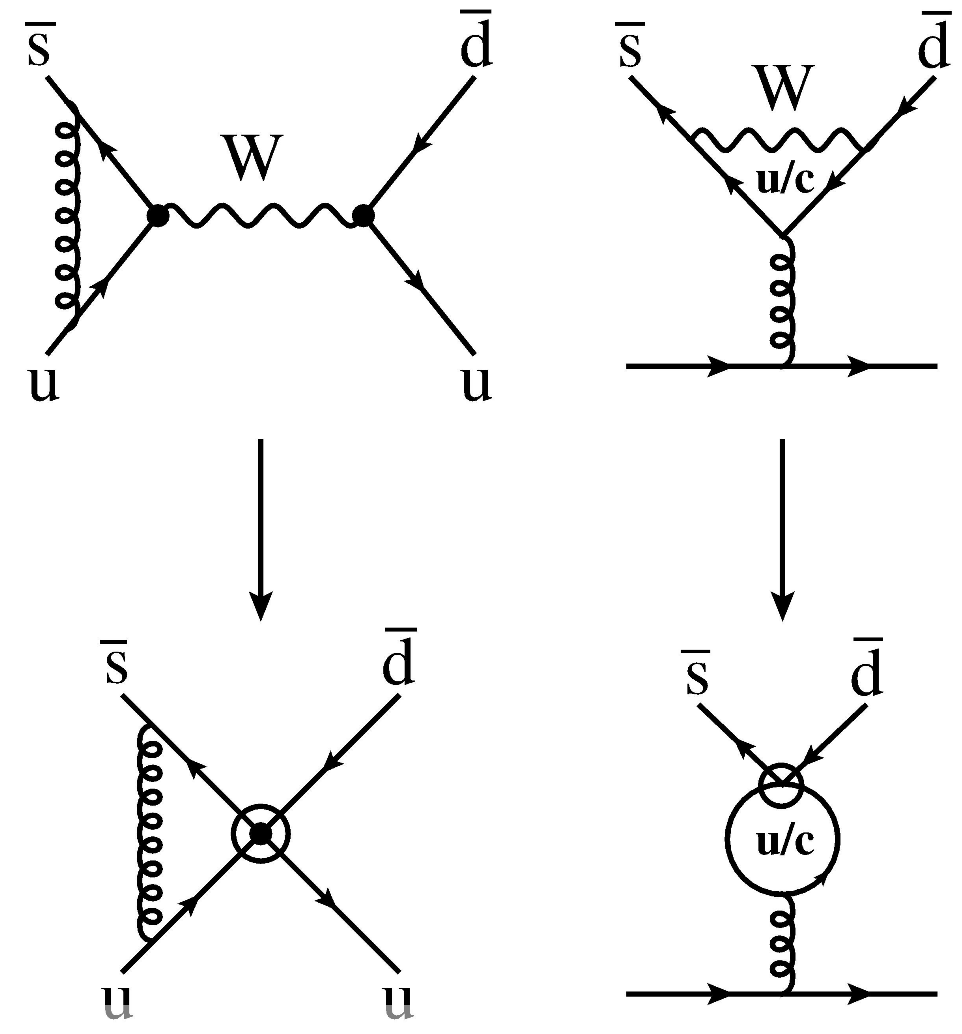
CP-violation effects neglected ($\frac{V_{td}V_{ts}^*}{V_{ud}V_{us}^*} \sim 10^{-3}$), keep active charm quark:

$$\mathcal{H}_{\mathrm{w}}^{\mathrm{eff}} = \frac{g_{\mathrm{w}}^2}{2M_W^2} V_{us}^* V_{ud} \sum_{\sigma=\pm} \{k_1^\sigma Q_1^\sigma + k_2^\sigma Q_2^\sigma\}$$

$$Q_1^\pm = (\bar{s}_{\mathrm{L}}\gamma_\mu u_{\mathrm{L}})(\bar{u}_{\mathrm{L}}\gamma_\mu d_{\mathrm{L}}) \pm (\bar{s}_{\mathrm{L}}\gamma_\mu d_{\mathrm{L}})(\bar{u}_{\mathrm{L}}\gamma_\mu u_{\mathrm{L}}) - [u \leftrightarrow c]$$

$$Q_2^\pm = (m_u^2 - m_c^2)\{m_d(\bar{s}_{\mathrm{L}} d_{\mathrm{R}}) + m_s(\bar{s}_{\mathrm{R}} d_{\mathrm{L}})\}$$

(do not contribute to physical $K \to \pi\pi$ transitions)



(penguin contributions cancel in GIM limit $m_c = m_u$)

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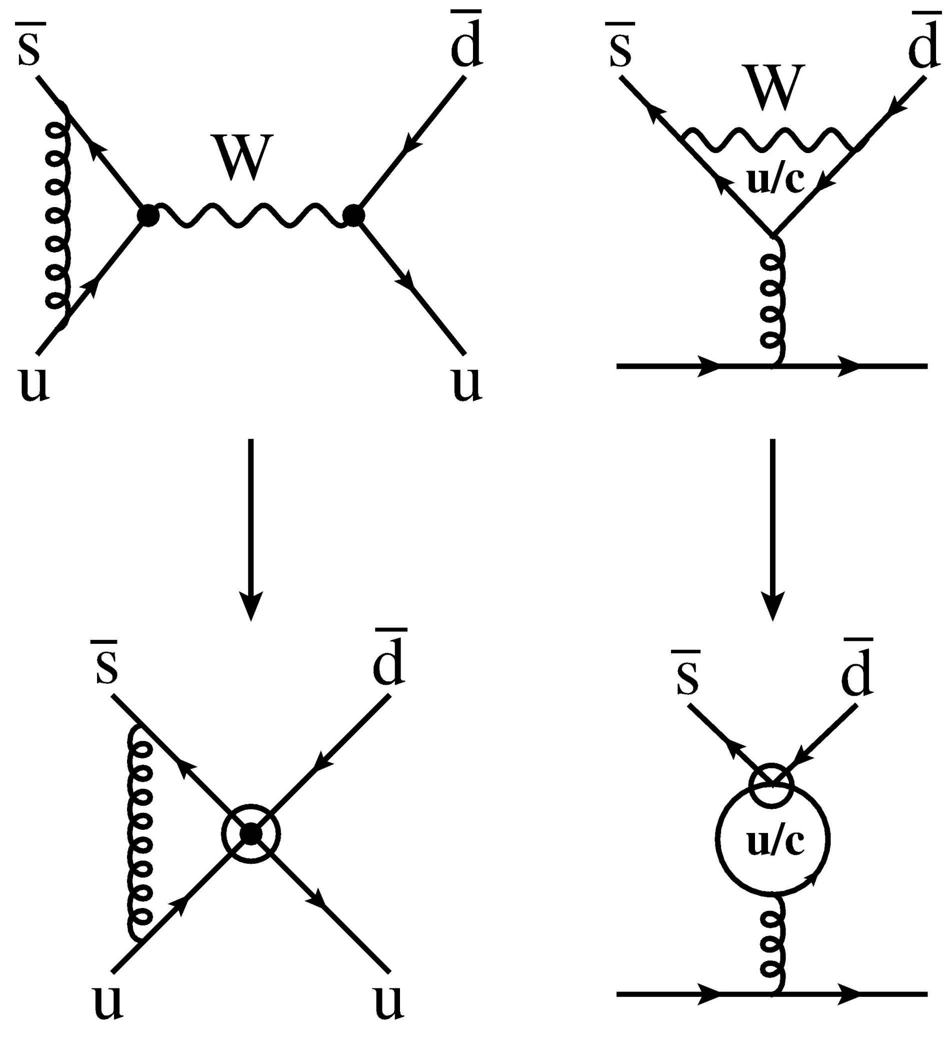


$$\left|\frac{A_0}{A_2}\right| = \frac{k_1^-(M_W)}{k_1^+(M_W)} \frac{\langle(\pi\pi)_{I=0}|Q_1^-|K\rangle}{\langle(\pi\pi)_{I=2}|Q_1^+|K\rangle} \qquad \frac{k_1^-(M_W)}{k_1^+(M_W)} \simeq 2.8$$

- bulk of effect should come from long-distance QCD contribution

  [Gaillard, Lee; Altarelli, Maiani 1974]

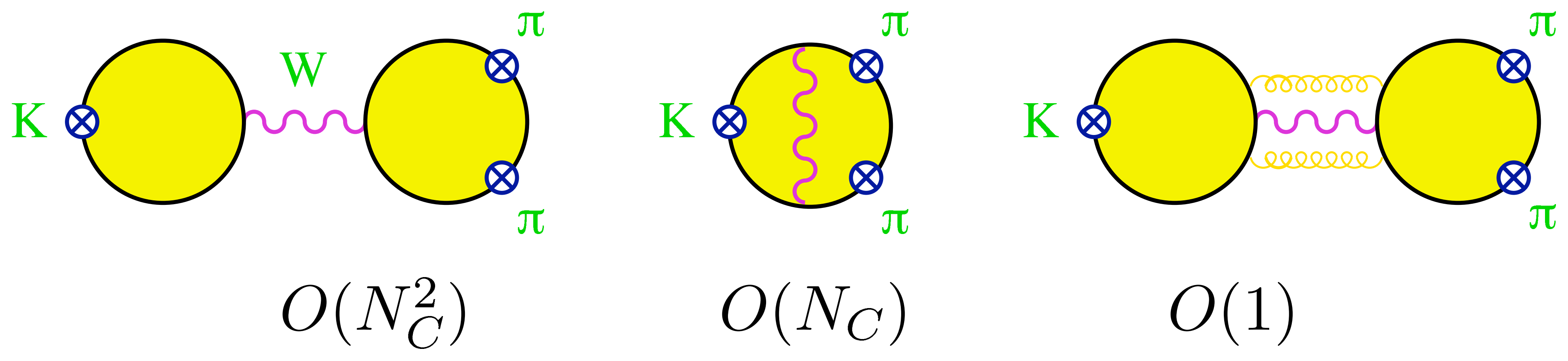
- reliable non-perturbative determination mandatory

  [Cabibbo, Martinelli, Petronzio; Brower, Maturana, Gavela, Gupta 1984]

weak decay and ΔI=1/2

approximate methods/effective theory

- spectacular failure of naive $1/N_c$ expansion



$$T[K^0 \to \pi^0\pi^0] \sim 0 \quad \Rightarrow \quad \left|\frac{A_0}{A_2}\right|_{N\to\infty} \sim \sqrt{2}$$

[Fukugita et al. 1977]
[Chivukula, Flynn, Georgi 1986]

- elaborate approaches that combine $1/N_c$, chiral perturbation theory+vector dominance, and quark-hadron duality claim (non-universal) success

[Aebischer, Bobeth, Buras 2020 (⊃ earlier)]
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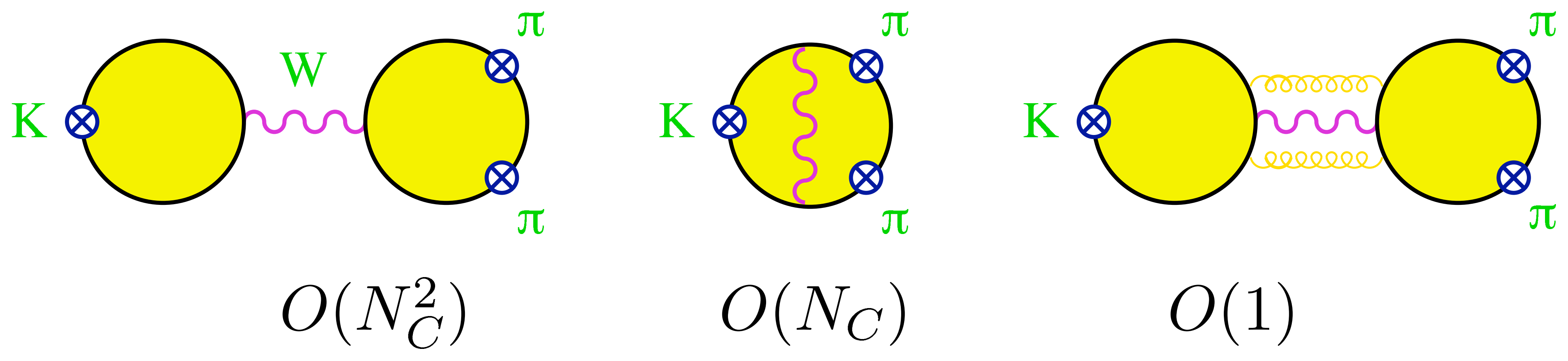

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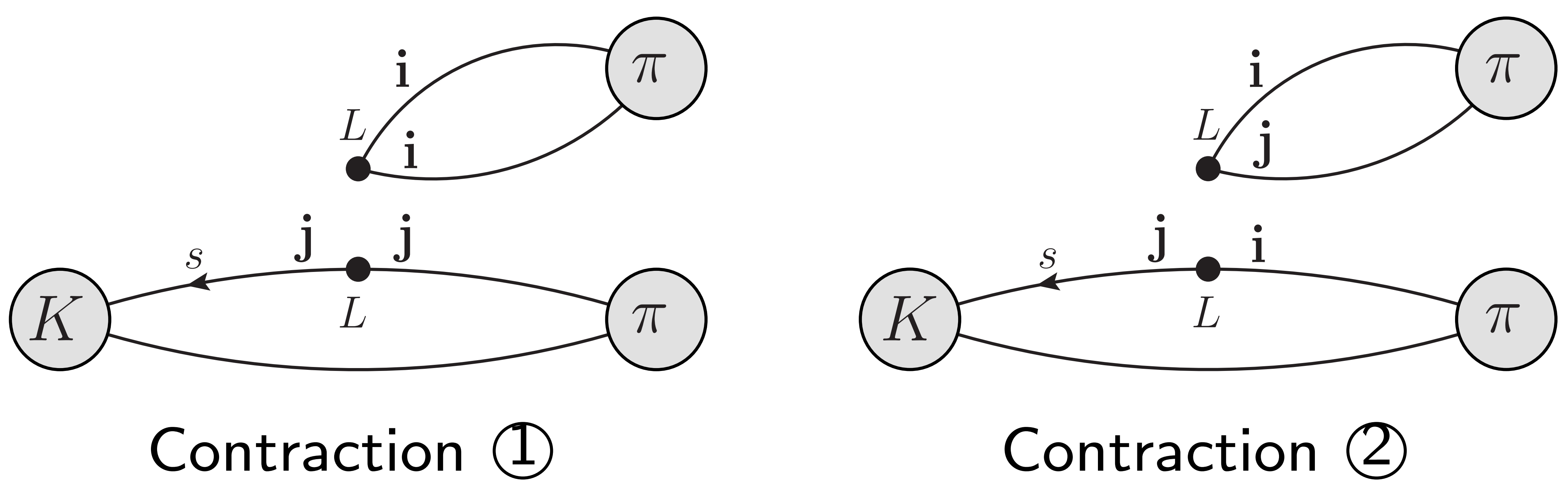
lattice QCD

far-reaching effort by RBC/UKQCD collaboration



- Naive factorisation approach: ② $\sim$ 1/3①
- Our computation: ② $\sim -0.7$①

"emerging understanding of the ΔI=1/2 rule"

[Boyle et al., PRL 110 (2013) 152001]

anatomy of ΔI=1/2

[Giusti, Hernández, Laine, Weisz, Wittig 2004]
[ChiPT: Hernández, Laine 2006]

several possible sources for ΔI=1/2 enhancement:

- physics at "intrinsic" QCD scale $\sim \Lambda_{\mathrm{QCD}}$
- physics at charm scale (penguins)
- final state interactions
- all of the above (no dominating "mechanism")

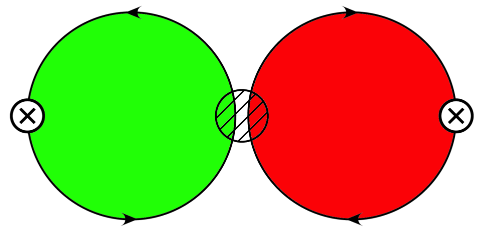
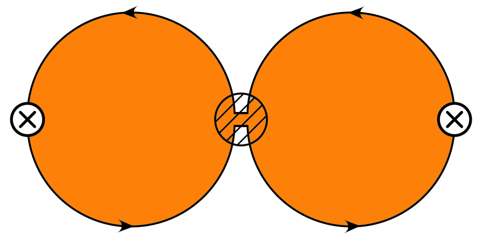
separate low-energy QCD and charm-scale physics: consider amplitudes as a function of charm mass for fixed u,d,s masses

$$\boxed{m_c = m_u = m_d = m_s \quad \longrightarrow \quad m_c \gg m_u = m_d \leq m_s}$$

implementation:

- active charm
- use fermions with good chiral properties (good renormalisation, arbitrarily low masses with GW)
- give up (too expensive) direct computation, use ChiPT ($\Rightarrow$ FSI captured at weak pion coupling only)

anatomy of ΔI=1/2 (GIM limit)

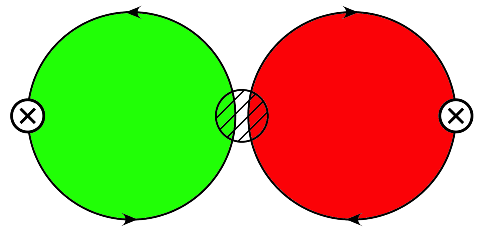
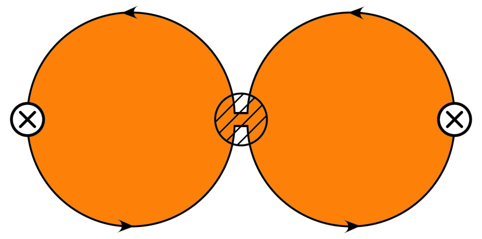
$g^{\pm} \propto A^{\pm} \propto$  $\mp$ 

Color-disconnected $O(N_c^2)$ Color-connected $O(N_c)$

$$\frac{A_0}{A_2} = \frac{1}{2\sqrt{2}}\left(1 + 3\frac{g^-}{g^+}\right) \xrightarrow[g^+=g^-]{\text{Large } N_c} \sqrt{2}$$

m_u=m_d=m_s limit: $B_K \propto A_2$

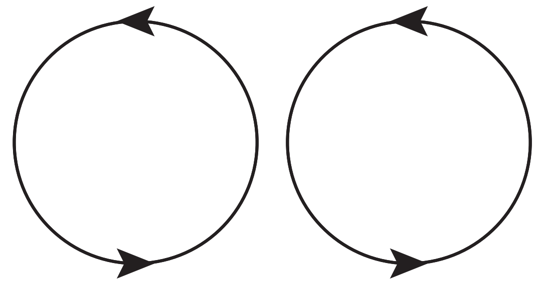
anatomy of ΔI=1/2 (GIM limit)

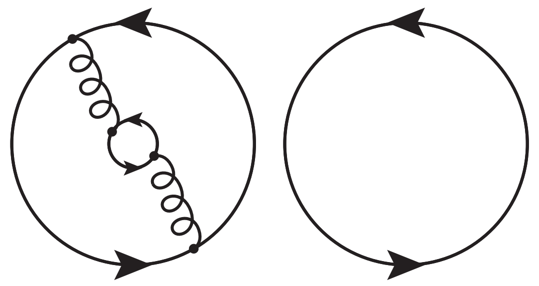
$g^{\pm} \propto A^{\pm} \propto$  $\mp$ 

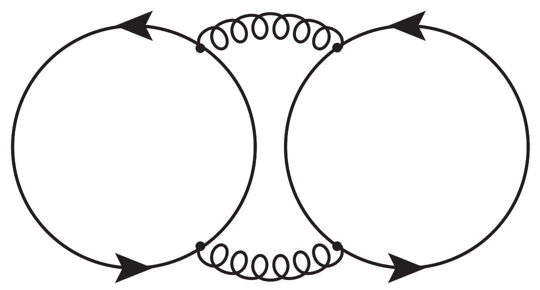
Color-disconnected $O(N_c^2)$ Color-connected $O(N_c)$

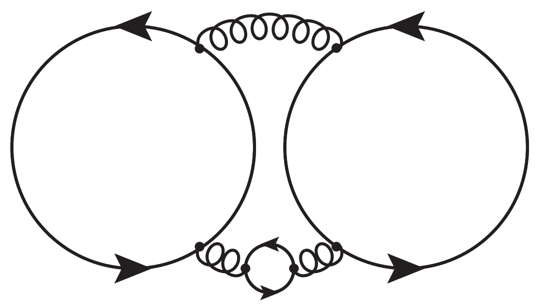
$$\frac{A_0}{A_2} = \frac{1}{2\sqrt{2}}\left(1 + 3\frac{g^-}{g^+}\right) \xrightarrow[g^+=g^-]{\text{Large } N_c} \sqrt{2}$$

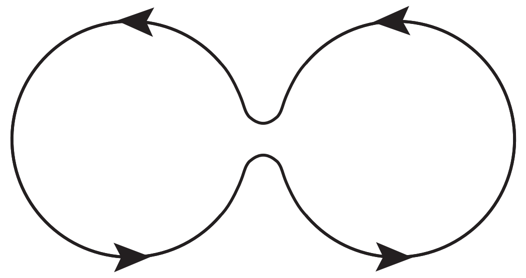
m_u=m_d=m_s limit: $B_K \propto A_2$

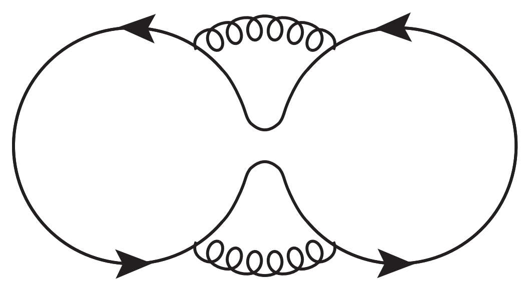
(a)  $\mathcal{O}(N_c^2)$

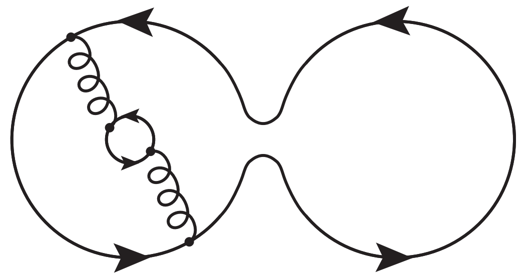
(b)  $\mathcal{O}(N_c N_f)$

(c)  $\mathcal{O}(N_c^0)$

(d)  $\mathcal{O}\left(\frac{N_f}{N_c}\right)$

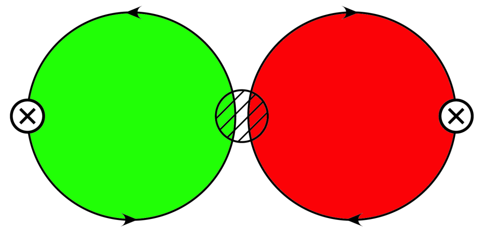
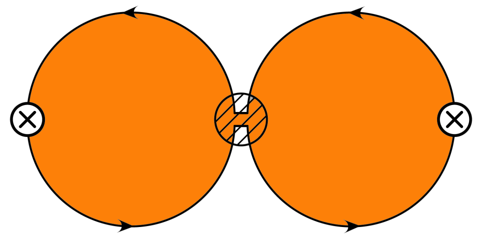
(e)  $\mathcal{O}(N_c)$

 $\mathcal{O}(N_c)$

(f)  $\mathcal{O}\left(N_f\right)$

$$A^{\pm} = 1 \pm \tilde{a}\frac{1}{N_c} \pm \tilde{b}\frac{N_f}{N_c^2} + \tilde{c}\frac{1}{N_c^2} + \tilde{d}\frac{N_f}{N_c^3} + \cdots$$

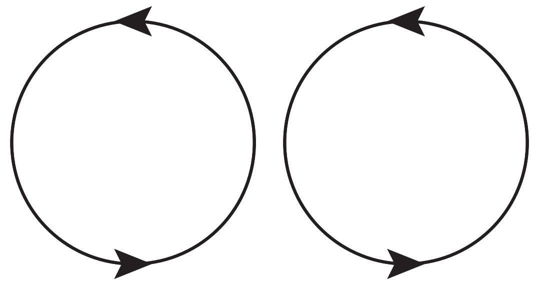
anatomy of ΔI=1/2 (GIM limit)

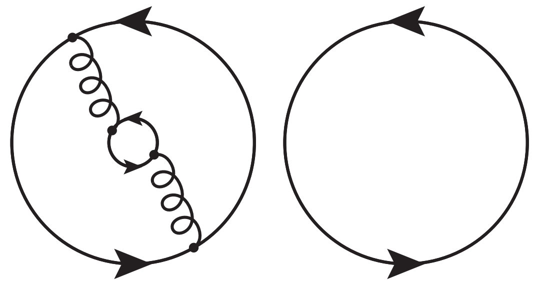
$g^{\pm} \propto A^{\pm} \propto$  $\mp$ 

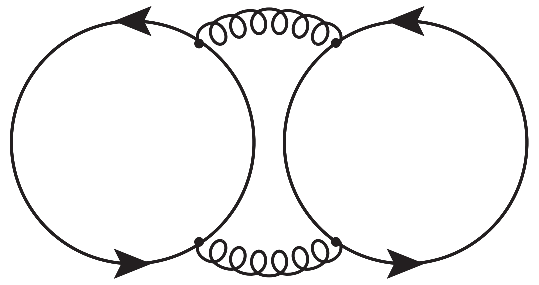
Color-disconnected $O(N_c^2)$ Color-connected $O(N_c)$

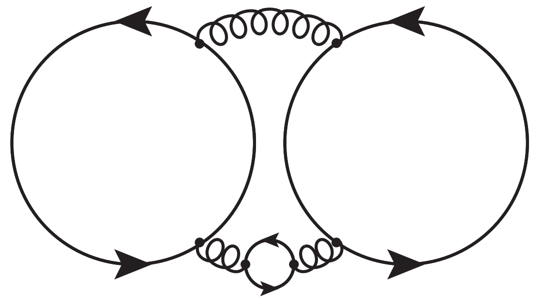
$$\frac{A_0}{A_2} = \frac{1}{2\sqrt{2}}\left(1 + 3\frac{g^-}{g^+}\right) \xrightarrow[g^+=g^-]{\text{Large } N_c} \sqrt{2}$$

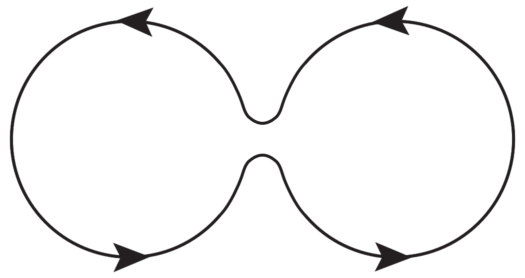
m_u=m_d=m_s limit: $B_K \propto A_2$

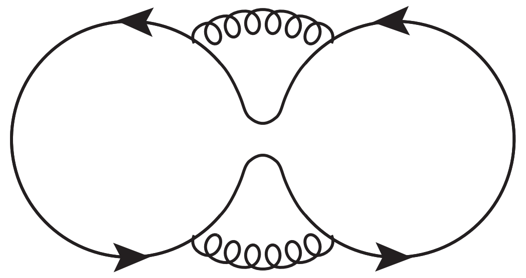
(a)  $\mathcal{O}(N_c^2)$

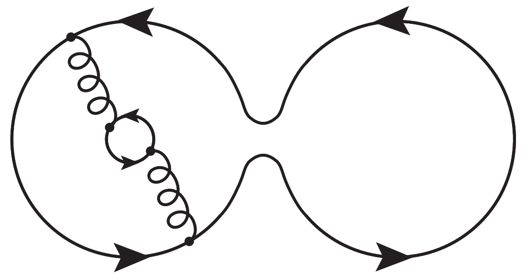
(b)  $\mathcal{O}(N_c N_f)$

(c)  $\mathcal{O}(N_c^0)$

(d)  $\mathcal{O}\left(\frac{N_f}{N_c}\right)$

(e)  $\mathcal{O}(N_c)$

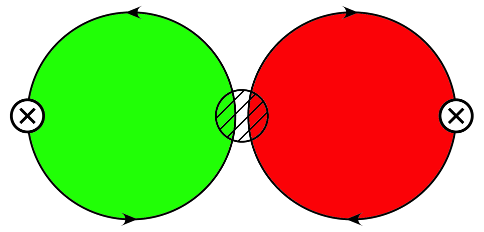
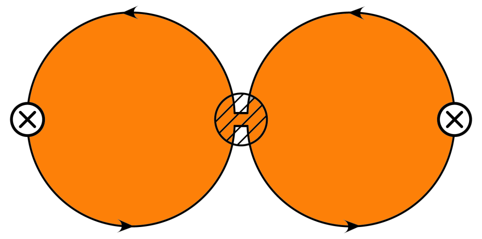
 $\mathcal{O}(N_c)$

(f)  $\mathcal{O}(N_f)$

anticorrelation

$$A^{\pm} = 1 \pm \tilde{a}\frac{1}{N_c} \pm \tilde{b}\frac{N_f}{N_c^2} + \tilde{c}\frac{1}{N_c^2} + \tilde{d}\frac{N_f}{N_c^3} + \cdots$$

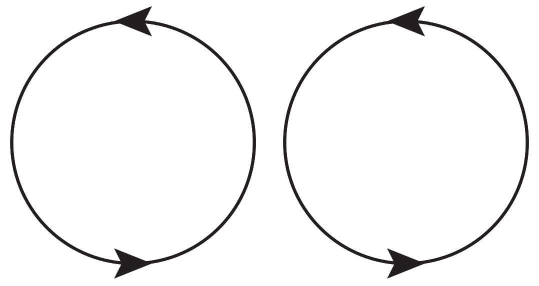
anatomy of ΔI=1/2 (GIM limit)

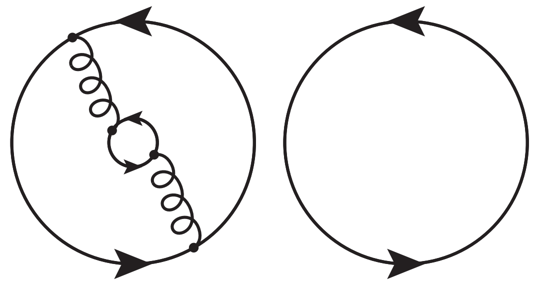
$g^{\pm} \propto A^{\pm} \propto$  $\mp$ 

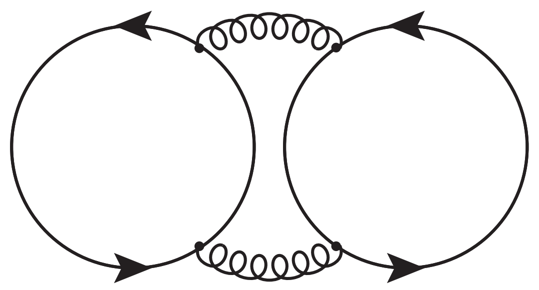
Color-disconnected $O(N_c^2)$ — Color-connected $O(N_c)$

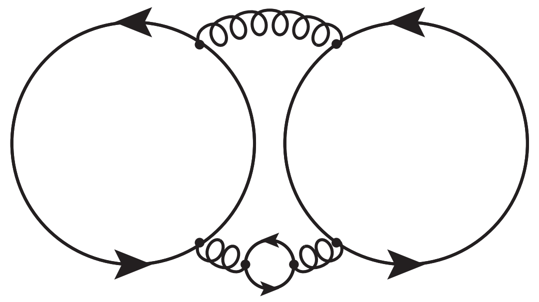
$$\frac{A_0}{A_2} = \frac{1}{2\sqrt{2}}\left(1 + 3\frac{g^-}{g^+}\right) \xrightarrow[g^+=g^-]{\text{Large } N_c} \sqrt{2}$$

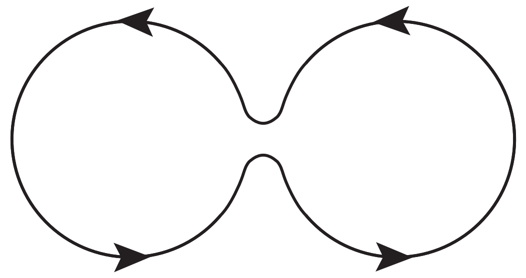
m_u=m_d=m_s limit: $B_K \propto A_2$

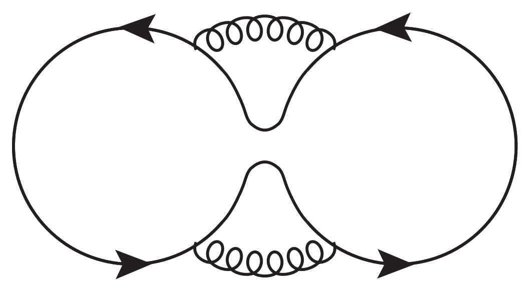
(a)  $\mathcal{O}(N_c^2)$

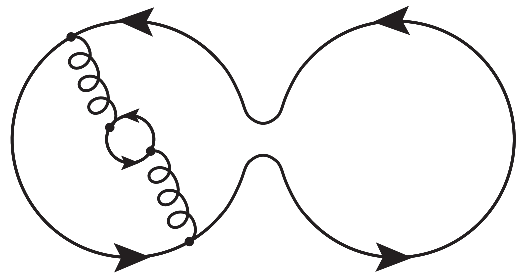
(b)  $\mathcal{O}(N_c N_f)$

(c)  $\mathcal{O}(N_c^0)$

(d)  $\mathcal{O}\left(\frac{N_f}{N_c}\right)$

(e)  $\mathcal{O}(N_c)$

 $\mathcal{O}(N_c)$

(f)  $\mathcal{O}(N_f)$

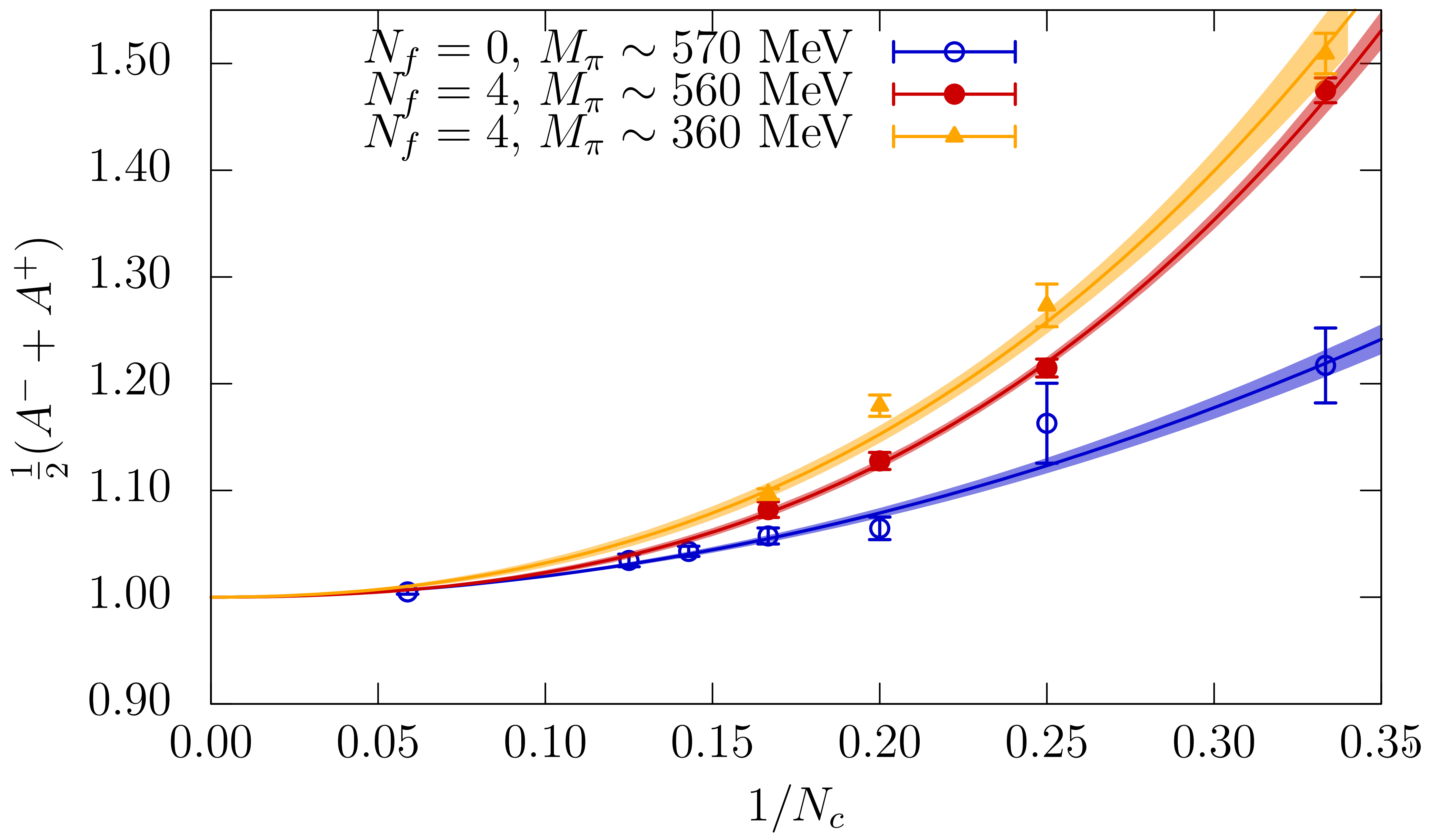
n.b. also chiral corrections are anticorrelated:

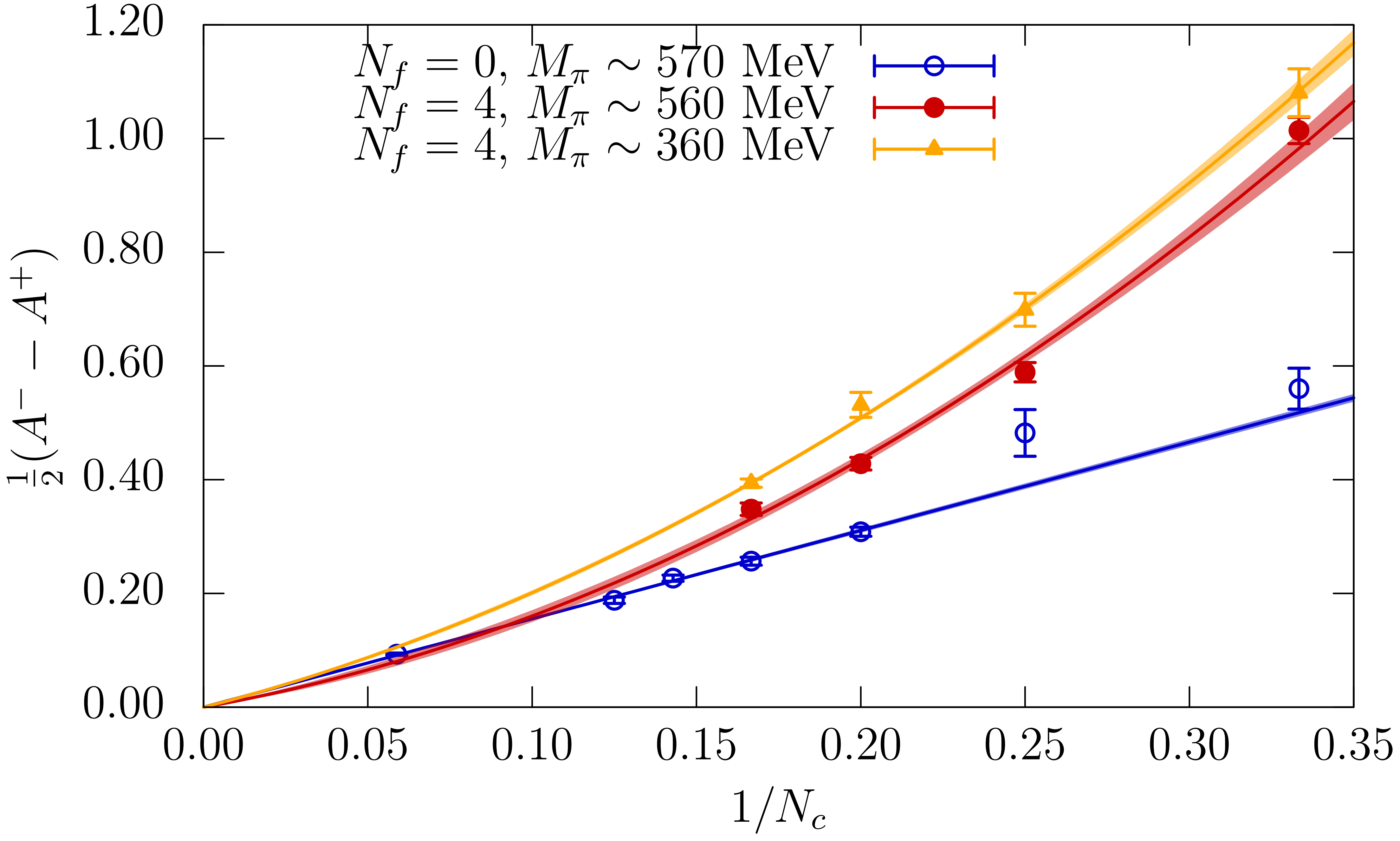
$$A^{\pm} = g^{\pm}\left[1 \mp 3\left(\frac{M_\pi}{4\pi F_\pi}\right)^2\left(\log\frac{M_\pi^2}{\mu^2} + L_{\pm}^r(\mu)\right)\right]$$

anatomy of ΔI=1/2 (GIM limit)

$$\frac{A^- + A^+}{2} = 1 + \tilde{c}\,\frac{1}{N_c^2} + \tilde{d}\,\frac{N_f}{N_c^3} + \ldots$$

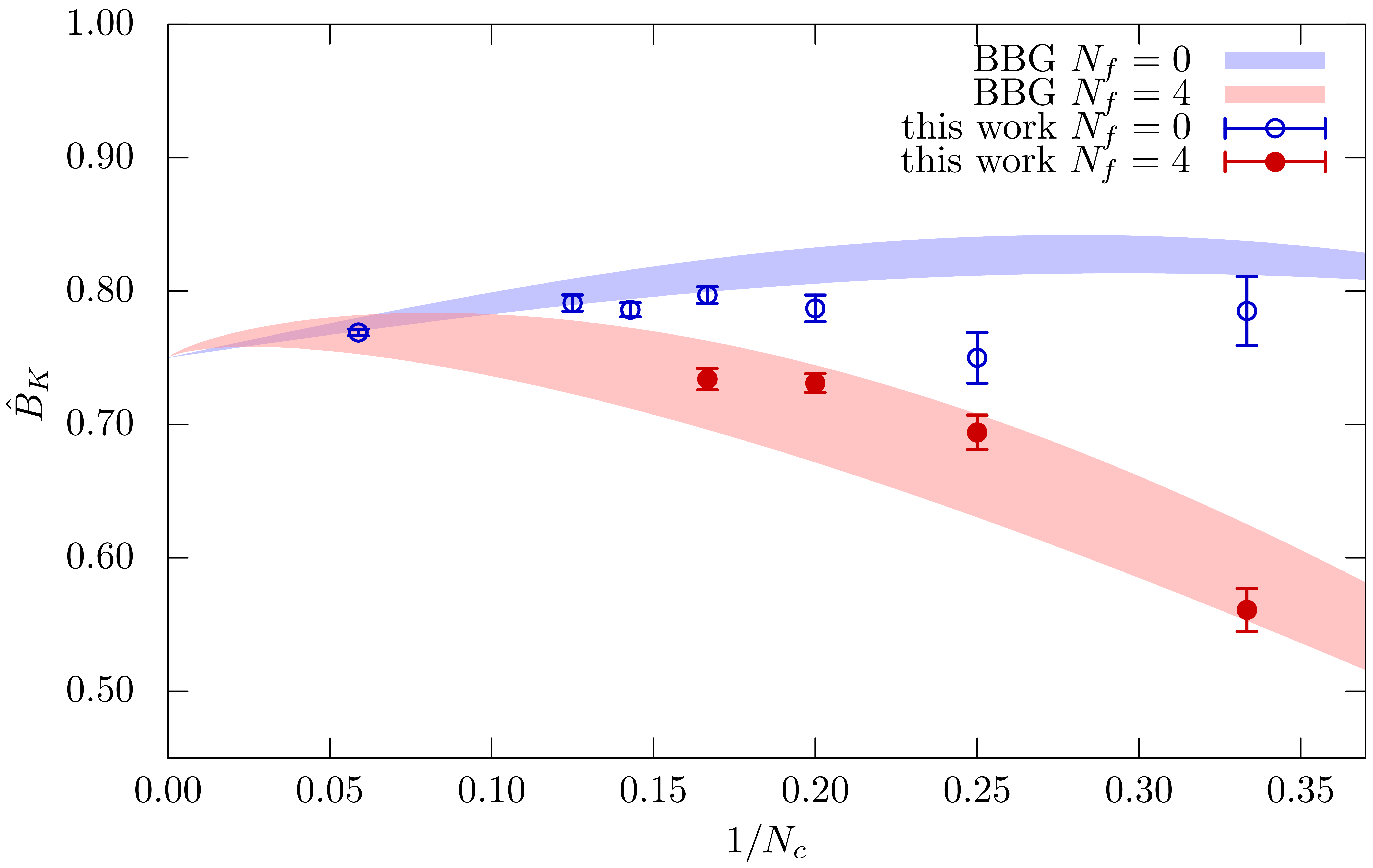
$$\frac{A^- - A^+}{2} = -\tilde{a}\,\frac{1}{N_c} - \tilde{b}\,\frac{N_f}{N_c^2} + \ldots.$$





- scaling with N_c confirms expectations, with natural $O(1)$ coefficients
- dynamical quarks enhance effect
- lighter quark masses enhance effect

anatomy of ΔI=1/2: B_K



anatomy of ΔI=1/2: conclusions

$$\boxed{\begin{aligned} &\hat{B}_K\Big|^{N_f=3}_{M_K=M_\pi} = 0.67(2)_{\rm stat}(6)_{Z^+}(3)_{\rm fit} \\ &{\rm Re}\,\frac{A_0}{A_2}\bigg|_{N_f=4} = 24(5)_{\rm stat}(4)_{\rm fit}(5)_{Z^\pm}(3)_{\rm NLO} \end{aligned}}$$

$\mathrm{Re}(A_0)/\mathrm{Re}(A_2) = 22.45(6)$ [expt]

$\mathrm{Re}(A_0)/\mathrm{Re}(A_2) = 19.9(2.3)(4.4)$

[RBC/UKQCD, PRD 102 (2020) 054509]

anatomy of $\Delta I = 1/2$: conclusions

- B_K displays a large N_f (in GIM limit), chiral dependence

- **ratio of decay amplitudes in GIM limit comes very close to the physical value (!)**

- handle on N_f dependence in principle allows us to make connection with other physical kinematics, but we are still missing a direct analysis of the m_c dependence

$$\boxed{\begin{aligned} \hat{B}_K\Big|^{N_f=3}_{M_K=M_\pi} &= 0.67(2)_{\rm stat}(6)_{Z^+}(3)_{\rm fit} \\ \mathrm{Re}\left.\frac{A_0}{A_2}\right|_{N_f=4} &= 24(5)_{\rm stat}(4)_{\rm fit}(5)_{Z^\pm}(3)_{\rm NLO} \end{aligned}}$$

$\mathrm{Re}(A_0)/\mathrm{Re}(A_2) = 22.45(6)$ [expt]

$\mathrm{Re}(A_0)/\mathrm{Re}(A_2) = 19.9(2.3)(4.4)$

[RBC/UKQCD, PRD 102 (2020) 054509]

- "mechanism budget":
  - short-distance $\longrightarrow$ x 3
  - physics at "intrinsic" QCD scale $\sim \Lambda_{\rm QCD}$ $\longrightarrow$ x 4 [gluons] x 2 [quarks] (N_f????)
  - physics at charm scale (penguins) $\longrightarrow$ x O(1) (???)
  - final state interactions $\longrightarrow$ small?

conclusions & outlook

- non-leptonic kaon decay remains an open problem... and a fertile ground to learn about strong interaction physics
  - indirect CP violation well under control
  - direct CP violation, isospin enhancement still witness claims of new physics

- lattice toolbox making steady progress
  - controlled quantitative predictions for amplitudes are at hand
  - the anatomy of the effect is ever better understood, **pure "low-energy" dynamics seems to play major role in enhancement**

- interesting spinoffs: qualitative understanding of meson interactions at low energies

- a theorist's paradise: field-theory, phenomenology, and computational physics all simultaneously at play!

backup

QCD @ large N_c

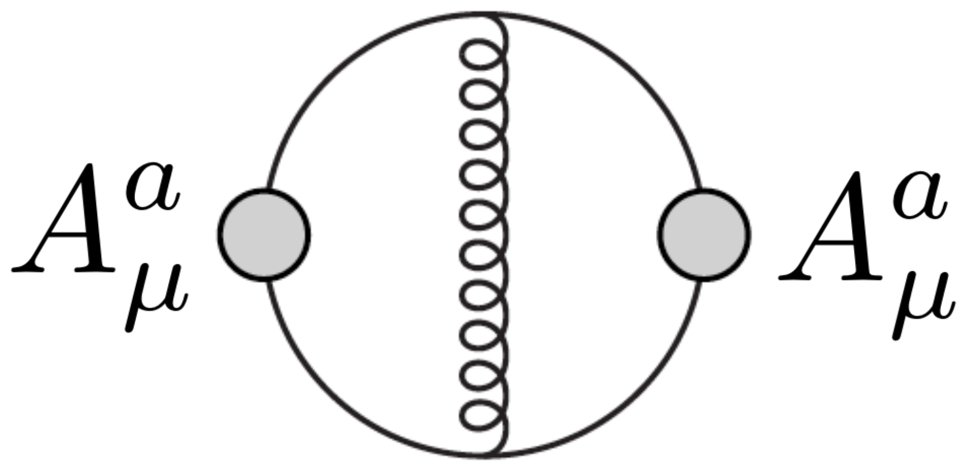
't Hooft's large N_c limit of QCD: $N_c \to \infty|_{\lambda = g^2 N_c = \text{fixed}}$ ['t Hooft 1974]

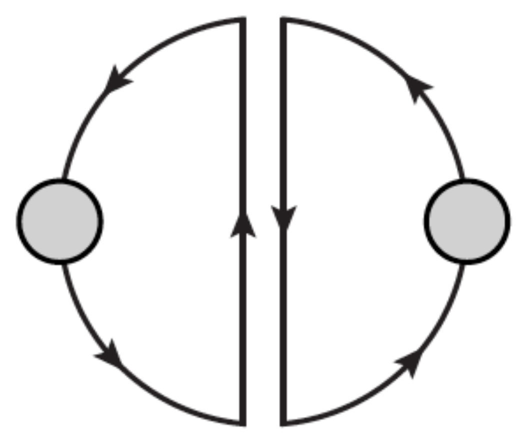
à la 't Hooft proper: $\frac{N_f}{N_c} \to 0 \;\Rightarrow\; m_{\eta'}^2 = m_\pi^2\,,\; U(N_f)$ chiral symmetry

à la Veneziano: $\frac{N_f}{N_c} = const \;\Rightarrow\; m_{\eta'}^2 \gg m_\pi^2\,,\; SU(N_f)$ chiral symmetry [Veneziano 1979] [also: Witten 1979]

- preserves asymptotic freedom
- captures most non-perturbative properties (confinement, chiral SSB, ...)
- leads to some *quantitative* non-perturbative predictions!

$$\langle 0|A_\mu^a(x) A_\mu^b(y)|0\rangle \propto F_\pi^2 \sim N_c$$





$g_s^2 N_c^2 \propto \mathcal{O}(N_c)$

QCD @ large N_c

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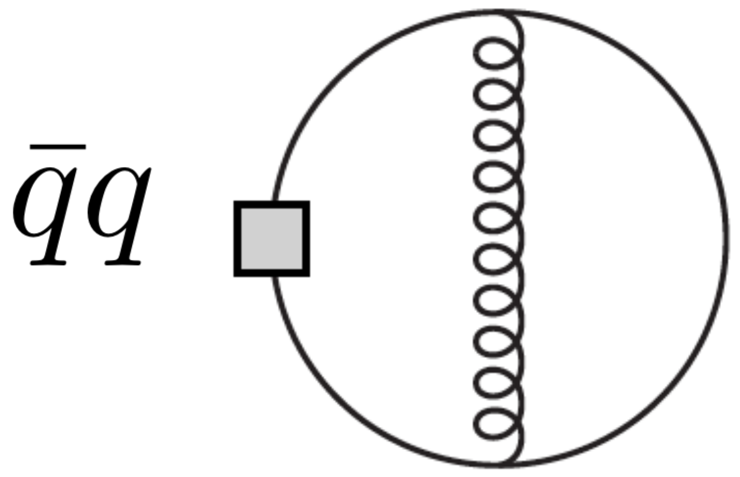
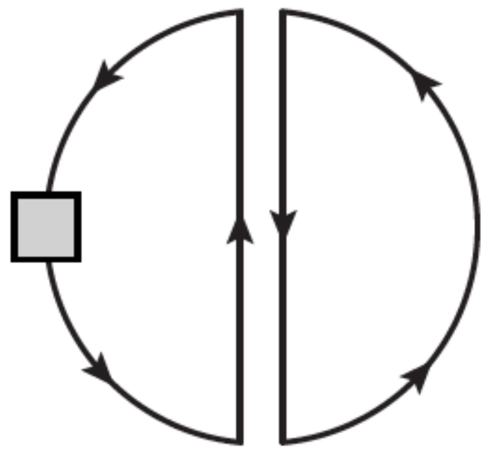
- preserves asymptotic freedom
- captures most non-perturbative properties (confinement, chiral SSB, ...)
- leads to some *quantitative* non-perturbative predictions!

$$\langle 0|\bar q q|0\rangle = -\Sigma \sim N_c$$





$g_s^2 N_c^2 \propto \mathcal{O}(N_c)$

lattice setup

quenched simulations in 16^3 lattices at (roughly) constant PS mass [Wilson+Wilson]

N_c	T/a	β	$am_{\rm PCAC}$	$am_{\rm PS}$	$R^{+}_{\rm bare}$	$R^{-}_{\rm bare}$
3	48	6.0175	-0.002(14)	0.2718(61)	0.774(21)	1.218(31)
4	48	11.028	-0.0015(11)	0.2637(39)	0.783(15)	1.198(19)
5	48	17.535	0.0028(9)	0.2655(31)	0.839(8)	1.145(12)
6	32	25.452	0.0013(7)	0.2676(28)	0.871(6)	1.125(7)
7	32	34.8343	-0.0034(6)	0.2819(19)	0.880(5)	1.122(5)

renormalisation (RI scheme) at scale around 2 GeV performed using one-loop P.T.

[Constantinou et al. 2011]
[Alexandrou et al. 2012]

perturbative two-loop RG running in RI to connect to RGIs

[Ciuchini et al. 1998]
[Buras et al. 2000]

dynamical simulations at varying PS mass and constant t_0 [Iwasaki+Clover]

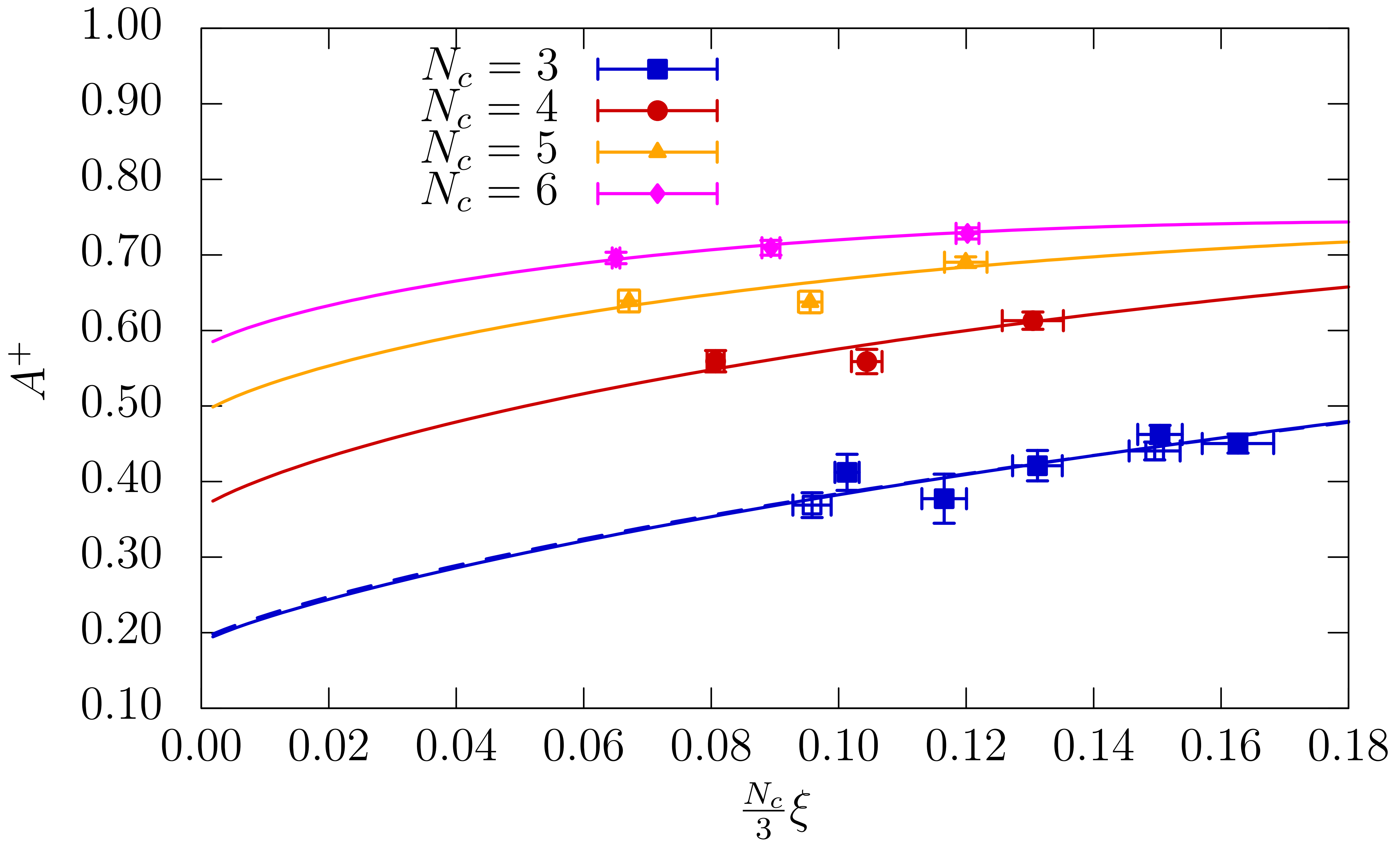
Ensemble	N_c	β	$c_{\rm sw}$	$T \times L$	$am_0^{\rm s}$
3A10	3	1.778	1.69	36×20	-0.4040
3A11				48×24	-0.4040
3A20				48×24	-0.4060
3A30				48×24	-0.4070
3A40				60×32	-0.4080
3B10	3	1.820	1.66	48×24	-0.3915
3B20				60×32	-0.3946
4A10	4	3.570	1.69	36×20	-0.3725
4A30				48×24	-0.3760
4A40				60×32	-0.3780
5A10	5	5.969	1.69	36×20	-0.3458
5A30				48×24	-0.3500
5A40				60×32	-0.3530
6A10	6	8.974	1.69	36×20	-0.3260
6A30				48×24	-0.3311
6A40				60×32	-0.3340

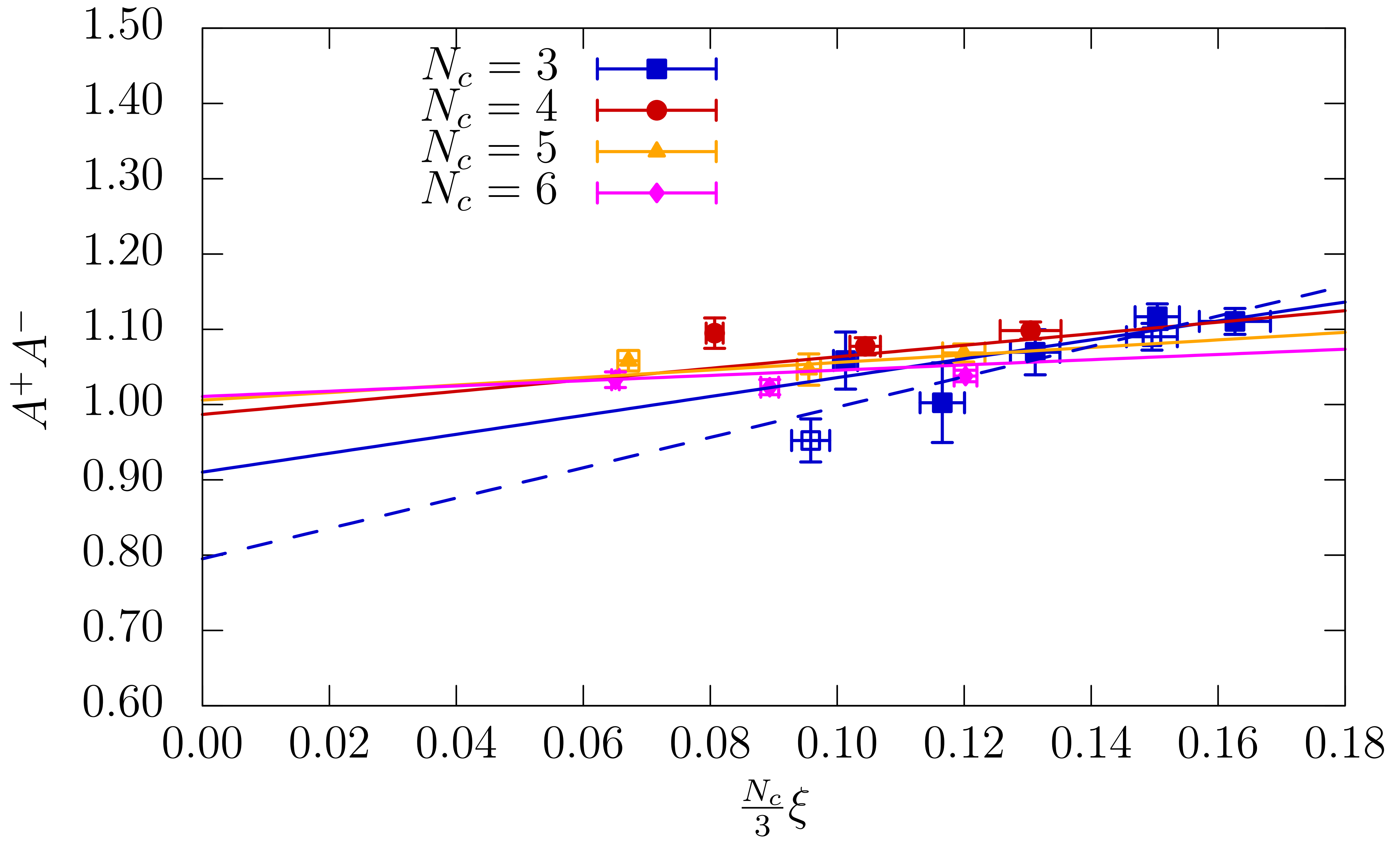
+ extra quenched points (N_c=8,17)

chiral and finite volume corrections

$$\mathrm{Re}\ \frac{A_0}{A_2}\bigg|_{M_\pi, M_D\to 0, M_K^{\mathrm{phys}}} = \frac{1}{2\sqrt{2}}\left(1+3\,\frac{g^-}{g^+}\right) + \frac{17}{12\sqrt{2}}\left(1+\frac{1}{17}\frac{g^-}{g^+}\right)\frac{M_K^2}{(4\pi F_K)^2}\log\frac{\Lambda_{\mathrm{eff}}^2}{M_K^2}$$

[extension to N_f=4 framework of Golterman, Leung PRD 56 (1997) 2950]





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[extension to N_f=4 framework of Golterman, Leung PRD 56 (1997) 2950]

$$\hat{B}_K^{QP} = \hat{B}_K\big|_{M_K = M_\pi}\left[1 + \frac{2}{3}\left(\frac{M_K}{4\pi F_K}\right)^2 \log\frac{\Lambda_{\mathrm{eff}}^{B_K}}{M_K}\right]$$

$$\bar{R}^\pm(L) = \bar{R}^\pm\left[1 \pm 6\sqrt{2\pi}\xi\frac{e^{-M_\pi L}}{(M_\pi L)^{3/2}}(M_\pi L - 4)\right]$$

[Hernández, Laine 2006]
[Colangelo, Dürr, Haefeli 2005]

meson scattering

Long-term goal: understand QCD at large N_c

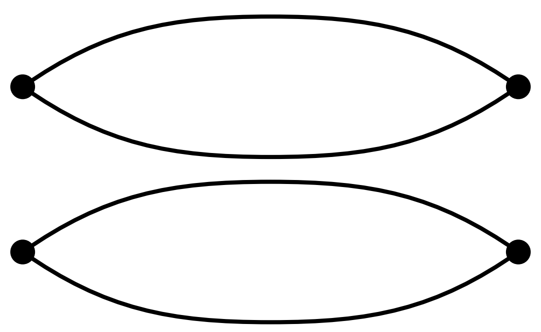
- Resonances $\rightarrow$ Stable ($\Gamma \sim 1/N_c$)
- Exotic states (tetraquarks?)
- $K \rightarrow (\pi\pi)_{I=0,2} \begin{cases} \text{Intrinsic QCD effects [Donini, et al. 2020]} \\ \text{Final state interactions} \end{cases}$

This work: $\pi\pi$ scattering at large N_c from lattice simulations

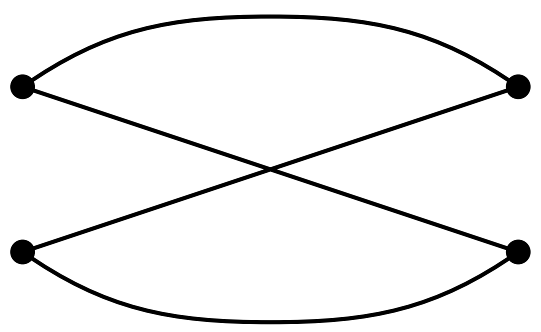
- $N_f = 4$ (u, d, s c) $\rightarrow$ 7 channels (4 with s-wave)

$$15 \otimes 15 = \underbrace{\mathbf{84}}_{\pi^+\pi^+} \oplus 45 \oplus \overline{45} \oplus \underbrace{\mathbf{20}}_{\pi^+ D_s^+ - K^+ D^+} \oplus \mathbf{15} \oplus 15 \oplus \mathbf{1}$$

$$\begin{aligned} C_{I=2} &= D - C \\ C_{AA} &= D + C \end{aligned}$$



D



C

- Match to Chiral Perturbation Theory (ChPT) to constrain Low Energy Coupling (LECs)

meson scattering

$\pi\pi$ scatering amplitudes for N_f flavours are known to NNLO [Weinberg 1979 Gasser, Leutwyler 1985 Bijnens, Lu 2011]

NLO, $N_\mathrm{f}=4$

$$M_\pi a_0^{I=2} = -\frac{M_\pi^2}{16\pi F_\pi^2}\left[1 - \frac{16M_\pi^2}{F_\pi^2}L_{I=2} + \frac{M_\pi^2}{32\pi^2 F_\pi^2}\left(\frac{13}{4}\ln\frac{M_\pi^2}{\mu^2} - \frac{3}{4}\right)\right]$$

$$M_\pi a_0^{AA} = \frac{M_\pi^2}{16\pi F_\pi^2}\left[1 - \frac{16M_\pi^2}{F_\pi^2}L_{AA} - \frac{M_\pi^2}{32\pi^2 F_\pi^2}\left(\frac{21}{4}\ln\frac{M_\pi^2}{\mu^2} + \frac{5}{4}\right)\right]$$

$$L_{I=2} = L^{(0)}N_\mathrm{c} + L^{(1)}_{I=2} + \ldots$$
$$L_{AA} = L^{(0)}N_\mathrm{c} + L^{(1)}_{AA} + \ldots$$

We have computed $\mathcal{M}_{I=2}$ and $\mathcal{M}_{AA}$ to NNLO in U(N_f) ChPT

$N_\mathrm{f}=4$

$$M_\pi a_0^{I=2} = -\frac{M_\pi^2}{16\pi F_\pi^2}\left\{1 - \frac{16M_\pi^2}{F_\pi^2}L_{I=2} + N_c^2 K^{(0)}_{I=2}\left(\frac{M_\pi^2}{F_\pi^2}\right)^2 + \frac{M_\pi^2}{32\pi^2F_\pi^2}\left[\frac{15M_\pi^2 - 13M_{\eta'}^2}{4(M_\pi^2 - M_{\eta'}^2)}\ln\frac{M_\pi^2}{\mu^2} + \frac{M_\pi^2 - 3M_{\eta'}^2}{4(M_\pi^2-M_{\eta'}^2)}\ln\frac{M_{\eta'}^2}{\mu^2} - \frac{1}{2}\right]\right\}$$

$$M_\pi a_0^{AA} = \frac{M_\pi^2}{16\pi F_\pi^2}\left\{1 - \frac{16M_\pi^2}{F_\pi^2}L_{AA} + N_c^2 K^{(0)}_{AA}\left(\frac{M_\pi^2}{F_\pi^2}\right)^2 - \frac{M_\pi^2}{32\pi^2F_\pi^2}\left[\frac{15M_\pi^2 - 21M_{\eta'}^2}{4(M_\pi^2 - M_{\eta'}^2)}\ln\frac{M_\pi^2}{\mu^2} + \frac{M_\pi^2 + 5M_{\eta'}^2}{4(M_\pi^2-M_{\eta'}^2)}\ln\frac{M_{\eta'}^2}{\mu^2} + \frac{3}{2}\right]\right\}$$

Large N_c or U(N_f) ChPT [Kaiser, Leutwyler 2000]:

- Leutwyler counting scheme

  $\delta \sim \mathcal{O}(m_q) \sim \mathcal{O}(M_\pi^2) \sim \mathcal{O}(k^2) \sim \mathcal{O}(N_\mathrm{c}^{-1})$

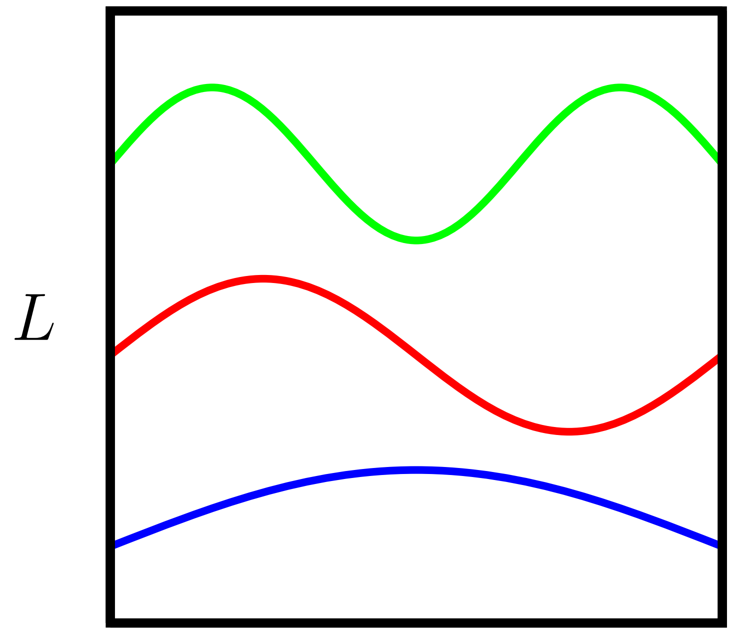
- $F_\pi \sim \sqrt{N_\mathrm{c}} \to$ Loop diagrams are NNLO

[slides from Jorge Baeza-Ballesteros' talk @ Lattice 2001]

meson scattering

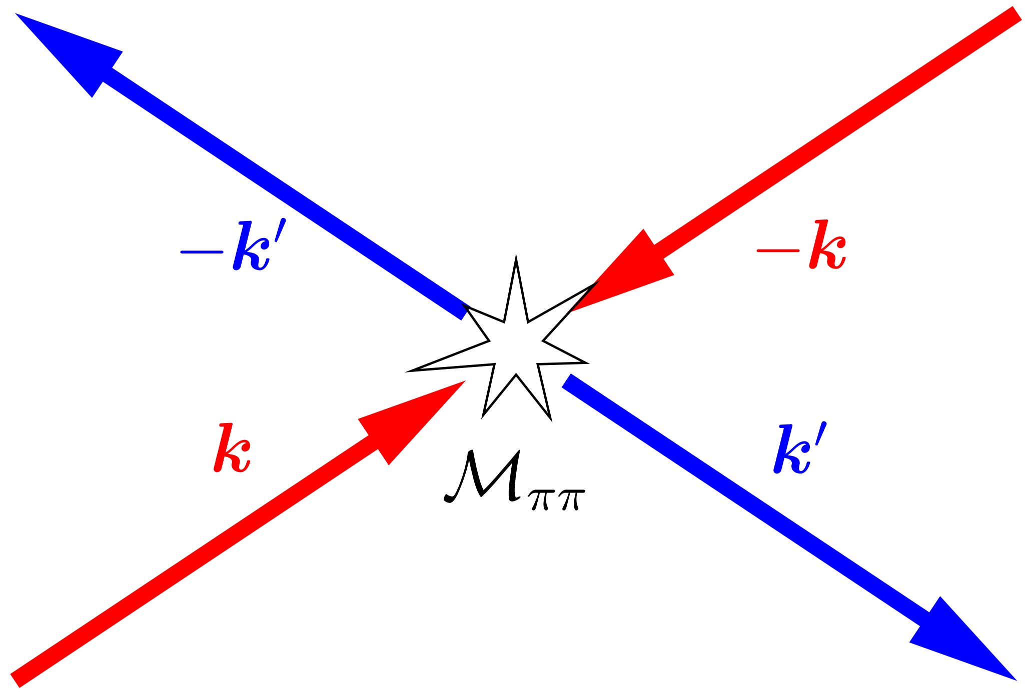
Finite-volume spectrum:

$$\delta E_{\pi\pi} = E_{\pi\pi} - 2M_\pi$$



Scattering properties:

$\mathcal{M}_{\pi\pi}$, $k\cot\delta_0$, a_0...

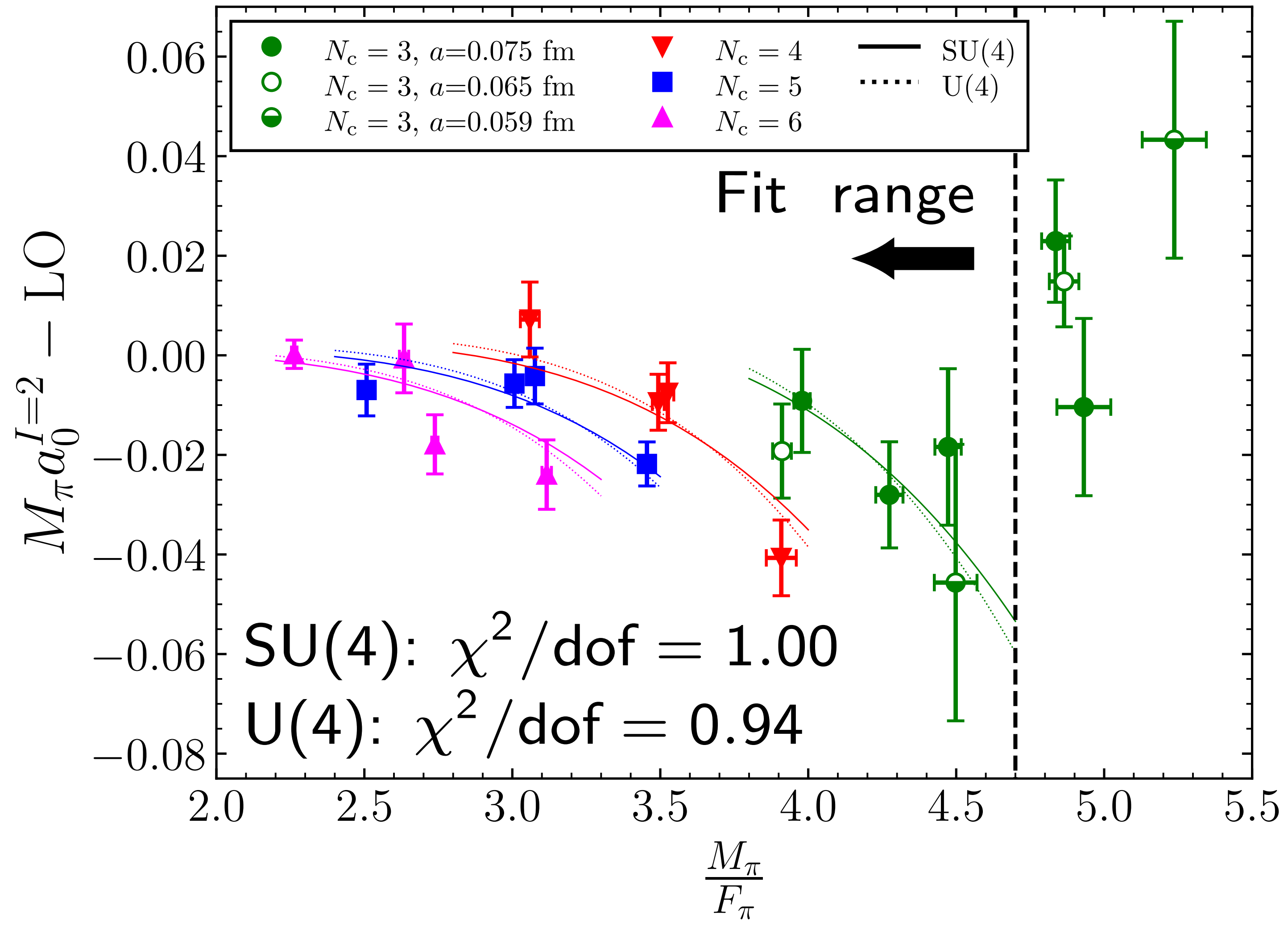


Lüscher's formalism [1986] $\longrightarrow$ $\boxed{k\cot\delta_0 = \frac{1}{\pi L}\mathcal{Z}\left(\frac{Lk}{2\pi}\right)}$

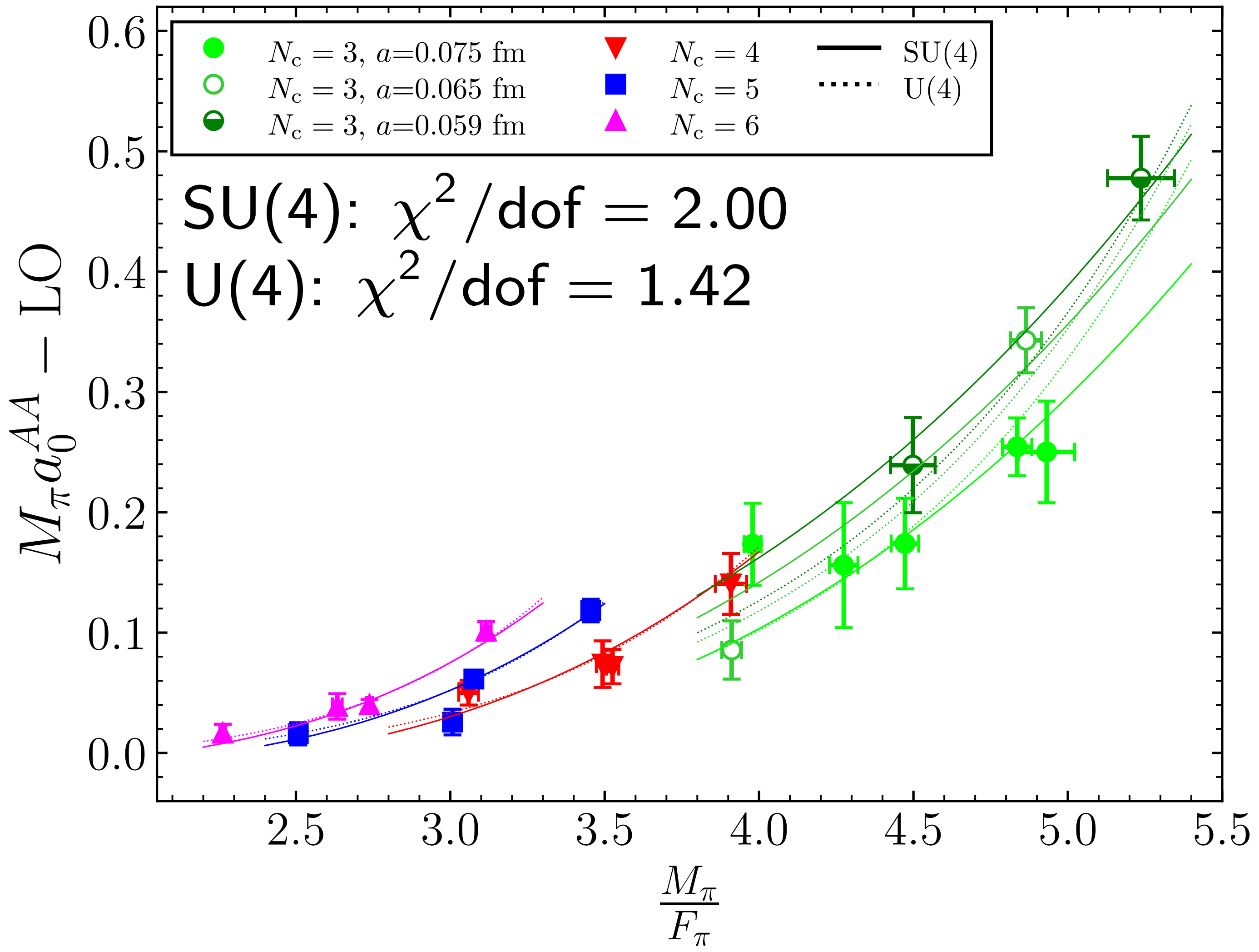
Threshold expansion

$$\delta E_{\pi\pi} = -\frac{4\pi a_0}{M_\pi L^3}\left[1 + c_1\left(\frac{a_0}{L}\right) + c_2\left(\frac{a_0}{L}\right)^2 \underbrace{+ c_3\left(\frac{a_0}{L}\right)^3 + \frac{2\pi r_0 a_0}{L^3} + \frac{\pi a_0}{M_\pi^2 L^3}}_{\mathcal{O}(L^{-6})\ \text{[Hansen, Sharpe 2017]}} + \ldots\right]$$

$I = 2$ channel



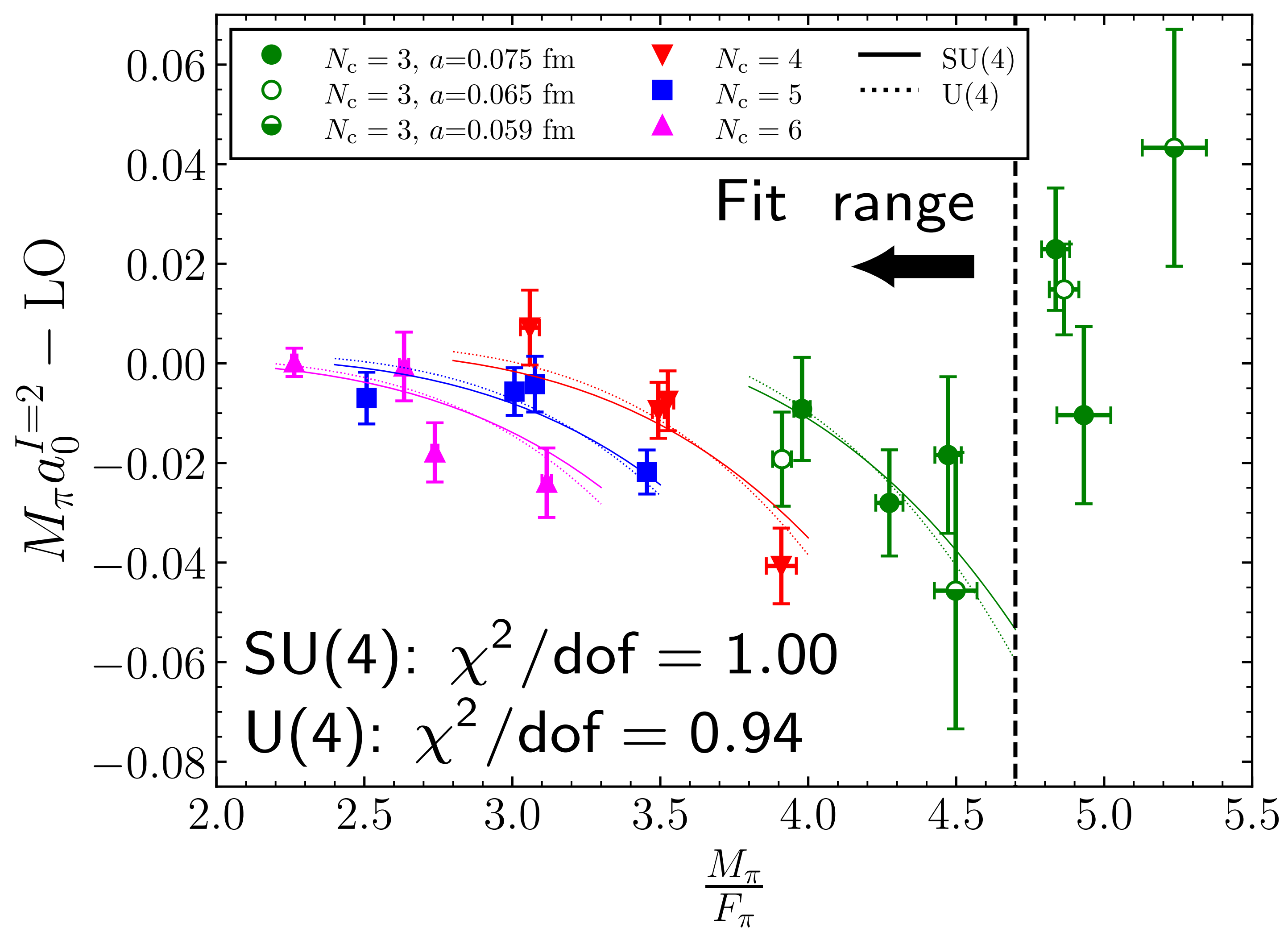
AA channel



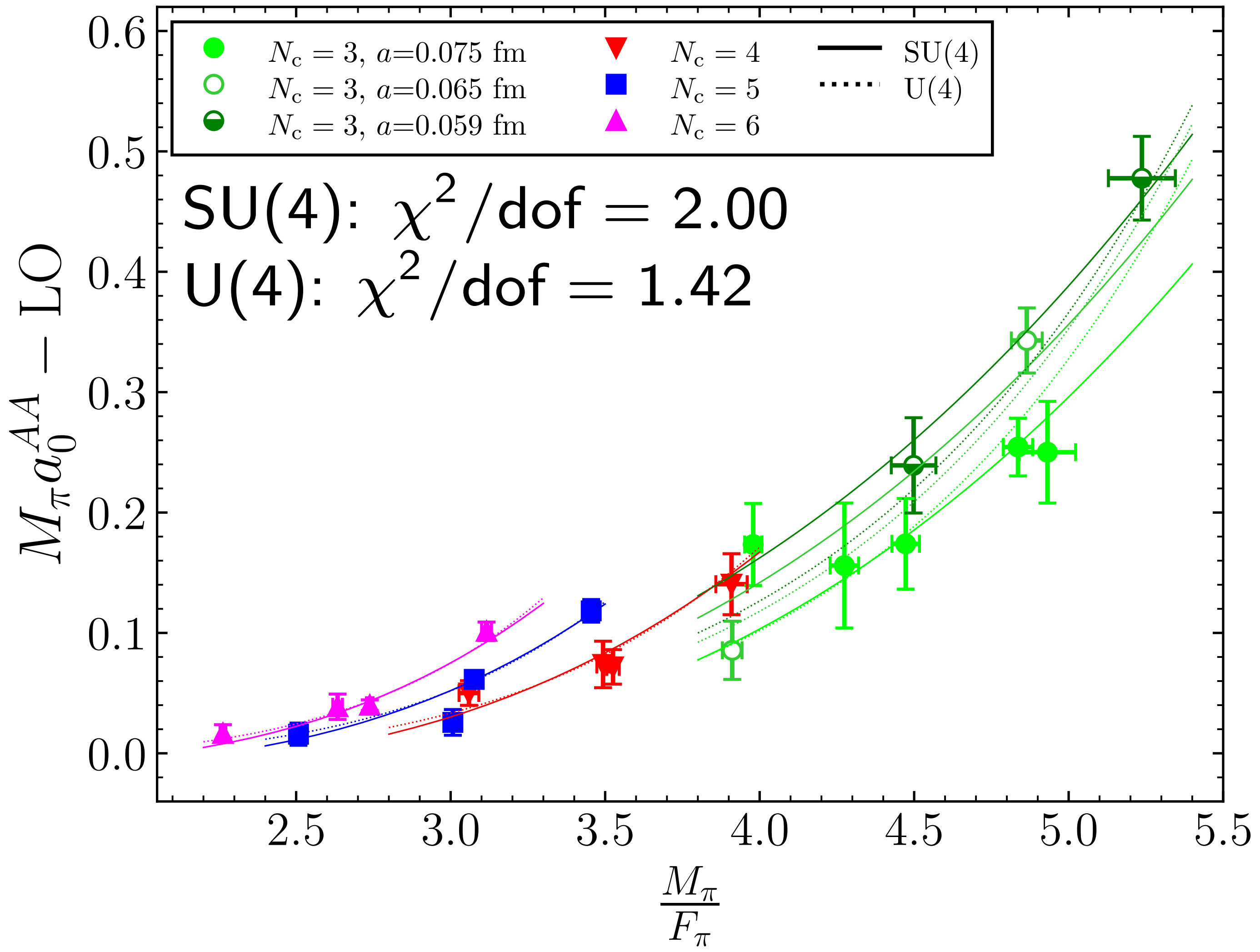
[slides from Jorge Baeza-Ballesteros' talk @ Lattice 2001]

meson scattering

$I = 2$ channel



AA channel



Match to ChPT to constrain LECs

SU(4) $\quad \dfrac{L_{I=2}}{N_c} \times 10^3 = -0.11(4) - \dfrac{1.43(16)}{N_c} \qquad \dfrac{L_{AA}}{N_c} \times 10^3 = -1.08(13) + \dfrac{2.2(3)}{N_c}$

U(4) $\quad \dfrac{L_{I=2}}{N_c} \times 10^3 = -0.10(7) - \dfrac{1.29(16)}{N_c} \qquad \dfrac{L_{AA}}{N_c} \times 10^3 = -0.6(4) + \dfrac{2.4(3)}{N_c}$

[slides from Jorge Baeza-Ballesteros' talk @ Lattice 2001]