The strong CP problem solved due to long-distance vacuum effects

Gerrit Schierholz

Deutsches Elektronen-Synchrotron DESY

With

Yoshifumi Nakamura

RIKEN Center for Computational Science, Kobe

RIKEN
Center for
Computational Science

Outline

Objective

Gradient flow

Confinement

Phase structure

Conclusions

Objective

- QCD allows for a CP-violating term $S_\theta = i\theta Q$ in the action, with $-\pi < \theta \leq \pi$. A nonvanishing value of θ would result in an electric dipole moment d_n of the neutron. The current experimental upper limit is $|d_n| < 1.8 \times 10^{-13} e\,\text{fm}$, which suggests that θ is anomalously small. This feature is referred to as the strong CP problem

- It is widely assumed that QCD is in a single (confinement) phase for $|\theta| < \pi$. The Peccei-Quinn axion solution, e.g., is realized by the shift symmetry $\theta \to \theta + \delta$

- However, it is known from the case of the massive Schwinger model that a θ term may change the phase of the system. Callan, Dashen and Gross have claimed that a similar phenomenon will occur in QCD. The statement is that the color fields produced by quarks and gluons will be screened by instantons for $|\theta| > 0$. 't Hooft has shown that due to the joint presence of gluons and monopoles a rich phase structure may emerge as a function of θ

- In this talk I will investigate the long-distance properties of the theory in the presence of the θ term, S_θ, and show that CP is naturally conserved in the confining phase

Gradient flow

To reveal the nonperturbative properties of the theory, we are faced with a multi-scale problem, involving the passage from the short-distance perturbative regime to the long-distance confining regime. The gradient flow provides a powerful framework for scale setting, and as such is a particular realization of the coarse-graining step of momentum space RG transformations Lüscher, Suzuki et al.

The gradient flow describes the evolution of fields as a function of flow time t. The flow of SU(3) gauge fields is defined by

$$\partial_t B_\mu(t,x) = D_\nu G_{\mu\nu}(t,x)\,, \quad G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu + [B_\mu, B_\nu]$$

where $B_\mu(t=0,x) = A_\mu(x)$ is the original gauge field of QCD. The renormalization scale μ is set by the flow time, $\mu = 1/\sqrt{8t}$ for $t \gg 0$.

The expectation value of the energy density $E(t,x) = \frac{1}{4}\, G^a_{\mu\nu}(t,x)\, G^a_{\mu\nu}(t,x)$ defines a renormalized coupling

$$g^2_{GF}(\mu) = \frac{16\pi^2}{3}\, t^2 \langle E(t)\rangle \,\Big|_{t=1/8\mu^2}$$

at flow time t in the gradient flow scheme Lüscher

For a start we may restrict our investigations to the Yang-Mills theory. If the strong CP problem is resolved in the Yang-Mills theory, then it is expected to be resolved in QCD as well. We use the plaquette action to generate representative ensembles of fundamental gauge fields on three different volumes

$$S = \beta \sum_{x,\,\mu<\nu} \Big(1 - \frac{1}{3}\,\mathrm{Re\,Tr}\; U_{\mu\nu}(x)\Big)$$

	16^4	24^4	32^4
#	4000	5000	5000

$\beta = 6.0 \quad a = 0.082\,\mathrm{fm}$

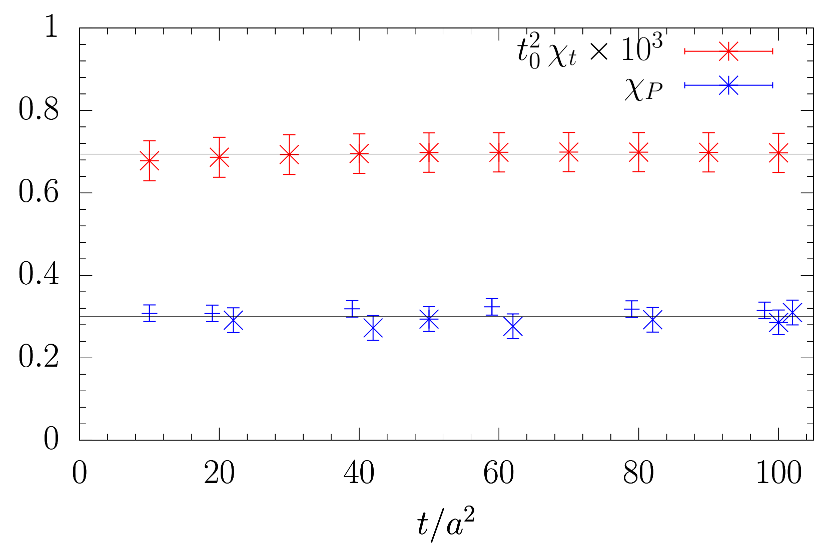
Physical quantities are independent of the RG scale. Two examples:

- Topological susceptibility

$$\chi_t = \frac{\langle Q^2\rangle - \langle Q\rangle^2}{V}$$

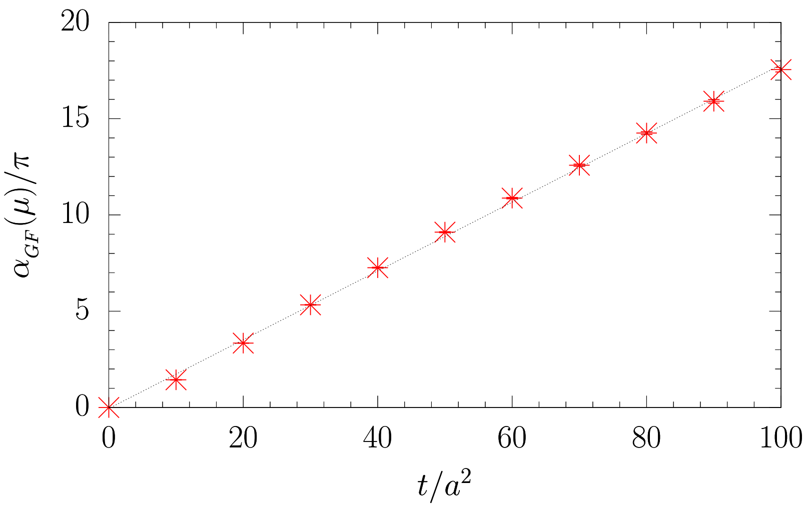
- Renormalized Polyakov susceptibility

$$\chi_P = \frac{\langle |P|^2\rangle - \langle |P|\rangle^2}{\langle |P|\rangle^2} \qquad P = \frac{1}{V_3}\sum_{\vec{x}} P(\vec{x})$$



Confinement

The gradient flow running coupling



$$\frac{\partial\, \alpha_{GF}(\mu)}{\partial\, \ln\, \mu} \equiv \beta_{GF}(\alpha_{GF})$$

$$\underset{\mu \ll 1\,\mathrm{GeV}}{=} -\, 2\, \alpha_{GF}(\mu)$$

$$\frac{\Lambda_{GF}}{\mu} = (4\pi b_0 \alpha_{GF})^{-\frac{b_1}{2\, b_0^2}} \exp\left\{ -\frac{1}{8\pi b_0 \alpha_{GF}} - \int_0^{\alpha_{GF}} d\alpha\, \frac{1}{\beta_{GF}(\alpha)} + \frac{1}{8\pi b_0 \alpha^2} + \frac{b_1}{2 b_0^2 \alpha} \right\}$$

$$\alpha_{GF}(\mu) \underset{\mu \ll 1\,\mathrm{GeV}}{=} \frac{\Lambda_{GF}^2}{\mu^2}$$

To make contact with phenomenology, it is desirable to transform the gradient flow coupling α_{GF} to a common scheme. A preferred scheme in the Yang-Mills theory is the V scheme

$$\frac{\Lambda_{GF}}{\Lambda_V} = \exp\left\{-\int_0^{\alpha_{GF}} d\alpha \frac{1}{\beta_{GF}(\alpha)} + \int_0^{\alpha_V} d\alpha \frac{1}{\beta_V(\alpha)}\right\}$$

$$\beta_V(\alpha_V) \underset{\mu \ll 1\,\mathrm{GeV}}{=} -2\,\alpha_V(\mu)$$

$$\alpha_V(\mu) \underset{\mu \ll 1\,\mathrm{GeV}}{=} \frac{\Lambda_V^2}{\mu^2}$$

$$\frac{\Lambda_V}{\Lambda_{\overline{MS}}} = 1.60\,, \quad \frac{\Lambda_{\overline{MS}}}{\Lambda_{GF}} = 0.534$$

The linear growth of $\alpha_V(\mu)$ with $1/\mu^2$ is commonly dubbed infrared slavery. The static quark-antiquark potential can be described by the exchange of a single dressed gluon

$$V(r) = -\frac{1}{(2\pi)^3}\int d^3\mathbf{q}\; e^{i\,\mathbf{qr}}\,\frac{4}{3}\,\frac{\alpha_V(q)}{\mathbf{q}^2 + i0} \underset{r \gg 1/\Lambda_V}{=} \sigma\, r$$

where $\sigma = \frac{2}{3}\Lambda_V^2$, giving the string tension $\sqrt{\sigma} = 445(19)$ MeV

$$\sqrt{t_0}\,\Lambda_{\overline{MS}} = 0.217(7)$$

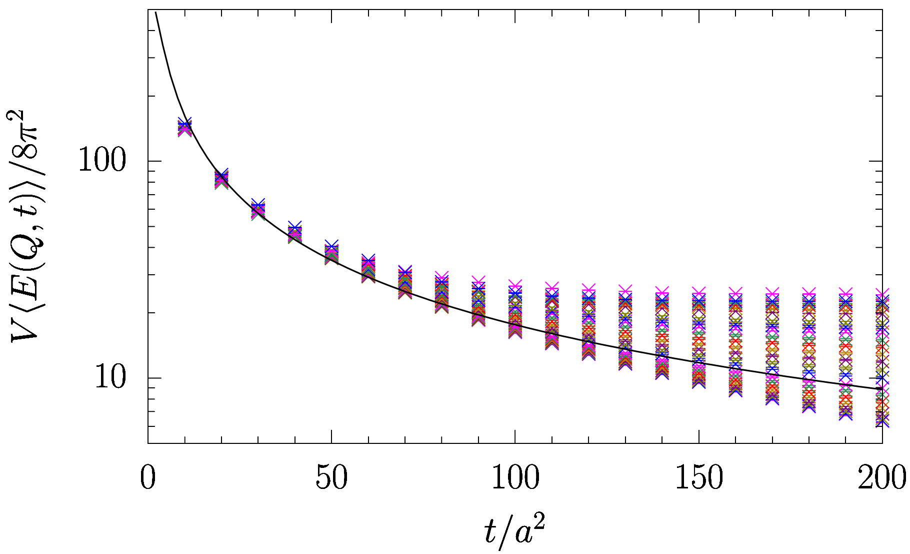
Literature:

$$\sqrt{t_0}\,\Lambda_{\overline{MS}} = 0.220(3)$$

arXiv:1905.05147

Phase structure

With increasing flow time the initial gauge field ensemble splits into effectively disconnected topological sectors of charge Q, at ever smaller flow time as β is increased Lefschetz thimble



$V\langle E(Q,t)\rangle/8\pi^2 \equiv S_Q \simeq |Q|$, while the ensemble average vanishes like $1/t$

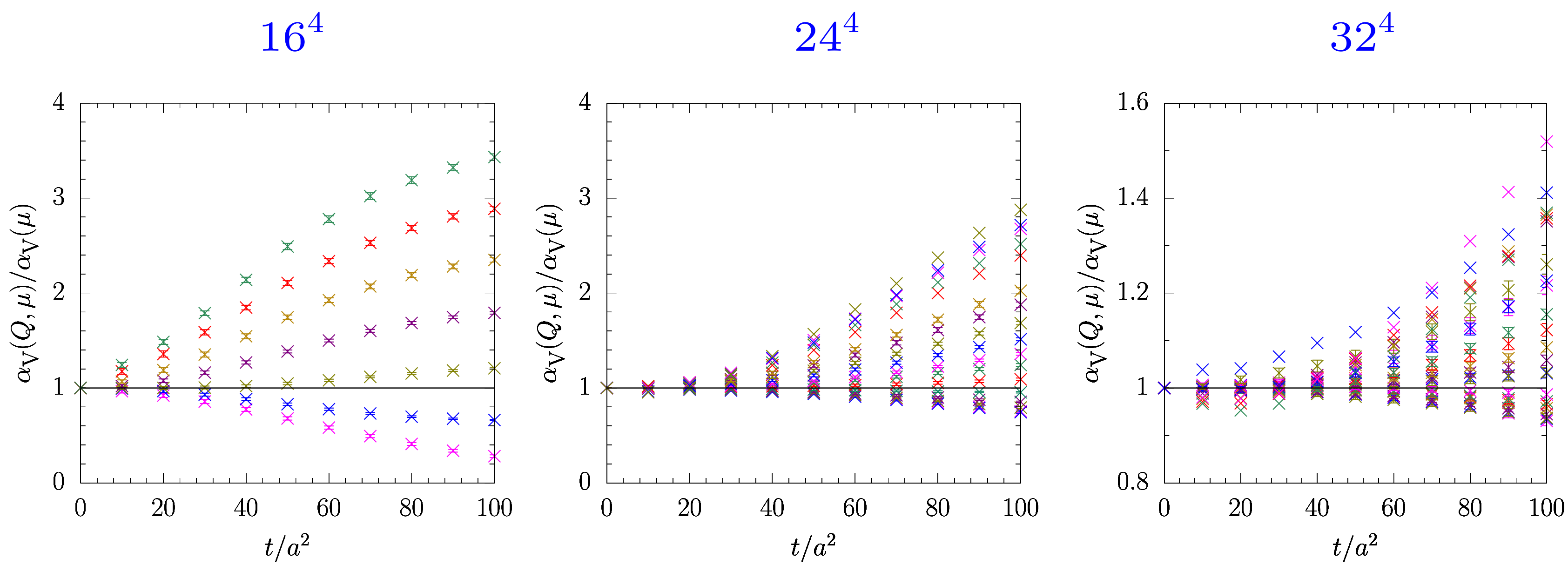
One is tempted to conclude that the vacuum is a dilute gas of instantons. However, this is not the case. We find a negative value for the 'kurtosis', $K = \langle Q^4\rangle_c/\langle Q^2\rangle_c$, on all lattice volumes, while $K = 1$ for a dilute instanton gas

Observables consistently show a clear dependence on Q. This is the reason for a nontrivial θ dependence when Fourier transformed to the θ vacuum

Running coupling α_V

If the general expectation is correct and the color fields are screened for $|\theta| > 0$, we should, in the first place, find that the running coupling constant is screened in the infrared

From $\langle E(Q,t)\rangle$ we obtain $\alpha_V(Q,\mu)$ in the individual topological sectors $|Q|$ from bottom to top



Interestingly, $\alpha_V(Q,\mu)$ vanishes in the infrared for $Q=0$, while the ensemble average $\alpha_V(\mu)$ is represented by $|Q| \simeq \sqrt{2\langle Q^2\rangle/\pi}$

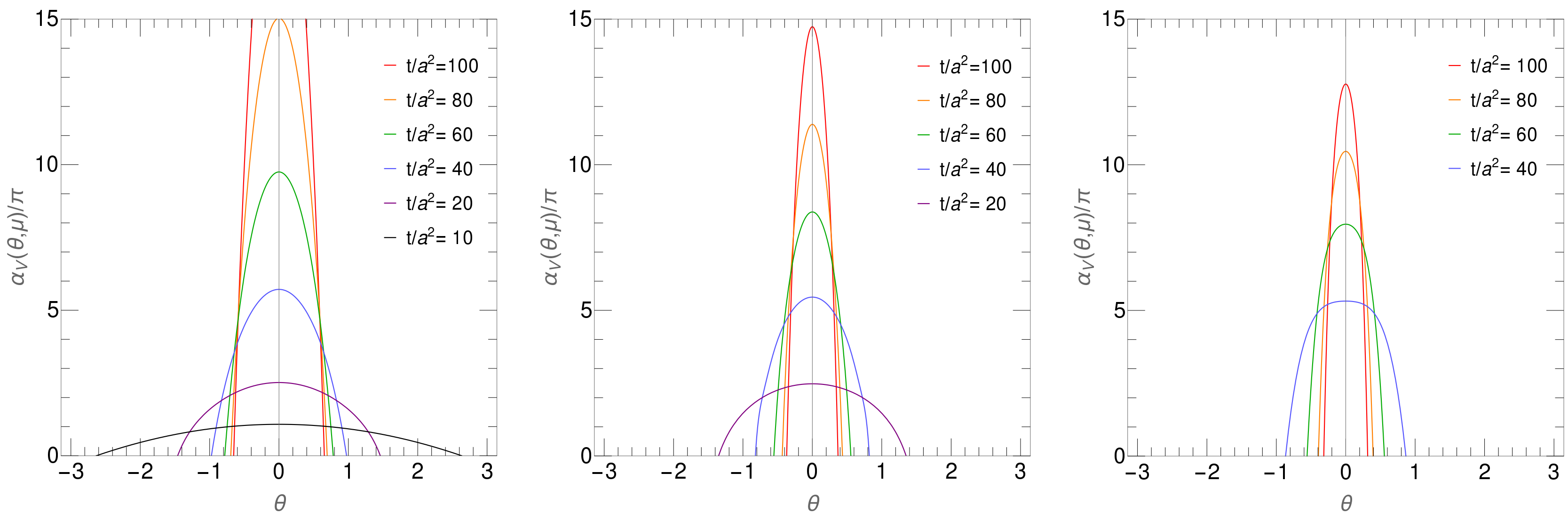
The transformation of $\alpha_V(Q,\mu)$ from Q to the θ vacuum is achieved by the discrete Fourier transform

$$\alpha_V(\theta,\mu) = \frac{1}{Z(\theta)} \sum_Q e^{i\theta Q} P(Q) \alpha_V(Q,\mu)$$

$$Z(\theta) = \sum_Q e^{i\theta Q} P(Q)$$

- $e^{i\theta Q} P_{\theta=0}(Q) = P_\theta(Q)$ 1502.02295
- Z_θ analytic at $\theta = 0$ Vafa-Witten
- Limits set by convergence of the Fourier sum

16^4 24^4 32^4



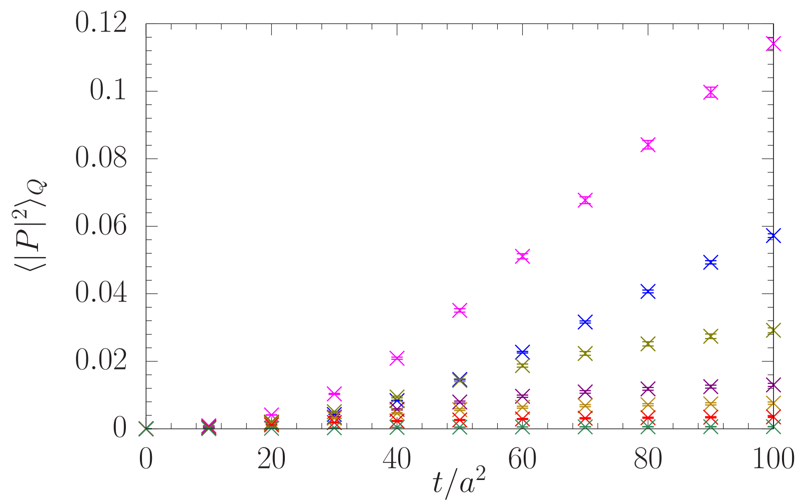
The color charge is totally screened for $|\theta| \gtrsim 0$ in the infrared, while it becomes gradually independent of θ as we approach the perturbative regime

Precision test by comparing different volumes

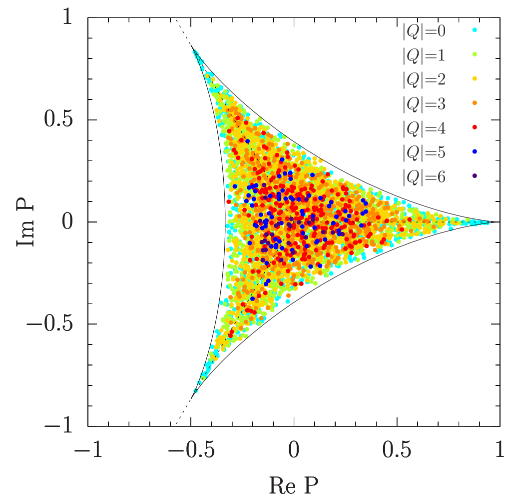
Polyakov loop

The Polyakov loop describes the propagation of a single static quark travelling around the periodic lattice

16^4



$t/a^2 = 60$



From $Q = 0$ (top) to 6 (bottom)

$\langle P \rangle = 0$ in each sector. That implies center symmetry throughout. P rapidly populates the entire theoretically allowed region for small values of $|Q|$, while it stays small for larger values of $|Q|$

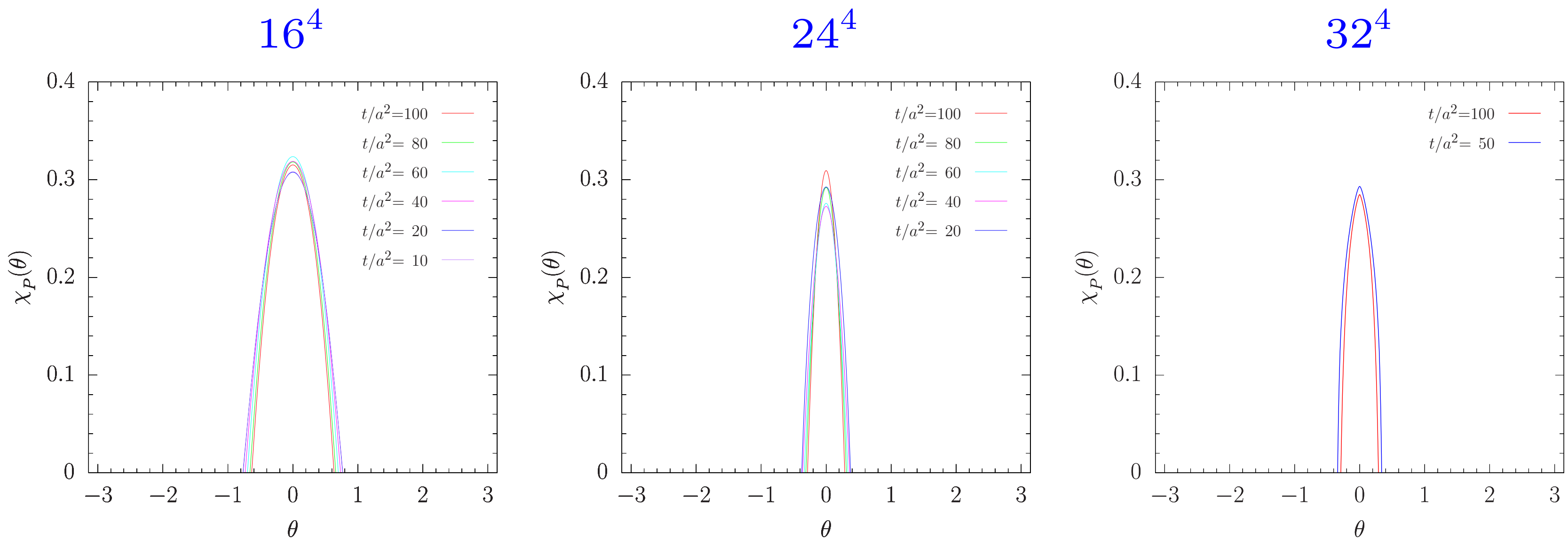
The transformation of the Polyakov loop expectation values to the θ vacuum is again achieved by the discrete Fourier transform

$$\langle |P|^2 \rangle_\theta = \frac{1}{Z(\theta)} \sum_Q e^{i\,\theta\,Q}\, P(Q) \langle |P|^2 \rangle_Q$$

$$\langle |P| \rangle_\theta = \frac{1}{Z(\theta)} \sum_Q e^{i\,\theta\,Q}\, P(Q) \langle |P| \rangle_Q$$

The the connected part of $\langle |P|^2 \rangle_\theta$ is described by the renormalized Polyakov loop susceptibility

$$\chi_P(\theta) = \frac{\langle |P|^2 \rangle_\theta - \langle |P| \rangle_\theta^2}{\langle |P| \rangle_\theta^2}$$



The Polyakov loop gets totally screened for $|\theta| \gtrsim 0$. The renormalized Polyakov loop susceptibility is independent of flow time t (even for $\theta \neq 0$!)

Mass gap

preliminary

$$\langle E^2\rangle = \frac{1}{T}\sum_t \langle E(0)E(t)\rangle$$

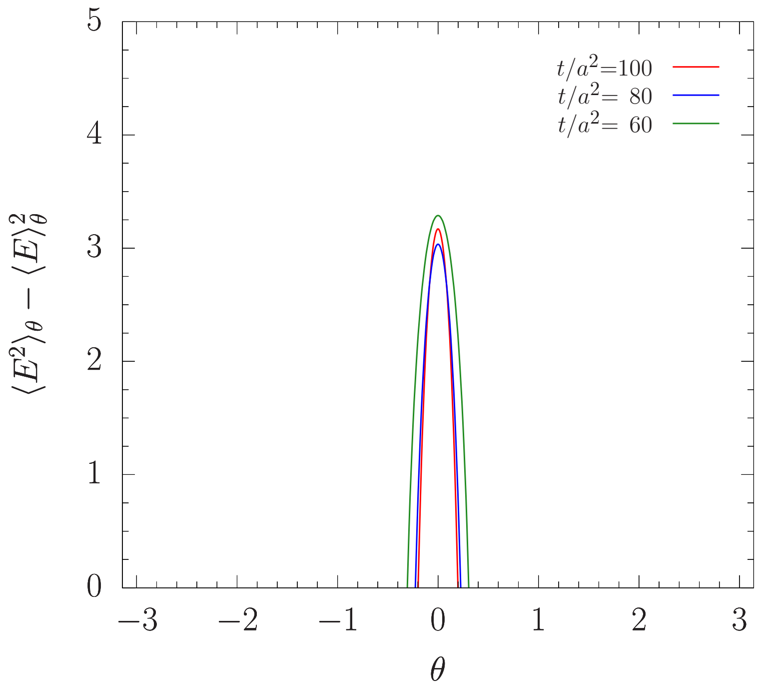
$$E(t) = \frac{1}{V_3}\sum_{\vec{x}} E(\vec{x},t)$$

$$\langle E^2\rangle - \langle E\rangle^2 = \sum_{n,t}\frac{1}{2m_n}\left|\langle 0|E|n\rangle\right|^2 e^{-m_n t}$$

$$\simeq \frac{1}{m^2_{0^{++}}}\left|\langle 0|E|0^{++}\rangle\right|^2 \propto \xi^2$$

Correlation length

24^4



$\langle E^2\rangle_\theta - \langle E\rangle_\theta^2$ Independent of flow time t

$\xi \simeq 0$ for $|\theta| \gtrsim 0$

No mass gap

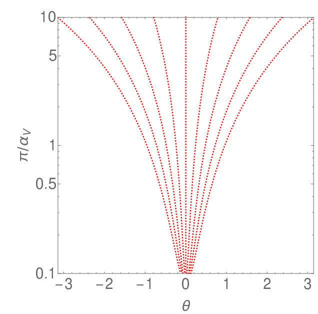
Conclusions

★ The numerical work is characterized by high statistics on three different volumes. Effectively it is a Monte Carlo sampling on Lefschetz thimbles. Comparing results on different volumes enabled us to control the accuracy of the calculation

★ The gradient flow proved a powerful tool for tracing the gauge field over successive length scales and showed its potential for extracting low-energy quantities. The novel result is that color charges are screened for $|\theta| > 0$ by nonperturbative effects, limiting the vacuum angle to $\theta = 0$ at macroscopic distances, which rules out any strong CP violation at the hadronic level

★ It is tempting to deriving RG flow equations for the inverse of the running coupling constant, $\pi/\alpha_V(\theta, \mu)$, by analogy with the quantum Hall conductivity

IR fixed point?



★ The nontrivial phase structure of QCD has far-reaching consequences for anomalous chiral transformations. Our results are incompatible with the axion extension of the SM, as the QCD vacuum will be unstable under the Peccei-Quinn chiral $U_{PQ}(1)$ transformation, realized by the shift symmetry $\theta \to \theta + \delta$, which thwarts the axion conjecture