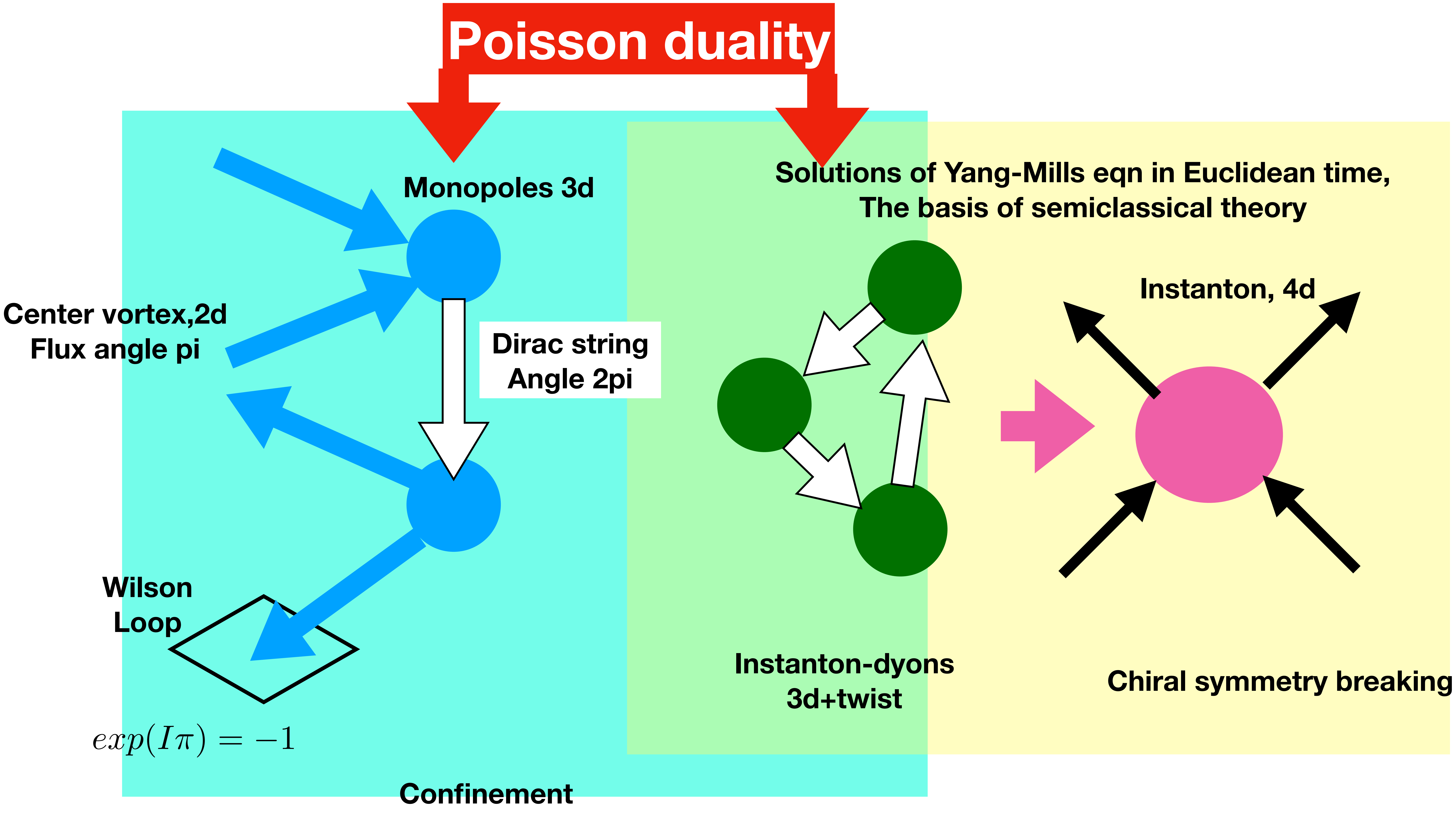
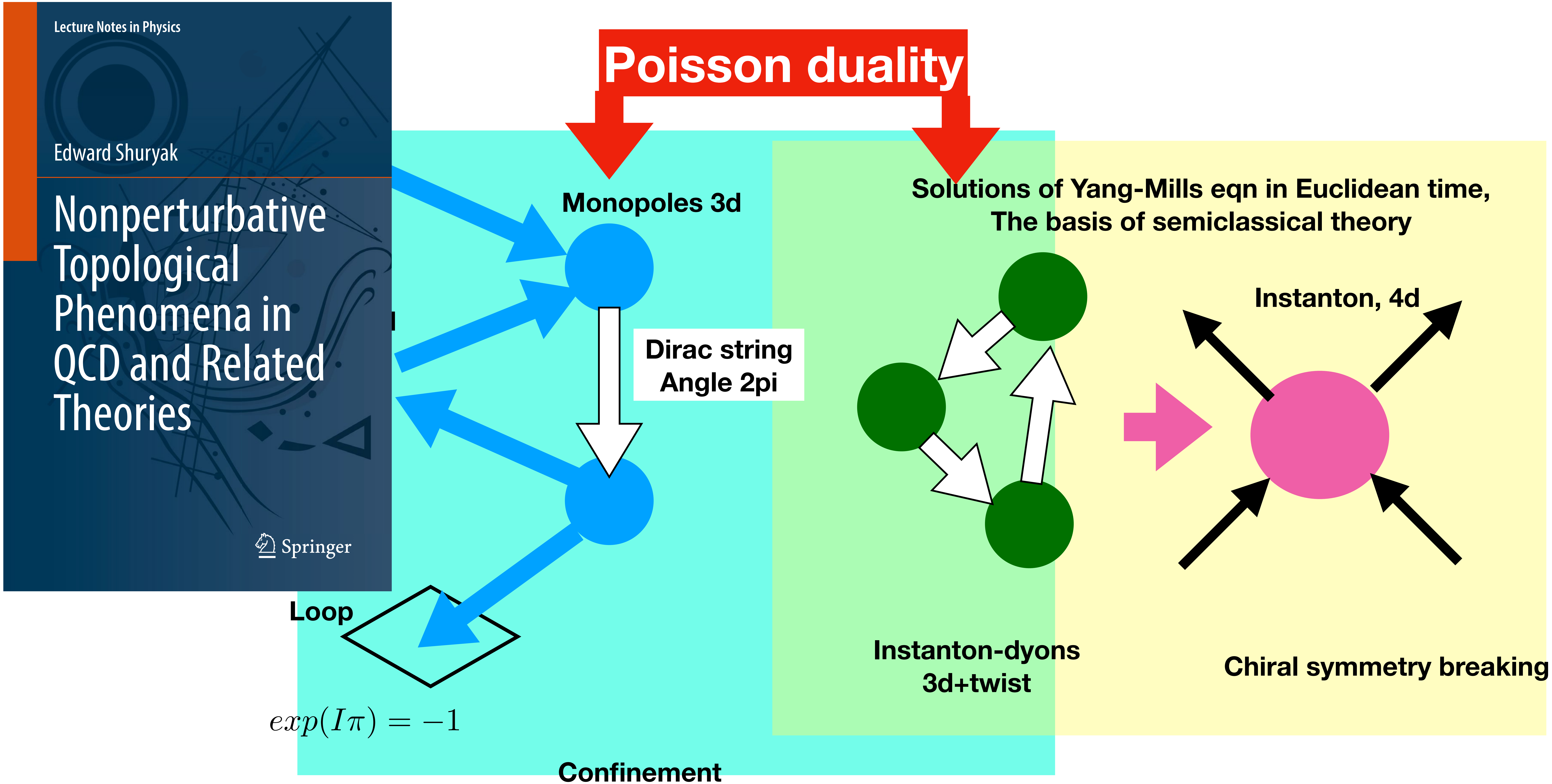
Instanton effects in hadronic structure

Edward Shuryak
Center for Nuclear Theory, Stony Brook

outline

- map of gauge topology
- Dirac zero modes and t'Hooft effective Lagrangian, Chiral symmetry breaking
- QCD correlation functions
- Light Front Wave Functions and antiquark flavor puzzle
- Mesonic formfactors at semi-hard Q^2
- Spin forces, from quarkonia to light mesons


Poisson duality
Monopoles 3d
Solutions of Yang-Mills eqn in Euclidean time,
The basis of semiclassical theory
Center vortex,2d
Flux angle pi
Dirac string
Angle 2pi
Instanton, 4d
Wilson
Loop
$exp(I\pi) = -1$
Instanton-dyons
3d+twist
Chiral symmetry breaking
Confinement


Lecture Notes in Physics
Edward Shuryak
Nonperturbative Topological Phenomena in QCD and Related Theories
Springer
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hadronic structure

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(too many to mention here):
(i) Mq (chiral symmetry breaking)
(ii) confining+Coulomb potentials
(iii) residual interactions (NJL,instantons)
(obviously done in the rest frame)

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Correlators in Euclidean time:
“instanton liquid models”
pro: chiral sim.breaking derived
numerical simulations in
lattice gauge theories:
pro: from first principles of QCD
confinement, spectra…
con: hard to get
PDF or light cone w.f.

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Light-front quantization:
pro: light front WFs and PDFs
con: mostly pQCD-based
(except recently)
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like chiral symmetry breaking
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Holographic QCD
hadrons are modeled by some fields
“in the bulk”
Veneziano limit in which both the number
of flavors and colors are large
$N_f, N_c \to \infty, N_f/N_c$ = fixed
Good spectra => Regge trajectories

historic introduction

Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961). doi:10.1103/PhysRev.122.345

NJL model:

$$G[\vec{\pi}^2 + \sigma^2]$$

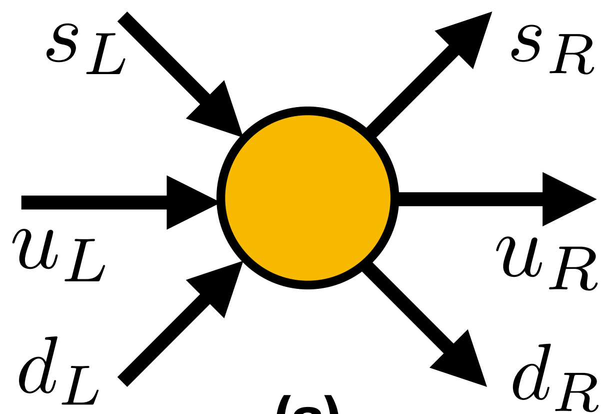
$$\vec{\pi} = (\bar{q}\vec{\tau}\gamma_5 q)$$

$$\sigma = (\bar{q}q)$$

$$G(|p| > \Lambda) = 0$$

NJL introduced the chiral symmetry and G large enough to break it spontaneously

Instantons: BPST and t' Hooft, 1975-76
new effective Lagrangian
it violates U(1) chiral symmetry
Turning left-handed to right handed



(a)

$$G[\vec{\pi}^2 + \sigma^2 - \vec{\delta}^2 - \eta'^2]$$

Instanton liquid model (ES 1982)
instead of G and Lambda of NJL
another two parameters
their values are such that
chiral symmetry gets broken

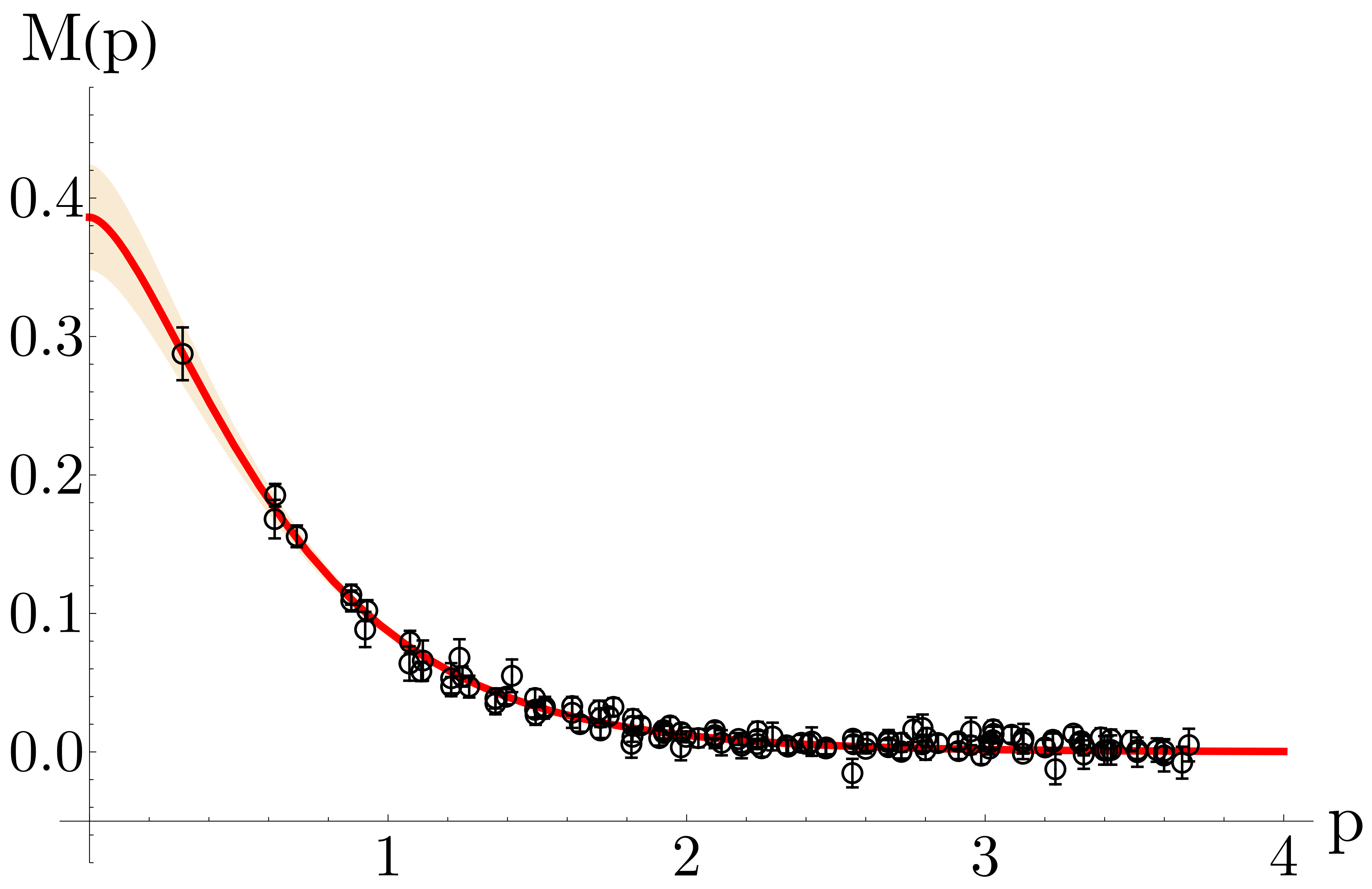
$$n_{inst} \approx 1\, fm^{-4}$$

$$\rho \approx 1/3\, fm$$

Interacting instanton liquid model 1990s
summed all orders of 't Hooft vertex
calculated correlation functions
good description of chiral symmetry breaking
no confinement

Fermion zero modes and t' Hooft effective Lagrangian

One quick example of
chiral symmetry breaking
In instanton liquid model
Quark effective mass
Versus virtuality
Points from lattice
Line from ILM

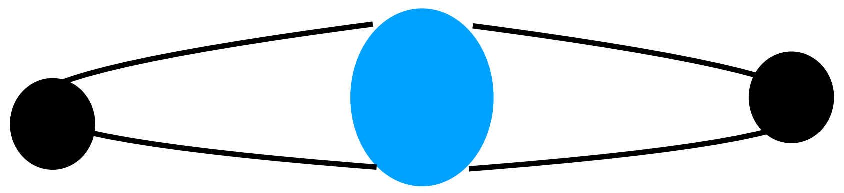


Correlation functions in the QCD vacuum

mesons: rho, pion and eta’

vectors: $\bar{q}\gamma_\mu q = \bar{q}_L\gamma_\mu q_L + \bar{q}_R\gamma_\mu q_R$

do not fit to topology-induced operator
which is (LR)(LR)+(RL)(RL) only
thus rho has no correction to H0



pion and eta’ do get corrections,
which are of opposite sign

$$\bar{u}\gamma_5 u = \bar{u}_L u_R - \bar{u}_R u_L$$

$$|\pi^0 > \sim (\bar{u}\gamma_5 u - \bar{d}\gamma_5 d)$$

$$|\eta > \sim (\bar{u}\gamma_5 u + \bar{d}\gamma_5 d)$$

let me use 2-flavor example here for simplicity

flavor structure
of the 4-quark operator
is flavor-nondiagonal:

so it appears in average
over the pion and eta’
with the opposite sign
making pion lighter and eta’ heavier

spectral densities => correlators

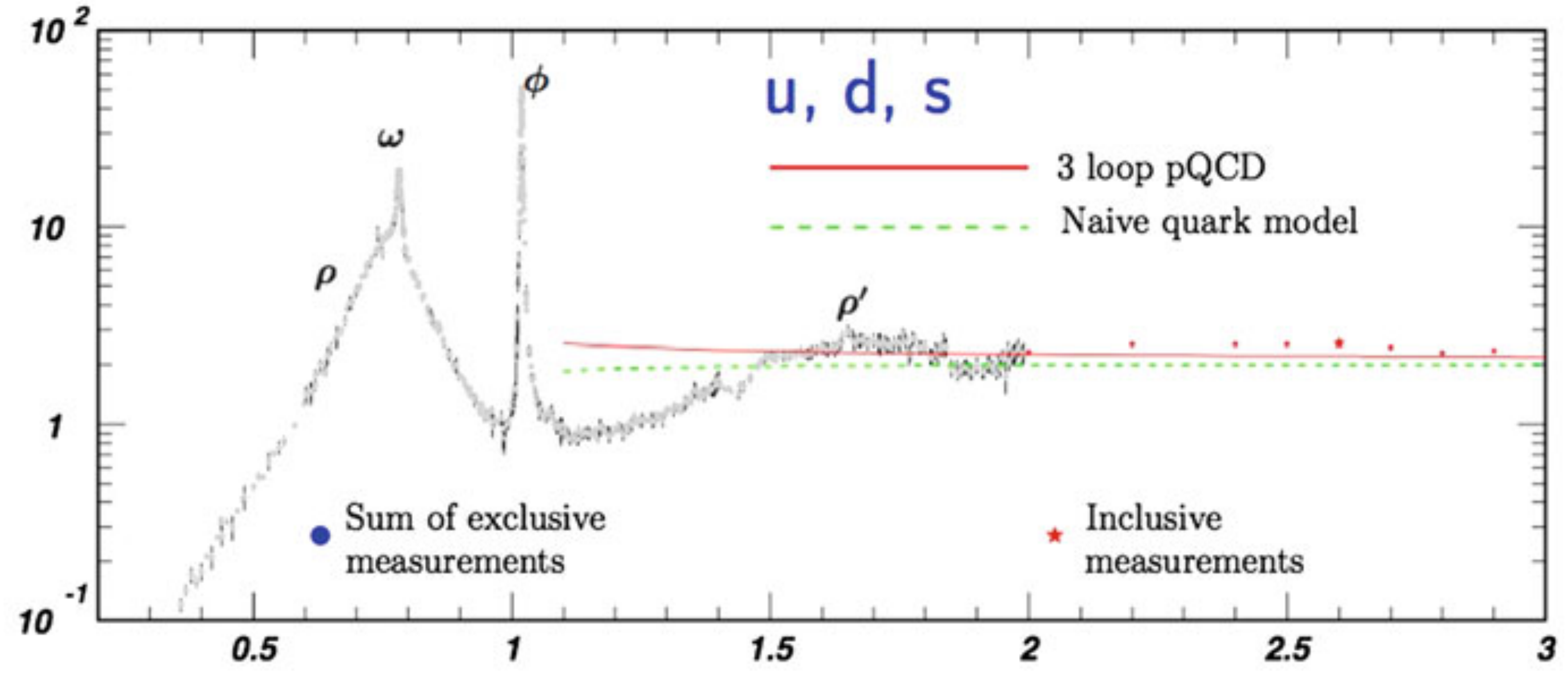
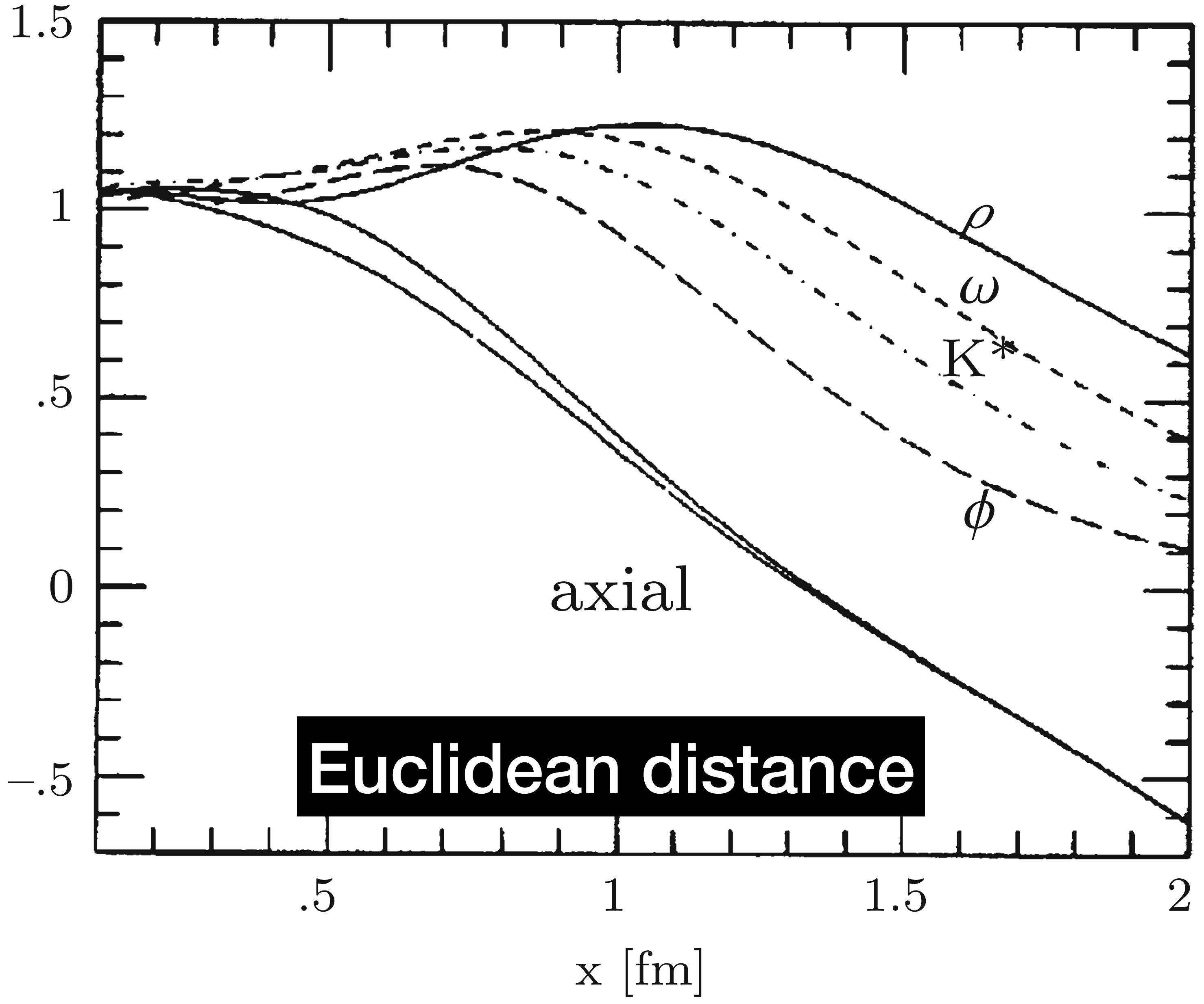


Fig. 9.4 The ratio of $R = \sigma(e^+e^- \to hadrons)/\sigma(e^+e^- \to \mu^+\mu^-)$ versus the total invariant mass of the hadronic system $\sqrt{s}$ in GeV

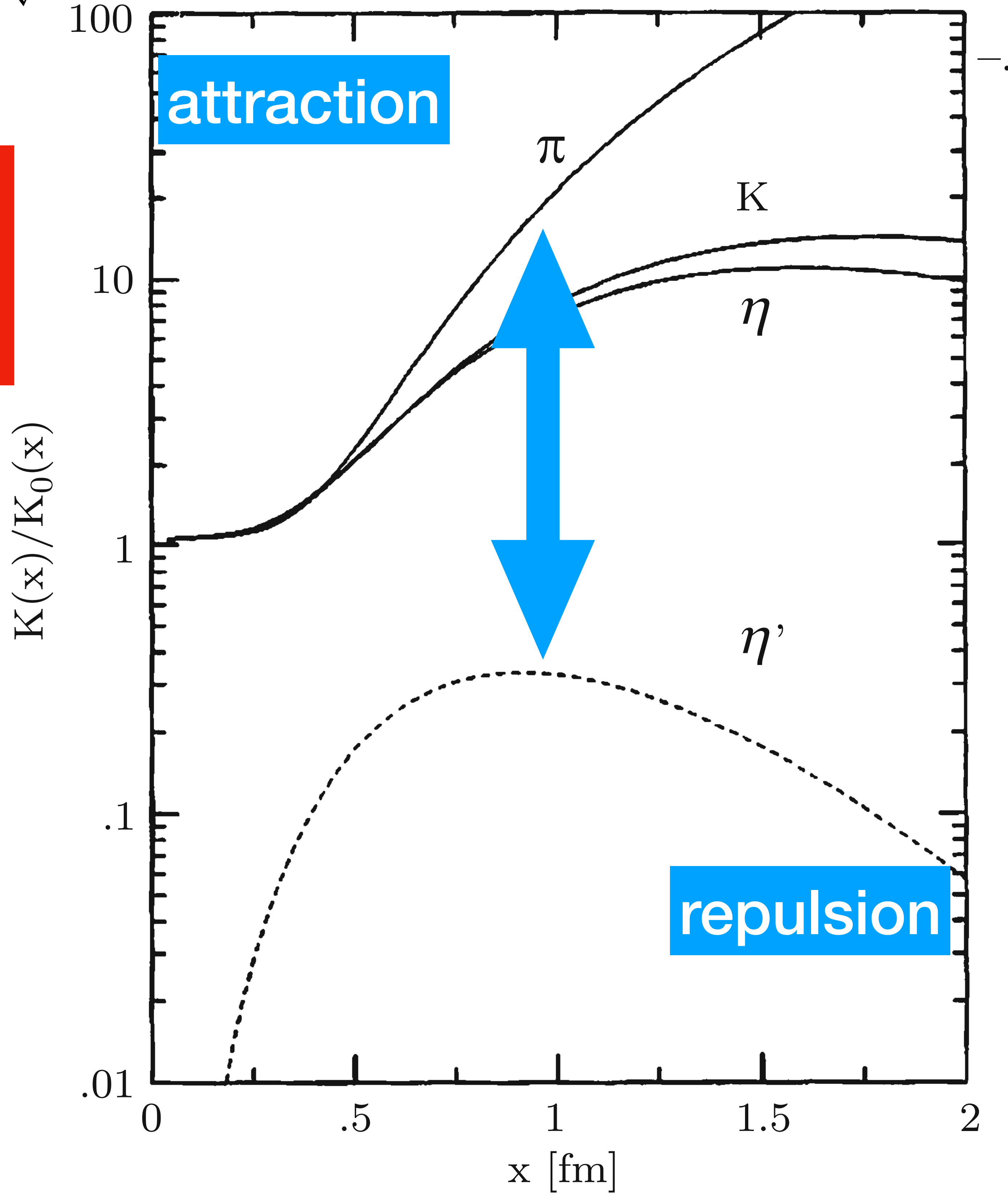
< J(x) J(0)>/(quark loop)



in light-light vector channel all pert. and nonpert corrections cancel till large distances

axial is split by chiral symmetry breaking effects

note that this is in log plot!



But pseudoscalar correlators show huge splittings, already at small distances!

Direct evidences for small-size instantons

Correlation functions in the QCD vacuum
Edward V. Shuryak *Rev.Mod.Phys.* 65 (1993) 1-46

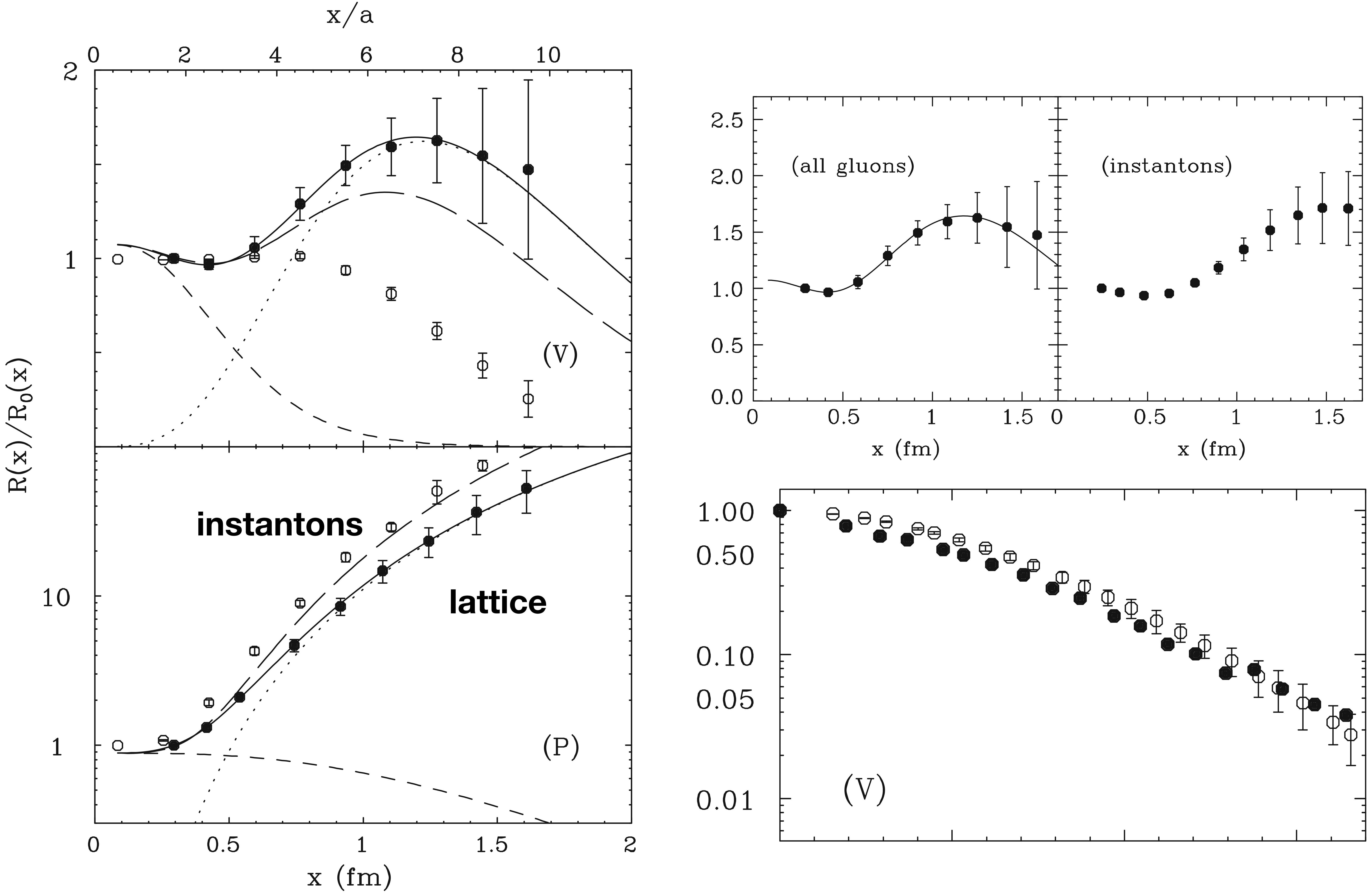


Fig. 9.10 The left panel shows the correlation functions for the vector (marked (V)) ρ channel and the pseudoscalar (marked (P)) π channel. The long-dashed lines are phenomenological ones, open and closed circles stand for RILM (Shuryak and Verbaarschot 1993b) and lattice calculation (Chu et al. 1994), respectively. The upper right panel compares vector correlators before and after "cooling". The lower part shows the same comparison for the ρ wave function; the closed and open points here correspond to "quantum" and "classical" vacua, respectively

instanton liquid model reproduces quantitatively not only PS but vector/axial correlators as well

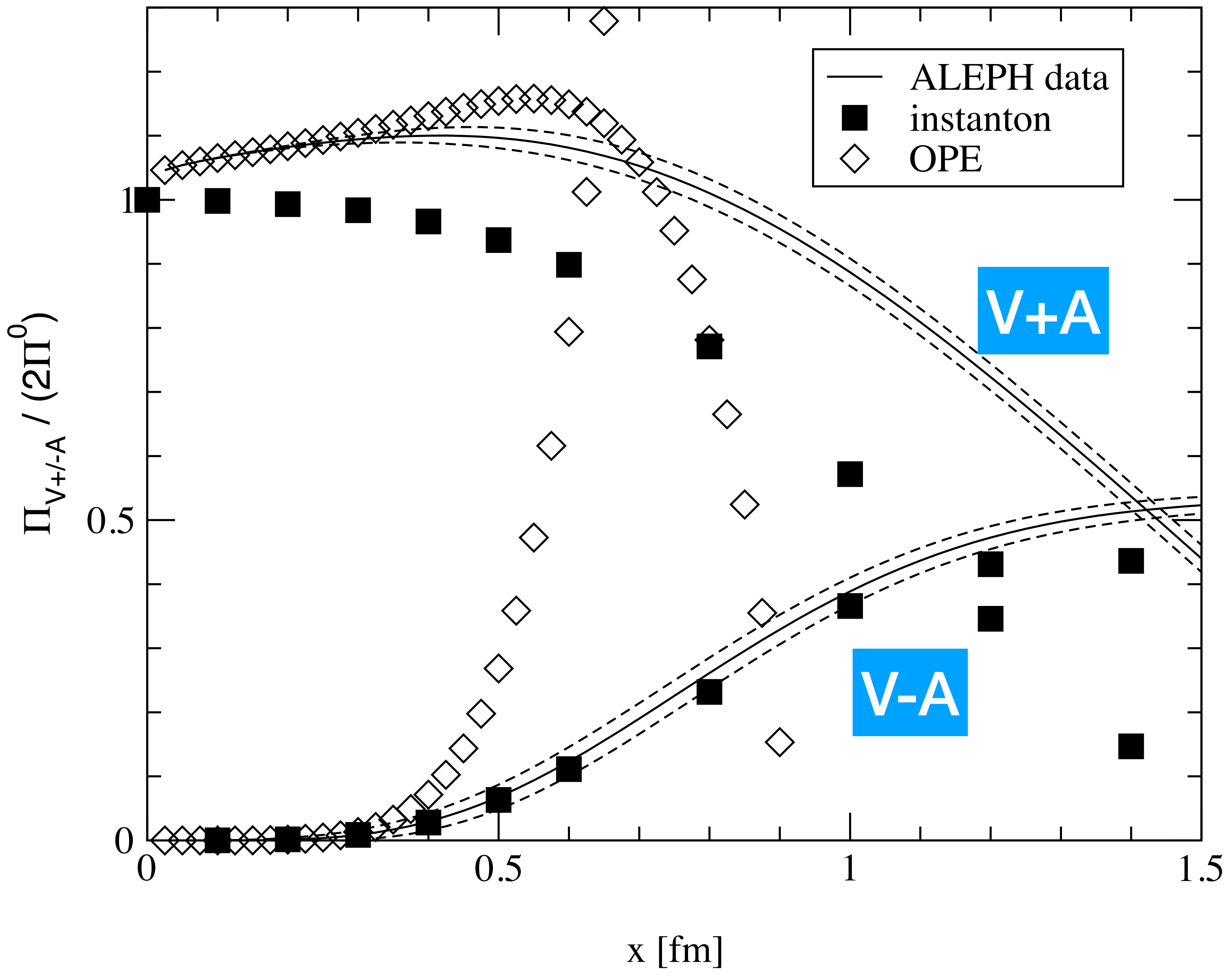


FIG. 2. Euclidean coordinate space correlation functions $\Pi_V(x) \pm \Pi_A(x)$ normalized to free field behavior. The solid lines show the correlation functions reconstructed from the ALEPH spectral functions and the dotted lines are the corresponding error band. The squares show the result of a random instanton liquid model and the diamonds the OPE fit described in the text.

Implications of the ALEPH tau lepton decay data for perturbative and nonperturbative QCD

Thomas Schäfer,ES

- *Phys.Rev.Lett.* 86 (2001) 3973-3976
- e-Print: hep-ph/0010116 [hep-ph]

$$\left(1 + \frac{\alpha_s}{\pi} + \ldots\right)$$

perturbative correction in V+A nicely complement instanton contribution

<G^2> OPE correction not seen…

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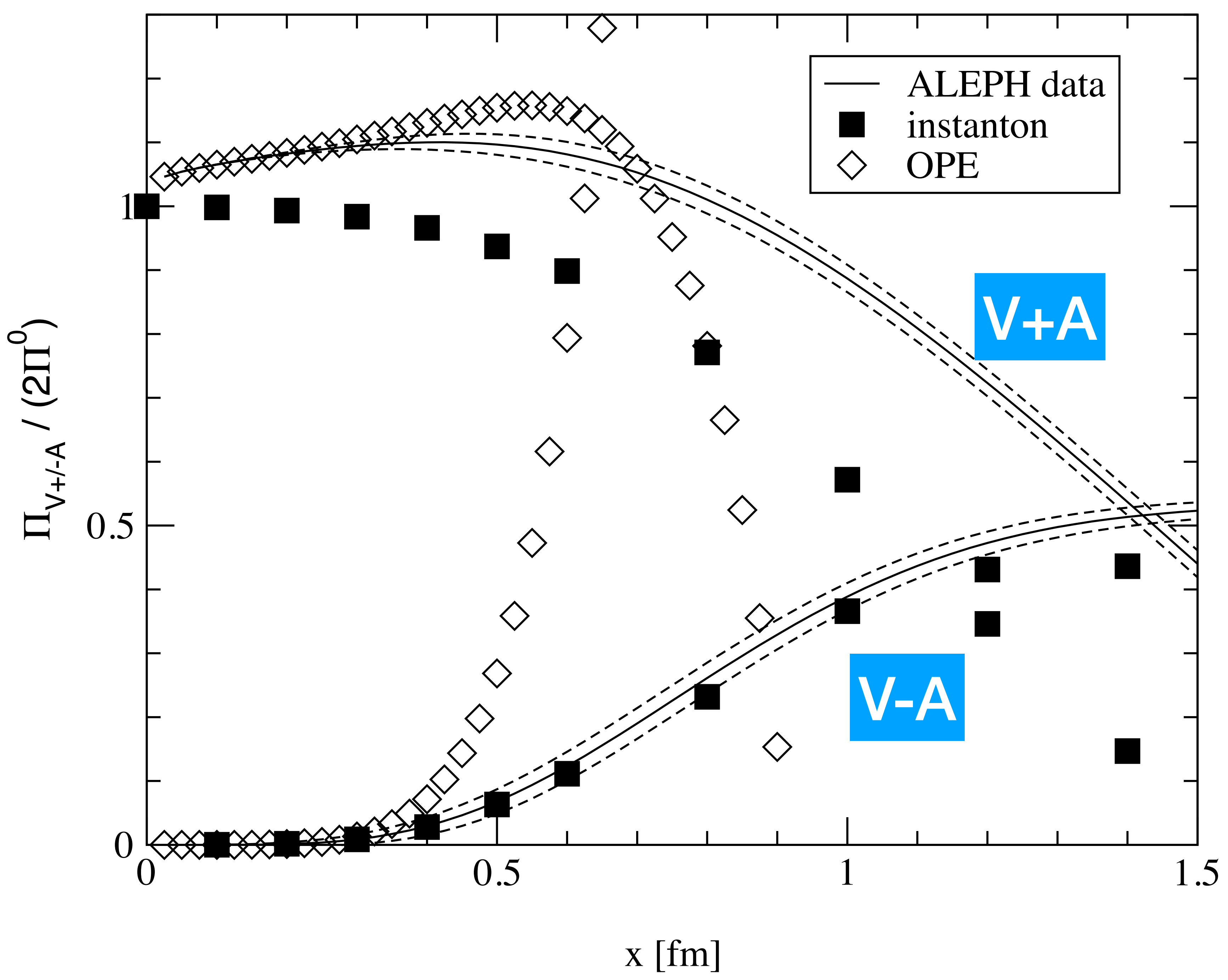

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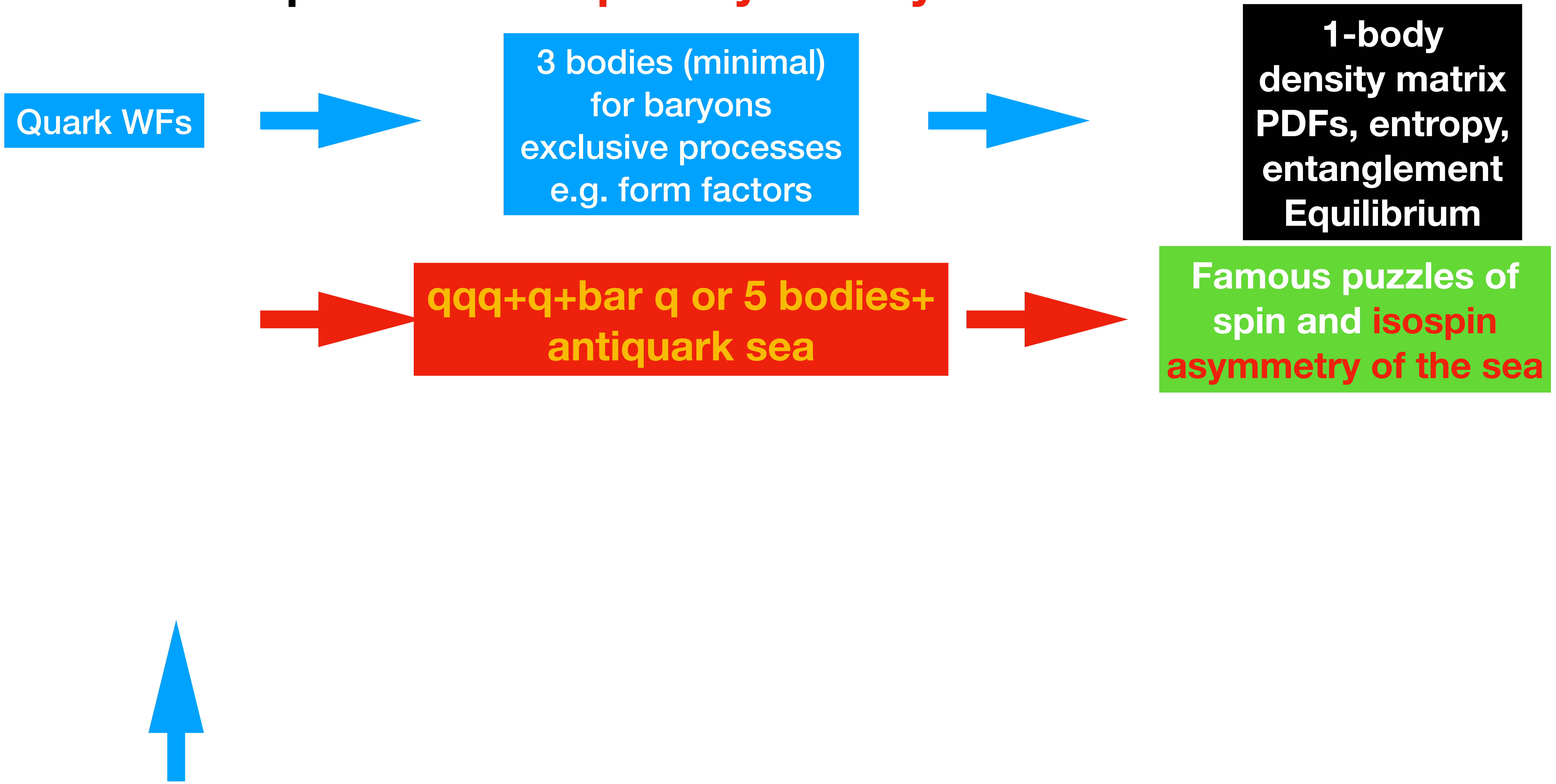
We conclude that the range of validity of the OPE in the vector channels is quite small, $x \lesssim 0.3$ fm. This means that there is essentially no "window" in which both the OPE is accurate and the correlation function is dominated by the ground state. Instantons, on the other hand, provide a quantitative tool at all distances. This is true even though the vector channels, because of the smallness of direct instanton effects, are generally considered to be the best system to study the OPE.

Light Front wave functions (LFWFs)

Meant to be defined at some "low normalization point"
\mu (1/\rho=0.6 GeV) at which gluon PDF =0
Only (constituent) quarks are there

Gluon radiation is to be done by evolution

motivation problem: isospin asymmetry of the ``sea”



D. F. Geesaman[1] and P. E. Reimer[2]
1812.10372

$$g \to q\bar{q}$$

is flavor
(and chirality) blind: why is
sea so asymmetric?

because glue is
not just gluons
there is gauge topology

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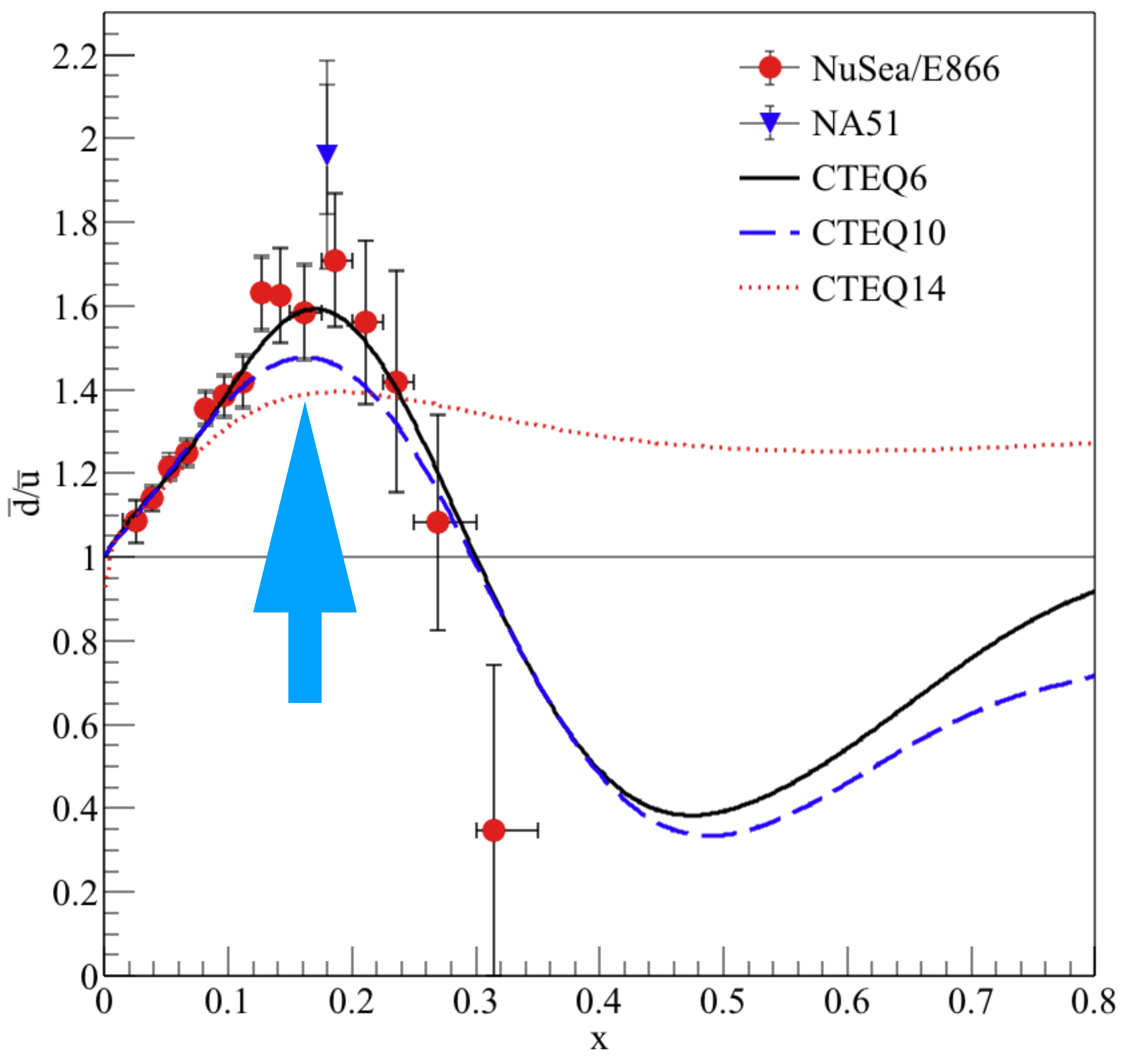
Quark WFs → 3 bodies (minimal) for baryons exclusive processes e.g. form factors → 1-body density matrix PDFs, entropy, entanglement Equilibrium

→ qqq+q+bar q or 5 bodies+ antiquark sea → Famous puzzles of spin and isospin asymmetry of the sea



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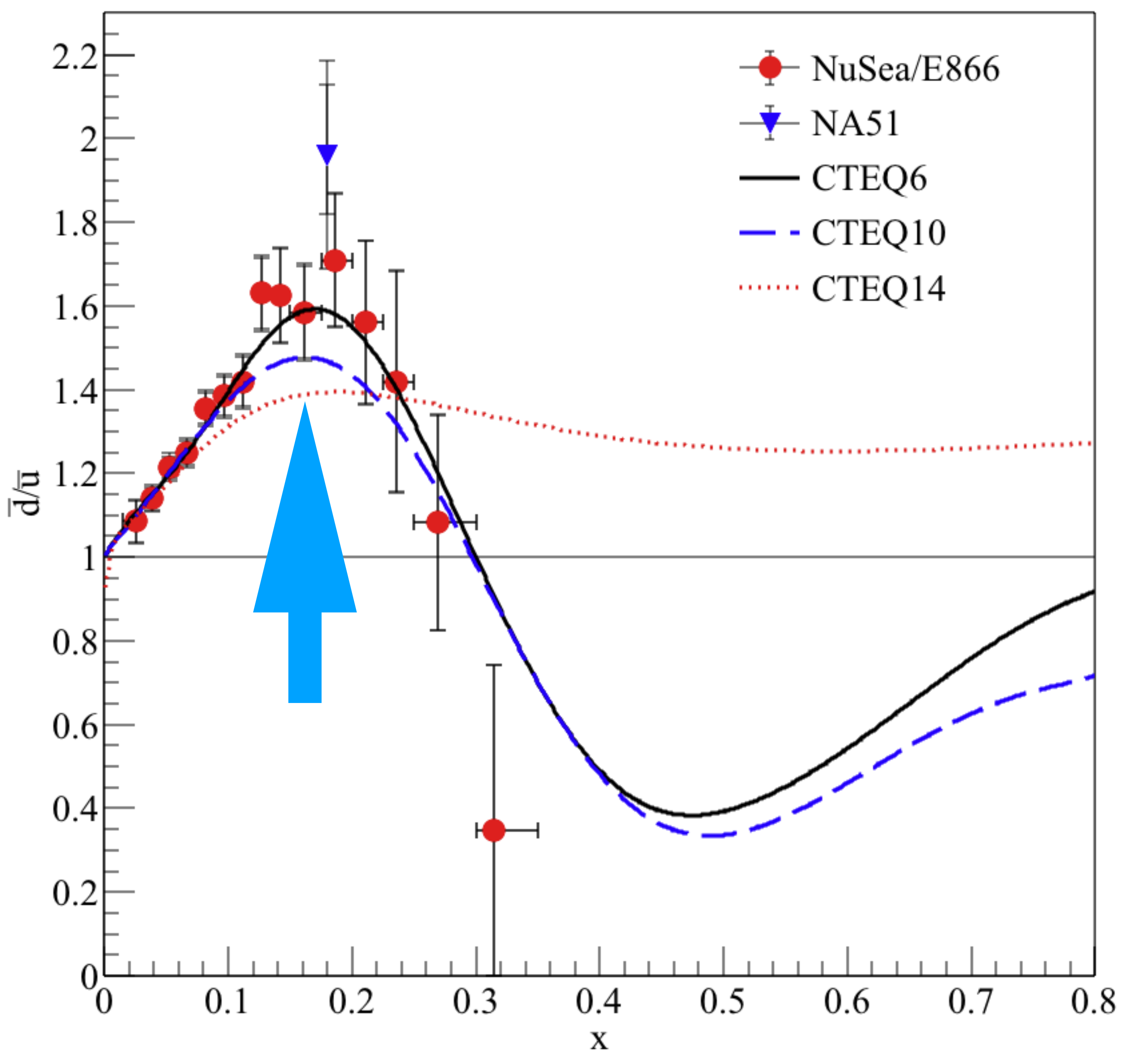

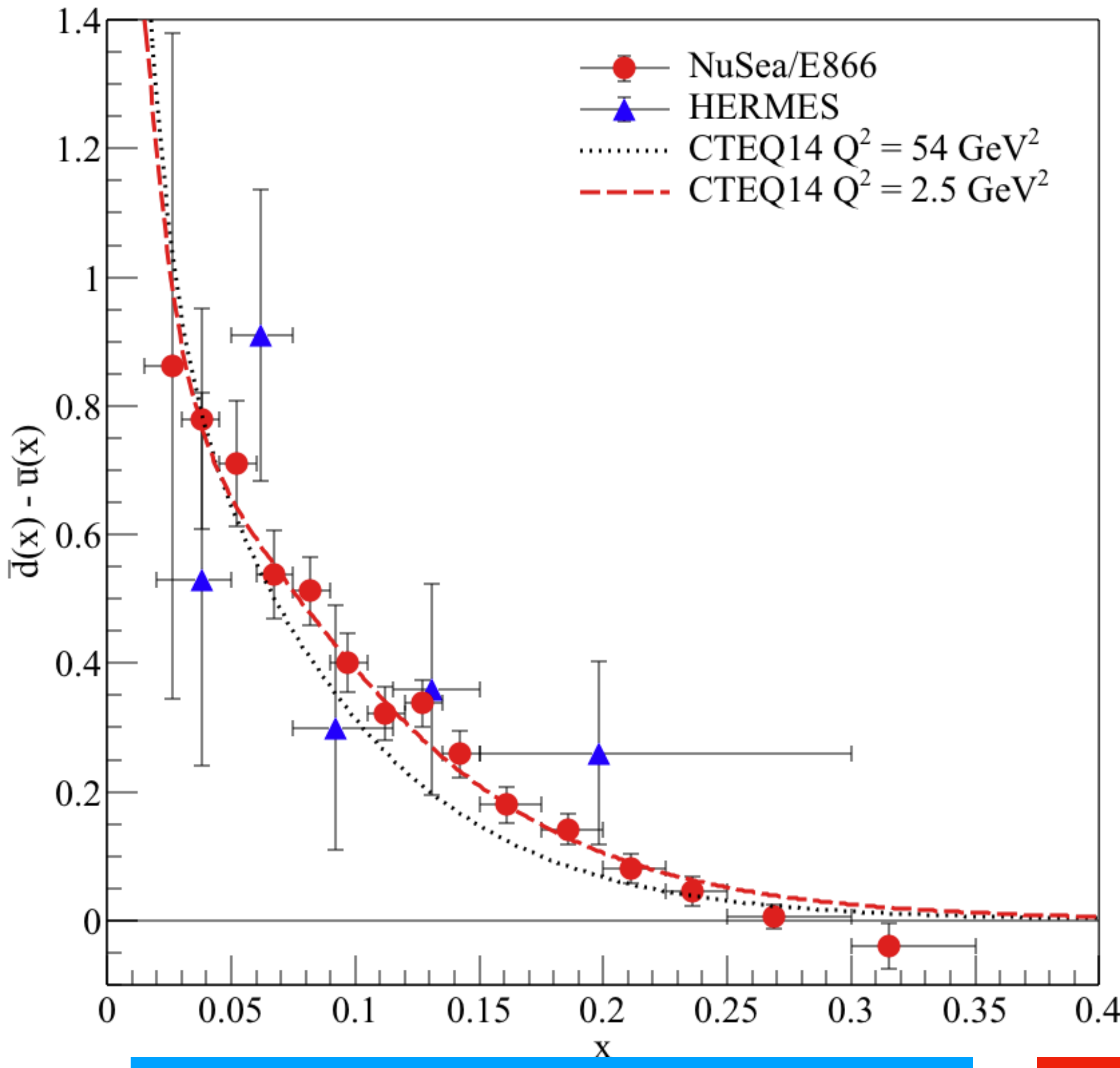

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Basis light front quantization for the charged light mesons with color singlet Nambu–Jona-Lasinio interactions

Shaoyang Jia* and James P. Vary†
Department of Physics and Astronomy, Iowa State University, Ames, Iowa 50011, USA
(Dated: March 14, 2019)

We apply the basis light front quantization (BLFQ) approach to describe the valence structures of the charged light meson ground states. Specifically, the light front wavefunctions of $\pi^\pm$, $\rho^\pm$, $K^\pm$, and $K^{*\pm}$ are obtained as the eigenvectors of the light front effective Hamiltonians with confinement potentials supplemented by the color singlet Nambu–Jona-Lasinio (NJL) interactions. We adjust our model such that the spectrum of these ground states and the charge radii of the pseudoscalar states agree with experimental results. We present the elastic form factors and parton distribution amplitudes (PDAs) as illustrations of the internal structures of the pseudoscalar pions and kaons in terms of valence quarks.

$$H = H_0 + H_{NJL}$$

4-fermion operators

const. quark mass

confinement

$$H_0 = \frac{\vec{\kappa}_\perp^2 + \mathbf{m}^2}{x} + \frac{\vec{\kappa}_\perp^2 + \overline{\mathbf{m}}^2}{1-x} + \kappa^4 x(1-x)\vec{r}_\perp^2$$

confinement

$$-\frac{\kappa^4}{(\mathbf{m}+\overline{\mathbf{m}})^2}\partial_x x(1-x)\partial_x,$$

Derivative in momenta
Are coordinates

They calculated, using certain functional basis,
the 2-body wave functions
for the pion and rho mesons
(also K,K*,phi)

the model has 3 parameters: m, kappa (I keep both the same)

My only parameter is the 4-quark coupling

Instanton size neglected rho=>0

Light-front wave functions of mesons, baryons, and pentaquarks with topology-induced local four-quark interaction

Edward Shuryak(*Phys.Rev.D* 100 (2019) 11, 114018 • e-Print: 1908.10270

what is new :

1.simplifications made, in particular different shapes of transverse momentum w.f. are ignored, only functions of x included

2.Only t' Hooft vertex (not all possible NJL ones)
Strange quarks ignored, only light included so far

3. 2-body mesons, rho and pions and eta'.
topology-induced 4-quark operator has coefficients 0,1,-1
a different functional basis (not diagonal for H0)

4. 3-body part of baryons: Delta(++) also has no 't Hooft operator, while the proton has it. It creates diquark-like ud correlations inside the proton

5. 5-body hadrons or pentaquark

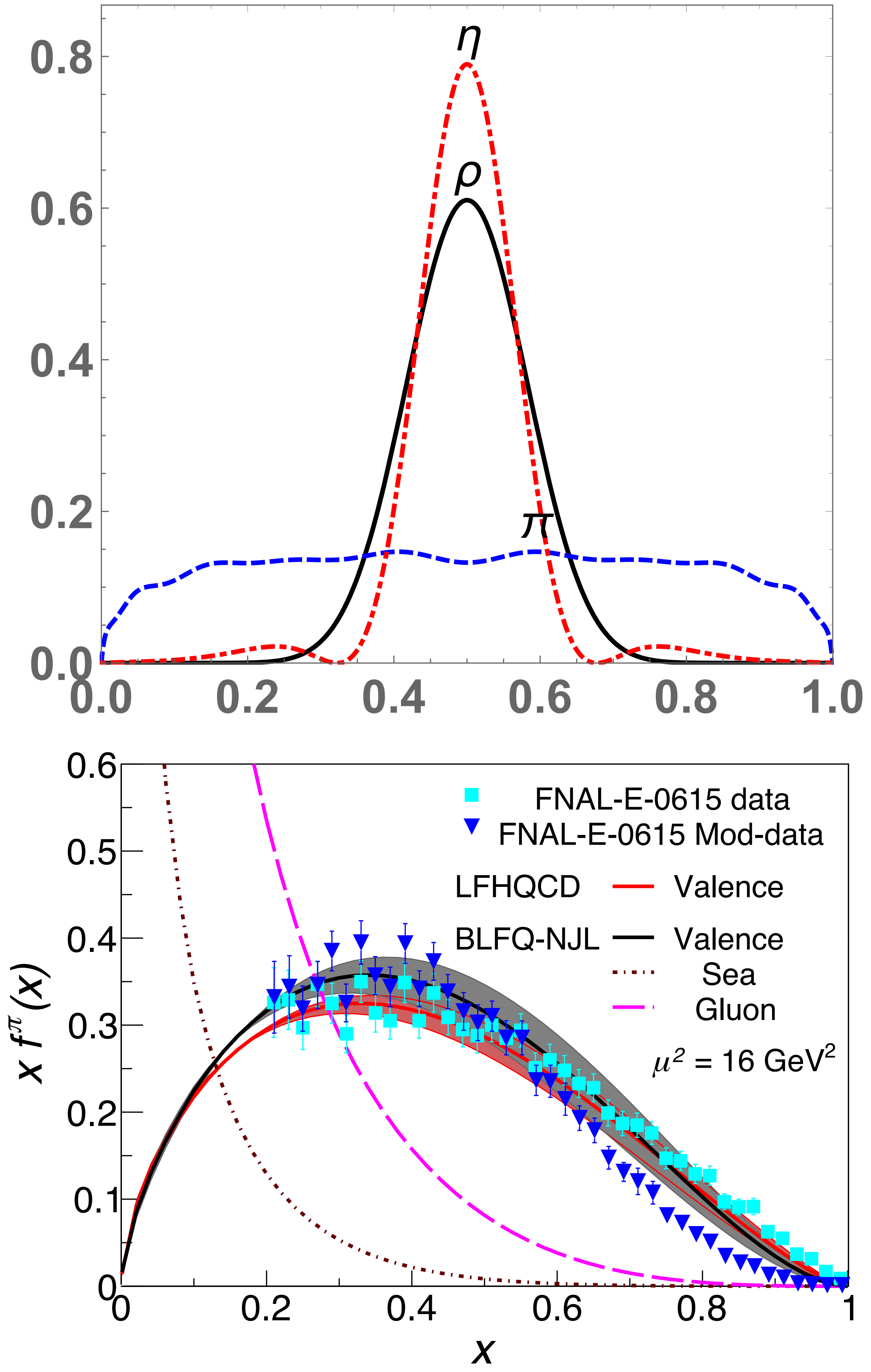
6. 5-body admixture to baryons added: antiquark appear and their distribution calculated. Problem of isospin sea asymmetry basically solved

H written as matrix in functional basis
(of even Jacobi polynomials)
and diagonalized

The only new parameter G
is fitted from vanishing pion mass
note the differences in distributions

the wave function squared
for 2-particles give PDF
without any integrations

**comparing to the data note,
that pions have 2,4,6 etc sectors,
unlike the calculation shown above**



LFWF

PDF for pion+

FIG. 3: Upper: momentum distribution for pion, rho and eta-prime mesons, calculated in the model. Lower (from [18]) comparison between the measured pion PDF (points) and the JV model (lines).

baryons: Delta(++) and the proton, as 3-quark states

Deltas

$$|\Delta^{++}\rangle \sim \psi_\Delta(x_i)|u^\uparrow(x_1)u^\uparrow(x_2)u^\uparrow(x_3)\rangle$$

no flavor-nondiagonal (uu)(dd) term at all!

$$H_{mass} = M_q^2\left(\frac{1}{x_1}+\frac{1}{x_2}+\frac{1}{x_3}\right)$$

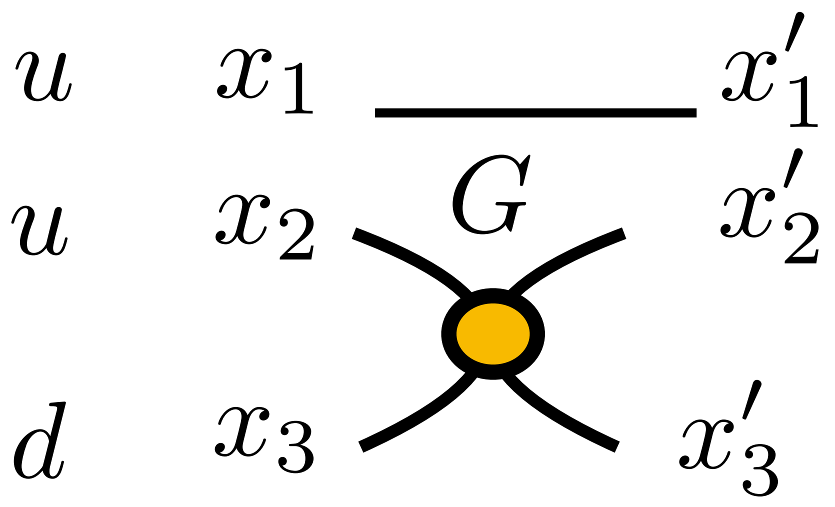
$$H_{conf} = -\frac{\kappa^4}{J(s,t)M_q^2}\left[\frac{\partial}{\partial s}J(s,t)\frac{\partial}{\partial s}+\frac{\partial}{\partial t}J(s,t)\frac{\partial}{\partial t}\right]$$

the mass of Delta follows with no new parameters, found to be in perfect agreement with data! the excited states not so good (confinement too schematic…)

Proton has a scalar diquark

$$|p\uparrow\rangle \sim \psi_p(x_i)\left(|u^\uparrow(x_1)u^\downarrow(x_2)d^\uparrow(x_3)\rangle\right.$$

$$\left.-|u^\uparrow(x_1)d^\downarrow(x_2)u^\uparrow(x_3)\rangle\right)$$



t' Hooft vertex does generate spin-0 (ud) diquarks
the coupling fitted to the correct proton mass,
the wave function and PDF are predictions

→ color superconductivity

Back to PDFs:

d quarks in Delta and p (from my w.f. which only has the 3-quark component)

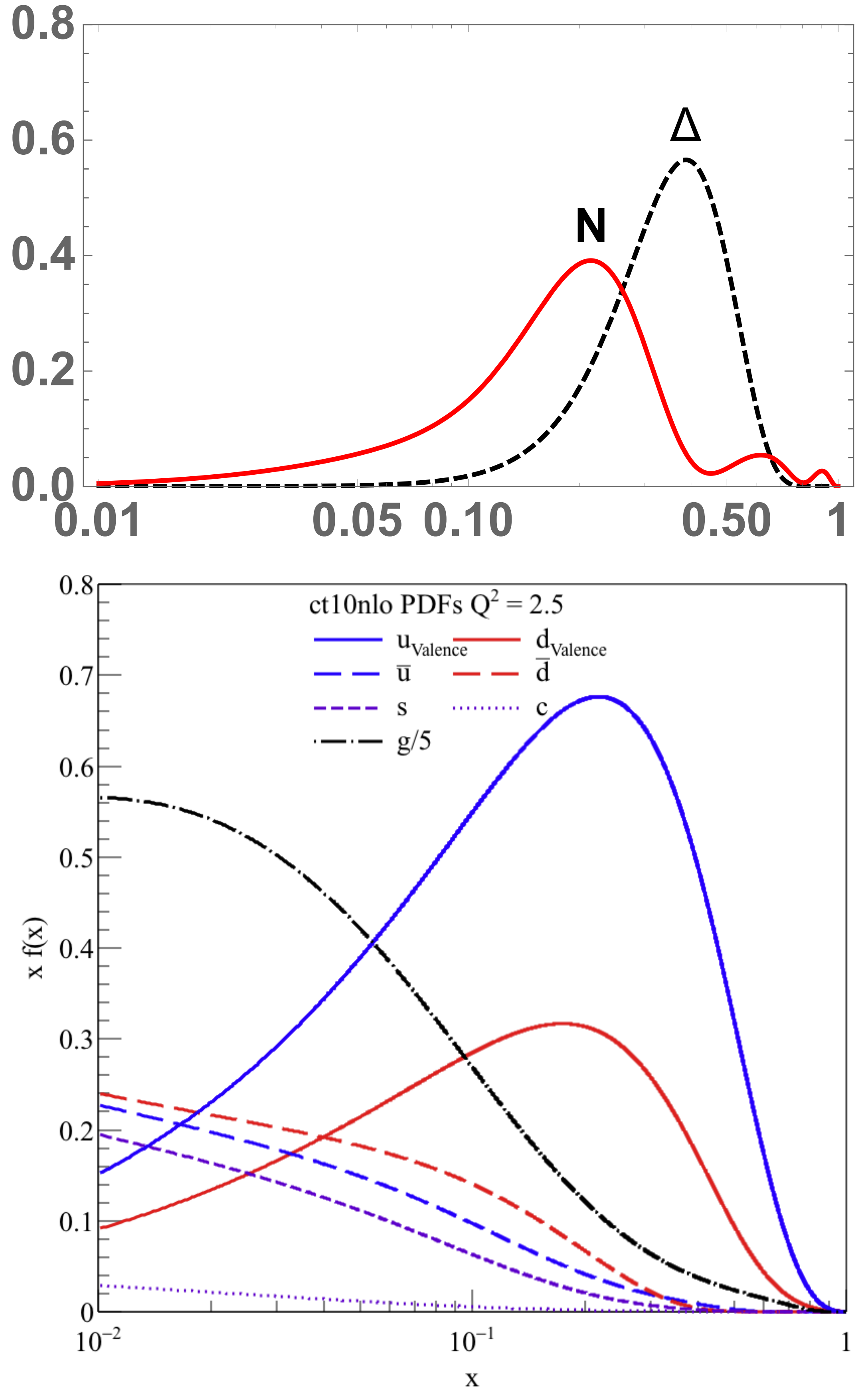


FIG. 5: Upper: our calculation of the d quark distribution in the Nucleon times x, $xd(x)$ (red, solid) and Delta (black, dashed) states. For comparison, the lower plot shows empirical structure functions (copied from [18]), where the valence $xd(x)$ is also shown in red.

Oscillations perhaps artefact due to cutoff On harmonics

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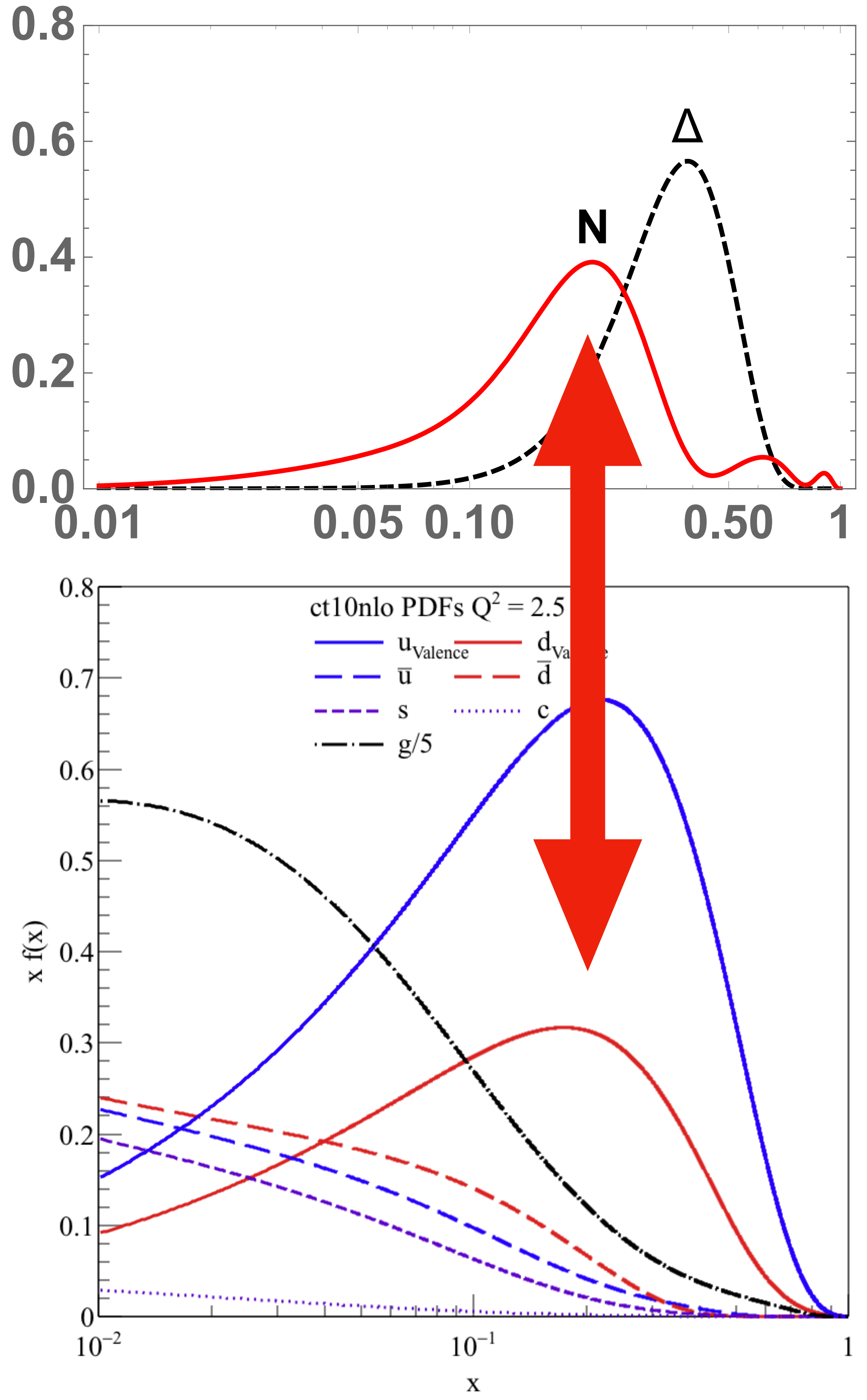

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5-quark systems: (no diquark interaction, only m and confinement)

$$M_{min.penta} = 2.13\, GeV$$

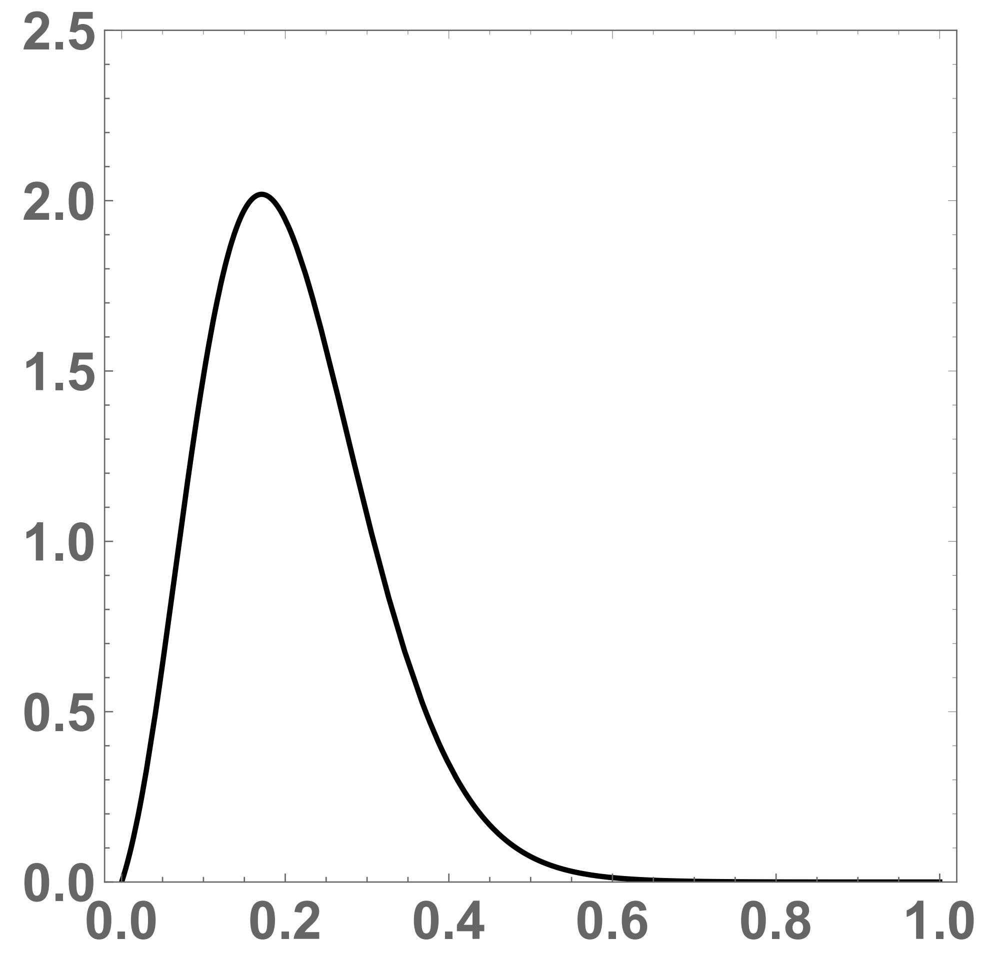

FIG. 8: Probability $P(X)$ to find a quark with momentum fraction X in the lowest pentaquark state, calculated from the wave function given in the Appendix. Note that in this WF the residual 4-quark interaction has not been yet implemented.

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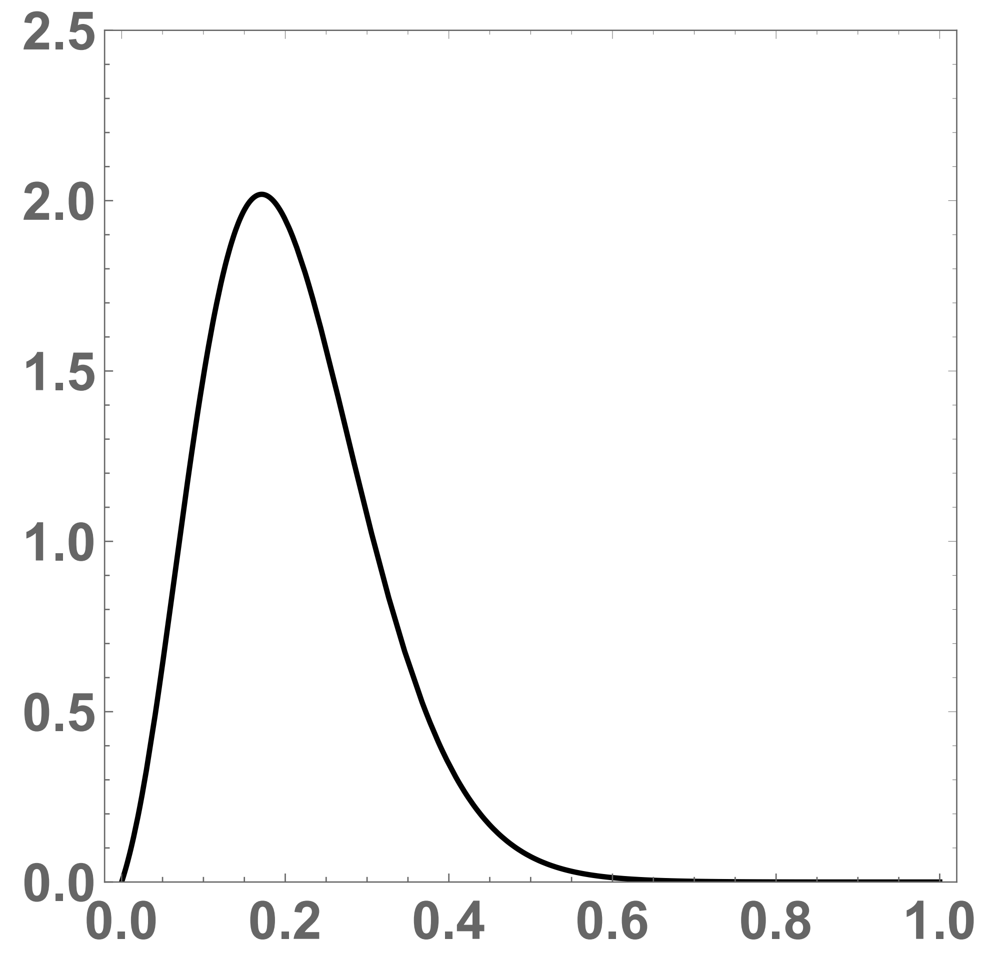
$$M_{min.penta} = 2.13\,GeV$$

To get this number in perspective, let us briefly remind the history of the light pentaquark search. In 2003 LEPS group reported pentaquark $\Theta^+ = u^2 d^2 \bar{s}$ with surprisingly light mass, of only $1.54\,GeV$, $0.6\,GeV$ lower than our calculation (and many others) yield.

Of course, so far the residual perturbative and NJL-type forces were not included. Quick estimates of the time (including mine [20]) suggested that since ud diquark has binding energy of $\Delta M_{ud} \approx -0.3\,GeV$ and the pentaquark candidate has two of them, one gets to "then observed" mass of $1.54\,GeV$.

Several other experiments were also quick to report observation of this state, till other experiments (with better detectors and much high statistics) show this pentaquark candidate does not really exist. Similar sad experimental status persists for all 6-light-quark dibaryons , including the flavor symmetric $u^2 d^2 s^2$ spin-0 state much discussed in some theory papers.



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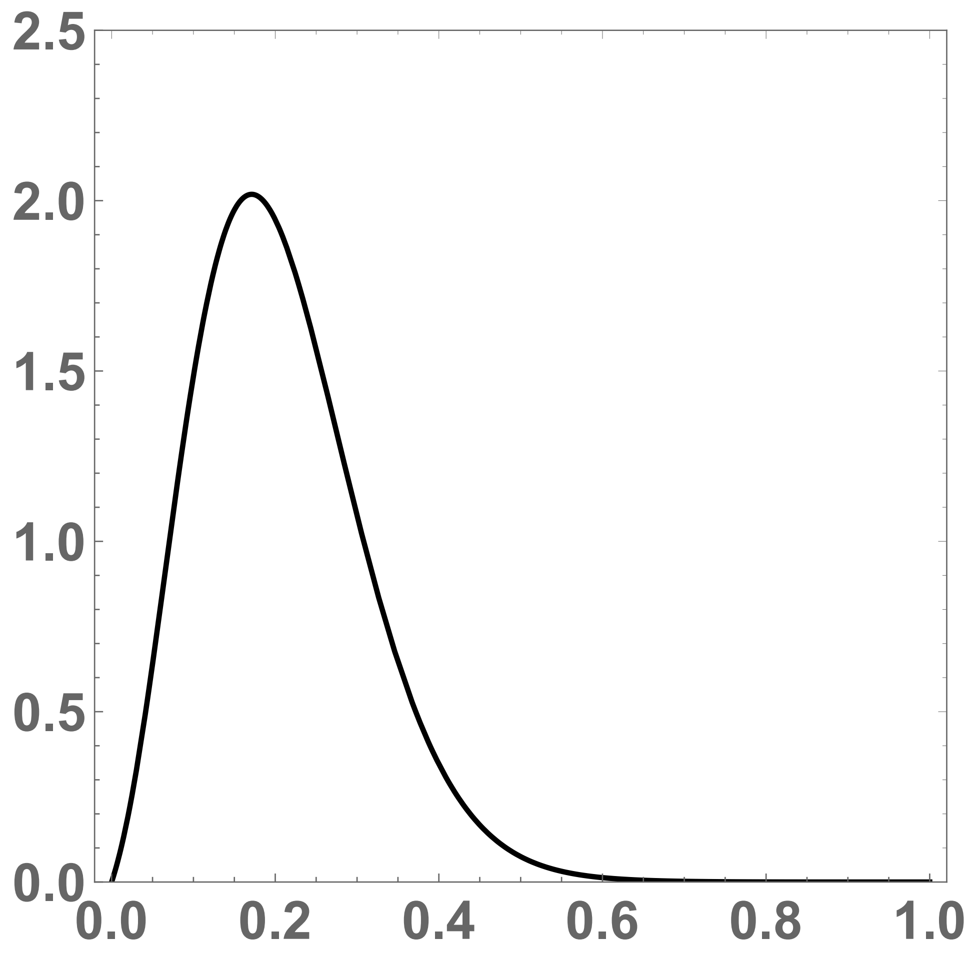

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Neither IILM nor lattice have found light pentas or dibaryons
Later explained by diquark mutual repulsion

THE 5-QUARK SECTOR OF THE BARYONS

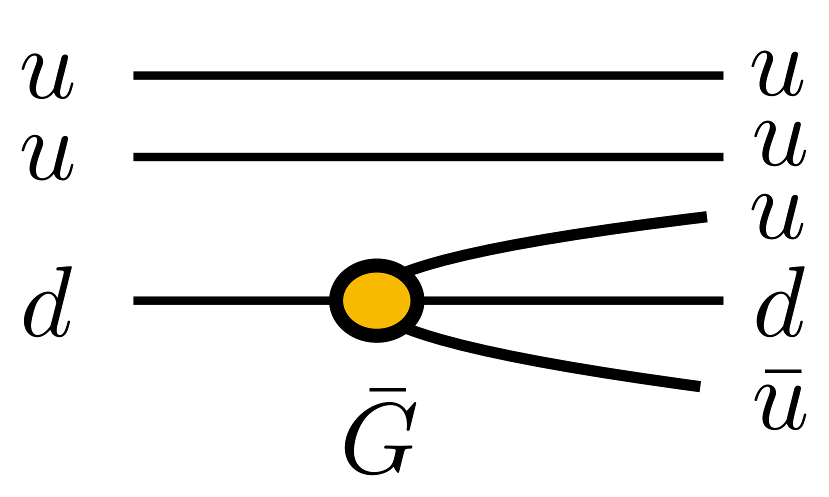

FIG. 9: The only diagram in which 4-quark interaction connects the 3 and 5 quark sectors, generating the $\bar{u}$ sea.

$$\psi_{tail}(s',t',u',w') = -\sum_i \frac{\langle N|H|5q,i\rangle}{M_i^2 - M_N^2}\psi_i(s',t',u',w')$$

mixing between N and All pentaquark calculated
It is hard to plot function of 4 variables…

As originally emphasized by Dorokhov and Kochelev [23], The 't Hooft topology-induced 4-quark interaction leads to processes

$$u \to u(\bar{d}d), \quad d \to d(\bar{u}u)$$

but not

$$u \to u(\bar{u}u), \quad d \to d(\bar{d}d)$$

which creates strong flavor asymmetry of the sea up to \bar d/\bar u =2

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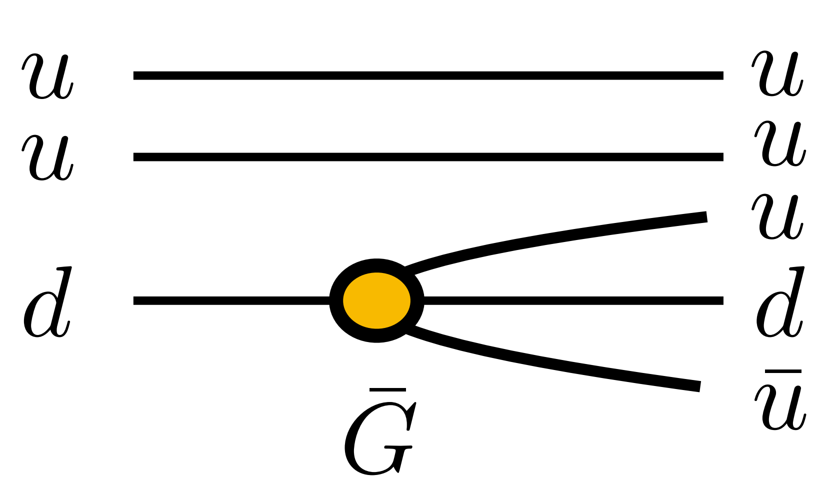

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$$\psi_{tail}(s',t',u',w') = -\sum_i \frac{\langle N|H|5q,i\rangle}{M_i^2 - M_N^2}\psi_i(s',t',u',w')$$

As originally emphasized by Dorokhov and Kochelev [23], The 't Hooft topology-induced 4-quark interaction leads to processes

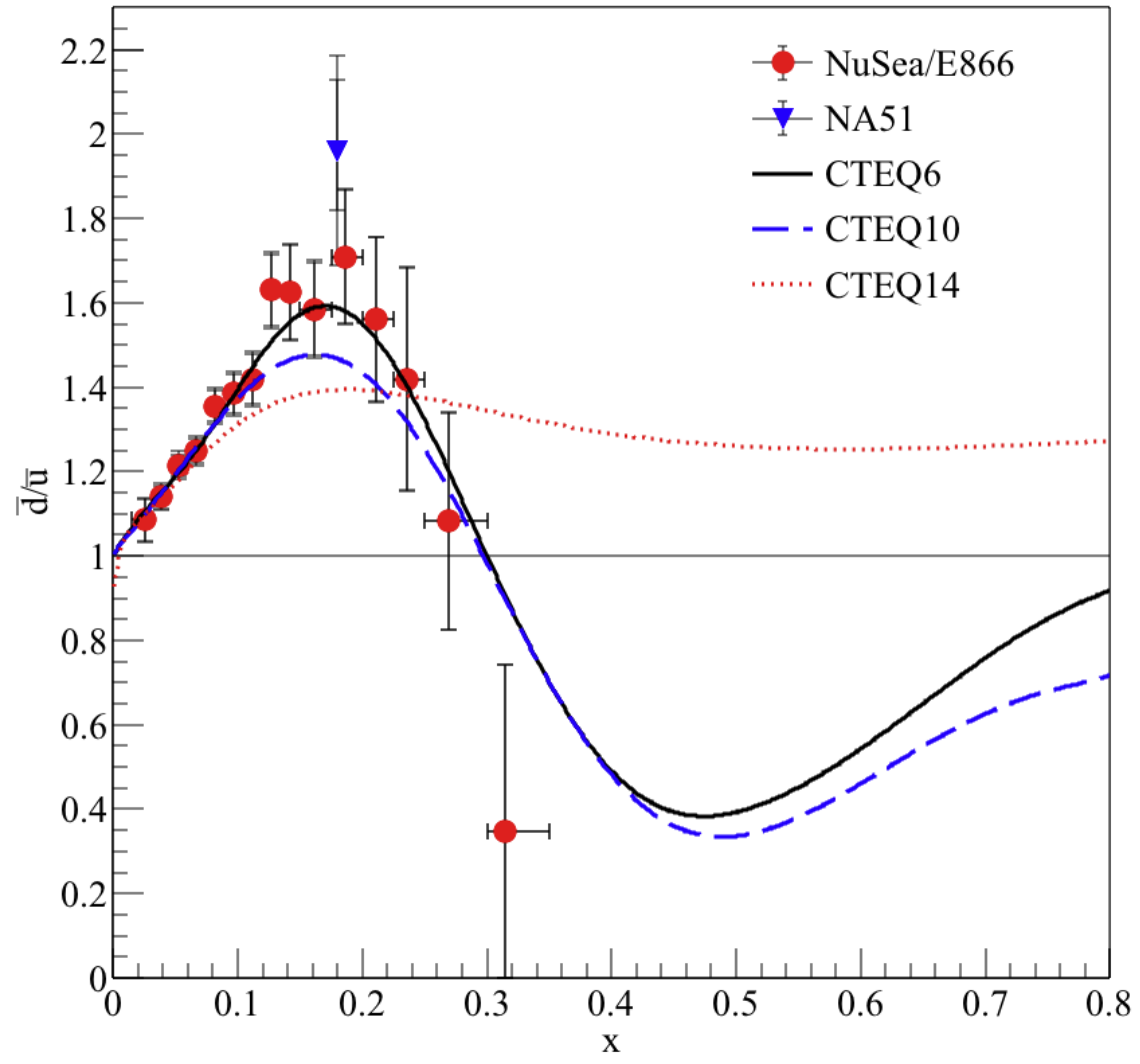
$$u \to u(\bar{d}d), \quad d \to d(\bar{u}u)$$

but not

$$u \to u(\bar{u}u), \quad d \to d(\bar{d}d)$$

which creates strong flavor asymmetry of the sea up to \bar d/\bar u =2

mixing between N and All pentaquark calculated
It is hard to plot function of 4 variables…



TOPOLOGY-INDUCED ANTIQUARK SEA

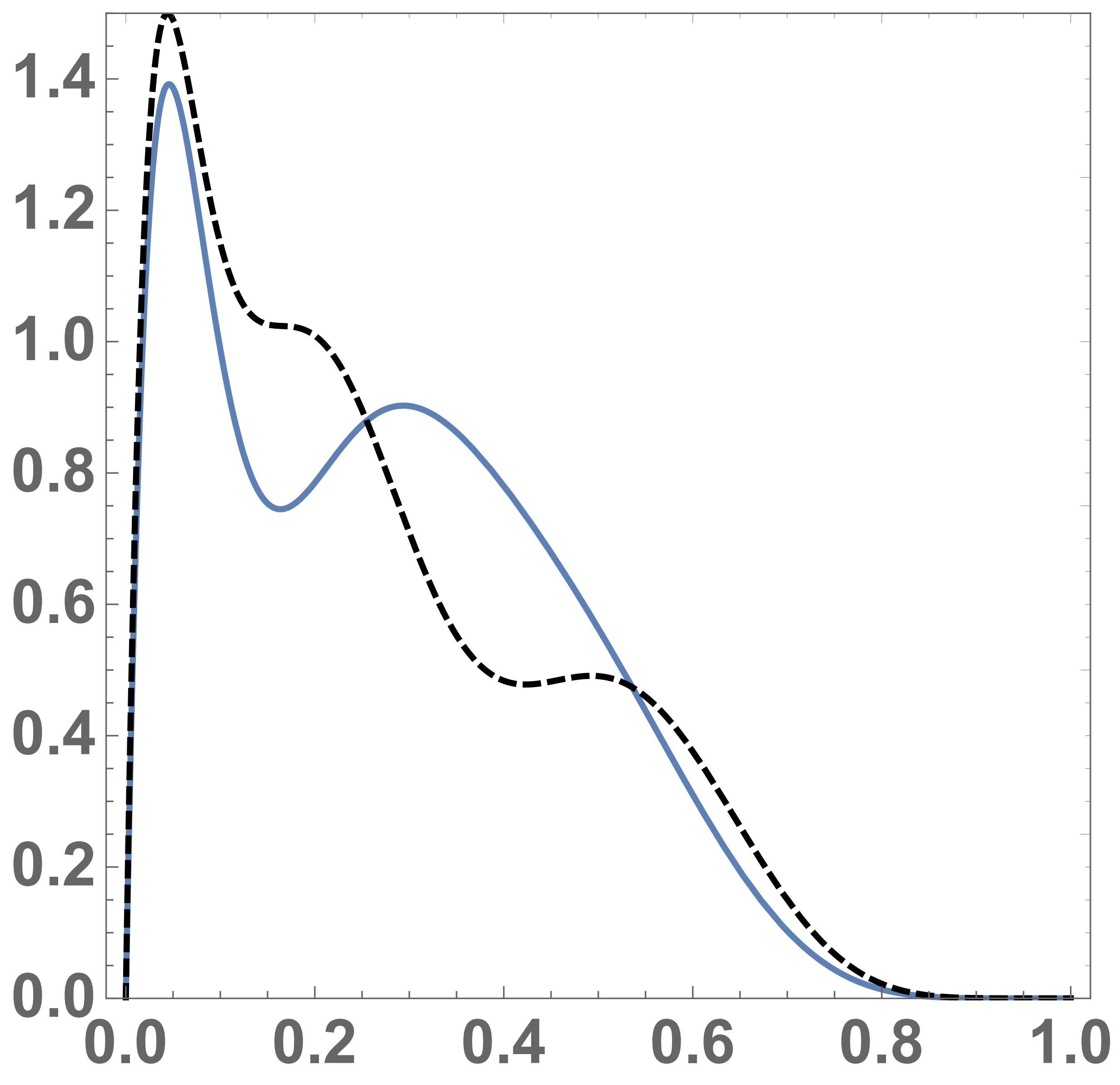
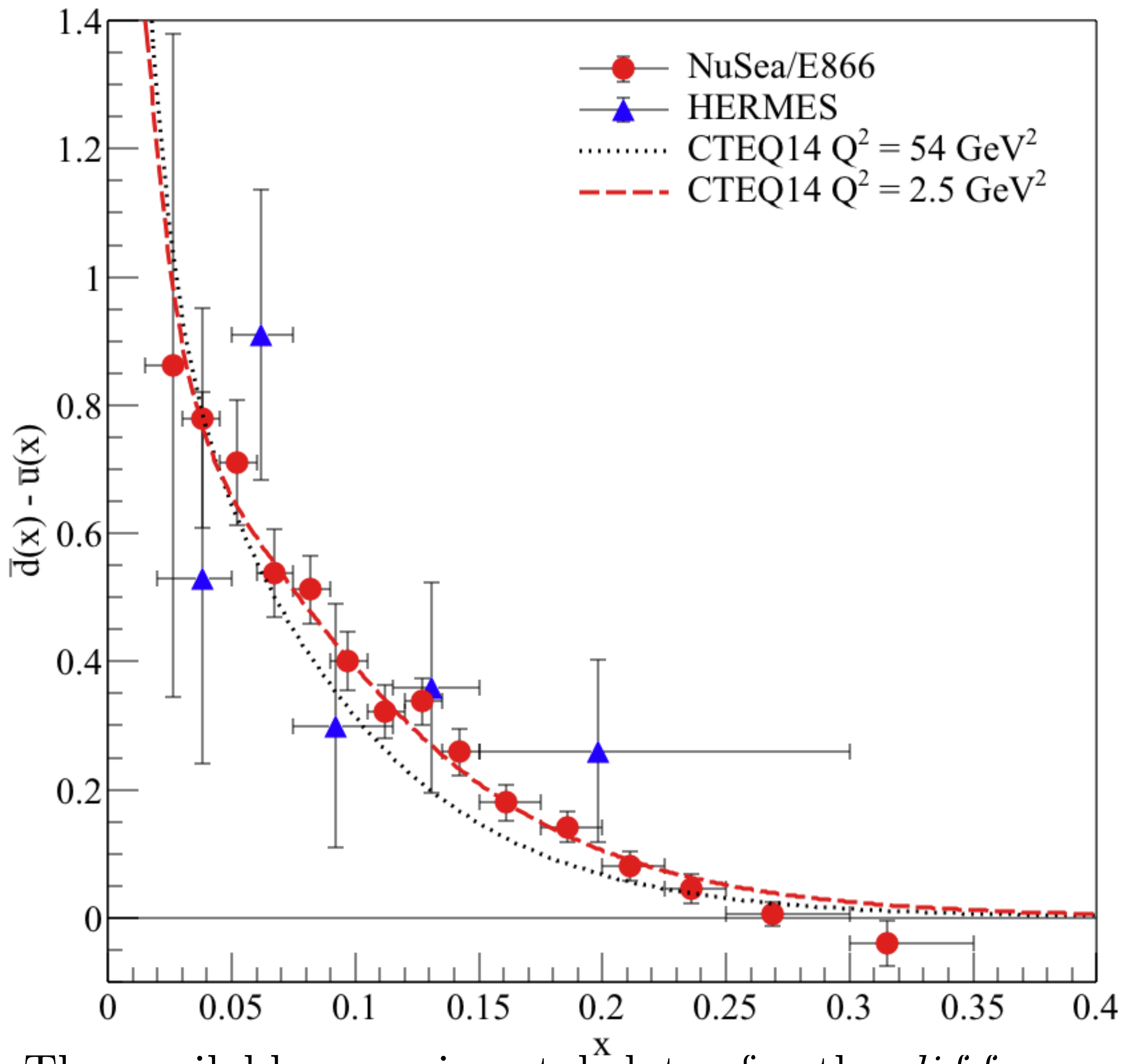


FIG. 10: The distribution over that $\bar{u}$ in its momentum fraction, for the Nucleon and Delta 5-quark "tails" (solid and dashed, respectively).

The "Kochelev mechanism" works semi-quantitatively: The magnitude and shape are basically correct

The isospin sea problem is thus declared solved

(sea spin problem is in work now)



The available experimental data, for the *difference* of the sea antiquarks distributions $\bar{d} - \bar{u}$ (from [18]) is shown in Fig.11. In this difference the symmetric gluon production should be cancelled out, and therefore it is sensitive only to a non-perturbative contributions.

Few comments: (i) First of all, the sign of the difference is indeed as predicted by the topological interaction, there are more anti-d than anti-u quarks;
(ii) Second, since 2-1=1, this representation of the data directly give us the nonperturbative antiquark production per valence quark, e.g. that of $\bar{u}$. This means it can be directly compared to the distribution we calculated from the 5-quark tail of the nucleon and Delta baryons, Fig.10.

“dense instanton liquid model” has two components

$$\kappa = \pi^2 (\frac{\rho}{R})^4 \approx 0.12$$ ES, 1982

dilute:
consists of well-separated instantons
their collectivised zero modes = quark condensate

inputs <Q^2> and <qbar q>

“molecular component” which is denser because overlapping I and bar-I has smaller action but it do not lead to near-zero Dirac eigenvalues!

Ilgenfritz,ES 1988,1993

current lattice studies with G^2,G^3 observables and gradient flow cooling (extrapolated to zero time) suggest kappa=O(1)

in the ff plots we used kappa=1 and this gets the data!

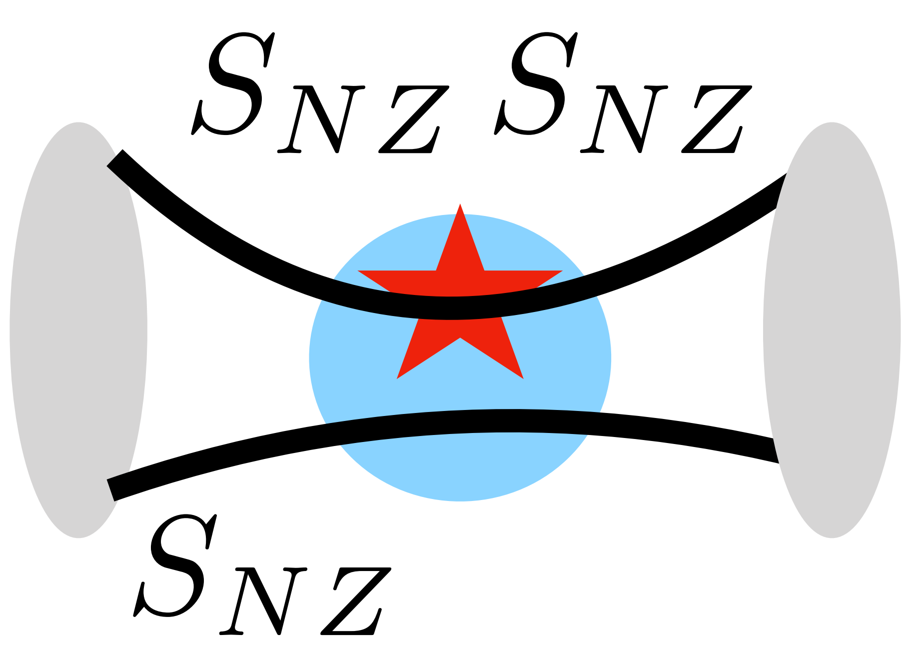
Including instanton effects in the hard block

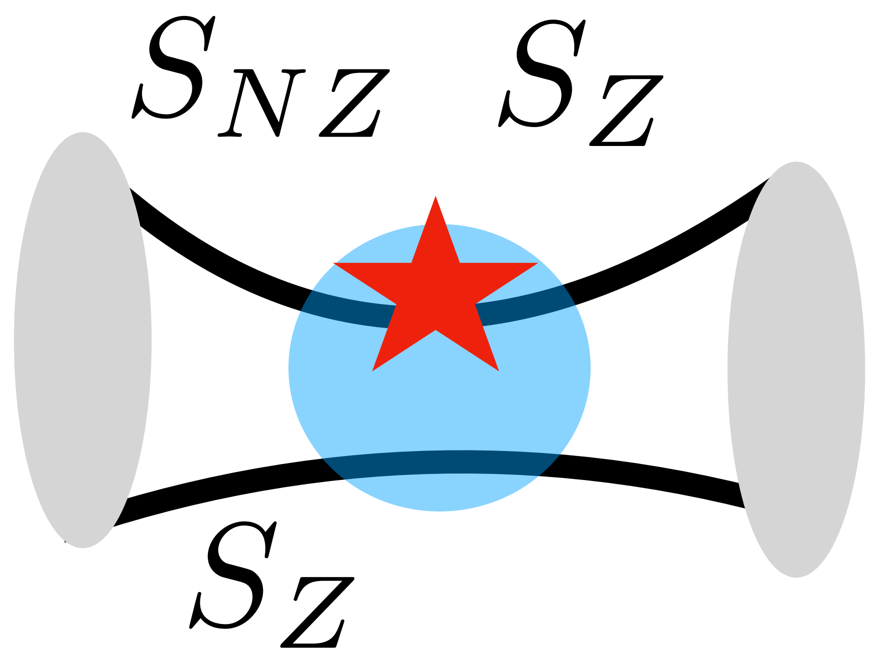
Quark propagators in the instanton field are
Known analytically
(for massless quarks only).

Performing LSZ “amputation” on them
One obtains **local hard block**

Integration over instanton location restore
4-momentum conservation

The results (exp(-Q\rho)) are averaged
Over instanton size distribution
And become power-like





Too many technical details
=> paper is about 50 pages
So I present result only

we calculated photon, scalar, graviton and dilaton FFs
for pion, rho and scalar a_0 (brother of eta’)
they are mostly dominated by diagram c (NZ modes)
or just strong instanton fields, not zero modes

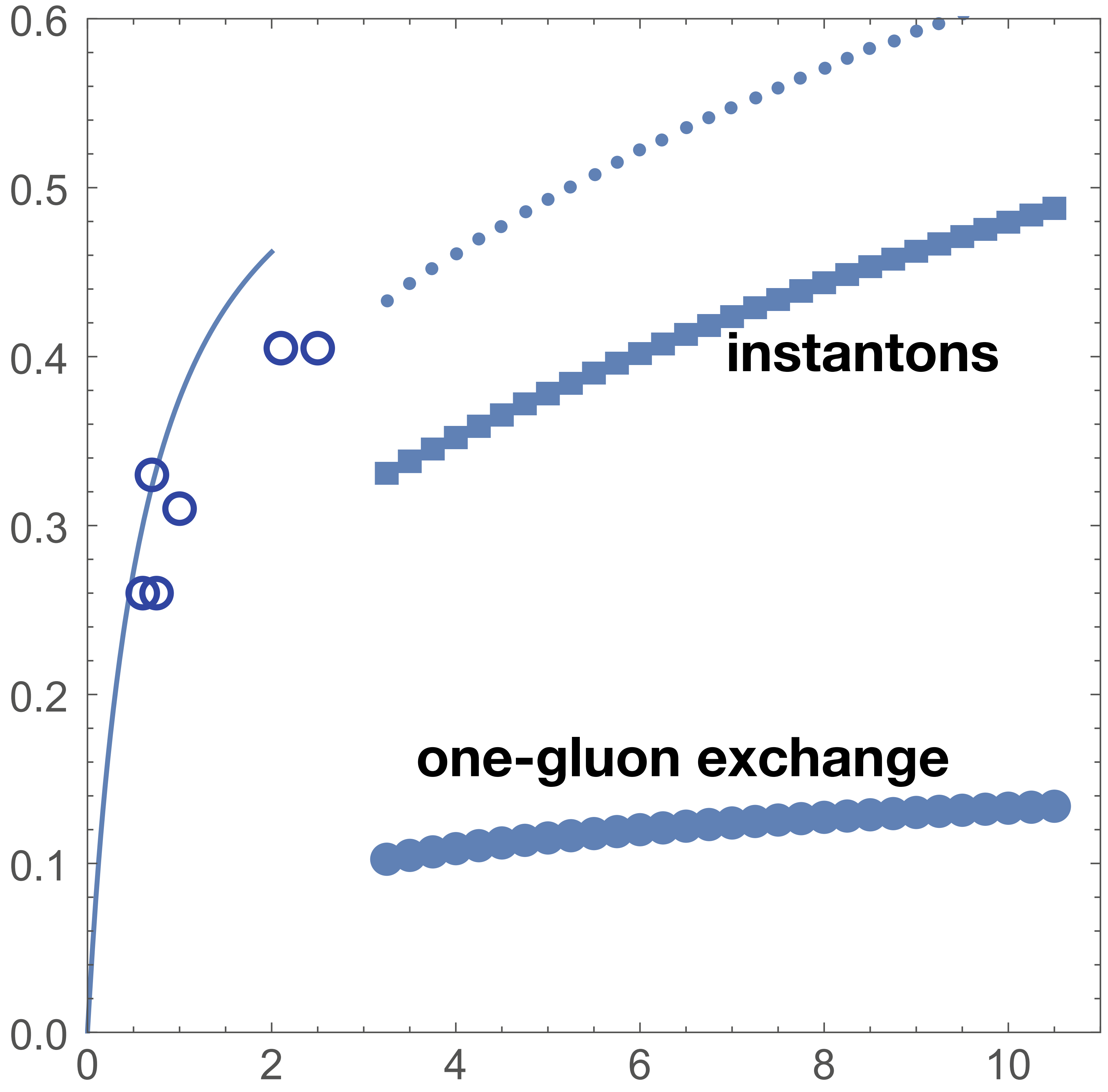


Fig. 10.9 The vector form factors of the pion times squared momentum transfer, $Q^2 F_\pi(Q^2)$ (GeV^2) versus Q^2(GeV^2). Closed discs show the total perturbative contribution. Squares correspond to the instanton contribution from nonzero mode propagators. The dotted line above is their sum. A curve in the l.h.s. is the usual dipole formula, and open points are from experimental measurements. We do not show data points at smaller Q^2, where they do agree with the dipole formula curve

diagram with 3 non-zero mode propagators

$$V_c^\pi = \epsilon_\mu(q)(p^\mu + p'^\mu)\,(e_u + e_{\bar d})\left[\frac{\kappa\pi^2 f_\pi^2 \chi_\pi^2}{N_c M_Q^2}\langle\rho^2 \mathbb{G}_V(Q\rho)\rangle \times \int dx_1 dx_2 \bar{x}_1 \left(\varphi_\pi^P(x_1)\varphi_\pi^P(x_2) - \frac{1}{36}\varphi_\pi'^T(x_1)\varphi_\pi'^T(x_2)\right)\right]$$

diagram containing zero mode propagators
(t’Hooft vertex)

$$V_d^\pi = -\epsilon_\mu(q)(p^\mu + p'^\mu)\,(e_u + e_{\bar d}) \quad (10.35)$$

$$\times\left[\left(\frac{1}{N_c^2(N_c+1)}\right)\frac{4\kappa\pi^2 f_\pi^2 \chi_\pi}{3M_Q^2}\left\langle\rho^2 \frac{K_1(Q\rho)}{Q\rho}\right\rangle \int dx_1 dx_2 \varphi_\pi^P(x_1)\varphi_\pi'^T(x_2)\right]$$

ES+ I.Zahed, in progress

Central and spin-dependent forces, in hadronic wave functions, at CM and Light Front

Instanton effects in central potentials

$$e^{-V_c(r)T} = \langle W(\vec{x}_1)W^+(\vec{x}_2)\rangle$$

spin forces are related to WGWG nonlocals

$$W = Pexp[ig\int dx^\mu A^a_\mu \hat{t}^a]$$

angle of color rotationalong a straight line is easy to calculatefor instanton fields

Callan et al 1978, Eichten, Feinberg 1981

$$V_{instanton}(r) = \frac{4\pi n_{\bar{I}+I}\rho^3}{N_c\rho} I(\frac{r}{\rho})$$

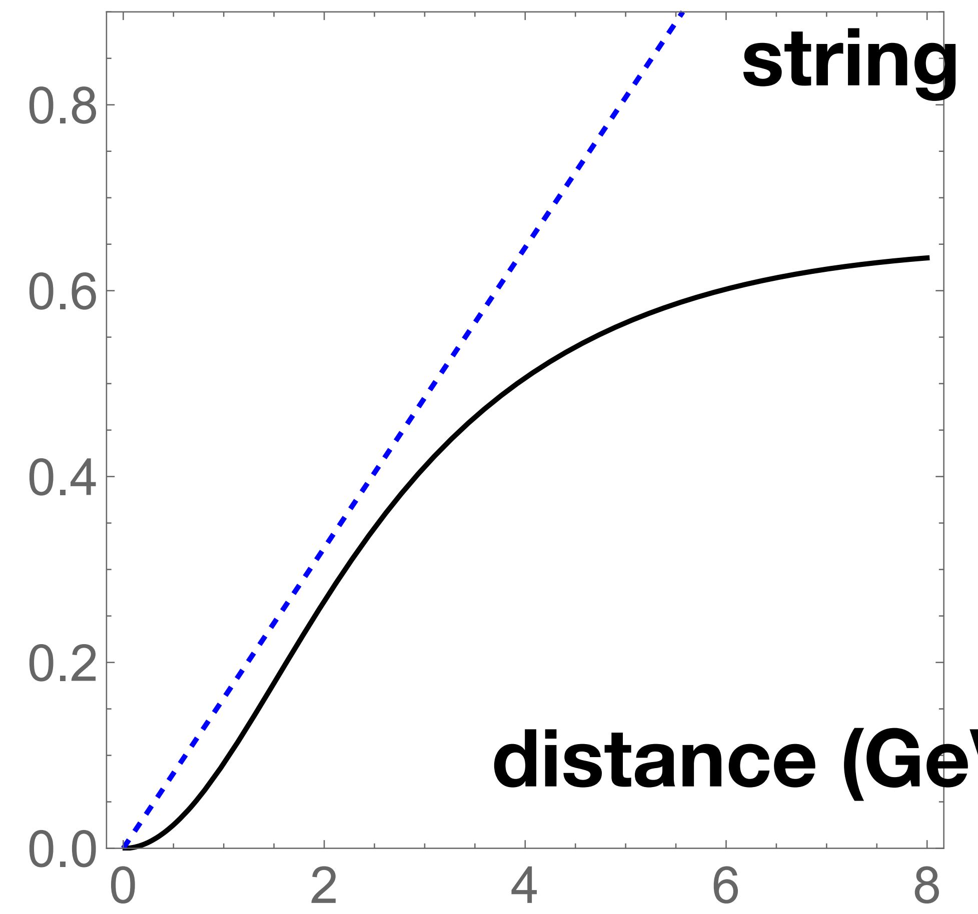
if "dense liquid" its magnitude is similar to Cornell

$$I(x) = \int_0^\infty dy y^2 \int_{-1}^1 dc[1 - cos(\alpha_1)cos(\alpha_2) - \frac{y+xc}{\sqrt{y^2+x^2+2xyc}} sin(\alpha_1)sin(\alpha_2)]$$

in which $c = cos(\phi)$, and two color rotation angles are

$$\alpha_1 = \pi\frac{y}{\sqrt{y^2+1}}, \quad \alpha_2 = \pi\sqrt{\frac{y^2+x^2+2xyc}{y^2+x^2+2xyc+1}}$$

$$n_{mol} + n_I + n_{\bar{I}} = 7.\,\mathrm{fm}^{-4} \quad R_{dense} \equiv n^{-1/4} = 0.61\, fm \approx 2\rho$$



as good as Cornell potential for

$$\Upsilon[1S], \eta_b[1S], \Upsilon[2S], \Upsilon[3S], \Upsilon[4S],$$

but for bb states with N>4 or ligher quark states, still one has to use linear

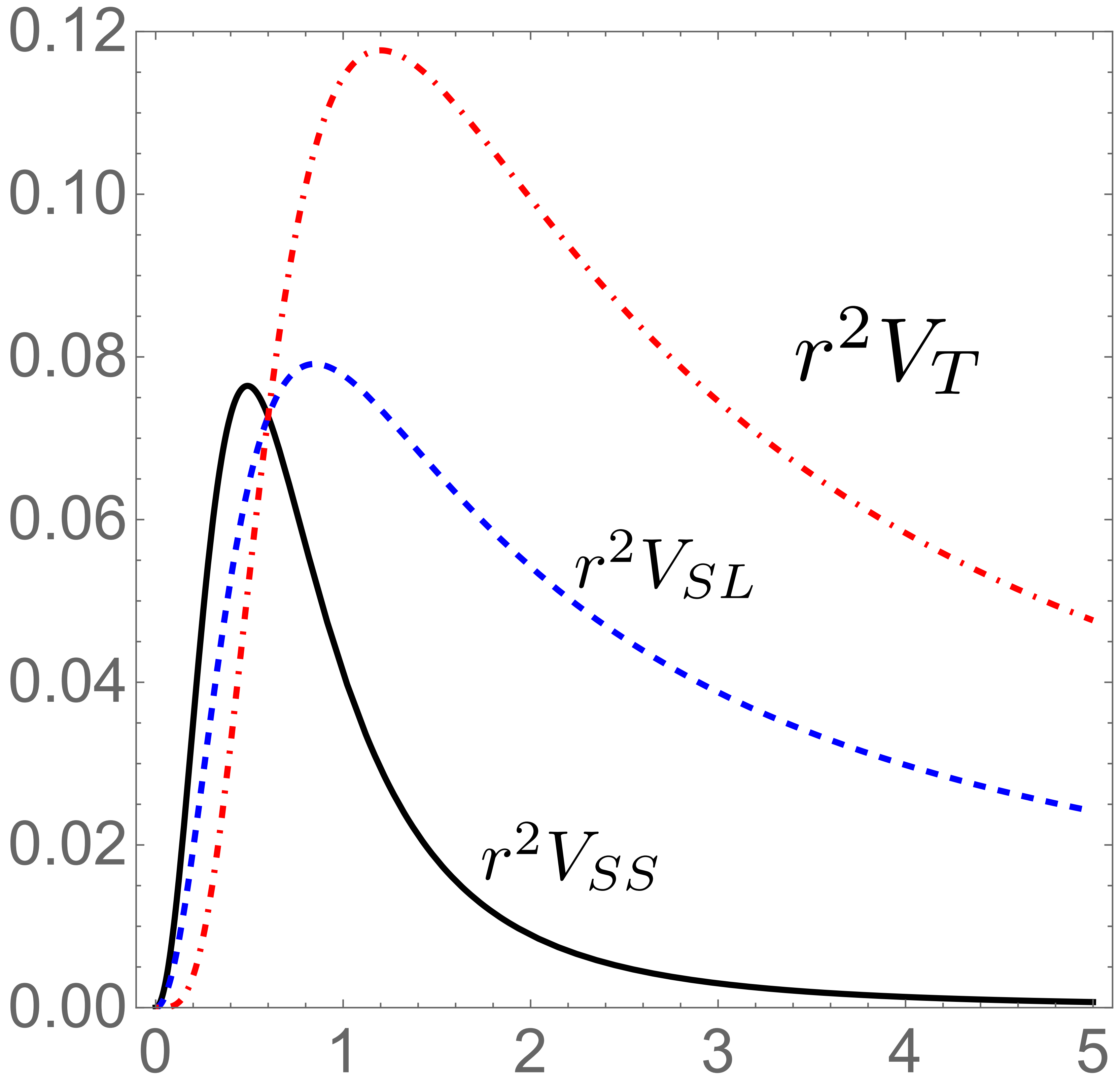
Instanton effects in spin-related potentials

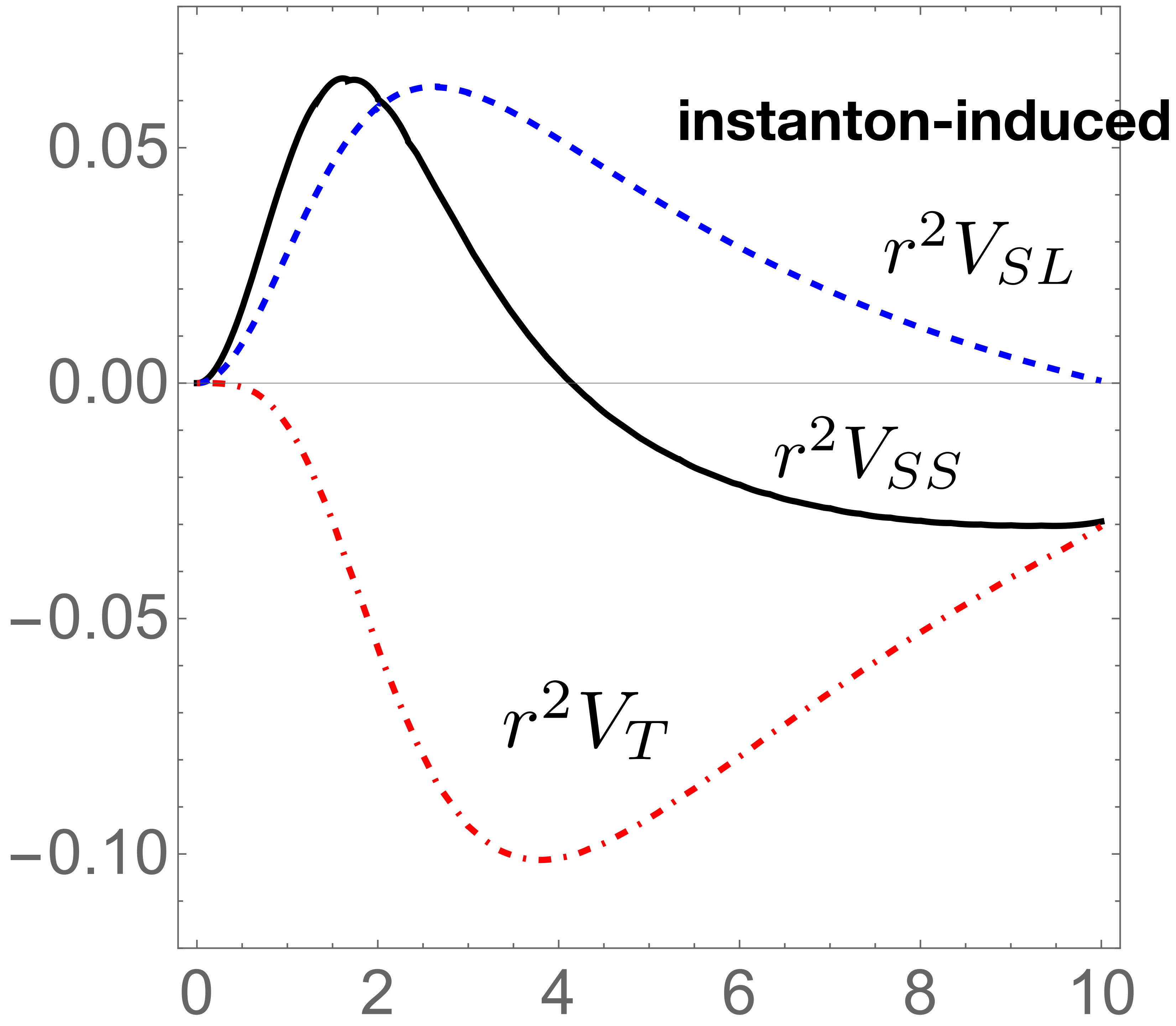
Wilson lines complemented by two field strengths => in general, 5 potentials for instantons related to Vc

E. Eichten and F. Feinberg, "Spin Dependent Forces in QCD," Phys. Rev. D **23**, 2724 (1981

$$V_{SD} = \left(\frac{S_Q \cdot L_Q}{2m_Q^2} - \frac{S_{\bar{Q}} \cdot L_{\bar{Q}}}{2m_{\bar{Q}}^2}\right)\left(\frac{1}{r}\frac{d}{dr}(V(r) + 2V_1(r))\right) + \left(\frac{S_{\bar{Q}} \cdot L_Q}{m_Q m_{\bar{Q}}} - \frac{S_Q \cdot L_{\bar{Q}}}{m_{\bar{Q}} m_{\bar{Q}}}\right)\left(\frac{1}{r}\frac{d}{dr}V_2(r)\right) + \frac{(3S_Q \cdot \hat{r} S_{\bar{Q}} \cdot \hat{r} - S_Q \cdot S_{\bar{Q}})}{3m_Q m_{\bar{Q}}}V_3(r) + \frac{1}{3}\frac{S_Q \cdot S_{\bar{Q}}}{m_Q m_{\bar{Q}}}V_4(r)$$

perturbative





Their sum explains lattice data for Vss and explains spin splittings rather well, except in light-light mesons

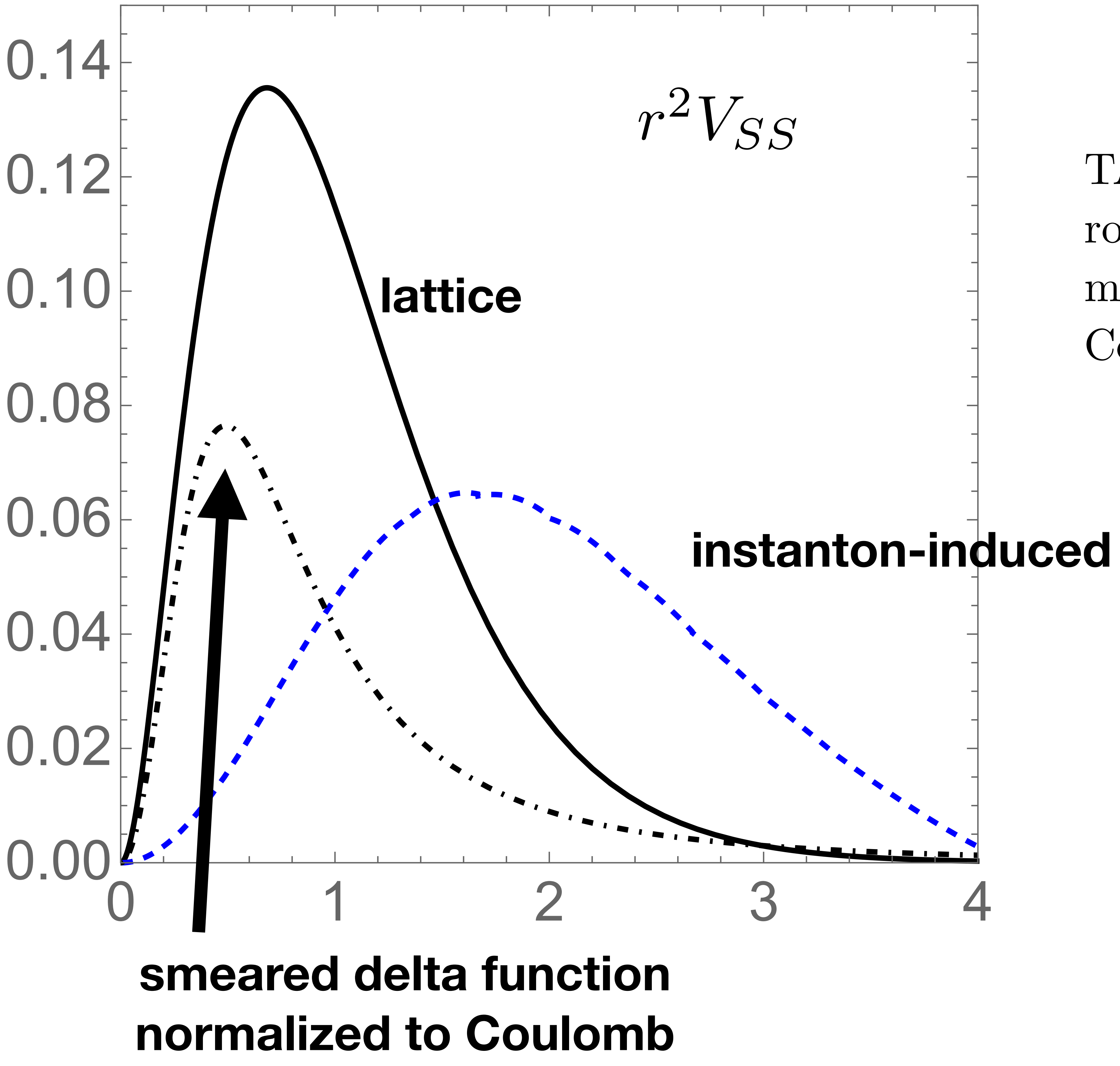


TABLE II. "Hyperfine" splittings of certain $L = 0$ mesons with $J = 1$ and $J = 0$. The first row of numbers shows the experimental values (MeV) (rounded to 1 MeV). The second gives matrix elements of the lattice-based spin-spin potential (19), the next two are those for (regulated) Coulomb and instanton-induced spin-spin forces.

flavors	$M_\Upsilon - M_{\eta_b}$	$M_{J/\psi} - M_{\eta_c}$	$M(D^*) - M(D)$	$M(K^*) - M(K)$	$M(\rho) - M(\pi)$
Exp	61.	116.	137.	398.	636.
$\langle V_{SS}^{lat}/3M_1M_2\rangle$	46.	108.	98.	170.	
$\langle \vec{\nabla}^2 V_C/3M_1M_2\rangle$	28.	58.	48.	82.	
$\langle \vec{\nabla}^2 V_{inst}/3M_1M_2\rangle$	7.	30.	48.	90.	

we also studied splittings of 1P states h,chi_0,chi_1,chi_2 and calculated matrix elements of VSS,VSL,VT also

massless pion is due to zero modes (t' Hooft Lagrangian)

correct mass of rho meson needs "molecular forces" to be included

Summary

- gauge topology objects: 2d wortices, 3d monopoles, 4d instantons are all related; e.g. Poisson duality for monopole-instanton-dyons
- Dirac zero modes and t'Hooft effective Lagrangian => instanton liquid explains chiral symmetry breaking and QCD correlation functions at intermediate distances, but not confinement
- LFWFs and antiquark flavor puzzle u-> u dbar d, u sbar s,
- not u ubar u
- New "molecular" components => explains mesonic formfactors at semi-hard Q^2, Central potential and spin forces, in quarkonia

Additional slides

Hadron formfactors at semi-hard Q^2. (1-10 GeV^2)

kinematics -> factorization

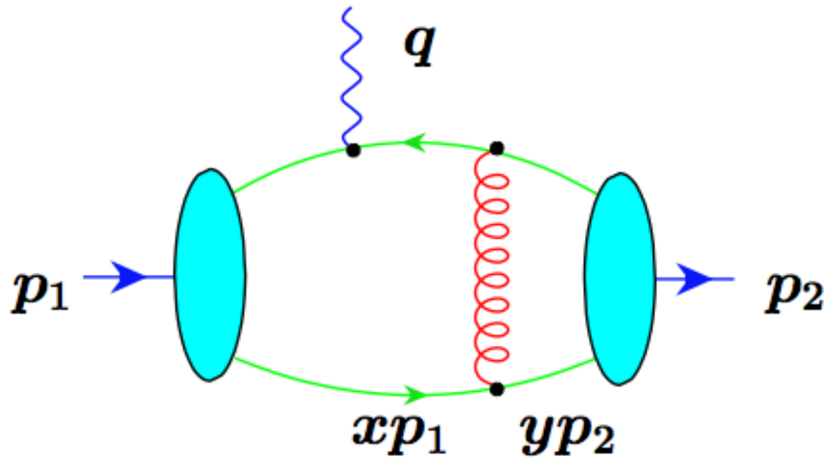
but hard block is not just perturbative

History of the exclusive QCD processes and the pion form factor

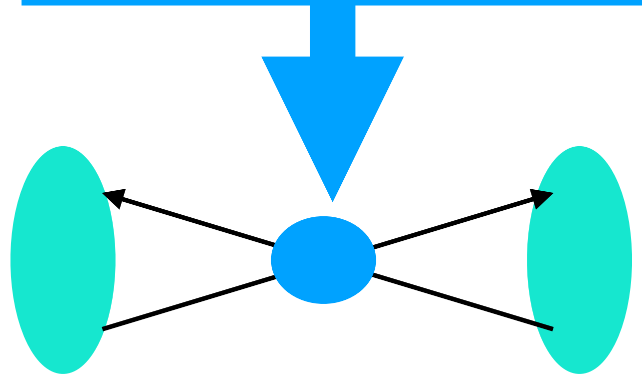
Asymptotic at large Q defined in 1970's
Brodsky-Lepage, Chernyak-Zhitnitsky, Radyushkin…

$$F_\pi^{\text{as (pQCD)}}(Q^2) = \frac{8\pi\alpha_s}{9}\int_0^1 dx \int_0^1 dy\, \frac{\varphi_\pi(x)\,\varphi_\pi(y)}{xyQ^2}\, f_\pi^2$$

f_pi^2 gives wave function at the origin
In the transverse plane



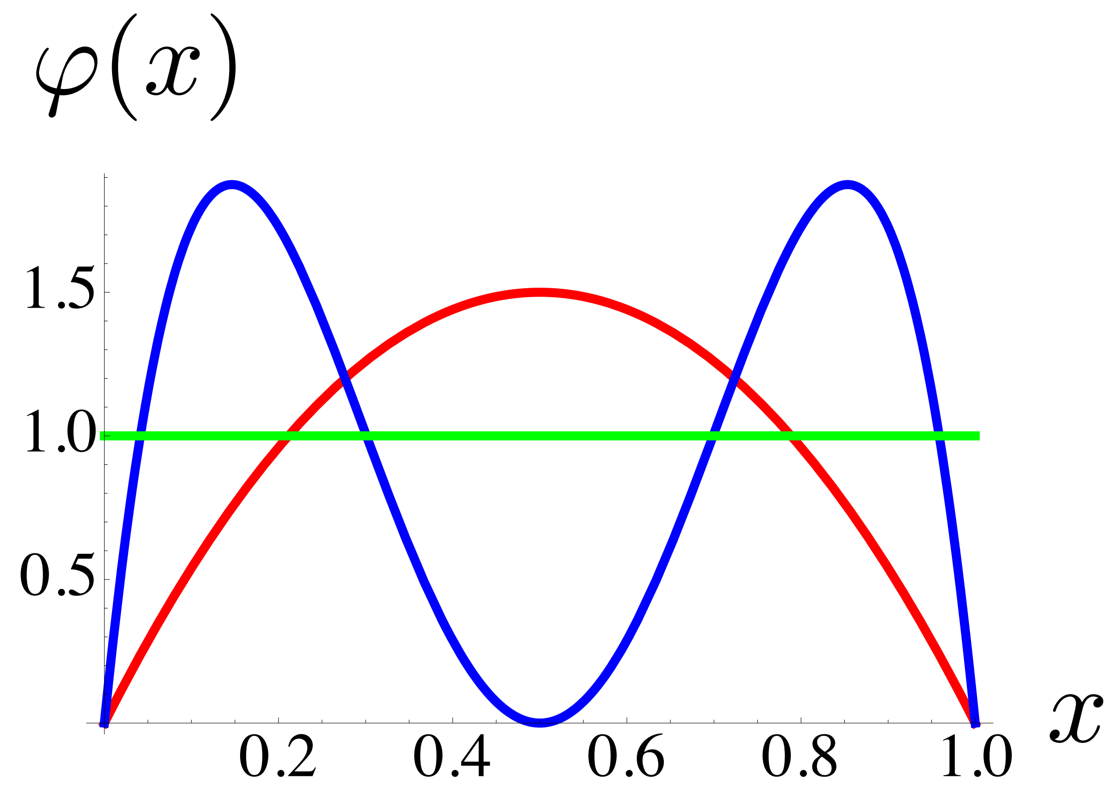
Hard block is pointlike
In transverse plane



unfortunately, for "asymptotic wave function"
It is well below the data for (F_pi Q^2)

One proposed way out:
Chernyak-Zhitnitsky
Wave function

Unfortunately it is not supported by
Any lattice or mode calculations



asymptotic (red), flat (green), Chernyak-Zhitnitsky (blue).

Another option: higher twist
Wave functions which partially helps

Our approach: include
nonperturbative qbar q scattering

additional comments: two puzzles which ILM explained

1.the "puzzle of strong breaking of the SU(3) flavor symmetry". Naively, in NJL-like models

$$m_s \sim 0.1\, GeV \ll \Lambda_{NJL} \sim 1\, GeV$$

yet SU(3)f symmetry is often broken by 100%

e.g. recent work by Regensburg group on PDF moments found completely different ones for N,Lambda,Sigma

$$M_f^* = \frac{2\pi^2}{3} |\langle \bar{q}_f q_f \rangle| \rho^2$$

$$\sim m_s$$

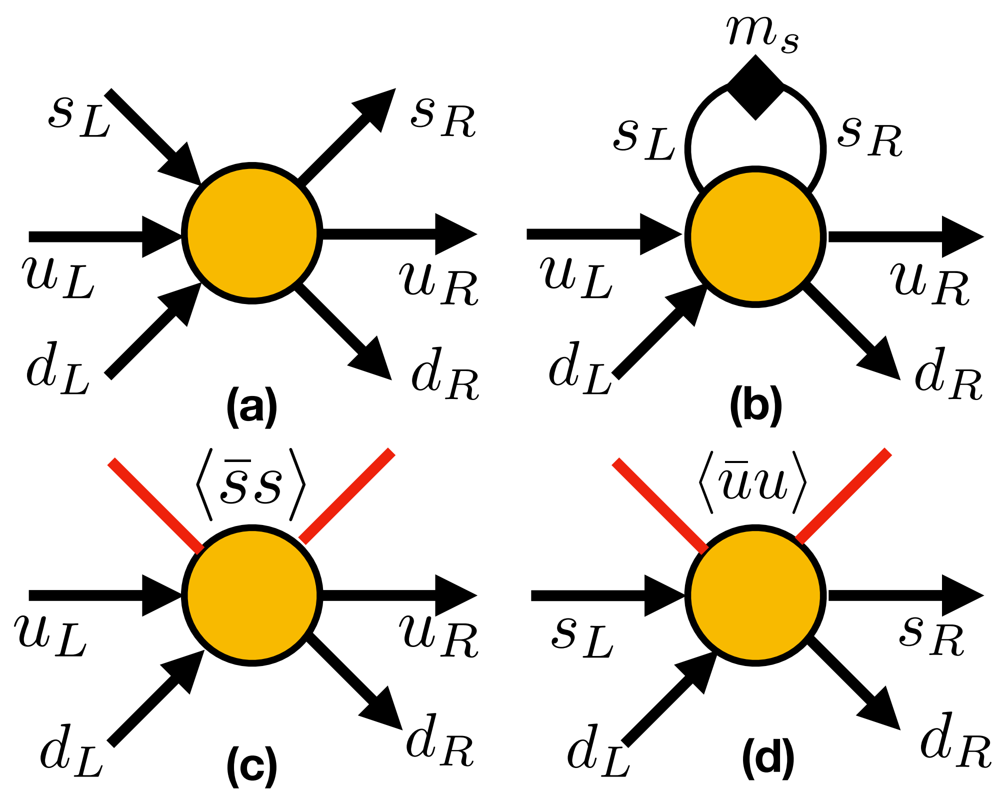


FIG. 1: Schematic form of the 6-quark 't Hooft effective Lagrangian is shown in fig (a). If quarks are massive, one can make a loop shown in (b), reducing it to 4-fermion operator. Note a black rhomb indicating the mass insertion into a propagator. We only show it for s quark, hinting that for u, d their masses are too small to make such diagram really relevant. In (c,d) we show other types of effective 4-fermion vertices, appearing because some quark pairs can be absorbed by a nonzero quark condensates (red lines).

$$m_{u,d} \to 0$$

$$\Delta\lambda \sim \rho^2/R^3 \sim 20\, MeV$$

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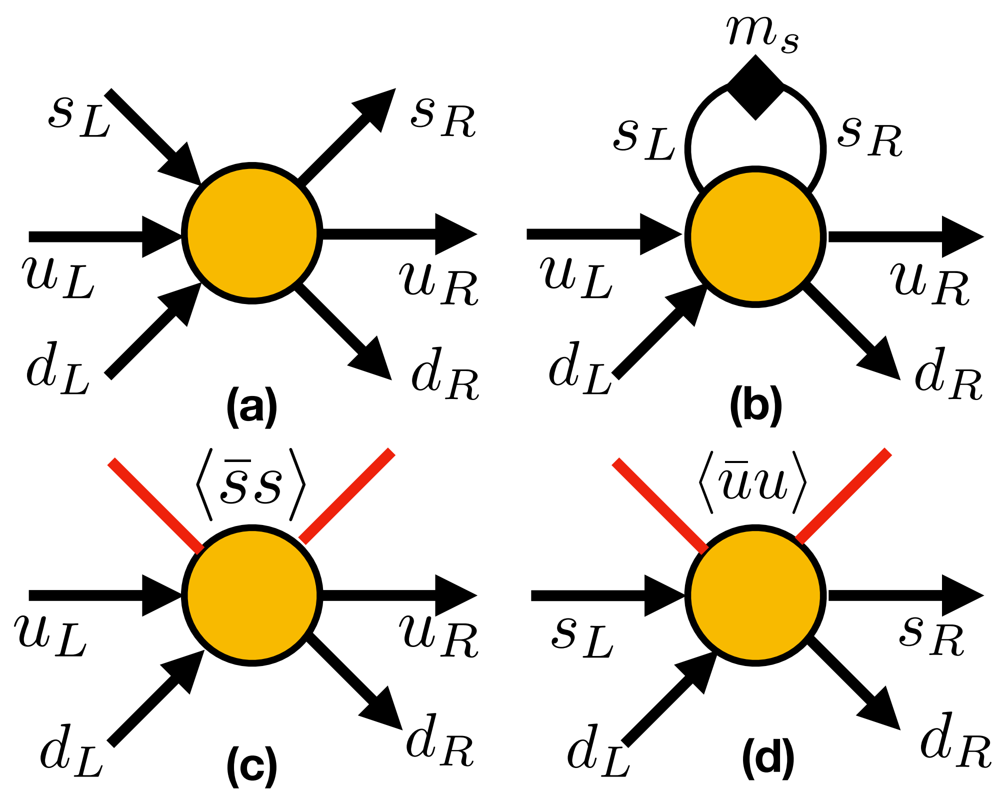

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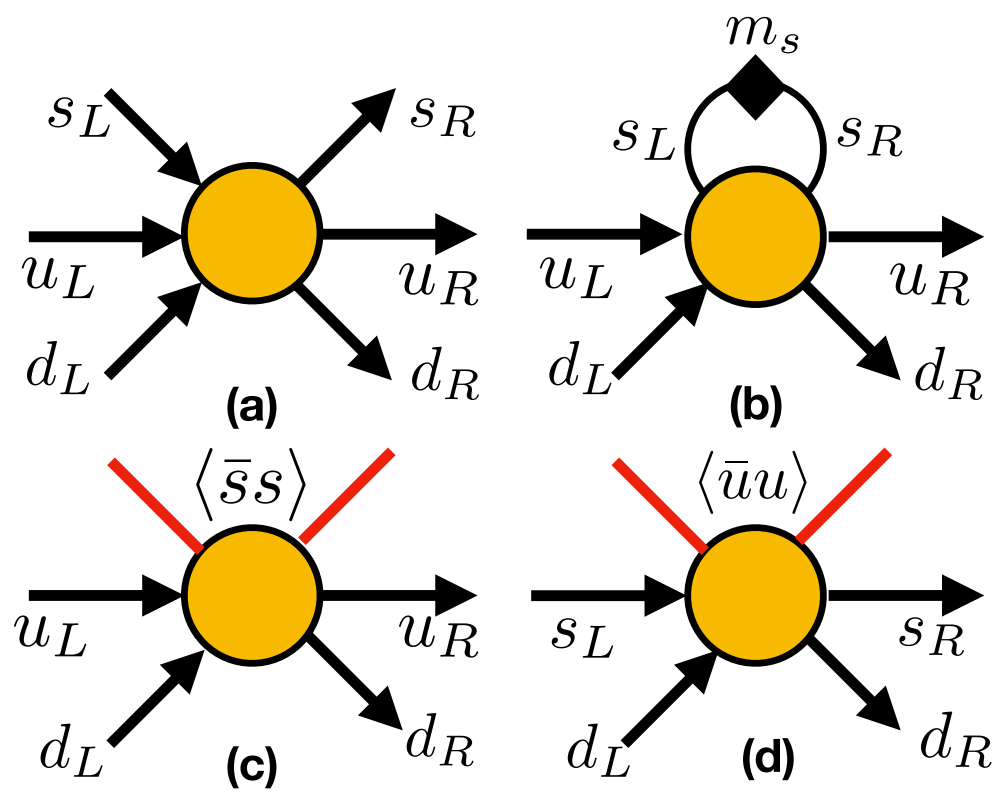

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2.the puzzle of nonlinear chiral extrapolation

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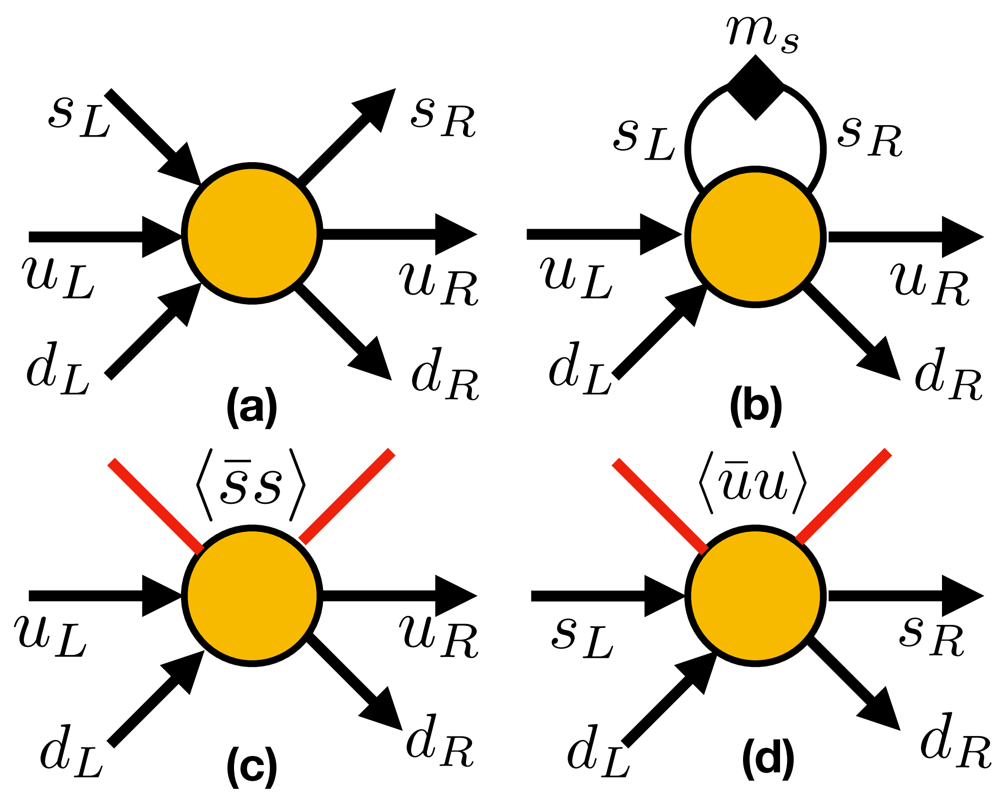

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2.the puzzle of nonlinear chiral extrapolation

$$m_{u,d} \to 0$$

$$\Delta\lambda \sim \rho^2 / R^3 \sim 20\, MeV$$

the chiral condensate, made of collectivized instanton zero modes, has a spread of Dirac eigenvalues proportional to overlaps of zero modes of individual instantons. To form a condensate, quark needs to hop from one instanton to the next. The overlap of their zero modes is surprisingly small because the topology ensemble is rather dilute

new functional basis for with N=2,3,5 bodies

$$\sum_{i}^{N} x_i = 1,$$

integration measure

N=2

$$s = x_1 - x_2$$

$$x_1 = \frac{1+s}{2}, \quad x_2 = \frac{1-s}{2}$$

$$\int dx_1 dx_2 \delta(x_1 + x_2 - 1) x_1 x_2 ... = \int_{-1}^{1} ds \frac{(1-s)(1+s)}{4} ...$$

functional basis: Jacobi polynomials $P_n^{1,1}(s)$

N=5

$$s = \frac{x_1 - x_2}{x_1 + x_2}, \quad t = \frac{x_1 + x_2 - x_3}{x_1 + x_2 + x_3},$$

enough for 3 bodies

$$u = \frac{x_1 + x_2 + x_3 - x_4}{x_1 + x_2 + x_3 + x_4}, \quad w = x1 + x2 + x3 + x4 - x5$$

all variables $\in [-1, 1]$

$$x_1 = \frac{1}{2^4}(1+s)(1+t)(1+u)(1+w)$$

$$x_2 = \frac{1}{2^4}(1-s)(1+t)(1+u)(1+w)$$

$$x_3 = \frac{1}{2^3}(1-t)(1+u)(1+w),$$

$$x_4 = \frac{1}{2^2}(1-u)(1+w) \qquad (15)$$

$$x_5 = 1 - x_1 - x_2 - x_3 - x_4 = \frac{1}{2}(1-w)$$

integration measure

$$\int \frac{dsdtdudw}{16777216}(1-s)(1+s)(1-t)(1+t)^3$$

$$\times(1-u)(1+u)^5(1-w)(1+w)^7 ...$$

functional basis: Jacobi polynomials

$$\psi_{lmnk}(s,t,u,w) \sim P_l^{1,1}(s) P_m^{1,3}(t) P_n^{1,5}(u) P_k^{1,7}(w)$$